Introduction to Energetic Particles Physics in Magnetic Fusion: Part II collective effects

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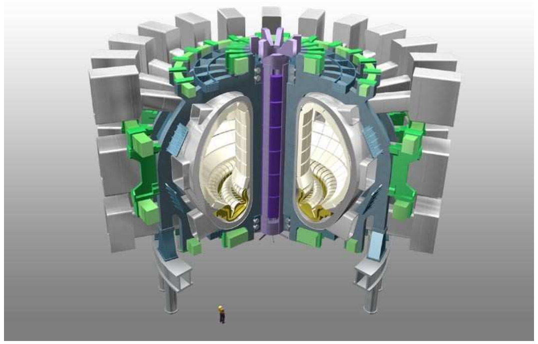
ITER (wikipedia) *NSTX (http://nstx.pppl.gov)* *Solar corona (wikipedia)*



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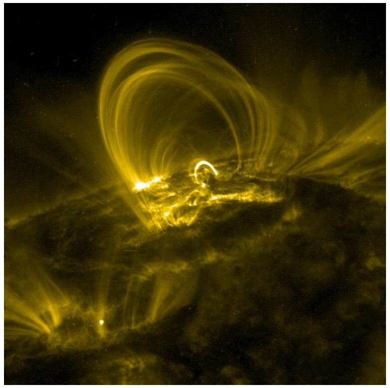

PPPL, Graduate Student AST558 Seminar, April 13, 2009

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Why study EP collective effects, instabilities

- Instabilities are expected to be responsible for Energetic Particle radial transport - how bad it is?
- Classically confined EPs are decoupled from the background plasma:
  - $\Rightarrow$ MHD and kinetic instabilities due to EPs can be tested
  - Fundamental plasma oscillations can be investigated
- Need to make predictions for burning plasmas
- Validated models can be applied for magnetosphere, solar corona heating, etc.

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EP physics/instabilities will be key issue for future tokamak reactor performance

In this lecture

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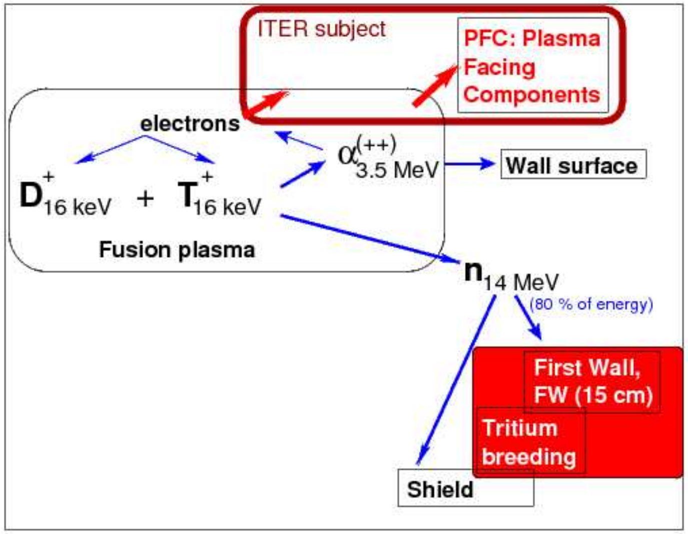
- Considerations for a reactor
- α-channeling
- Alfvénic waves
- EP driven instabilities
- Open issues

ITER is the first machine targeting the α-heating regime



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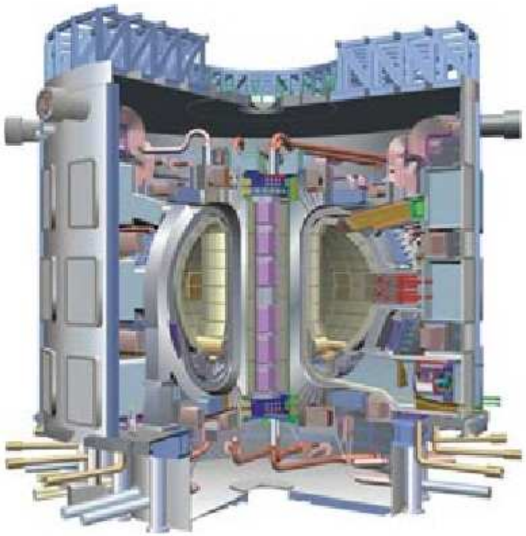

Inconsistent with high T_i regime, α - channeling.

Present confinement time is $\tau_E \simeq 1 - 3sec$.

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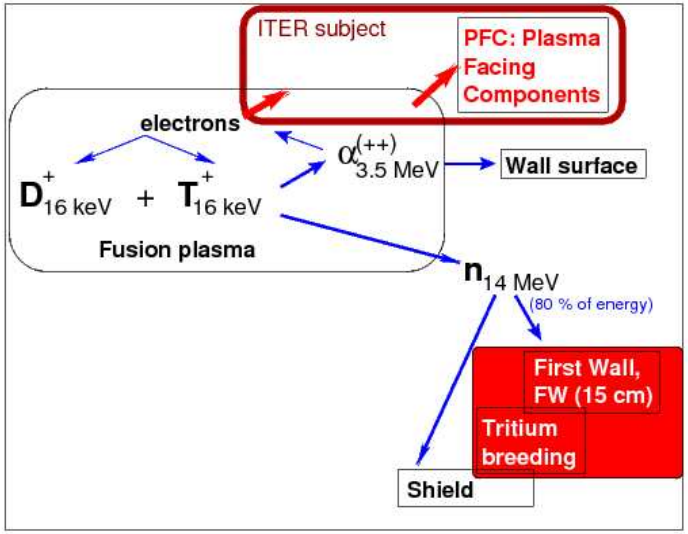

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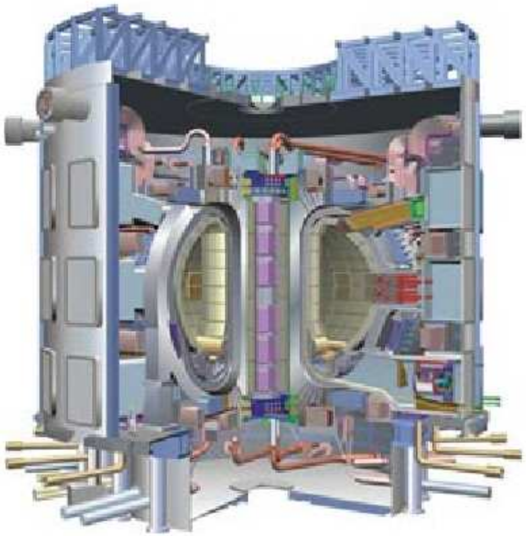

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Present confinement time is $\tau_E \simeq 1 - 3sec$.

Full T burn-up, high τ_E is not consistent with present tokamak concept. Li wall tokamak design (Zakharov, APS'06) may help to suppresses temperature gradient driven turbulence and have strong τ_E enhancement.

How much alphas do we need in a reactor? another “non-traditional” view

How do we design high performance tokamak reactor, high $n\tau_E$?

How much alphas do we need in a reactor? *another “non-traditional” view*

How do we design high performance tokamak reactor, high $n\tau_E$?

Suppose we improved the plasma confinement according to the Lawson criteria in ITER-like reactor

Can we design high power reactor? - hardly

How much alphas do we need in a reactor? another “non-traditional” view

How do we design high performance tokamak reactor, high $n\tau_E$?

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Fundamental problem limiting performance can be *good α confinement*:

$$\frac{Q_{pl}}{\tau_E} = P_{\alpha conf} = \frac{P_{reactor}}{5},$$

where $Q_{pl} = \beta B^2 V$ is the plasma energy content, reactor power $P_{reactor} = P_\alpha + P_n = 5P_\alpha$, τ_E is energy confinement time
(Zakharov, Gorelenkov’05-’07)

Understanding wave-particle interactions helps to design controllable future tokamak - reactor

1. Present tokamak concept: goal is to confine EPs
   - find regimes with good confinement
2. High T_i mode: α channeling improves tokamak performance up to $2P_f$.
   - find regimes with near threshold instability, controlled confinement
   - will discuss in some detail here (next)
3. Li wall concept - suppression plasma turbulence
   - find regimes with strong losses $\Rightarrow$ mitigate heat flux to the walls
   - offers prospects of high reactor performance

Effect of instabilities on energetic ions depends on frequency

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Assume that perturbed quantities are

$$\mathbf{E} = -\nabla\phi = \mathbf{E}_0 e^{-i\omega t + im\theta}.$$

Then particle energy change is

$$\frac{\partial \mathscr{E}_\alpha}{\partial t} = e_\alpha \left(-i\omega\right)\phi$$

and particle radial displacement comes from $\mathbf{E}\times\mathbf{B}$ drift in the magnetic field

$$\mathbf{v}_r = \frac{c}{B^2}\mathbf{B}\times\nabla_\theta\phi = \frac{c}{B^2}\mathbf{B}\times\left(\left(i\frac{m}{r}\right)\nabla\theta\right)\phi = -\frac{icm}{Br}\phi\nabla r.$$

Or

$$\frac{\Delta r}{\Delta\mathscr{E}_\alpha} = \frac{cm}{e_\alpha Br\omega} = \frac{m}{m_\alpha\omega_{c\alpha}r\omega}.$$

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- Low frequency instabilities ($< 200kHz$) may induce radial particle transport
- High frequency instabilities affect velocity space transport.

How do we manage alphas in a tokamak reactor: traditional vs. non-traditional view

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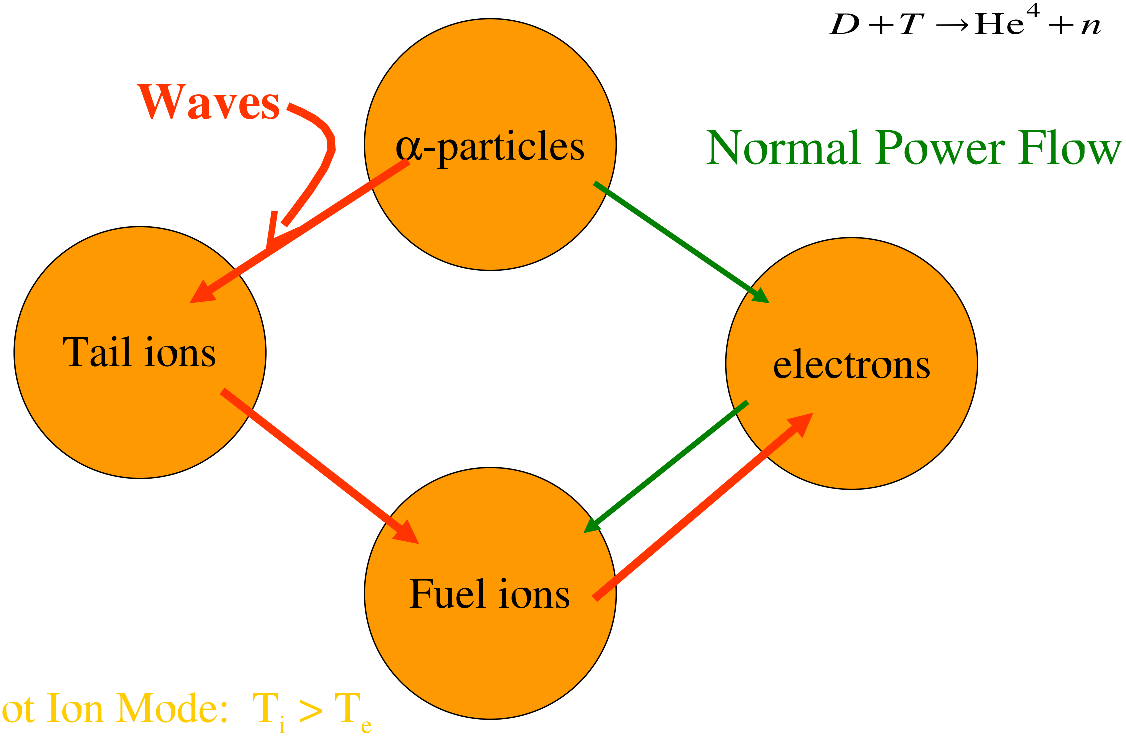
Advantages of “α-Channeling”:
Control of (1) power flow in a reactor and (2) He ash

$$D + T \rightarrow \mathrm{He}^4 + n$$



Waves

α-particles

Normal Power Flow

Tail ions

electrons

Fuel ions

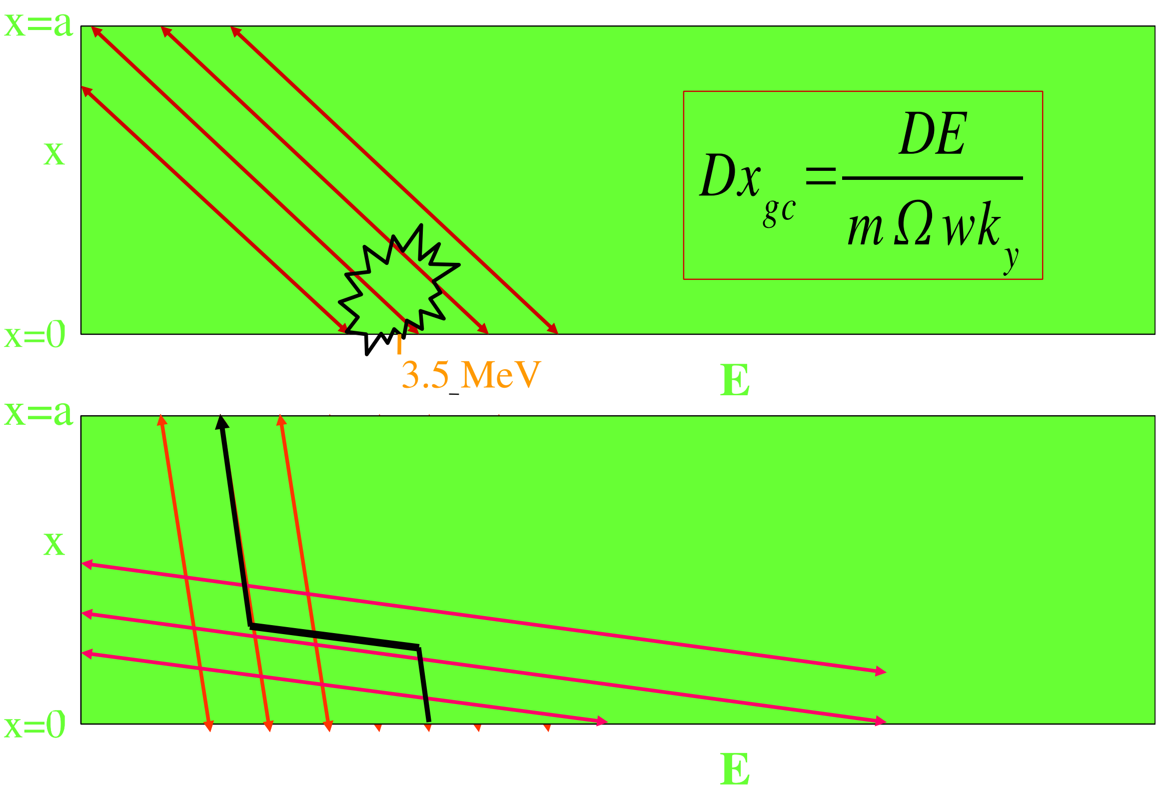
Get Hot Ion Mode: $T_i > T_e$
75% of α power to ions $\Rightarrow P_f \rightarrow 2\,P_f$

N.Fisch, J.Rax, PRL’92

Two waves are needed to achieve “full” α channeling

$$\Delta r = \frac{\Delta\mathcal{E}_\alpha m}{m_\alpha \omega_{c\alpha} r \omega}$$

Diffusion Paths with 2 Waves: Highly Constrained Stochastic Motion



$$Dx_{gc} = \frac{DE}{m\,\Omega\, w k_y}$$

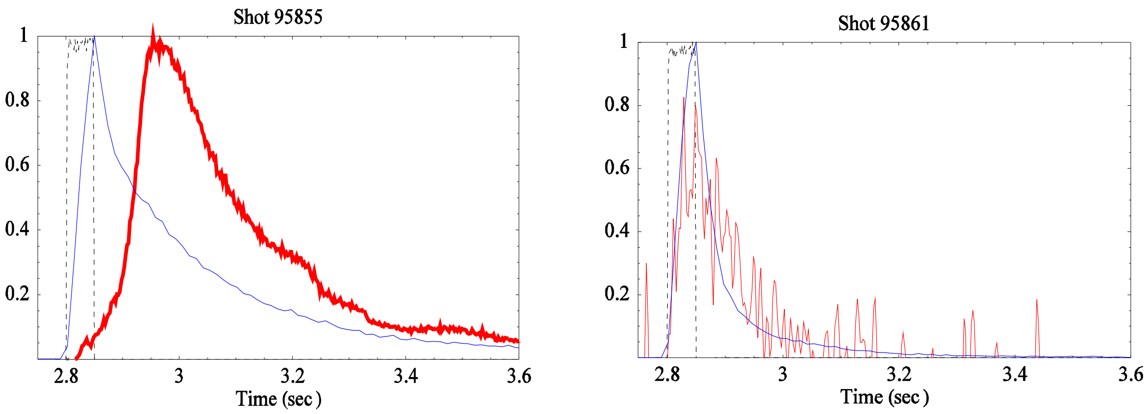
M.Herrmann, N.Fisch, PRL’92

TFTR showed evidence of strong wave-particle coupling: helps to design α channeling

Beam (black), neutrons (blue), & losses (red) [Figures from Herrmann (1998)]

With $P_{rf} = 3.2$ MW

With $P_{rf} = 1.0$ MW



Energy diffusion coefficient of lost particles estimated to be $D_\varepsilon \sim 25$ MeV^2/sec.

$> 100keV$ (> injection energy) beam ion Energy diffusion is 50 times the value which follows from ray tracing $\Rightarrow$
there is plasma amplification of the applied RF fields.
Need to control which waves to excite.

D.Clark, N.Fisch, Phys. Plasmas’00.

Waves excited by Energetic Particles

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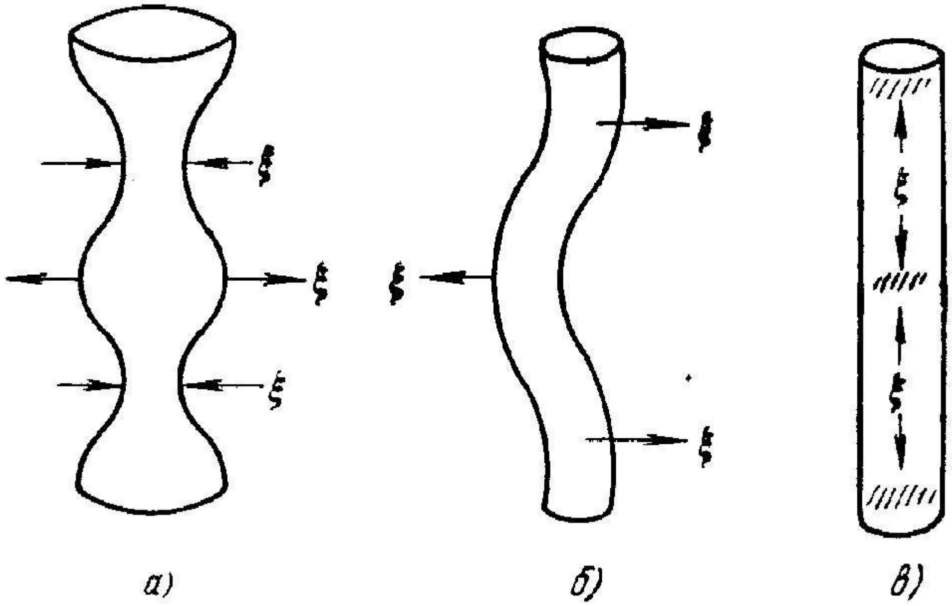
Three types of plasma oscillations can be of interest

compr. Alfvén shear Alfvén sound waves



Shear Alfvén wave dispersion in homogeneous plasma

Comes from MHD and incompressible plasma displacement $\nabla\cdot\vec{\xi}_{\perp}=0$ and $\xi_{\|}=0$

$$\rho_i\frac{dv}{dt}=\frac{1}{4\pi}\left[\left[\nabla\times\tilde{\mathbf{B}}\right]\times\mathbf{B}\right]$$

and

$$\frac{d\tilde{\mathbf{B}}}{dt}=\left[\nabla\times\left[\mathbf{v}\times\mathbf{B}\right]\right].$$

$\Rightarrow$ (using $\mathbf{B}\nabla=B\partial/\partial z$)

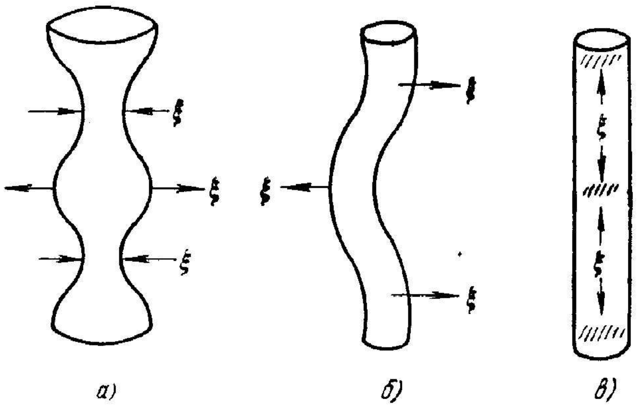
$$\frac{\partial^2}{\partial^2 t}\xi_{\perp}=v_A^2\frac{\partial^2}{\partial^2 z}\xi_{\perp}$$

or (using $k_{\|}^2=-\partial^2/\partial^2 z$)

$$\left(\omega^2-v_A^2k_{\|}^2\right)\xi_{\perp}=0$$

What do these oscillations do?

compr. Alfvén shear Alfvén sound waves



$$\omega_{cA}^2 = v_A^2 k^2 \gg \omega_A^2 = v_A^2 k_\parallel^2 \gg \omega_s^2 = v_s^2 k_\parallel^2$$

Typical effects on particles

1. compr. Alfvén - energy diffusion
2. shear Alfvén - space diffusion - more urgent need for ITER
3. sound waves - space diffusion, but strongly damped on ions

Shear Alfvén continuum in nonuniform plasma equilibrium

Operate with ∇ on

$$\left(\frac{\omega^2}{v_A^2}-k_\parallel^2\right)\xi_\perp=0$$

which has a vector form and make use of $\mathbf{v}_E=\frac{c}{B_0^2}\mathbf{B}_0\times\nabla\phi$

$$\left(\frac{\omega^2}{v_A^2}-k_\parallel^2\right)\nabla\phi=0.$$

We find

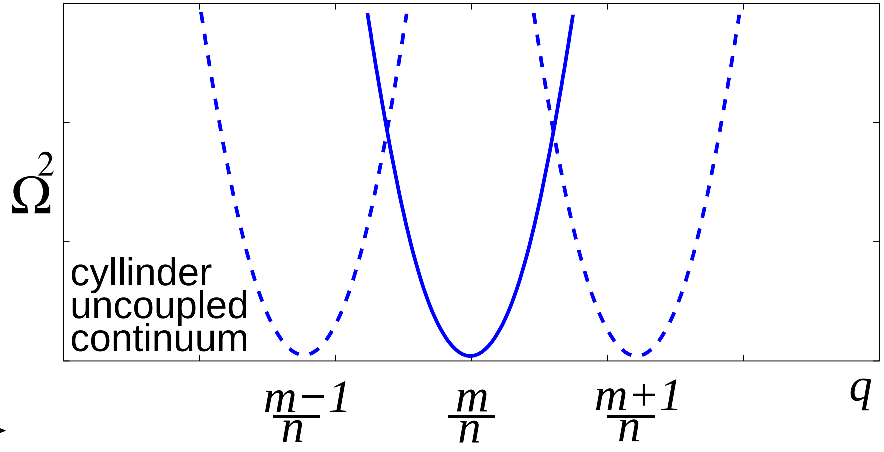
$$\frac{1}{r}\frac{\partial}{\partial r}\left(\frac{\omega^2}{v_A^2}-k_\parallel^2\right)r\frac{\partial}{\partial r}\phi+\left(\frac{\omega^2}{v_A^2}-k_\parallel^2\right)\frac{1}{r^2}\frac{\partial^2}{\partial\theta^2}\phi+Q\phi=0.$$

Some other terms are included in Q, such as ballooning term.
Each radial point of $\omega_A^2(r)=v_A^2k_\parallel^2(r)$ corresponds to singularity and represents the continuum.

Shear Alfvén continuum schematics (cylinder, zero β)

Magnetic field variation shapes the continuum through the safety factor profile $q = B_\varphi r / B_\theta R$,

$$\hat{k}_\parallel \phi = -i\vec{B}\cdot\vec{\nabla}\sum_m \phi_m e^{-i\omega t + im\theta - in\varphi} = \left(\frac{m}{qR} - \frac{n}{R}\right)\phi$$



$B = B(\theta)$, $\omega_A^2(r) = v_A^2 k_\parallel^2 \Rightarrow$

Rational surfaces corresponds to $k_\parallel = 0$.

If harmonics are coupled continuum gaps appear.

Why continuum gaps are important?

Continuum gaps are common phenomena.

Example: energy spectrum gaps in valence electrons in a periodic potential well of the crystal lattice.

- Higher order effects are tested as coupling is described by small parameters toroidicity, β: great test for theories,
- global modes may exist with frequency inside gaps
- if modes with frequencies in the gaps exist they have low damping: no continuum damping from kinetic, singular mode.

Shear Alfvén continuum schematic (torus)

Again take the eigenmode equation:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(\frac{\omega^2}{v_A^2}-k_\parallel^2\right)r\frac{\partial}{\partial r}\phi+\left(\frac{\omega^2}{v_A^2}-k_\parallel^2\right)\frac{1}{r^2}\frac{\partial^2}{\partial\theta^2}\phi+Q\phi=0.$$

If poloidal/toroidal harmonic expansion is applied

$$\phi=\sum_m\phi_m exp\left[-i\omega t+im\theta-in\varphi\right],$$

geometrical effects couple harmonics:

$$v_A^2k_\parallel^2=B\left(r,\theta\right)^2k_\parallel^2/4\pi\rho\Rightarrow$$

toroidicity, ellipticity, triangularity, curvature, pressure

Cheng,Chance’86,K.L.Wong’91,W.Heidbrink’91

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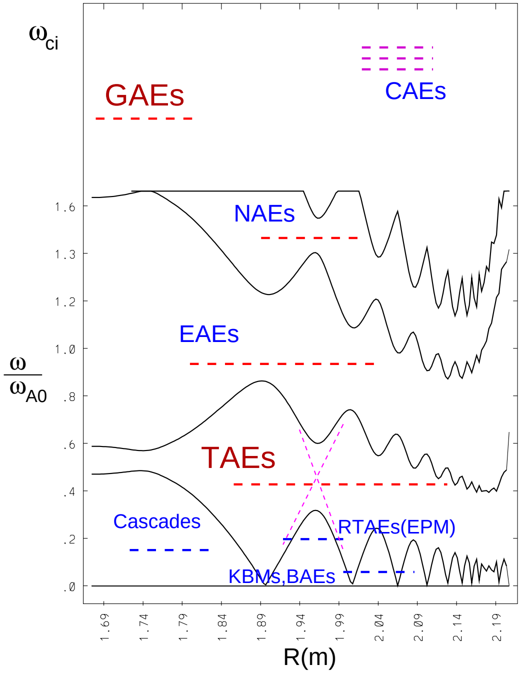
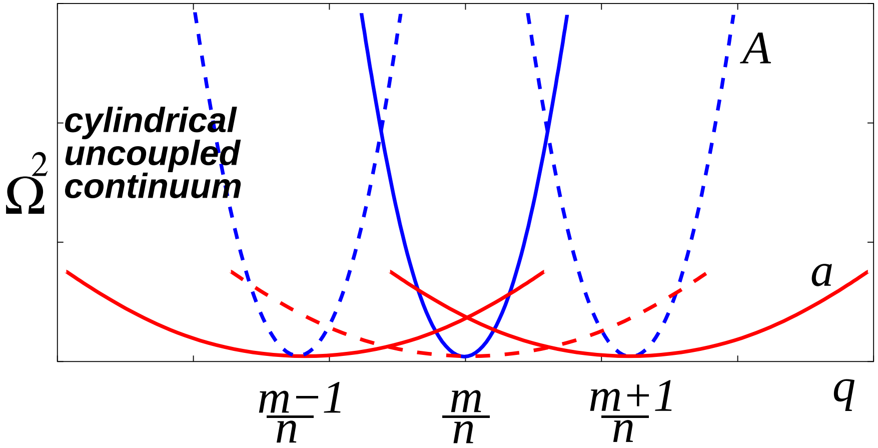


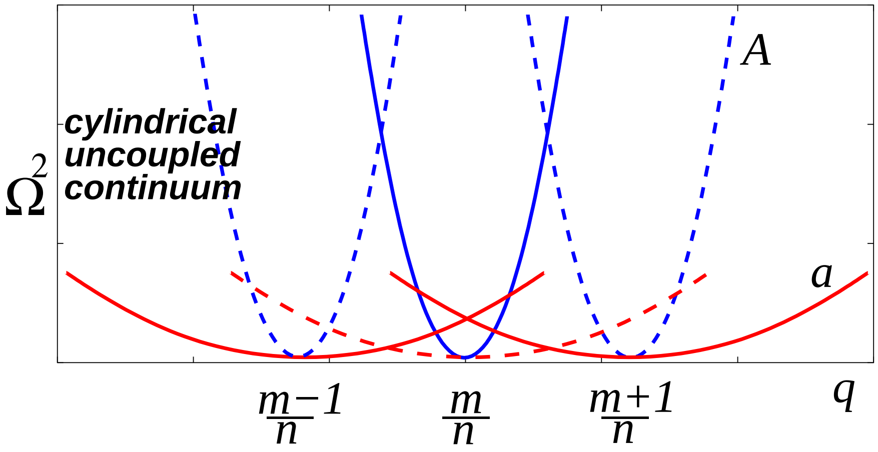
Finite plasma pressure introduces new gaps: Beta - induced Alfvén and acoustic gaps

- Alfvén (A) continuum at low frequency: $\omega^2 = v_A^2 k_{0,\pm1}^2$
- Acoustic (a) branch $\Omega^2 = \gamma\beta v_A^2 k_{0,\pm1}^2 / 2\,(1+\delta)$

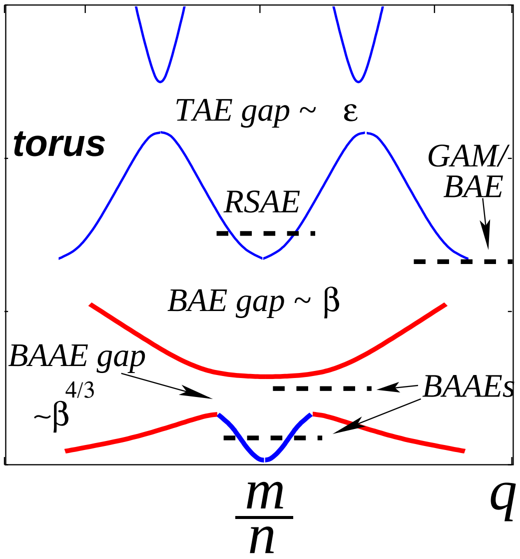


Finite plasma pressure introduces new gaps: Beta - induced Alfvén and acoustic gaps

- Alfvén (A) continuum at low frequency: $\omega^2 = v_A^2 k_{0,\pm1}^2 \; / \left(1+2q^2\right)$ (modified)
- Acoustic (a) branch $\Omega^2 = \gamma\beta v_A^2 k_{0,\pm1}^2 / 2\left(1+\delta\right)$ is coupled via $m \pm 1$ sidebands with modified Alfvén continuum (m harmonic) due to geodesic curvature and pressure.



$\Rightarrow$



Chu'92, Vander Holst'01, Gorelenkov'07

A remarkable agreement between TAE, RSAE mode structure in DIII-D and NOVA modeling (ideal MHD)

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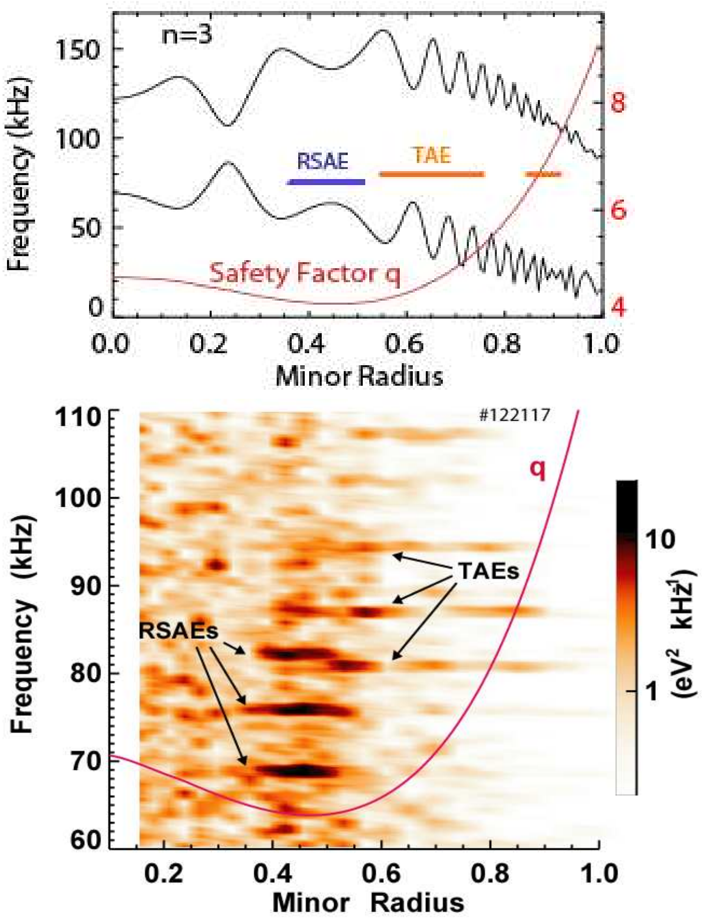


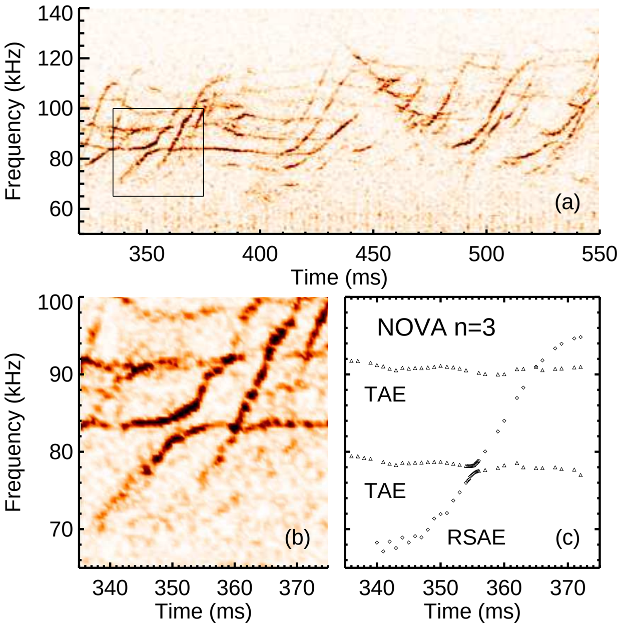
- Various gap modes reside in an effective waveguides, such as caused by q_{min} presence
- RSAEs are localized near continuum extrema points
  - RSAEs have one dominant poloidal harmonic
- Both RSAE and TAE structures agree quantitatively with ideal MHD (NOVA simulations)

Van Zeeland'06 PRL, '07 PhysPlasmas.

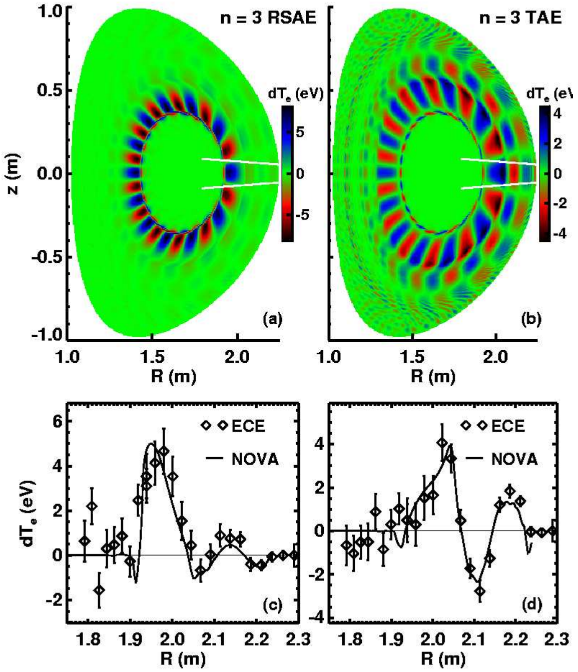
Ideal MHD TAE structure

Internal TAE/RSAEs mode structures measured by ECE show excellent agreement with ideal MHD predictions *(NOVA calculations, Van Zeeland, PRL ’06)*

RSAE & TAE frequencies crossover



RSAE & TAE structures



It could be that earlier works *(Carolipio’01, Heidbrink’97)* dealt with other modes such as EPMs in high fast ion beta plasma.

EP driven instabilities

Three conditions should be satisfied:

- source of free energy must be present
- particles should be at resonance
- drive should be higher than the damping

Free energy sources:

- pressure radial gradient (universal drive), low frequency
- velocity space gradient, high frequency
  - inversed velocity distribution
  - velocity anisotropy

As an example we construct the universal instability drive

$$\frac{\gamma}{\omega} = K^{-1}\left[\Delta\mathscr{E}\beta_\alpha \frac{\partial f\left(v_{res}\right)}{\partial\mathscr{E}_\alpha} + \frac{\Delta r}{r}\frac{\partial\beta_\alpha}{\partial\ln r} f\left(v_{res}\right)\right] \sim -\omega\beta_\alpha f\left(v_{res}\right) + \frac{m}{\omega_c}\frac{\partial\beta_\alpha}{\partial\ln r} f\left(v_{res}\right).$$

Condition for slow ramp up of the alpha source should be met.

Theory predicts that the most unstable mode number scales with the machine size

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Among possible Alfvén eigenmodes, AEs, toroidicity-induced AE (TAEs) modes are likely to limit fusion product confinement in BP.

Low frequency modes are mostly responsible for fast ion radial transport: TAEs, EPMs, BAEs.

High frequency modes are responsible for phase space particle diffusion: GAEs, CAEs, ICE.

Theory:
n range of most unstable TAEs determined by Finite Orbit Width (FOW) effects: (Berk PL,'92, Fu PF, '92, Breizman'95, Candy'95)

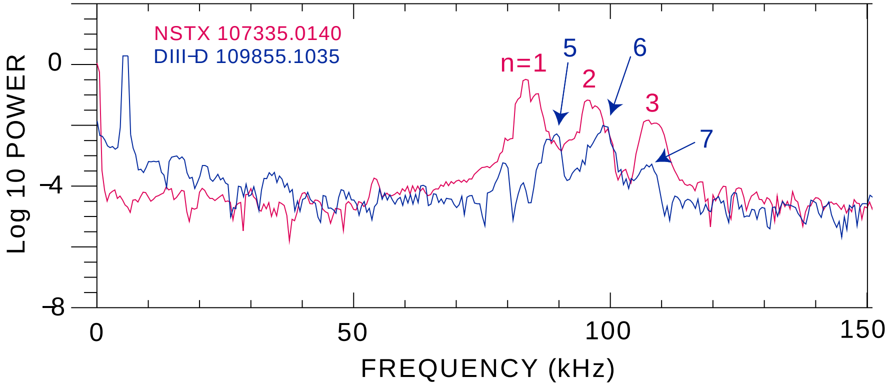
$$k_\perp \Delta_b \simeq \frac{nq^2\rho_b}{r} \sim 1 \Rightarrow \frac{r}{R} n_{max} < n < n_{max} \simeq \frac{r\omega_{c\alpha}}{q^2 v_A}$$

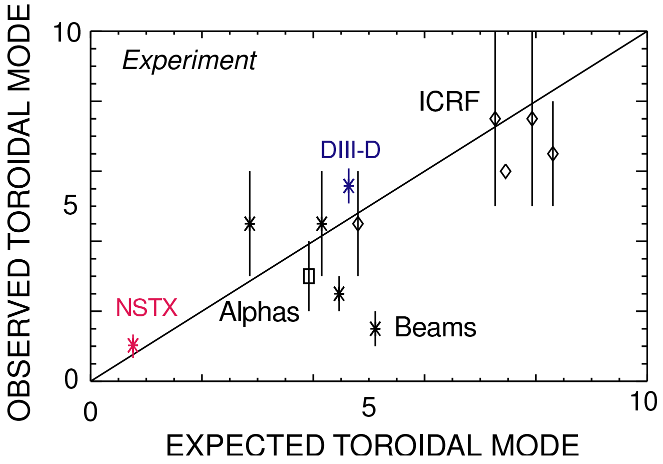
NOVA results agree with theory (Gorelenkov PoP,'99)

Experimental observations confirm theory predictions

DIII-D/NSTX similarity experiments were designed to confirm theory predictions *(W. Heidbrink, PPCF '03)*:

- The $a = 0.8m$ radius in NSTX and DIII-D but different $R = 1m$ vs $R = 1.7m$, $B = 0.6T$.
- Use similar NBI features: injection geometry, energy, trapped to passing particle ratio.





Most unstable mode number (larger amplitude at the edge) scales as

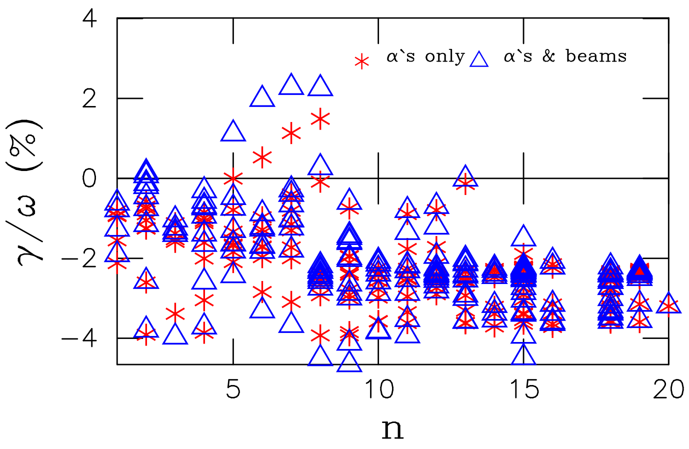
$$n \sim a/q^2$$

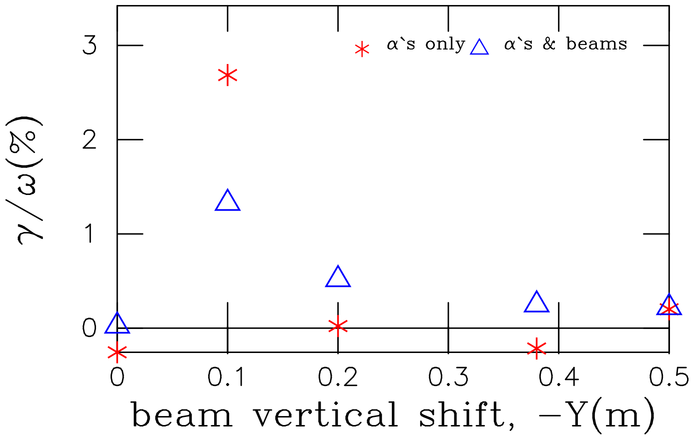
Trend of unstable TAE's n-dependence is recovered by NOVA/NOVA-K codes

🟥 no damping, ◆ - with damping

- TAEs excitation thresholds are reproduced for medium-n numbers
- NOVA-K with isotropic d.f. does not predict observed unstable modes
- Main damping mechanisms are:
  - ion Landau damping (dominant in ITER), radiative damping
- Stabilization of TAEs at high end of n range is due to FOW effects and higher damping
- **Helps to validate predictive capabilities of NOVA-K *(Gorelenkov'03 APS).***

Numerical simulations of AE stability in ITER





$1MeV$ tangentially injected beam ion drive is comparable with the $\alpha-$drive.

- lowering NBI energy to $0.5MeV$ significantly reduces drive.
  - $0.5MeV$ energy is enough for good beam penetration.
- medium, high-n's are unstable $n = 6 - 13$.
- most unstable modes are localized at $r/a \sim 0.5$ and avoid central ion Landau damping.
- TAE control is possible via NBI aiming.

DIII-D recent beam driven mode XP (W.W. Heidbrink, IAEA’06)

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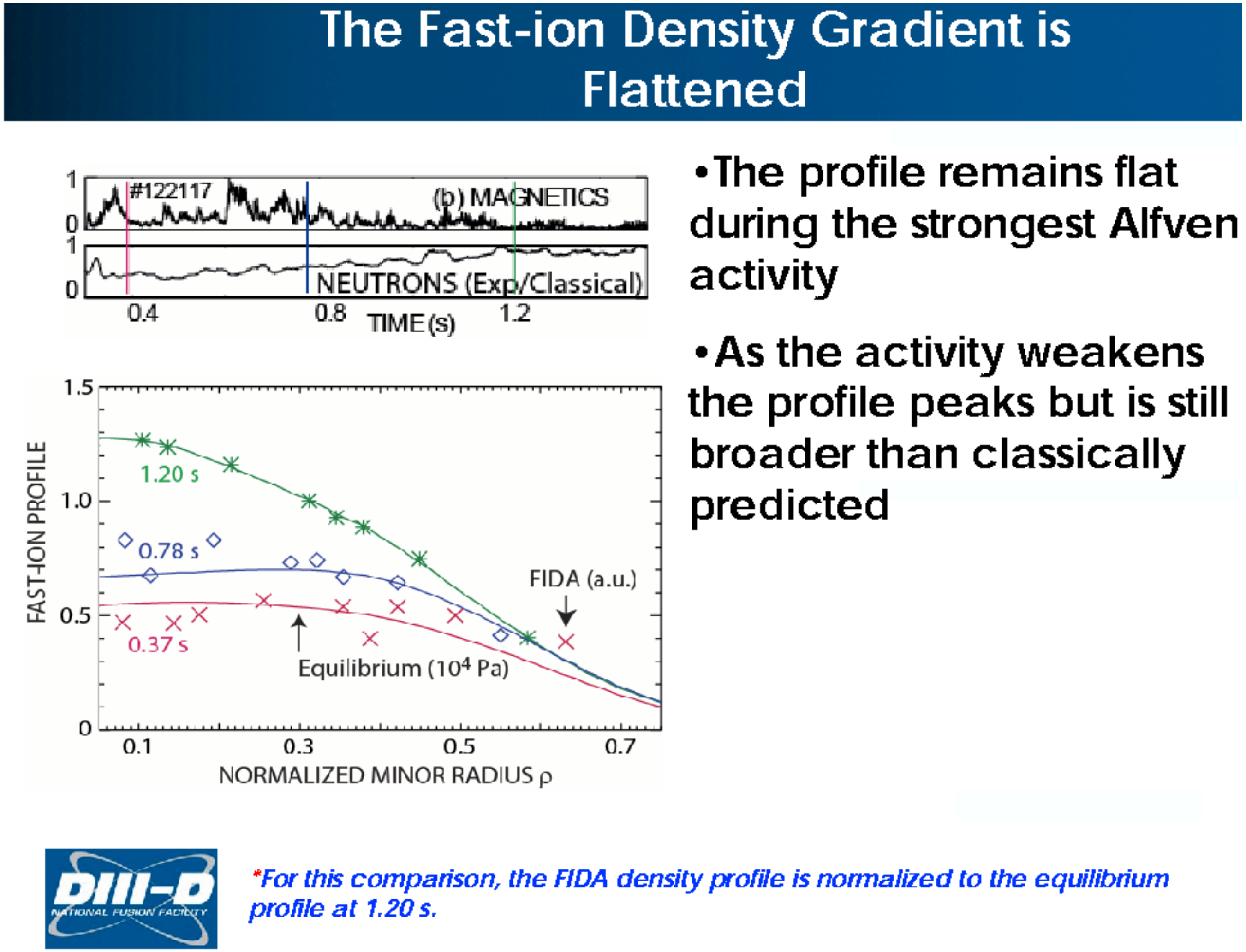


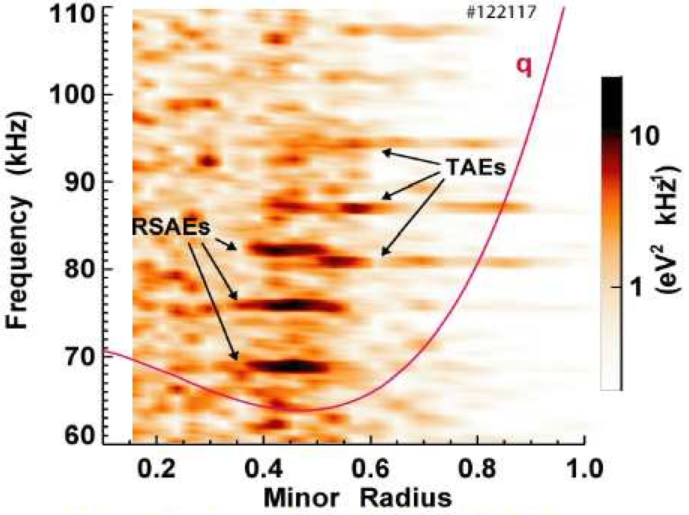
The Fast-ion Density Gradient is Flattened

- The profile remains flat during the strongest Alfven activity
- As the activity weakens the profile peaks but is still broader than classically predicted

**For this comparison, the FIDA density profile is normalized to the equilibrium profile at 1.20 s.*

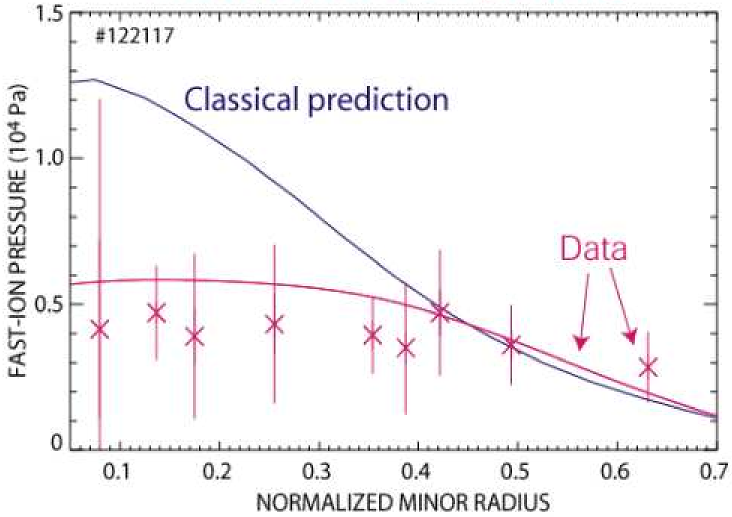
Quantitative calculations of EP transport are unsuccessful

Radial dT profile during beam injection into DIII-D



Van Zeeland, PRL 97 (2006) 135001

Radial fast-ion profile



Heidbrink, PRL 99 (2007) in press

- **Measured mode structure agrees well with MHD model**
- **Input these wave fields into an orbit-following code**
- **Calculate much less fast-ion transport than observed**
- **What's missing?**

APS-DPP 11/07

The Status

Linear theory

- well developed, zoo of modes
- theory is validated
- problems exist: kinetic effects, damping mechanisms are not tested, comprehensive codes are not available

Non-linear theory

- Nonlinear/transport with multiple instabilities should be #1 priority:
  - Develop realistic reduced models
  - Realistic, global nonlinear codes.
- Experiments on the “sea of Alfven modes” driven transport should be done on present day machines.

Further slides

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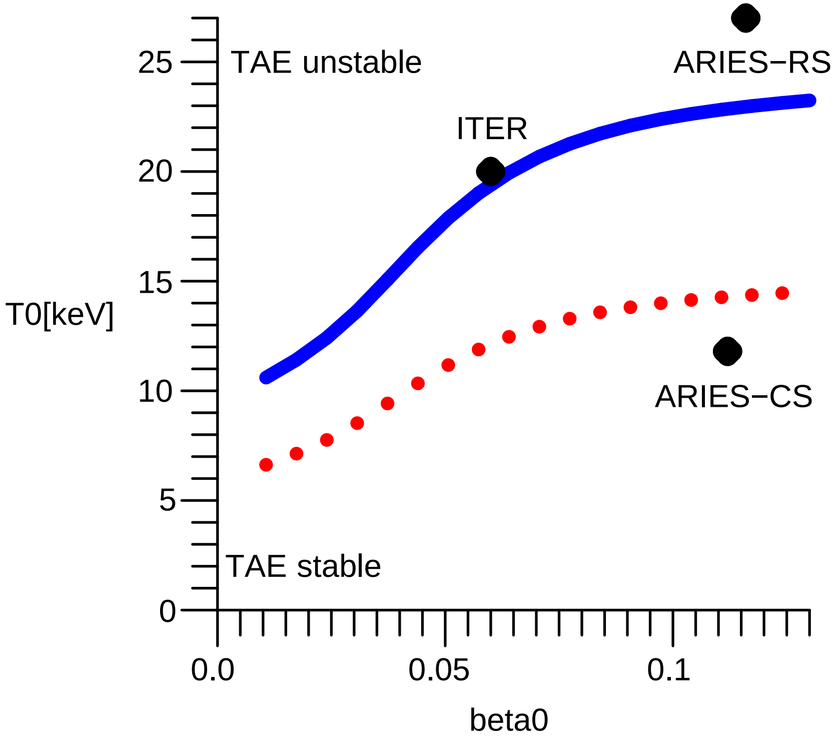
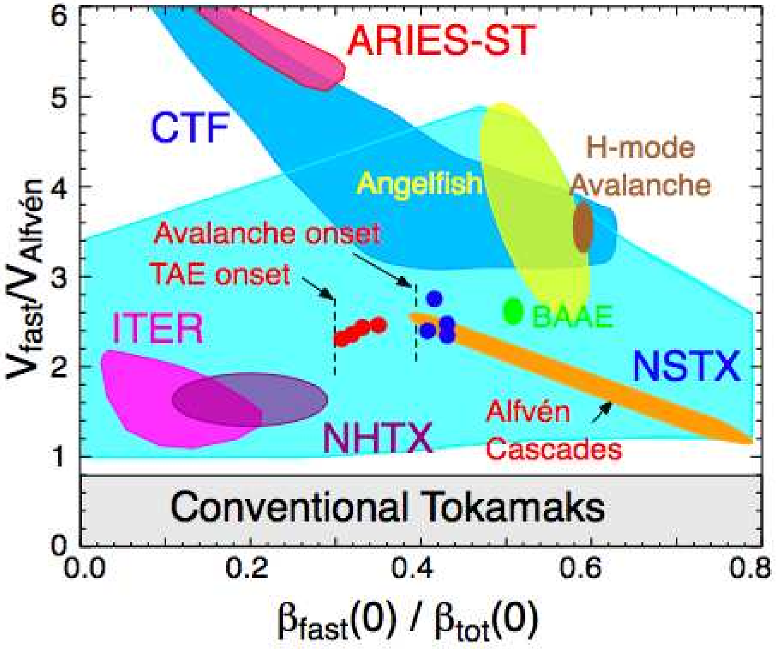
Projection to ITER and other BPs designs

Analytical model for TAE stability in BPs



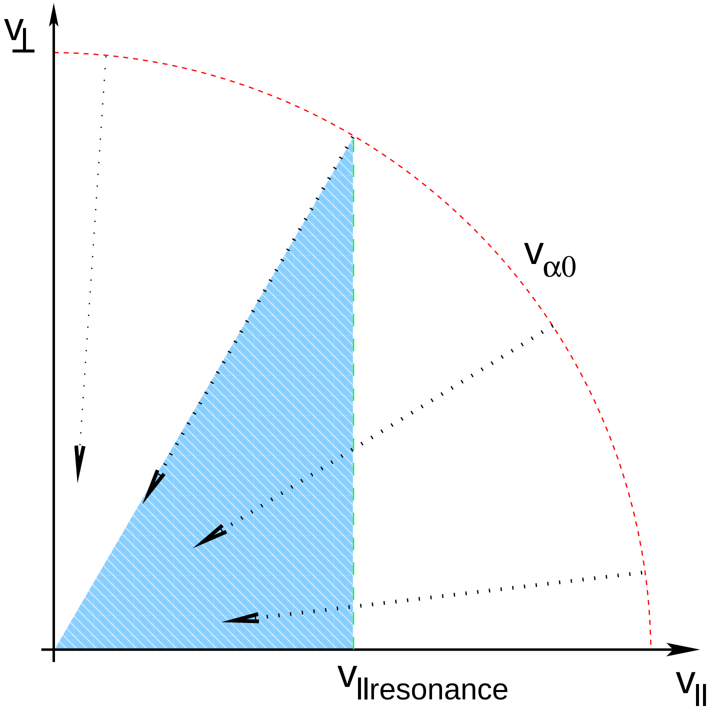
- Stability diagram in $\beta_{pl}(0) - T_i(0)$ plane (local theory)
  - source of alphas comes from background plasma, not external heating $S_\alpha \sim \langle\sigma v\rangle \Rightarrow \frac{-\partial\beta_\alpha}{\partial\ln r} \simeq \frac{7}{2}\beta_\alpha\frac{-\partial\ln T}{\partial\ln r}$
  - theory (dashed curve), normalized to NOVA (solid curve)
  (Gorelenkov, NF'03)
- α's slowing down d.f., ion Landau and trapped electron collisional dampings.
- Projection to DEMO, $\beta_{pl0} \simeq 10\%$, $T_{i0} = 45keV$: strongly unstable TAEs.

Relation of present day plasma regimes to the burning plasmas projections



- Most of the present day experiments are in different regimes than in BP, ITER
- NSTX overlap some EP relevant parameters: β_{EP}/β_{total}, v_{EP}/v_A.
- But ρ_{EP}/a are different $\Rightarrow$ should be addressed in XP and theory.

What to expect? How much of alphas population affected?



Maximum effect from instabilities with $v_{\|}$ resonance

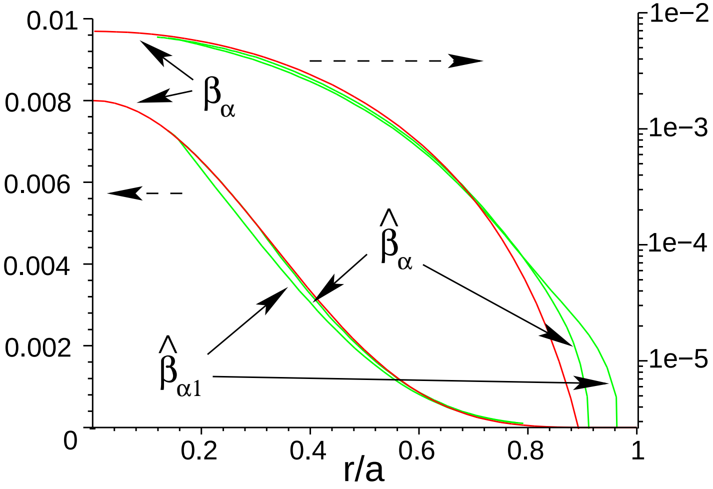
Fraction of effected alpha power (Kolesnichenko '80)

$$P_{\alpha res} = P_\alpha \left(v_{\alpha 0} - v_{\|}\right) v_{\|}/v_{\alpha 0}^2 \leq 25\%$$

Other particles are not interacting with such instabilities

What are expected effects on alpha profiles (normal shear, qualitative theory)

PPPL



Maintain the critical gradient profiles:

$$\frac{\partial \beta_{\alpha cr}}{\partial r} = -\frac{\gamma_{iL}+\gamma_{ecoll}}{\gamma\prime_{\alpha}}, \quad \gamma_{\alpha\prime} = \gamma_{\alpha} / \left(\partial \beta_{\alpha} / \partial r\right)$$

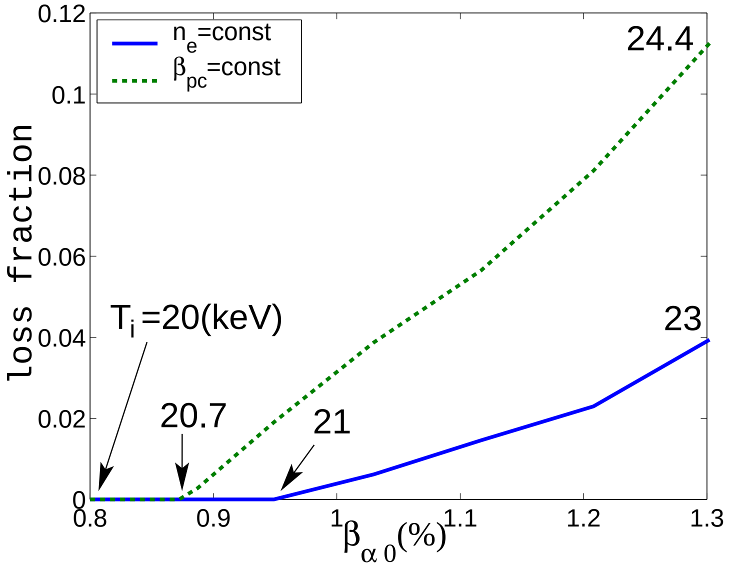
Use the phase space particle conservation law

$$\int_0^a r\left(\beta_\alpha - \hat\beta_\alpha\right) dr = 0$$

- *α's are redistributed, but keep critical beta profile*
- *β_α is equilibrium profile, $\hat\beta_\alpha$ is final profile with near threshold gradient*
- *$\hat\beta_{\alpha 1}$ is at critical beta at 0.7 of theoretical beta critical*

This model can also predict losses of alphas

PPPL



Increasing alpha beta increases losses.
RULES OF THUMB

1% alpha's central beta is threshold

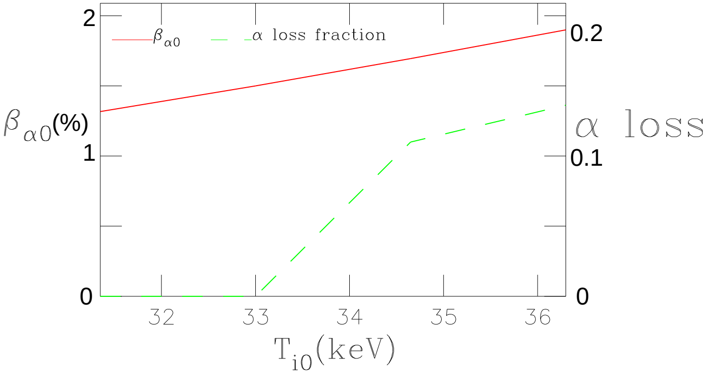
5% of all alphas is tolerable for ITER (Putvinskii, NF '99)

TAE effects will be benign in nominal regular shear scenario according to the quasilinear diffusion model.

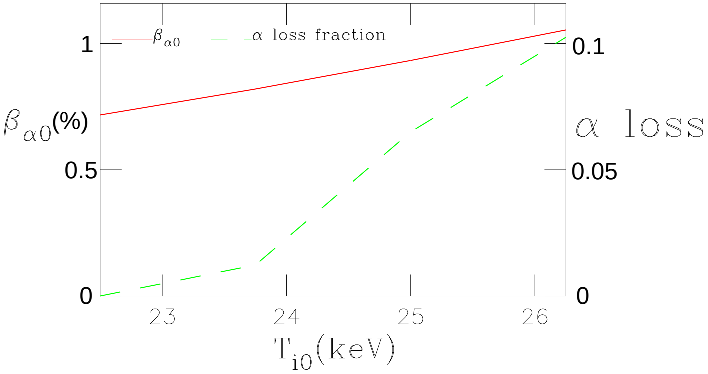
Losses are calculated at fixed beta with ion temperature within $20 < T_{i0}(keV) < 24$ ($\beta_{\alpha 0} \sim T_{i0}^{5/2}$) and density $20 < T_{i0}(keV) < 23$ ($\beta_{\alpha 0} \sim T_{i0}^{7/2}$).

Losses of alphas in hybrid and AT plasmas

Hybrid $T_{i0} = 33keV$



Reversed shear $T_{i0} = 25keV$



- Constant plasma beta scan $\beta_\alpha \sim T_i^{5/2}$.
- Quasilinear/local TAE theory predicts ITER operating point to be near TAE threshold
- Rather benign effects at nominal parameters.

Tone down statement:
Need to develop a selfconsistent quasilinear model.