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Division of Plasma Physics
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Category Number and Subject ________________

☐ Theory ☐ Experiment

<u>Generalization of PPPL Transport Analysis Code TRANSP for Time Dependent Magnetic Flux Geometry</u>,*

D. MCCUNE, R. GOLDSTON, R. MCCANN, R. WIELAND, <u>Princeton University</u>--The TRANSP time dependent transport analysis code has been modified to permit analysis of data from plasmas with time dependent geometry, i.e., plasmas which move, change shape, or enclose varying amounts of toroidal flux. The measured shape of the plasma boundary vs. time is specified as input data for TRANSP. An MHD equilibrium solver periodically reevaluates the interior flux surface equilibrium using pressure and current profiles provided by TRANSP. A numeric integrator is used to accurately evaluate the metric associated with the new equilibrium. Ion and electron energy balance are solved incorporating new compression terms due to time rate of change of plasma geometry. A generalized expression for poloidal field diffusion is used to determine the plasma current profile, voltage profile, and poloidal field energy balance.

The coordinate system and transport equations used in the new version of TRANSP will be shown. Examples of use of the new code for analysis of TFTR compression and/or PBX plasmas will be presented.

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☐ Prefer Poster Session

☐ Prefer Oral Session

☐ No Preference

☐ Special Requests for placement of this abstract:

☐ Special Facilities Requested (e.g., movie projector)

Submitted by: D. McCune
(signature of APS member)

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(same name typewritten)

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(address)

This form, or a reasonable facsimile, plus *Two Xerox Copies* must be received NO LATER THAN Noon, Friday, July 13, 1984, at the following address:

Division of Plasma Physics Annual Meeting
c/o Ms. Barbara Sarfaty
Princeton Plasma Physics Laboratory
P. O. Box 451
Princeton, New Jersey 08544

# CONTENTS

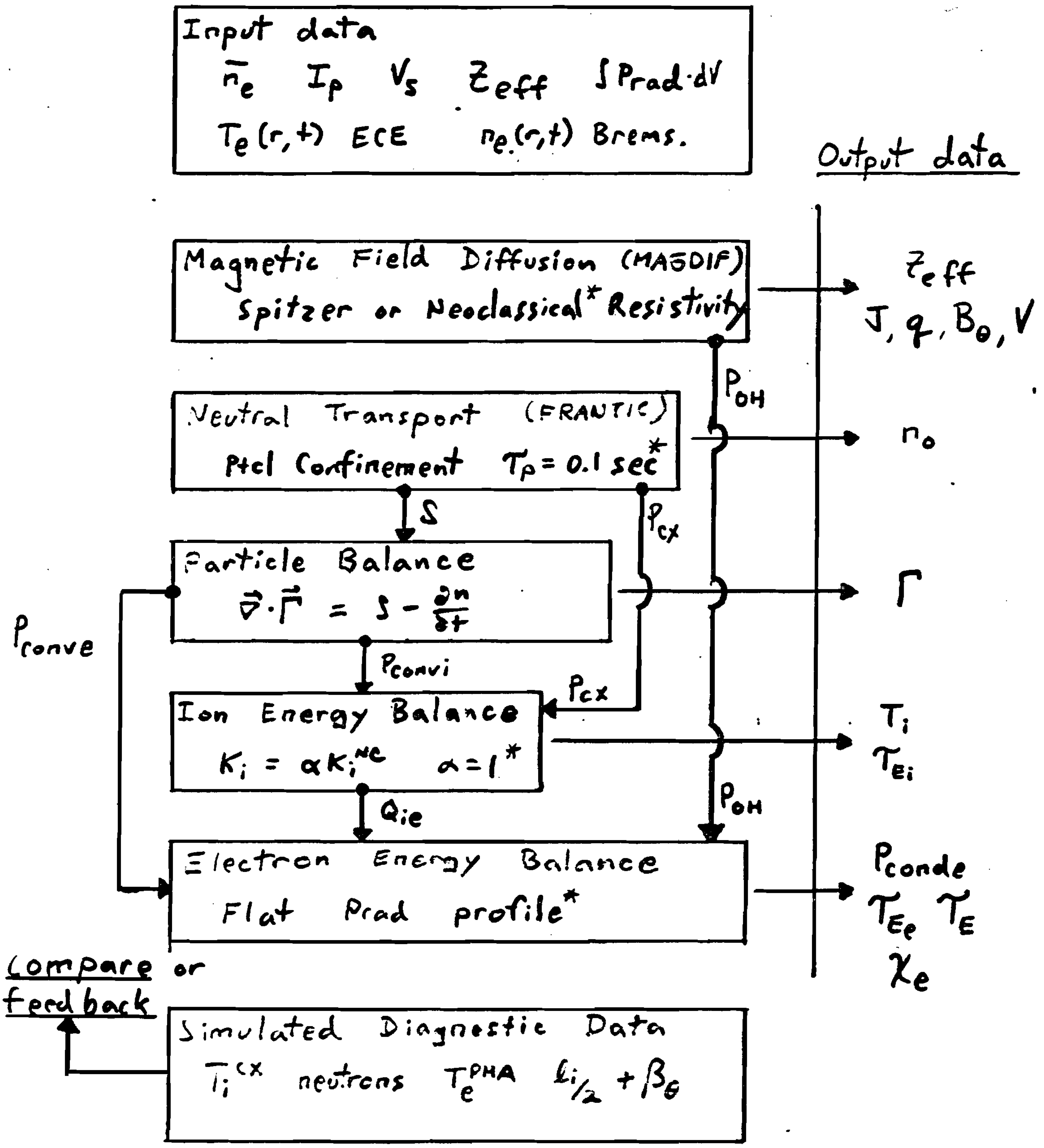
# * INTRODUCTION *

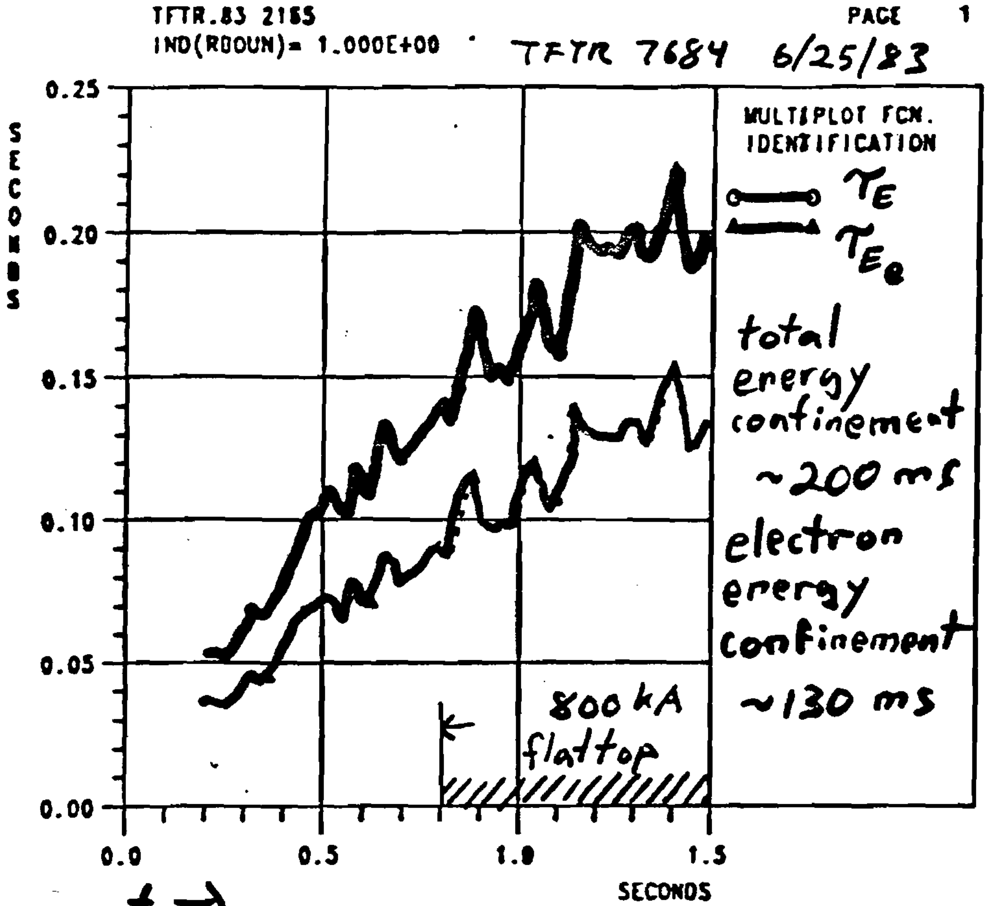
The PPPL TRANSP TIME DEPENDENT TRANSPORT ANALYSIS CODE is used to infer the transport and confinement properties of tokamak plasmas directly from the measured diagnostic data of tokamak plasma experiments.

As an example the TRANSP run configuration and a sample result from the TFTR summer 1983 ohmic plasma experiments are shown below:

# TRANSPORT ANALYSIS for TFTR OH-HEATED DISCHARGE

* = assumed for initial TFTR run series






ENERGY CONFINEMENT VS. TIME

Energy confinement times are deduced from Plasma parameters and the energy balances:

$U_e$ = elec energy  $U_i$ = ion energy

$$\tau_{E_e} = U_e / (P_{conde} + P_{conve} + P_{rad})$$

$$\tau_E = \frac{U_e + U_i}{P_{conde} + P_{conve} + P_{rad} + P_{condi} + P_{convi} + P_{cx}}$$

with

$P_{cond}$ = conduction  $P_{conv}$ = convection

$P_{rad}$ = radiation  $P_{cx}$ = charge exchange

# FREE EXPANSION (TFTR)

## *METHOD OF ANALYSIS

### COORDINATE TRANSFORM TO

to a relative flux label is useful if the amount of flux enclosed in the system is a function of time. The transformation is straight-forward and permits modeling the expanding or shrinking system with a fixed number of grid points.

TRANSP SUPPORTS ANALYSIS OF
plasmas with ohmic heating, neutral beam injection, RF heating, pellet injection, COMPRESSION, FREE EXPANSION, and NON-CIRCULAR GEOMETRIES.

Code features to support the CAPITALIZED items are new this year and are the subject of THIS POSTER.

# TRANSP Coordinate System
## for
## Arbitrary Time-Changing 2-D Magnetic Geometry
## to
## Analyze PBX and TFTR Compression Experiments

D. McCune
R. Goldston
1/84

- toroidal flux function and label:

$$\Phi(\rho) = \pi\rho^2 B_0$$

- $\rho$ is the absolute flux label;

$$\Phi_{lim} = \pi\rho_{lim}^2 B_0 = \text{flux enclosed by plasma}$$

- the relative flux label

$$\xi = \rho/\rho_{lim} = (\Phi/\Phi_{lim})^{1/2}$$

is used in TRANSP as the fundamental spatial coordinate.

# TRANSP Coordinate System

Advantages of the relative flux label coordinate $\xi$:

- time-invariant range.

  $\Phi_{lim}$ and $\rho_{lim}$ vary in time but $\xi_{lim} \equiv 1$ always.

  In code, fixed $\xi$ zones always cover plasma.

- independent of choice of $B_0$ (cf. $\rho$ coord.)

  definition of $B_0$ is a problem in compression experiments.

- coordinate transformations between absolute flux ($\rho$) and relative flux ($\xi$) are straightforward.

  e.g. let $\dot{\lambda} = \frac{1}{2\Phi_{lim}} \frac{d}{dt}\Phi_{lim}$

  then $$\left.\frac{\partial}{\partial t}\right|_\rho = \left.\frac{\partial}{\partial t}\right|_\xi - \xi\dot{\lambda}\frac{\partial}{\partial \xi} \quad \ldots$$

$\longrightarrow$ solve transport equations on $\xi$-grid

$\longrightarrow$ plot results on $\rho$-grid to show physical significance.

ALL TRANSP EQUATIONS ARE WRITTEN IN TERMS OF THE RELATIVE TOROIDAL FLUX LABEL. "$\xi$".

*APPLICATION

# INPUTS TO TRANSP -- FREE EXPANSION

- o Plasma position and minor radius
- o External toroidal field
- o Electron temperature profile
- o Electron density profile
- o Total plasma current
- o li/2 + beta
- o Total neutron emission

```
===========
|all data|
|vs. time|
===========
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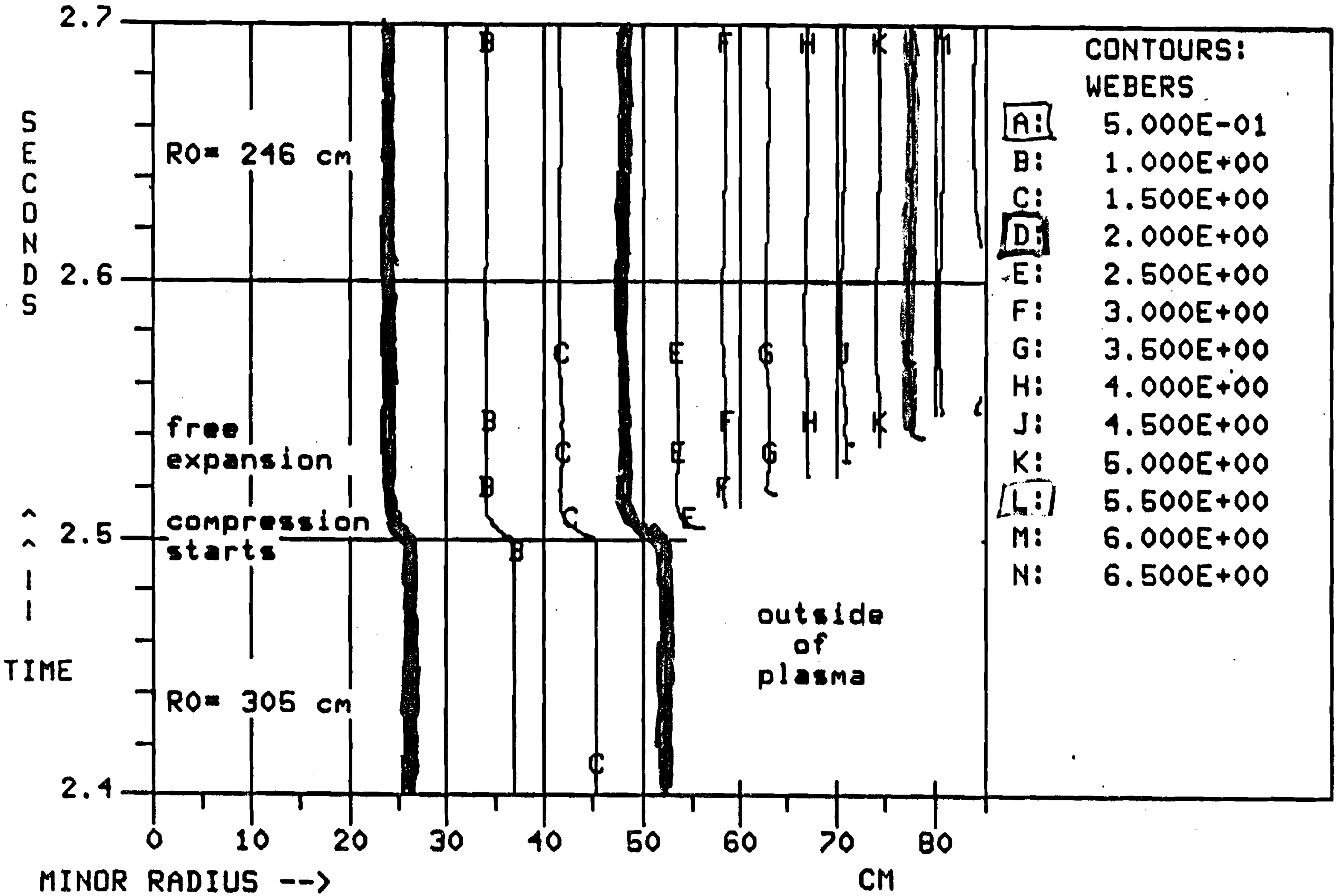
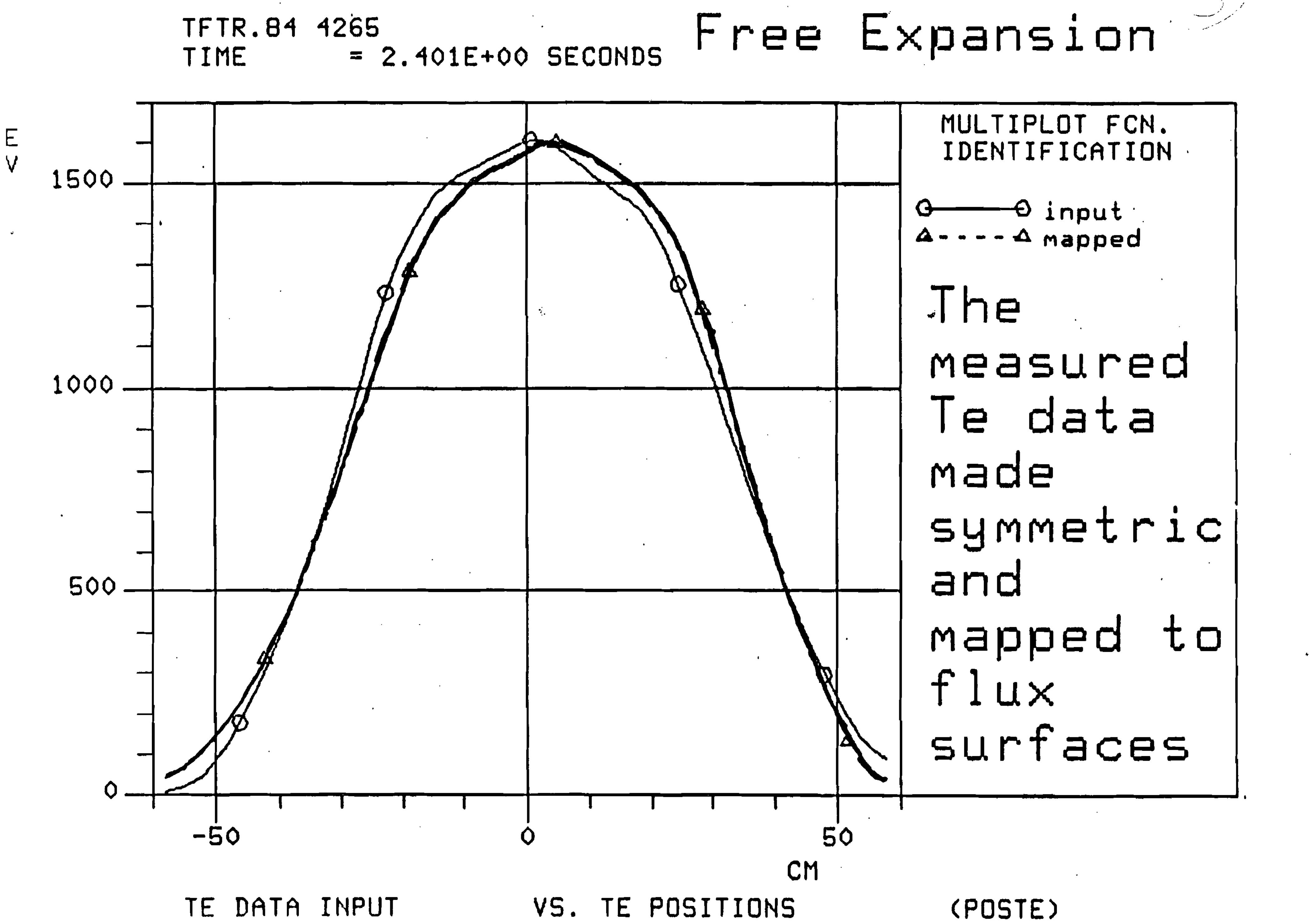
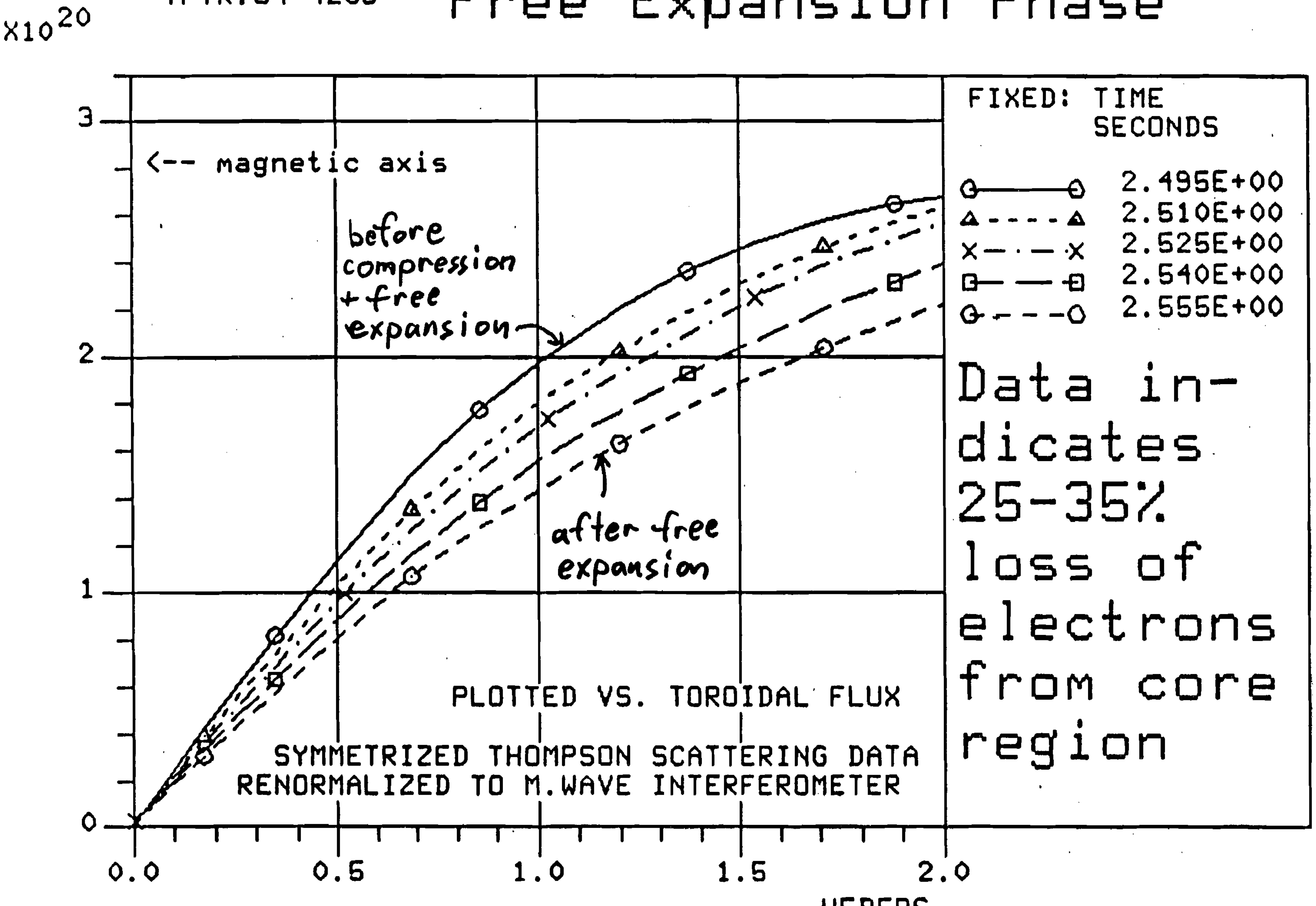
# TFTR summer 1984 free expansion shot



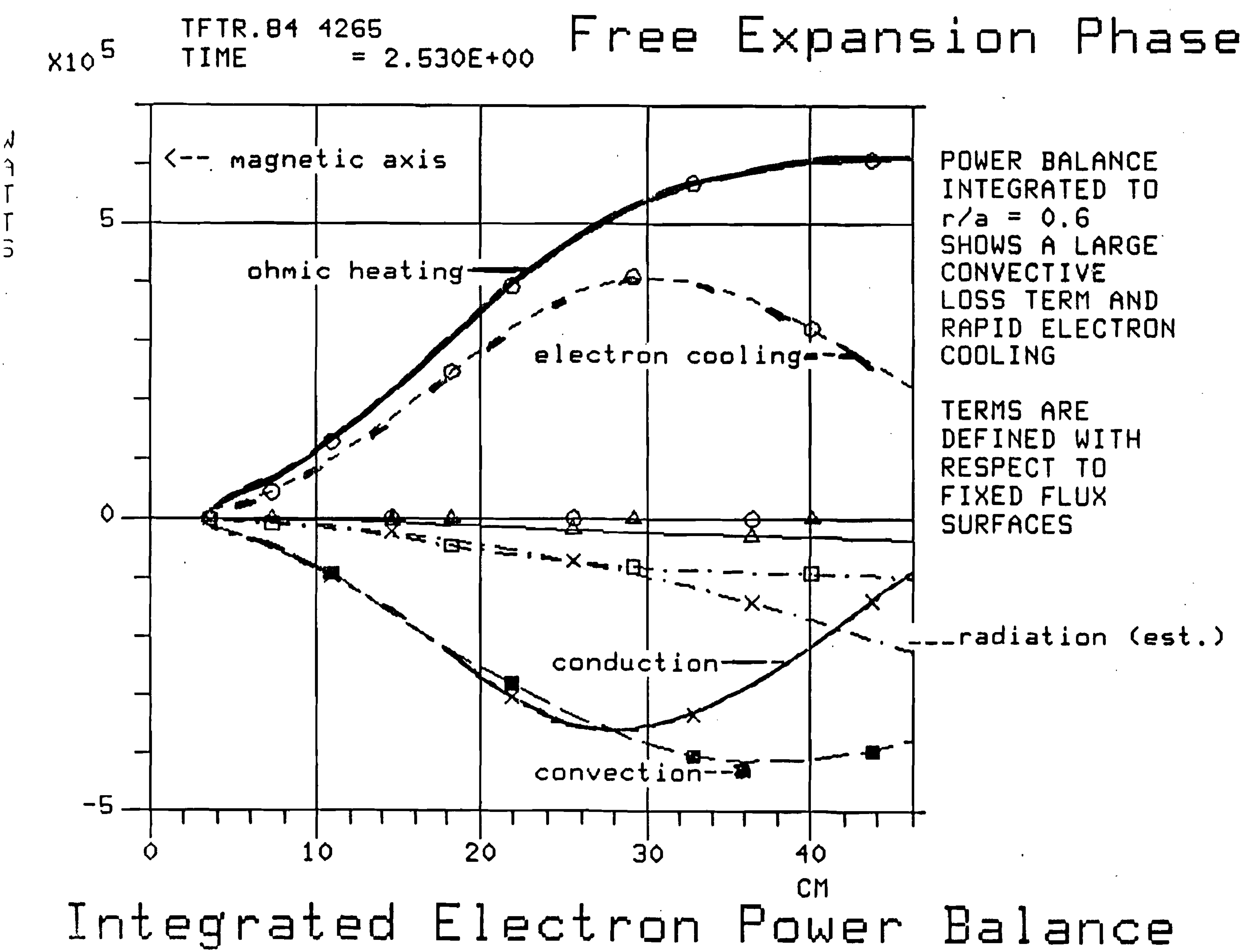

Location of Flux Surfaces


TFTR.84 4265
TIME = 2.401E+00 SECONDS
Free Expansion
EV
1500
1000
500
0
-50
0
50
CM
MULTIPLOT FCN. IDENTIFICATION
input
mapped
The measured Te data made symmetric and mapped to flux surfaces
TE DATA INPUT VS. TE POSITIONS (POSTE)

# Free Expansion Phase



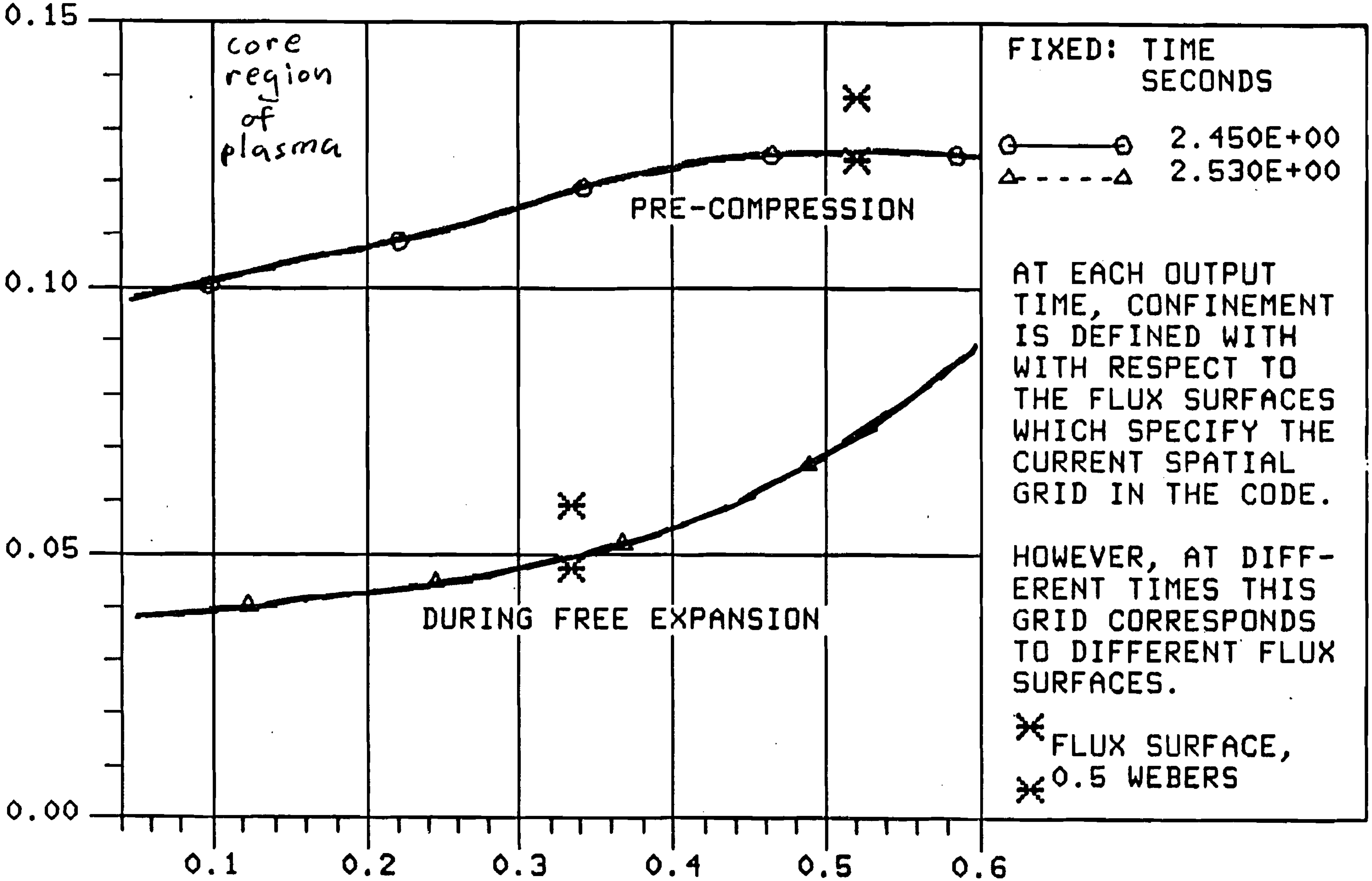

Volume Integrated Electron Density



Free Expansion Phase
TFTR.84 4265
TIME = 2.530E+00
X10 5
WATTS
<-- magnetic axis
ohmic heating
electron cooling
conduction
convection
radiation (est.)
5
0
-5
0
10
20
30
40
CM
POWER BALANCE INTEGRATED TO r/a = 0.6 SHOWS A LARGE CONVECTIVE LOSS TERM AND RAPID ELECTRON COOLING
TERMS ARE DEFINED WITH RESPECT TO FIXED FLUX SURFACES
Integrated Electron Power Balance

TFTR.84 4265

# Free Expansion




Electron Energy Confinement vs. r/a

# COMPRESSION (TFTR)

## *METHOD OF ANALYSIS

COMPRESSION OR DECOMPRESSION terms are required in the thermal energy balance equations if the shape or position of flux surfaces are to vary in time. The poloidal field diffusion equation should be cast in terms of a flux coordinate so that it may follow the motion of flux surfaces during compression

# TRANSP

## 1 1/2 Particle and Energy Balance Equations

# DETAILS

# Energy Balance — adiabatic compression of Maxwellian plasma

In the absence of sources and sinks, the ideal gas law applies:

@ fixed $\rho$, $\frac{3}{2} nT \left(\frac{\partial V}{\partial \rho}\right)^{5/3} = \text{const.}$

(V is volume)

$$\frac{\partial}{\partial t} \left\{ \frac{3}{2} nT \left(\frac{\partial V}{\partial \rho}\right) \left(\frac{\partial V}{\partial \rho}\right)^{2/3} \right\}\Bigg|_\rho = 0$$

Seperating and multiplying thru by $(\partial V/\partial \rho)^{-5/3}$,

$$\underbrace{\left(\frac{\partial V}{\partial \rho}\right)^{-1} \frac{\partial}{\partial t} \left\{ \frac{3}{2} nT \left(\frac{\partial V}{\partial \rho}\right) \right\}\Bigg|_\rho}_{\text{time rate of change of energy density}} = \underbrace{-\left(\frac{\partial V}{\partial \rho}\right)^{-1} nT \frac{\partial}{\partial t}\left(\frac{\partial V}{\partial \rho}\right)\Bigg|_\rho}_{\text{compression power density}}$$

Conversion to $\xi$ grid yields

$$\left(\frac{\partial V}{\partial \xi}\right)^{-1} \left\{ \frac{\partial}{\partial t}\Big|_\xi \left(\frac{3}{2} nT \frac{\partial V}{\partial \xi}\right) - \dot{\ell} \frac{\partial}{\partial \xi}\left(\xi \cdot \frac{3}{2} nT \frac{\partial V}{\partial \xi}\right) \right\}$$

$$= -\left(\frac{\partial V}{\partial \xi}\right)^{-1} nT \left\{ \frac{\partial}{\partial t}\Big|_\xi \left(\frac{\partial V}{\partial \xi}\right) - \dot{\ell} \frac{\partial}{\partial \xi}\left(\xi \frac{\partial V}{\partial \xi}\right) \right\}$$

The full energy balance, of course, requires addition of source and sink terms.

# Particle Balance

vs. $\rho$ (flux surface grid)

$$\left.\frac{\partial n}{\partial t}\right|_\rho = S(\rho) - \langle \vec{\nabla} \cdot \vec{\Gamma} \rangle$$

vs. $\xi$ (normalized grid)

$$\left.\frac{\partial n}{\partial t}\right|_\xi - \xi \dot{\ell} \frac{\partial n}{\partial \xi} = S(\xi) - \langle \vec{\nabla} \cdot \vec{\Gamma} \rangle$$

in TRANSP the electron and ion source functions $S$ are output of neutral beam heating and thermal neutral transport models.

$\frac{\partial n}{\partial t}$ is deduced from electron density and $Z_{eff}$ measurement and quasi-neutrality.

Thus the surface average ptcl. flux

$$\langle \Gamma_r \rangle = \frac{1}{S} \int_0^\xi dV \cdot \left( S + \xi \dot{\ell} \frac{\partial n}{\partial \xi} - \left.\frac{\partial n}{\partial t}\right|_\xi \right)$$

may be calculated. The integral is done numerically using zone volumes $\Delta V_j$ and surface areas $S_j$ from the current geometry.

Energy Balance – thermal plasma species

In the $\xi$ grid,

$$\left(\frac{\partial V}{\partial \xi}\right)^{-1}\left\{\frac{\partial}{\partial t}\Big|_{\xi}\left(\frac{3}{2}nT\frac{\partial V}{\partial \xi}\right) - \dot{\xi}\frac{\partial}{\partial \xi}\left(\xi\cdot\frac{3}{2}nT\frac{\partial V}{\partial \xi}\right)\right\}$$ energy gain

$$= -\left(\frac{\partial V}{\partial \xi}\right)^{-1} nT \left\{\frac{\partial}{\partial t}\Big|_{\xi}\left(\frac{\partial V}{\partial \xi}\right) - \dot{\xi}\frac{\partial}{\partial \xi}\left(\xi\frac{\partial V}{\partial \xi}\right)\right\}$$

compression

$$-\left(\frac{\partial V}{\partial \xi}\right)^{-1}\frac{\partial}{\partial \xi}\left\{S\langle q_{cond}\rangle_S + S\langle q_{conv}\rangle_S\right\}$$ ($\vec{\nabla}\cdot$ conduction + convection)

S – surface area / avg.

$$+\langle Q_{ie}\rangle_V + \langle Q_{\alpha}\rangle_V - \langle Q_{ex}\rangle_V + \langle Q_{beam}\rangle_V + -\langle Q_{rad}\rangle \ldots$$

ion-electron coupling; neutral ionization; charge exchange; neutral beam; radiation, etc...

With heat conduction

$$\langle q_{cond}\rangle = \chi\cdot n\langle\vec{\nabla}T\rangle = \chi\cdot n\,\frac{\partial T}{\partial \xi}\cdot\langle|\vec{\nabla}\xi|\rangle$$

conduction based on thermal diffusivity

Energy Balance (continued)

And convection

$$\langle q_{conv} \rangle_s = \alpha \cdot \frac{5}{2} T \langle \Gamma_r \rangle_s$$

$\langle \Gamma_r \rangle_s = n \langle v_r \rangle_s$ is the "radial" or cross-surface averaged particle flux calculated from the particle balance. $\alpha$ is an adjustable model parameter. NOTE: ion-electron coupling terms of the form $\vec{v} \cdot \vec{\nabla} P = \langle v_r \rangle_s \cdot \partial P / \partial \xi \langle |\vec{\nabla} \xi| \rangle$ may also optionally be included in the convection model.

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The ion energy balance is used in TRANSP to advance the ion temperature profile. Conduction is defined from $\chi_i = \gamma \chi_i^{NC}$ where $\chi_i^{NC}$ is neoclassical heat conductivity and $\gamma$ is an anomolous multiplier. If a measured indicator of ion temperature is available (e.g. neutron data), $\gamma$ may automatically and dynamically be adjusted as a function of time to match that data.

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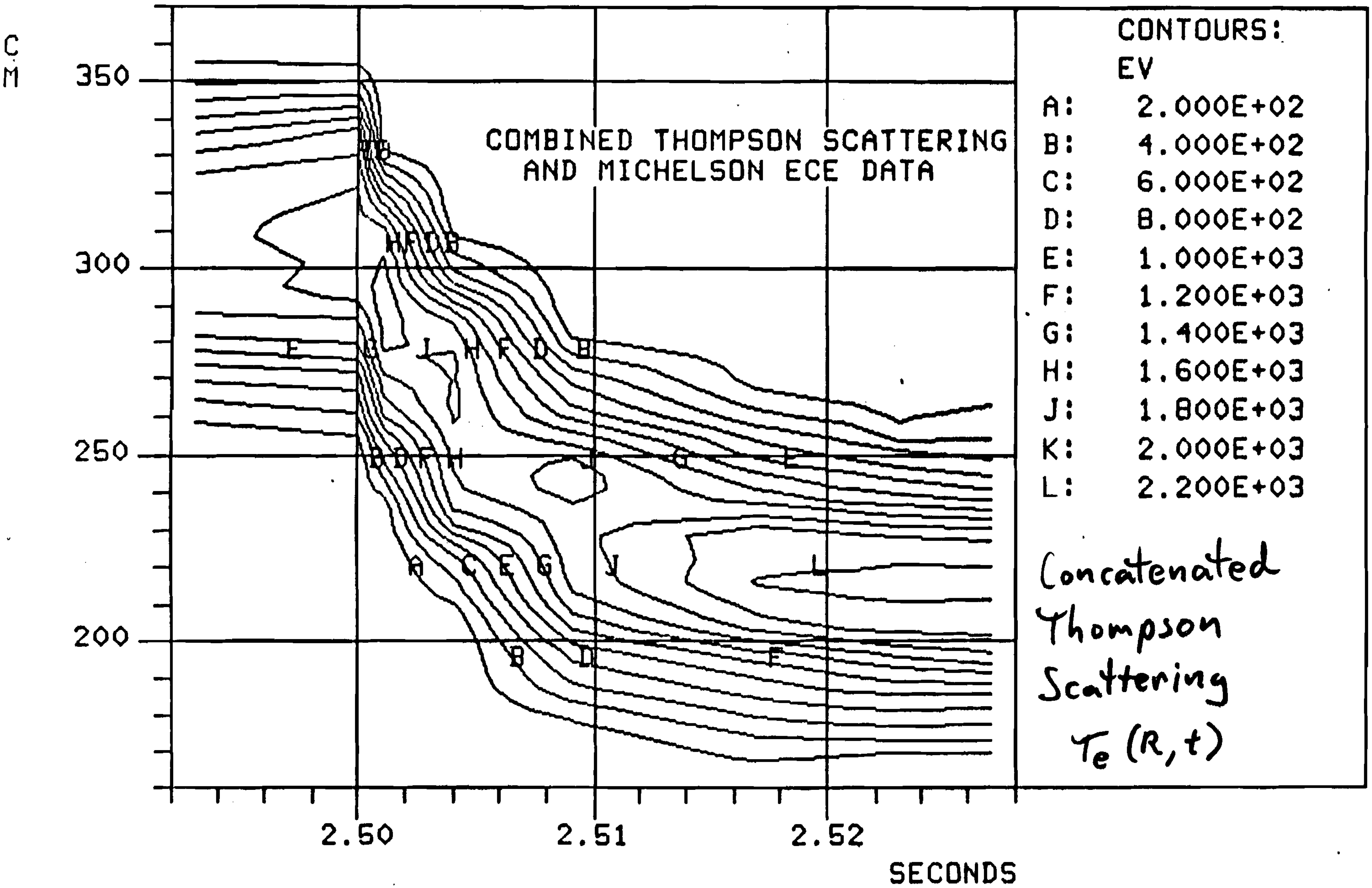
In the TRANSP electron energy balance, all terms are taken as known except conduction. (Electron temperature, density, and radiation are measured). The equation is solved to yield an experimental value for $\chi_e$, the electron thermal diffusivity

# INPUTS TO TRANSP-- COMPRESSION

- o Plasma position and minor radius
- o External toroidal field
- o Electron temperature profile
- o Electron density profile
- o Total plasma current
- o li/2 + beta
- o Total neutron emission

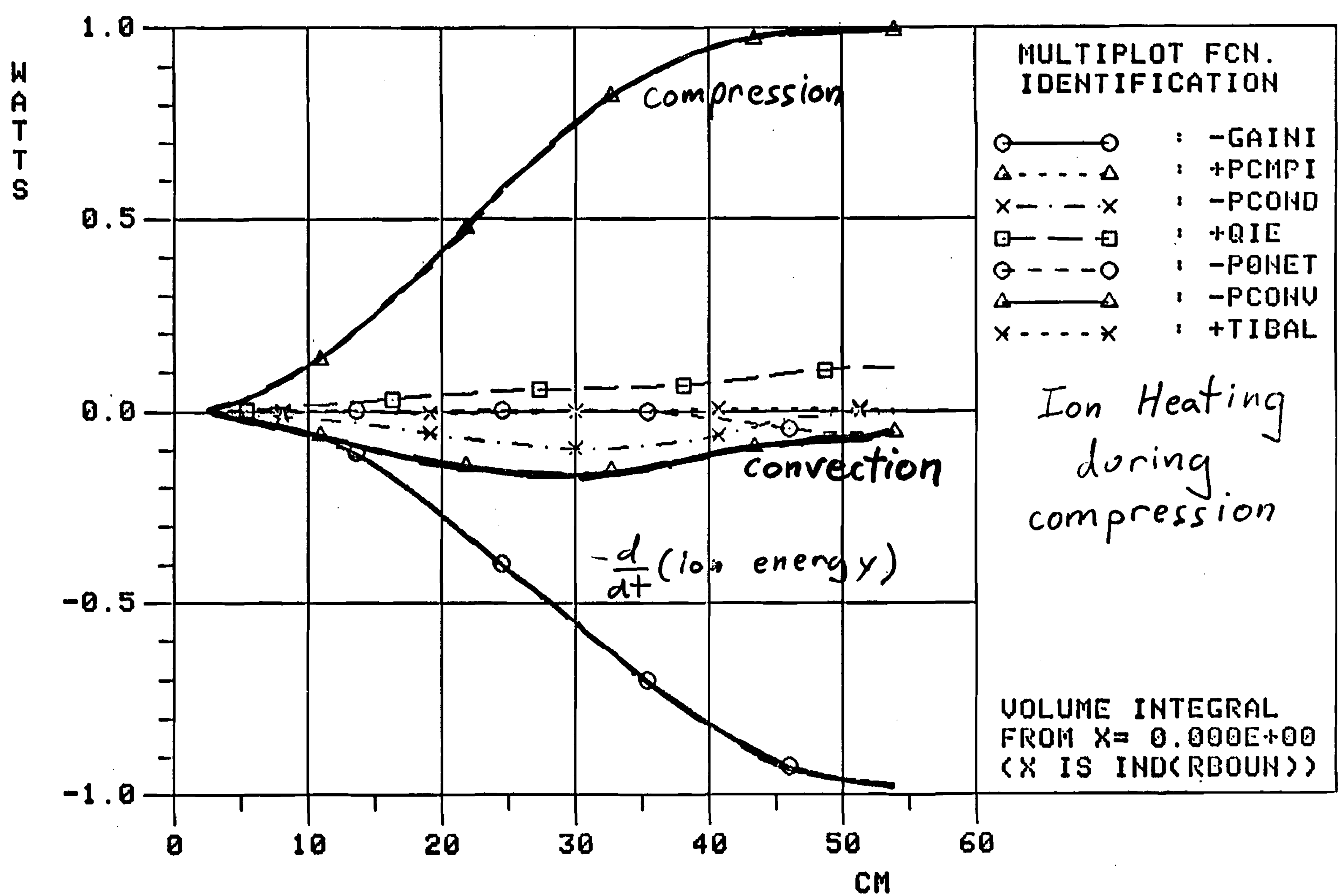
===========
|all data|
|vs. time|
===========

# TFTR summer 1984 compression data



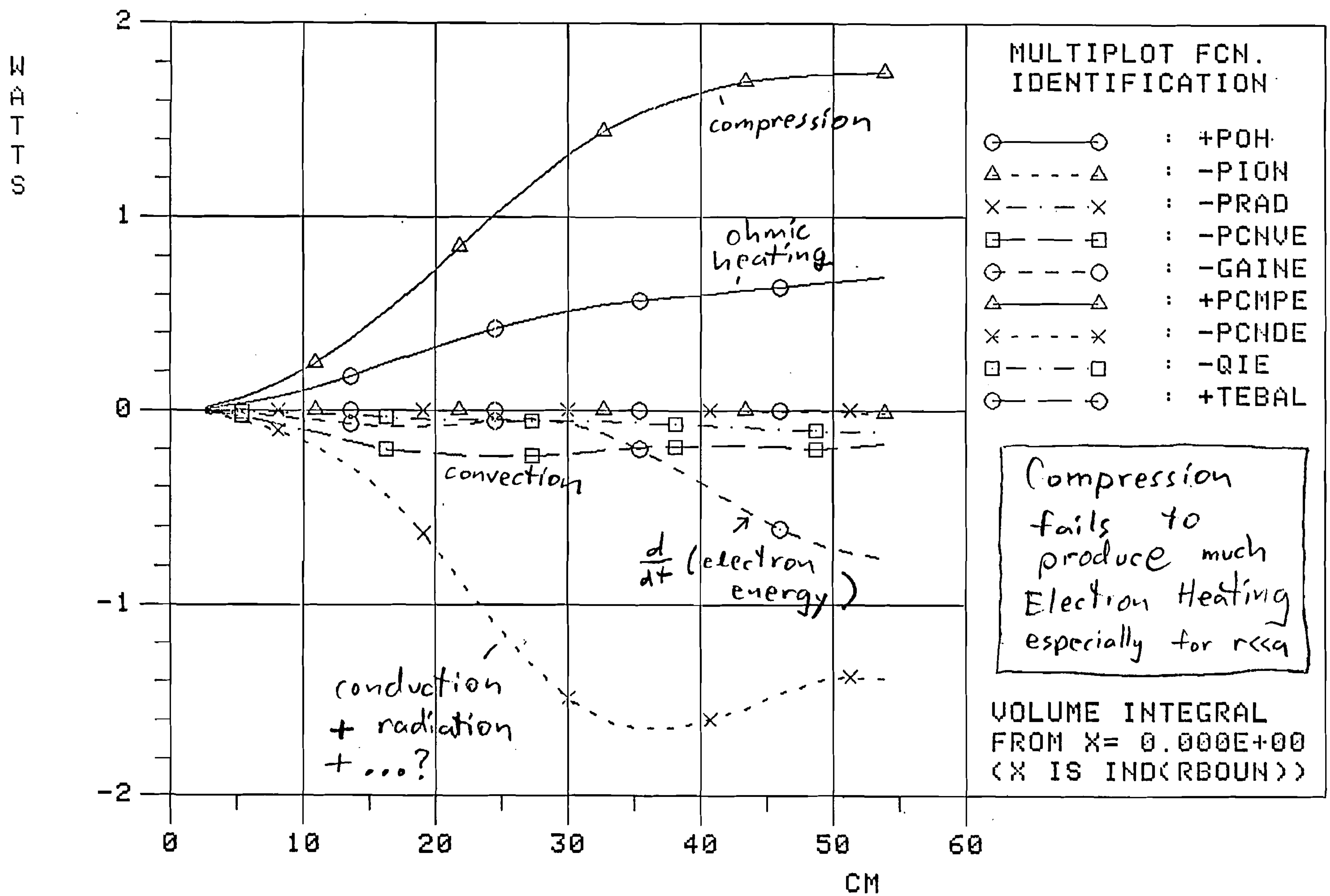

# Electron Temperature vs. R and time

TFTR.84 4211 AVERAGED IN TIME PAGE 23
TIME = 2.510E+00 STANDARD AVG +/- 7.500E-03 SECONDS



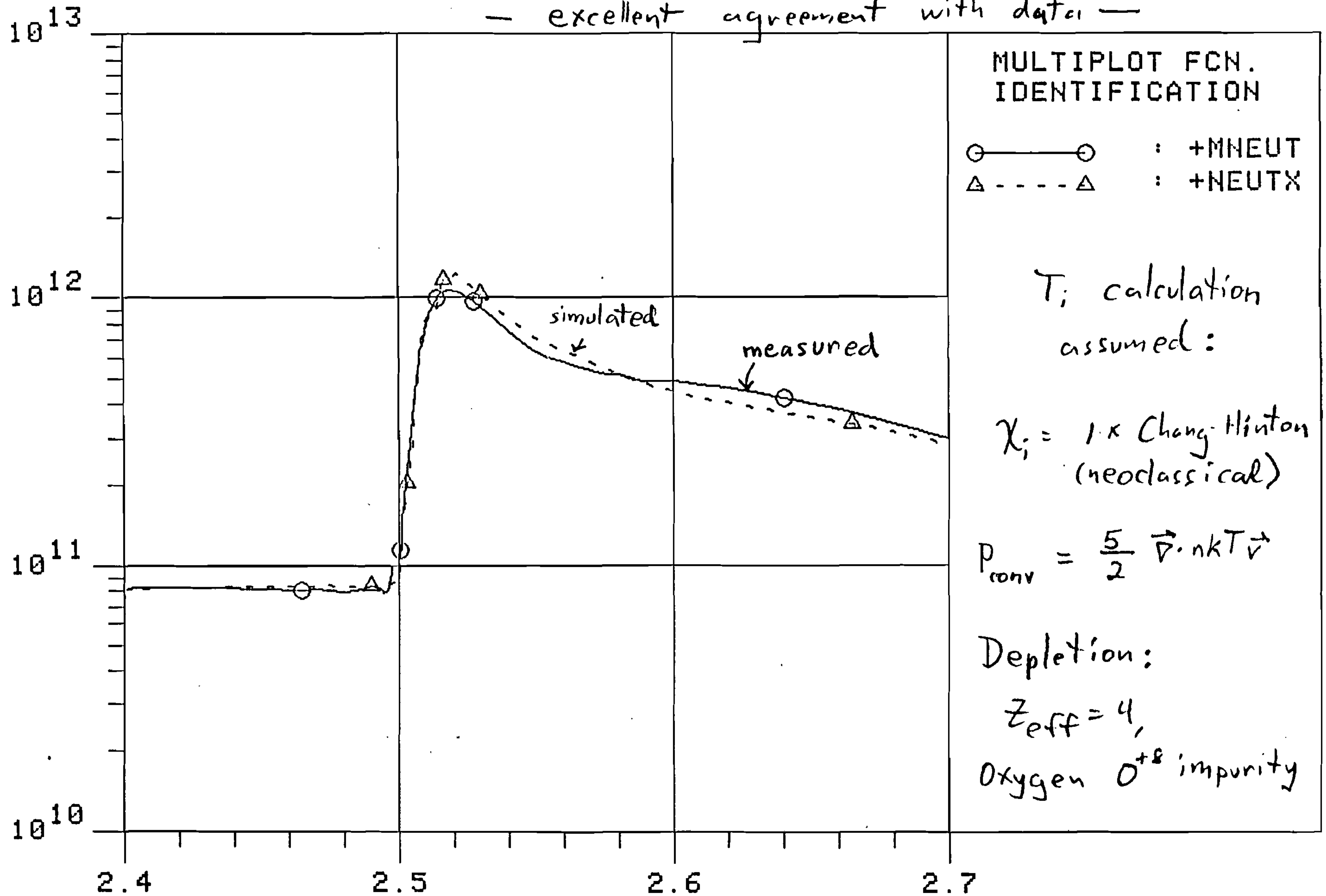

ION POWER BALANCE VS. RADIUS (RBOUN)

TFTR.84 4211 AVERAGED IN TIME PAGE 30
TIME = 2.510E+00 STANDARD AVG +/- 7.500E-03 SECONDS




ELECTRON POWER BAL. VS. RADIUS (RBOUN)

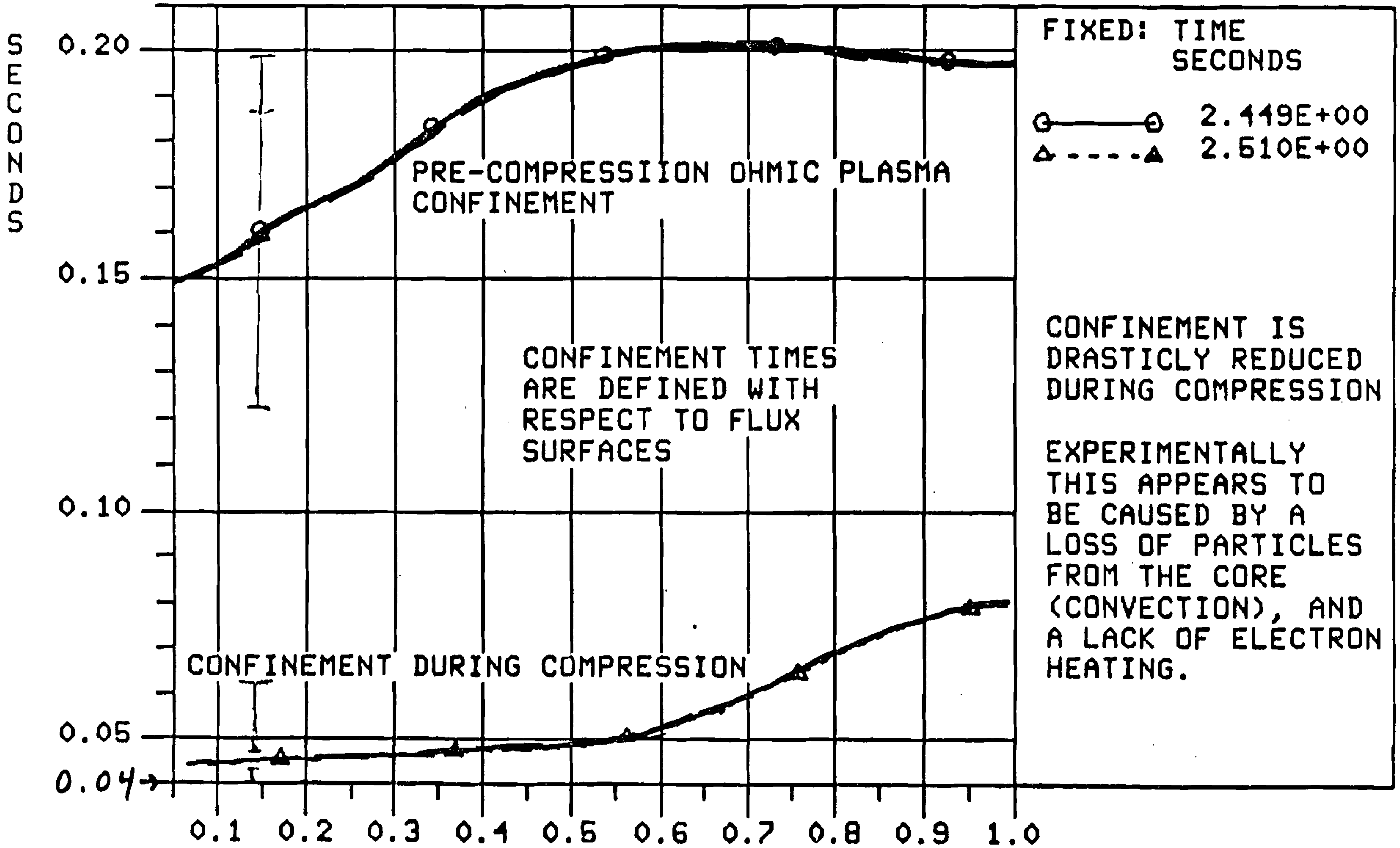
TRANSP simulation of neutron flux, based on $T_i$ from ion power balance.
— excellent agreement with data —




NEUTRON EMISSION VS. TIME

679

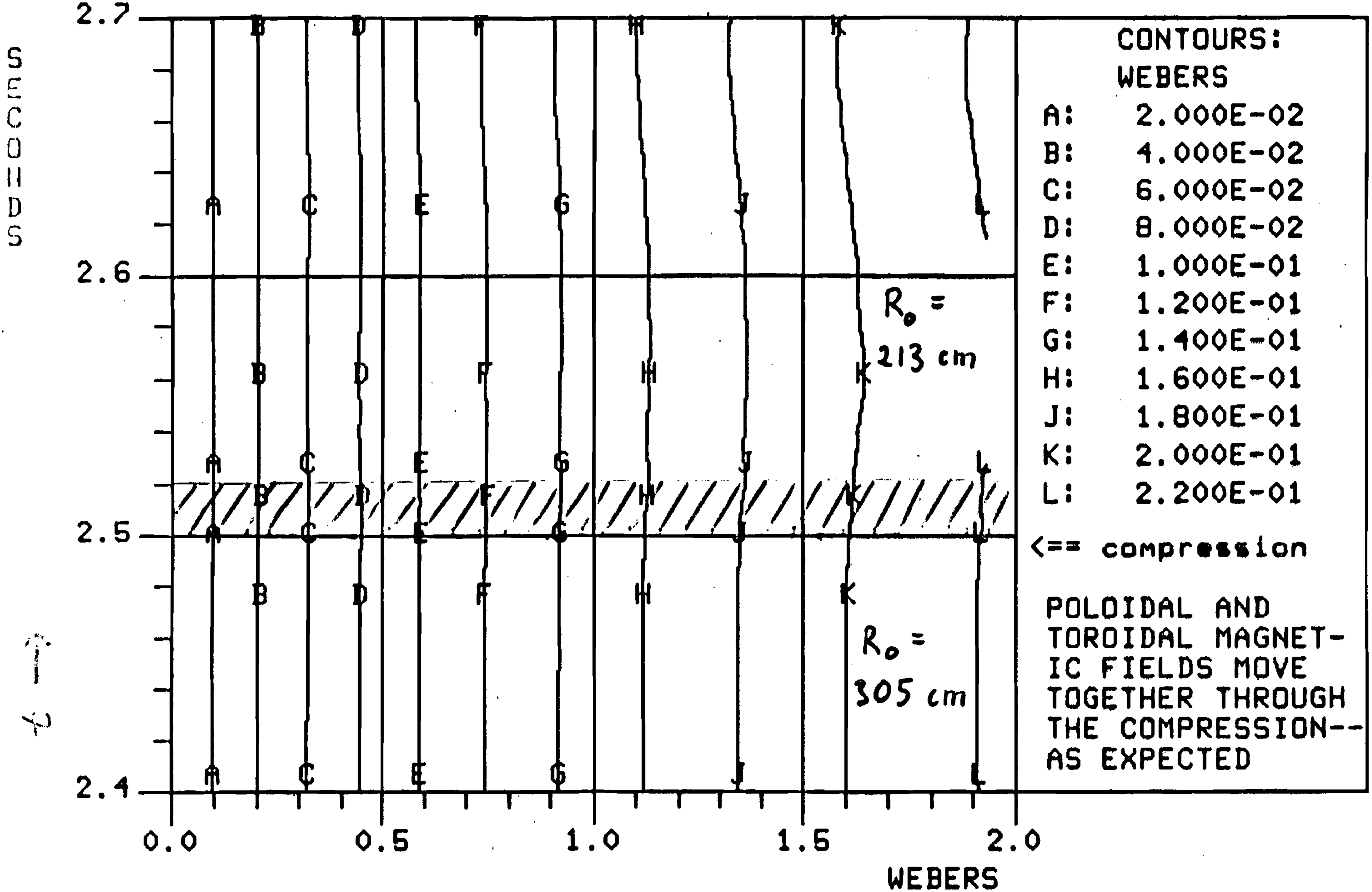
TFTR summer 1984 compression expmt




Energy Confinement Time vs. r/a

# TFTR compression

CHECK POLOIDAL FIELD DIFFUSION EQUATION




Poloidal Flux vs. Toroidal flux & t

# NON-CIRCULAR CROSS SECTIONS (PBX)

# *METHOD OF ANALYSIS

THE 2D GRAD-SHAFRANOV EQUATION IS solved by the MHD equilibrium code EQ2D which uses a Fourier moments representation for the flux surfaces. EQ2D is the reduced version of the 3D MHD equilibrium code developed at ORNL by S. Hirshman, J.C. Whitson H. Meier, et. al.

# Steepest-descent moment method for three-dimensional magnetohydrodynamic equilibria

reference

S. P. Hirshman
*Oak Ridge National Laboratory, Oak Ridge, Tennessee 37830*

J. C. Whitson
*UCC-ND Computer Sciences, Oak Ridge, Tennessee 37830*

(Received 13 June 1983; accepted 23 August 1983)

An energy principle is used to obtain the solution of the magnetohydrodynamic (MHD) equilibrium equation $\mathbf{J}\times\mathbf{B}-\nabla p=0$ for nested magnetic flux surfaces that are expressed in the inverse coordinate representation $\mathbf{x}=\mathbf{x}(\rho,\theta,\zeta)$. Here, $\theta$ and $\zeta$ are poloidal and toroidal flux coordinate angles, respectively, and $p=p(\rho)$ labels a magnetic surface. Ordinary differential equations in $\rho$ are obtained for the Fourier amplitudes (moments) in the doubly periodic spectral decomposition of $\mathbf{x}$. A steepest-descent iteration is developed for efficiently solving these nonlinear, coupled moment equations. The existence of a positive-definite energy functional guarantees the monotonic convergence of this iteration toward an equilibrium solution (in the absence of magnetic island formation). A renormalization parameter $\lambda$ is introduced to ensure the rapid convergence of the Fourier series for $\mathbf{x}$, while simultaneously satisfying the MHD requirement that magnetic field lines are straight in flux coordinates. A descent iteration is also developed for determining the self-consistent value for $\lambda$.

## I. INTRODUCTION

The global analysis of finite-aspect-ratio, high-beta, three-dimensional (3-D) toroidal configurations with complex external coil configurations of the type envisioned for fusion reactor applications generally requires numerical methods. The variational formulation of magnetohydrodynamic (MHD) equilibria[1,2] provides a mathematically efficient prescription for treating the truncation or closure of an approximate finite-series solution of the nonlinear equilibrium equations. Also inherent in any energy principle is an iteration scheme for obtaining the solution of this truncated set of equations, which is based on seeking the minimum energy state.

The practical application of variational principles for obtaining numerical equilibria has progressed recently, so that there are currently fully 3-D codes based on either Eulerian[3] or Lagrangian[4] formulations. Both of these methods are numerically inefficient in comparison with moment methods that have been previously applied to two-dimensional (2-D) problems arising in systems with an ignorable spatial coordinate[5] or that result from averaging 3-D equilibria.[6] This has prompted the present formulation of 3-D moment equilibria, as well as an alternate approach[7] based on the variational principle of Grad.[2]

The moment expansion of the plasma equilibrium results in a finite set of coupled, nonlinear, ordinary differential equations for the Fourier amplitudes of the inverse mapping[8] $\mathbf{x}=\mathbf{x}(\rho,\theta,\zeta)$, where $(\rho,\theta,\zeta)$ are flux coordinates, $\rho$ labels the flux surfaces (constant pressure contours), and $\theta$ and $\zeta$ are poloidal and toroidal angle variables, respectively. In the present paper, a steepest-descent procedure is developed for solving the nonlinear moment equations that arise in MHD equilibrium problems. This is the Fourier space formulation of the numerical scheme used in Ref. 4.

The success of moment methods is attributable in part to the rapid convergence of the Fourier series for the inverse equilibrium coordinates. In the present formulation, this convergence property is ensured by introducing a renormalization parameter (Sec. II) to distinguish between the geometric and the magnetic poloidal angles (the latter describes straight magnetic field lines).

The MHD energy principle[1] is used in Sec. III to obtain the equilibrium equations in a conservative form. It is shown that the variational moment equations correspond to the spectral coefficients of the covariant components of the MHD force. In Sec. IV, the Fourier decomposition of the inverse mapping is introduced, and the steepest-descent method of solution for the moment amplitudes is derived. The boundary conditions at the magnetic axis and at the plasma edge are discussed in Sec. V, and the descent algorithm is generalized to include a vacuum region surrounding the plasma. The moment representation of an analytic 2-D equilibrium is given in Sec. VI to clarify the role of the poloidal angle $\theta$. Some details of the numerical techniques used to solve the inverse equations are given in Sec. VII. A Galerkin method for treating the magnetic axis and plasma shift is described in Sec. VIII, and some numerical results are presented in Sec. IX.

The equilibria calculated here have a single magnetic axis. By applying magnetic perturbations of the form $\mathbf{B}=\nabla\times A_{\parallel}\mathbf{B}_0$, where $A_{\parallel}=\Sigma_{m,n}A_{mn}(\rho)\exp[i(m\theta-n\zeta)]$, it is possible to investigate the stability of these equilibria to a more general class of (tearing) perturbations.

## II. EQUILIBRIUM EQUATIONS IN FLUX COORDINATES

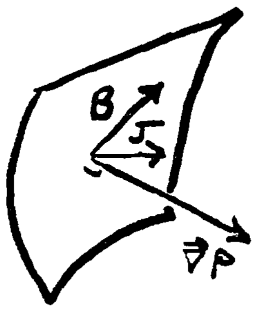
The equations describing MHD equilibrium of a static (no fluid flow), isotropic plasma are the force balance equation and Ampere's and Gauss's laws:

# MHD equilibrium — Flux Surface Constants

1½ d TRANSP <u>assumes</u> that the plasma is well described as a sequence of nested flux surfaces satisfying

$$\vec{J} \times \vec{B} = \vec{\nabla} P$$

with no "islands", seperatrices, ergotic regions or other singularities.




In this scheme, $\vec{\nabla} P$ points perpendicular to the flux surface, while the $\vec{J}$ and $\vec{B}$ vectors lie in the surface.

The following quantities are taken to be constants on a flux surface:

$P, n, T$ — plasma pressure, density, temperature

$\Phi = \int \vec{B}_T \cdot d\vec{A}$ — toroidal flux

$\Psi = \frac{1}{2\pi} \int \vec{B}_p \cdot d\vec{A}$ — poloidal flux

$\iota = 1/q = 2\pi \, \partial\Psi / \partial\Phi$ — rotational transform

$(R\, B_T)$ — major radius x toroidal field

$V = 2\pi (R E_T)$ — loop voltage

GEOMETRIC INFORMATION based on the MHD equilibrium (Jacobian) is condensed into a handful of 1d functions which are all that are needed to solve the transport and poloidal field equations. These functions are evaluated by a highly accurate numeric integ-tator (R. McCann) each time a new MHD equilibrium is calculated. These geometric quantities are summarized below:

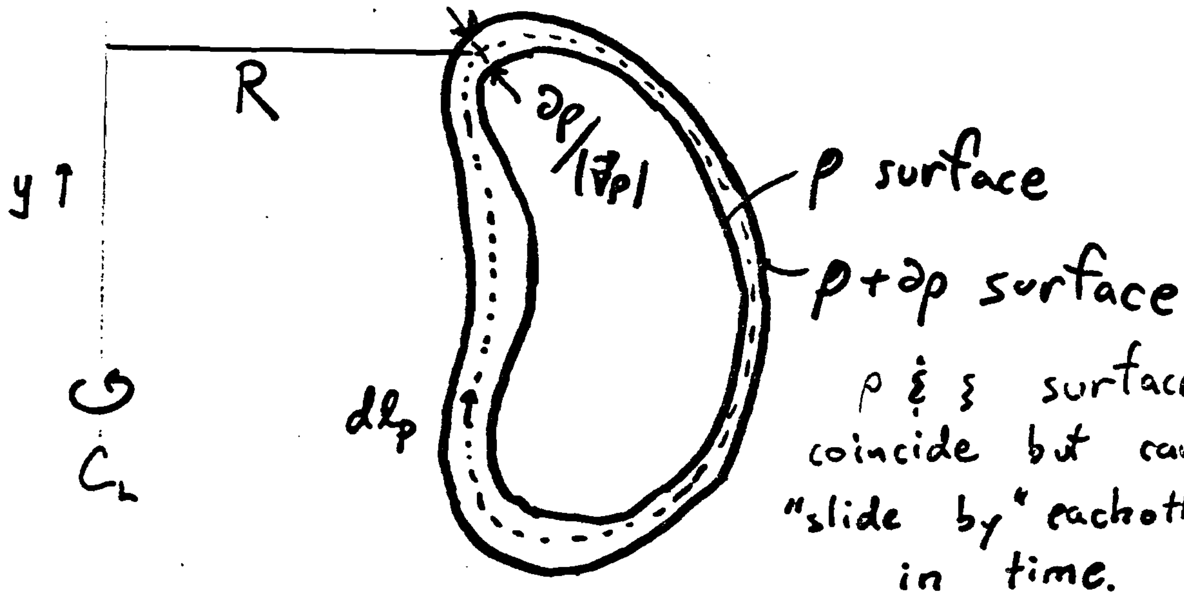
| Geometric Quantity | | Formula (Concentric Toroids, radius $R_0$) |
| --- | --- | --- |
| $\Delta V_j$ | zone volume | $2\pi R_0 \, 2\pi r \, dr$ |
| $\Delta A_j$ | zone cross-sectional area | $2\pi r \, dr$ |
| $S_j$ | surface area | $2\pi R_0 \, 2\pi r$ |
| $L_j$ | poloidal circumference | $2\pi r$ |
| $\langle \lvert \vec{\nabla} \xi \rvert \rangle \frac{\partial}{\partial \xi}$ | gradient (operator; $\langle \lvert \vec{\nabla} \xi \rvert \rangle$ is evaluated) | $\frac{\partial}{\partial r}$ (dr evaluated) |
| $\langle 1/R \rangle$ | zone average $\frac{1}{R}$ * | $\frac{1}{R_0}$ |
| $\langle 1/R^2 \rangle$ | zone average * | $\frac{1}{R_0 \sqrt{R_0^2 - r^2}} \cong \frac{1}{R_0^2}$ |
| $\langle \lvert \vec{\nabla} \xi \rvert^2 / R^2 \rangle$ | zone average * | $\frac{1}{a^2 R_0 \sqrt{R_0^2 - r^2}} \cong \frac{1}{a^2 R_0^2}$ <br> $a$ = minor radius |

*useful in poloidal field equation

# DETAILS

Working with $\rho$ & $\xi$ coordinates ...






$\rho$ & $\xi$ surfaces coincide but can "slide by" eachother in time.

$$\boxed{\partial\rho = \rho_{lim}\, \partial\xi}$$

differential volume element

$$\frac{1}{\rho_{lim}} \frac{\partial V}{\partial \xi} = \frac{\partial V}{\partial \rho} = 2\pi \oint dl_p \cdot \frac{R}{|\vec\nabla\rho|}$$

differential volume average of $f$

$$\langle f \rangle_V = \left(\frac{\partial V}{\partial \rho}\right)^{-1} \cdot 2\pi \oint dl_p \cdot f \cdot \frac{R}{|\vec\nabla\rho|} = \left(\frac{\partial V}{\partial \xi}\right)^{-1} 2\pi \oint dl_p \cdot f \cdot \frac{R}{|\vec\nabla\xi|}$$

In TRANSP the real space locations of flux surfaces are described by a Fourier moments expansion output by an MHD equilibrium solver:

$$\left.\begin{aligned} R(\rho,\theta) &= R_0(\rho) + \sum_{j=1}^{n} R_j(\rho)\cos(j\theta) \\ y(\rho,\theta) &= \sum_{j=1}^{n} y_j(\rho)\sin(j\theta) \end{aligned}\right\} (1)^*$$

(approximate solution to $\vec J \times \vec B = \vec\nabla P$ with externally specified boundary)

*up-down symmetric

A very accurate numeric integrator package[1] is used to convert the moments (1) into the quantities $\langle f \rangle_V$ needed by TRANSP to solve the transport and poloidal field equations.

TRANSP works with a grid of N discrete flux zones (typically N=20). Thus quantities $\langle f \rangle_V$ need to be integrated accross finite zone width as well as around a surface.

For zone j, spanning toroidal flux $\xi_j \to \xi_{j+1}$,

Zone volume:

$$\Delta V_j = \int_{\xi_j}^{\xi_{j+1}} d\xi \cdot \left(\frac{\partial V}{\partial \xi}\right) = \int_{\xi_j}^{\xi_{j+1}} d\xi \cdot \oint dl_p \cdot \frac{2\pi R}{|\vec{\nabla}\xi|}$$

(sample use: integrate ptcl or energy density)

Zone cross-sectional area (poloidal plane)

$$\Delta A_j = \int_{\xi_j}^{\xi_{j+1}} d\xi \oint dl_p \cdot \frac{1}{|\vec{\nabla}\xi|}$$

(sample use: integrate toroidal current density)

---

[1] R.T. McCann PPPL

Working with $\rho$ & $\xi$ coordinates . . . 

Two useful surface quantities

Surface area of flux toroid

$$S_j = 2\pi \oint R \, d\ell_p$$

(sample use:

$$\langle \vec{\nabla} \cdot \Gamma_r \rangle_j = \left( S_{j+1} \cdot \langle \Gamma_r \rangle_{j+1} - S_j \langle \Gamma_r \rangle_j \right) / \Delta V_j$$

divergence of a cross-surface flux)

Poloidal path length

$$L_j = \oint d\ell_p$$

(sample use: $|\vec{B}_p| = \frac{1}{R} \frac{\partial \psi}{\partial \xi} |\vec{\nabla} \xi|$ ;

$$\langle B_p \rangle_v = 2\pi \left( \frac{\partial V}{\partial \xi} \right)^{-1} \oint d\ell_p \cdot B_p \cdot \frac{R}{|\vec{\nabla} \xi|}$$

$$= 2\pi \left( \frac{\partial V}{\partial \xi} \right)^{-1} \left( \frac{\partial \psi}{\partial \xi} \right) \cdot L_j \quad )$$

average poloidal magnetic field, from $B_p = \frac{1}{R} |\vec{\nabla} \psi| = \frac{1}{R} \frac{\partial \psi}{\partial \xi} |\vec{\nabla} \xi|$

$\frac{\partial \psi}{\partial \xi}$ is a flux surface constant.

Working in $\rho$ $\xi$ $\zeta$ coordinates ...

Volume average of $|\nabla\xi|$

$$\langle|\vec{\nabla}\xi|\rangle = \frac{1}{\Delta V_j}\cdot\int_{\xi_j}^{\xi_{j+1}} d\xi \oint d\ell_p \cdot 2\pi R$$

(use: if $f = f(\xi)$, $\vec{\nabla}f = \frac{\partial f}{\partial\xi}\vec{\nabla}\xi$, so

$$\langle|\vec{\nabla}f|\rangle = \frac{\partial f}{\partial\xi}\langle|\vec{\nabla}\xi|\rangle$$

This provides a means of defining zone averaged gradients to combine with diffusivities, etc. in the transport equations).

Volume average $\langle 1/R\rangle$

$$\langle 1/R\rangle = \frac{1}{\Delta V_j}\cdot 2\pi\int_{\xi_j}^{\xi_{j+1}} d\xi \oint d\ell_p \cdot \frac{1}{|\vec{\nabla}\xi|} = \frac{2\pi\Delta A_j}{\Delta V_j}$$

(use: $RE_T$ = loop voltage = a flux surface constant. $\therefore \langle E_T\rangle = RE_T\cdot\langle 1/R\rangle$ )

Average toroidal electric field.

working with $\rho$ & $\xi$ coordinates ... 

Volume average of $\langle 1/R^2 \rangle$

$$\langle 1/R^2 \rangle = \frac{1}{\Delta V_j} \cdot \int_{\xi_j}^{\xi_{j+1}} d\xi \cdot \oint dl_p \cdot \frac{2\pi}{R|\vec{\nabla}\xi|}$$

(sample use: average toroidal field energy density $\langle B_T^2/2\mu_0 \rangle = \frac{(RB_T)^2}{2\mu_0} \langle 1/R^2 \rangle$

$(RB_T)$ is a flux surface constant )

Volume average $\langle |\vec{\nabla}\xi|^2/R^2 \rangle$

$$\langle |\vec{\nabla}\xi|^2/R^2 \rangle = \frac{1}{\Delta V_j} \int_{\xi_j}^{\xi_{j+1}} d\xi \oint dl_p \cdot \frac{2\pi |\vec{\nabla}\xi|}{R}$$

(sample use: average poloidal field energy density $\langle B_p^2/2\mu_0 \rangle$

$$= \frac{1}{2\mu_0} \left(\frac{\partial\psi}{\partial\xi}\right)^2 \langle |\vec{\nabla}\xi|^2/R^2 \rangle,$$

using $B_p = \frac{1}{R}|\vec{\nabla}\psi| = \frac{1}{R}\frac{\partial\psi}{\partial\xi}|\vec{\nabla}\xi|$

and $\frac{\partial\psi}{\partial\xi}$ is a flux surface constant ) working with $\rho$ + $\xi$ coordinates ... (6)

## Summary

Evaluation of the aforementioned geometric factors $\Delta V_j$, $\Delta A_j$, $S_j$, $L_j$, $\langle|\vec{\nabla}\xi|\rangle_j$, $\langle 1/R\rangle_j$, $\langle 1/R^2\rangle_j$, and $\langle|\vec{\nabla}\xi|^2/R^2\rangle_j$, with sufficient accuracy for time differentiation, enables solution of the 1½ d generalized transport and poloidal field equations.

The transport and field equations are developed on a flux surface ($\rho$) grid. But in TRANSP they are solved on the normalized $\xi = \rho/\rho_{lim} = (\Phi/\Phi_{lim})^{1/2}$ grid which is time invariant (bdy: $\xi_{lim} \equiv 1$). The coordinate transformation is straightforward:

$$\partial\rho = \rho_{lim}\,\partial\xi \qquad |\vec{\nabla}\rho| = \rho_{lim}\,|\vec{\nabla}\xi| \qquad \frac{\partial}{\partial\rho} \rightarrow \frac{1}{\rho_{lim}}\frac{\partial}{\partial\xi}$$

defining $\dot{l} = \frac{1}{\rho_{lim}}\frac{d\rho_{lim}}{dt} = \frac{1}{2\Phi_{lim}}\frac{d\Phi_{lim}}{dt}$, then

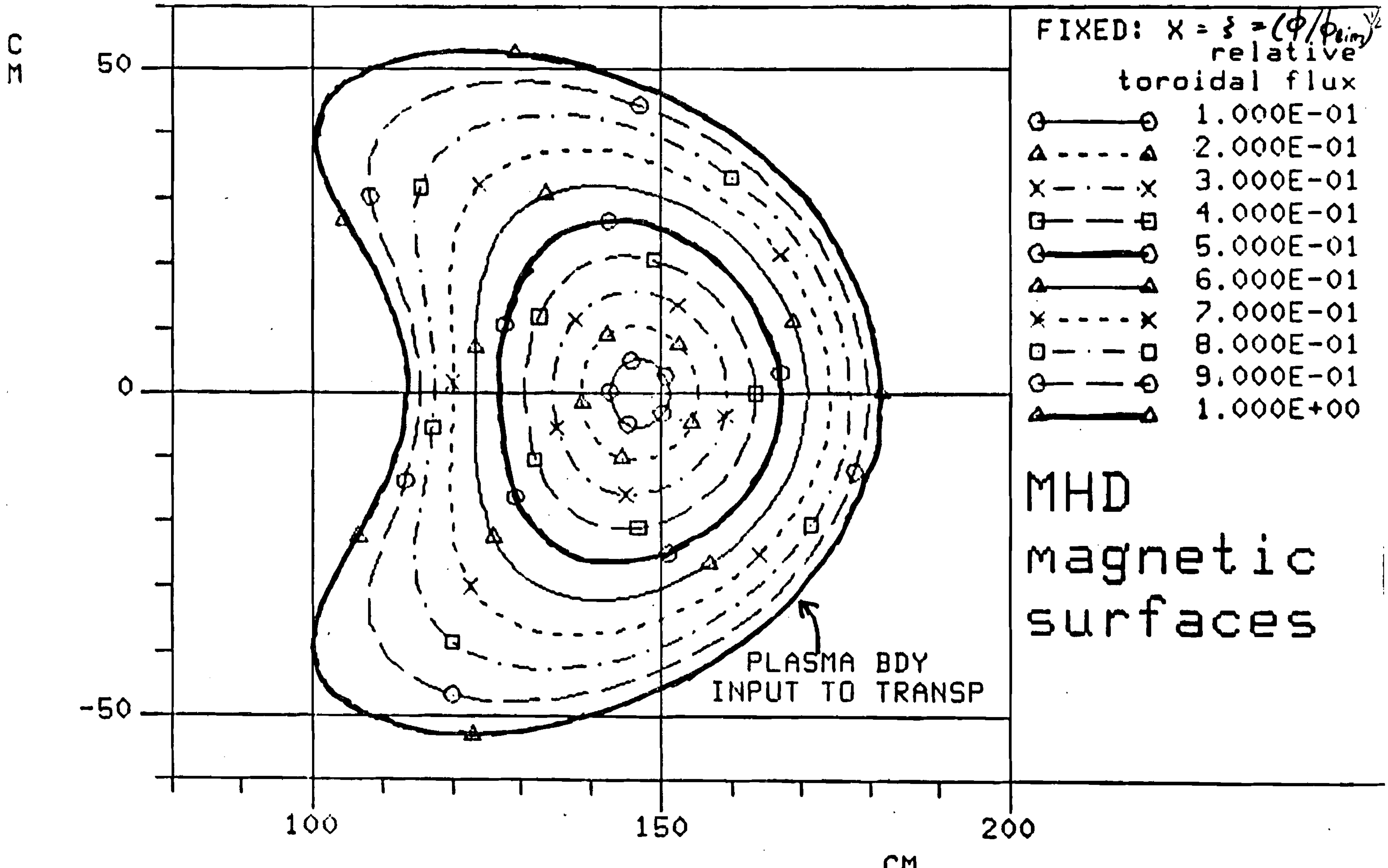
$$\left.\frac{\partial}{\partial t}\right|_\rho \longrightarrow \left.\frac{\partial}{\partial t}\right|_\xi - \xi\,\dot{l}\,\frac{\partial}{\partial\xi}$$ for time derivatives.

# *APPLICATION

TRANSP is being used to study the confinement properties of ohmic "beans" in PBX. In the near future it will be possible to use TRANSP in conjuction with neutral beam heated PBX "bean" experiments.

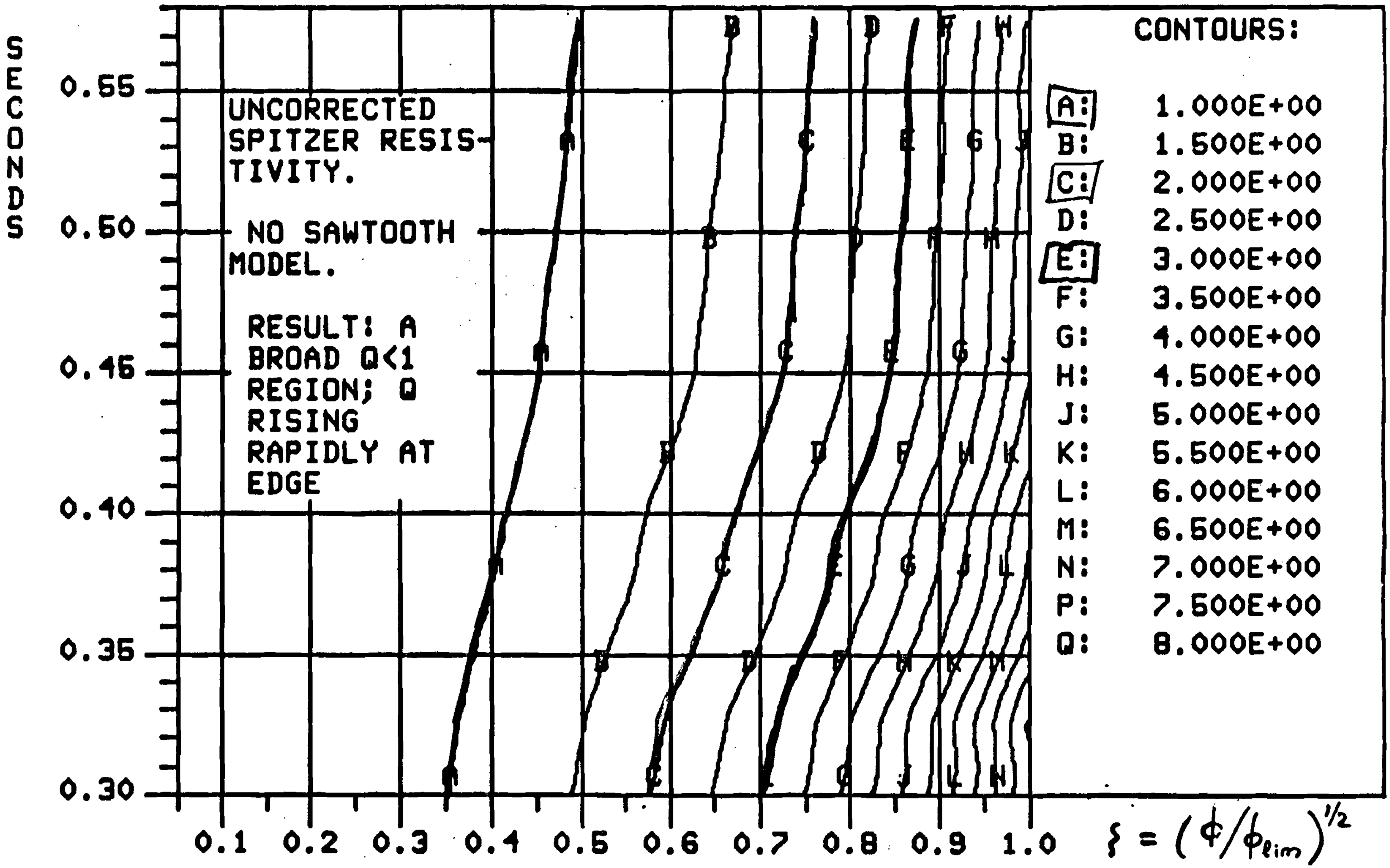
PBX.84 1300
TIME = 7.000E-01 SECONDS

# PBX ohmic bean



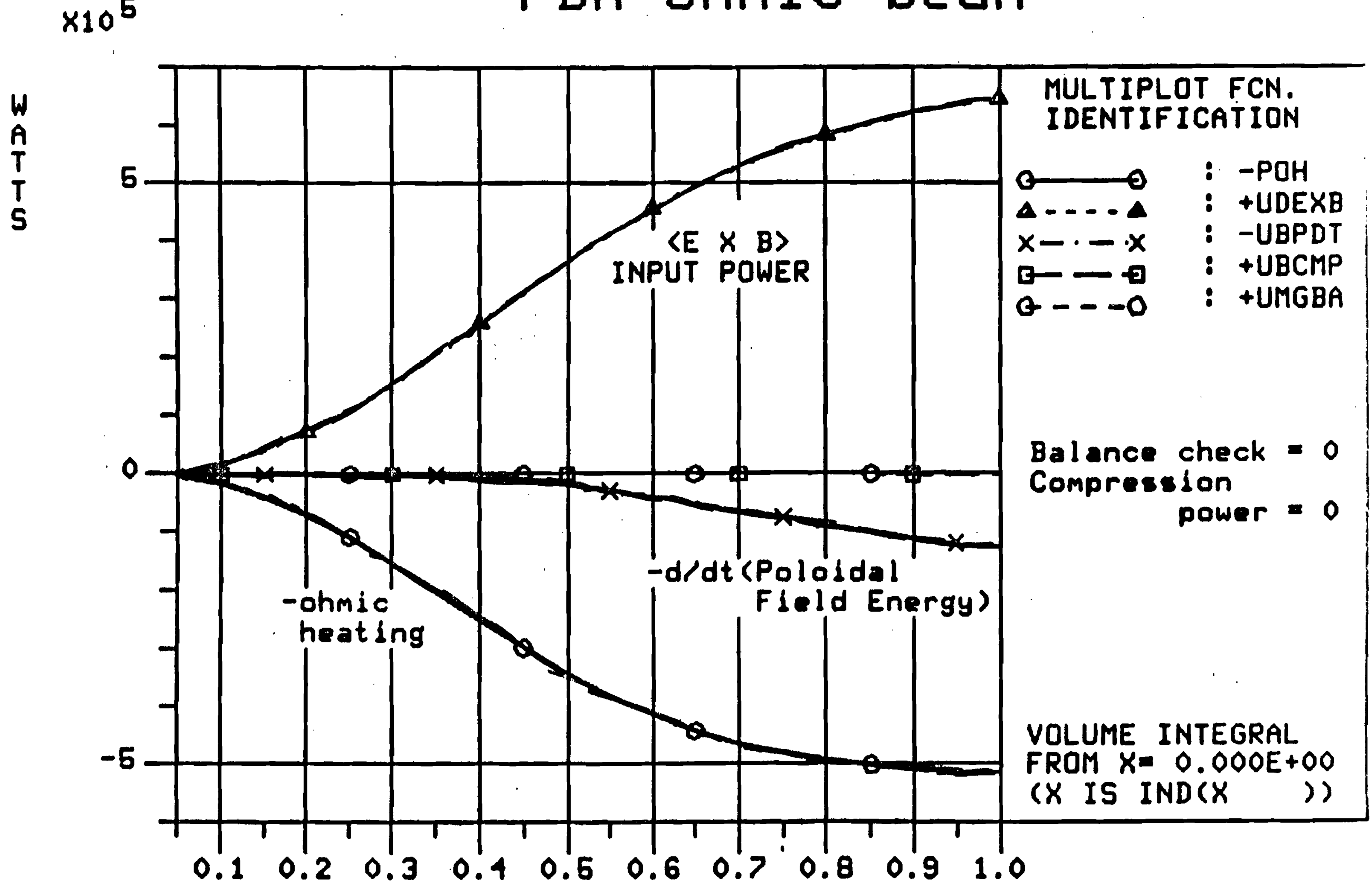

FLUX SURFACE CONTOURS: (R,Y) MOMENTS EXPANSION

# PBX ohmic bean




Q profile vs. relative flux, t

# PBX ohmic bean




Poloidal Field Energy Balance VS REL. FLUX LABEL

Integrated from the magnetic axis $\xi = (\phi/\phi_{lim})^{1/2}$

# ** SUMMARY **

TRANSP HAS BEEN UPGRADED TO SUPPORT ANALYSIS OF PLASMAS WHICH:

- enclose variable amounts of toroidal flux
- undergo compression
- have flux surfaces of non-circular cross-section

# APPENDIX:

Derivation of the Poloidal Field Diffusion Equation in 2d Magnetic Flux Geometry. Conversion to TRANSP relative flux coordinates.

# TRANSP 1 1/2 d

## MAGNETIC FIELD DIFFUSION EQUATION

in arbitrary time dependent magnetic flux geometry.

Original Derivations

Hinton & Hazeltine Rev. Modern Physics <u>48</u> p. 239 (1976)

Also in GA-A16178, the ONETWO transport code write-up.

Here the TRANSP version is presented.

# TRANSP MAGNETIC FIELD DIFFUSION EQUATION SOLVER: MAGDIF

initial TFTR runs assume neoclassical resistivity, concentric circular flux surfaces, no bootstrap current. 1-d eq.$^{n}$, no MHD.

measured data: ECE $T_e(r,t)$
plasma current $I_p(t)$ surface voltage $V_s(t)$
1 mm interferometer $\bar{n}_e(t)$
Bremstrahlung $n_e(r,t)$ (if avail.) or parabolic $n_e(r)$ assumed.

Algorithm:

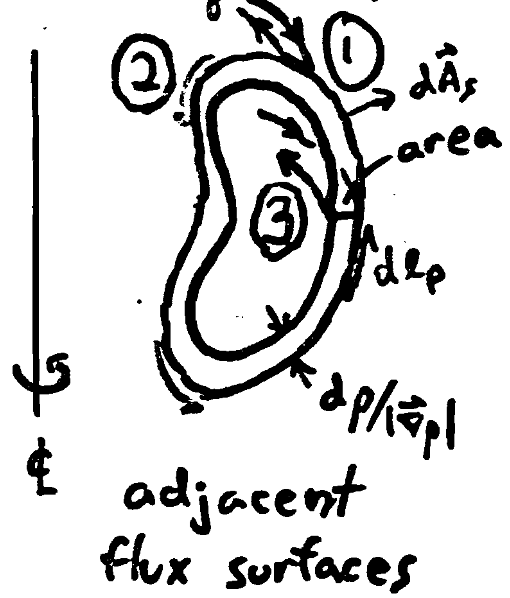
Previous timestep $Z_{eff}$, $B_\theta(r)$ + measured data
↓
→ Calculate Resistivity
↓
Advance MAGDIF $\dot{B}_\theta$ eq.$^{n}$ matching $I_p(t)$ exactly
↓
Compare calculated & measured surface voltage
Acceptable agreement?

NO ↓ Adjust $Z_{eff}$ (return to Calculate Resistivity)

YES ↓ Timestep complete.
OUTPUTS: $B_\theta(r)$, $V(r)$, $q(r)$, $P_{OH}(r)$
MAGDIF/resistivity $Z_{eff}$

# Derivation of Magnetic Diffusion Equation in Time Dependent Magnetic Flux Geometry — a Sketch.

## I. Consequences of Faraday's Law




$$\oint \vec{E}\cdot d\vec{\ell} = \frac{1}{c}\frac{\partial}{\partial t}\int \vec{B}\cdot d\vec{A}$$

① $\vec{B}$ lies within flux surfaces

$\therefore \frac{\partial}{\partial t}\int \vec{B}\cdot d\vec{A}_s = \int \dot{\vec{B}}\cdot d\vec{A}_s = 0$ for any surface S which is part of a flux surface.

$\therefore V_\ell = 2\pi R E_T$ = loop voltage = const. on a flux surface ; $V_\ell = V_\ell(\rho)$
R = dist. from centerline.

② $\oint E_p \, d\ell_p$
$$= \frac{1}{c}\frac{\partial}{\partial t}\Phi = 0$$

The poloidal E field loop integrates to zero because $\Phi$ is constant in time on a flux surface

③
$$\frac{\partial V_\ell}{\partial \rho} = \frac{2\pi}{c}\frac{\partial}{\partial t}\left(\frac{\partial \Psi}{\partial \rho}\right)$$

where $\Psi = \frac{1}{2\pi}\int \vec{B}_p \cdot d\vec{A}$ is poloidal flux. Note that since $\vec{B}_p$ is tangent to the flux surfaces and $\vec{\nabla}\cdot\vec{B} = 0$, $\left(\frac{\partial \Psi}{\partial \rho}\right)$ is a flux surface constant.

I. Consequences of Faraday's Law (cont.)

It will prove convenient to express Faraday's Law ③ in terms of the average $\langle \vec{E}\cdot\vec{B}\rangle_V$

Definition: The flux surface differential volume average of $f$,

$$\langle f\rangle_V = \left(\frac{\partial V}{\partial\rho}\right)^{-1}\cdot \oint dl_p \left(\frac{2\pi R}{|\vec{\nabla}\rho|}\cdot f\right)$$

We get

$$\langle \vec{E}\cdot\vec{B}\rangle_V = \langle E_T B_T\rangle_V + \langle E_p B_p\rangle_V$$

$$= \left\langle \frac{V_\ell}{2\pi R}\cdot\frac{(RB_T)}{R}\right\rangle_V + \left\langle E_p\cdot\frac{|\vec{\nabla}\rho|}{R}\frac{\partial\psi}{\partial\rho}\right\rangle_V$$

$$= \frac{(RB_T)\,V_\ell}{2\pi}\langle 1/R^2\rangle_V + 2\pi\left(\frac{\partial\psi}{\partial\rho}\right)\left(\frac{\partial V}{\partial\rho}\right)^{-1}\cancelto{0}{\oint dl_p\cdot E_p}$$

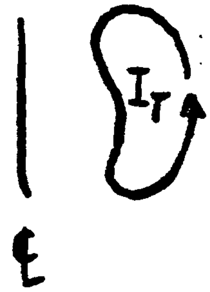
($RB_T$ is a flux surface constant due to ~~Ampere's Law, see below.~~ by ②)

thus

$$V_\ell = \frac{\langle\vec{E}\cdot\vec{B}\rangle\cdot 2\pi}{(RB_T)\,\langle 1/R^2\rangle_V}$$

$\langle 1/R^2\rangle_V$ is a useful geometric factor evaluated by TRANSP

II. Ampere's Law (continued).




(2) The toroidal component of Ampere's Law gives

$$\oint B_p \cdot dl_p = \frac{4\pi}{c} I_T$$

Using $B_p = \frac{1}{R}|\vec{\nabla}\psi| = \frac{1}{R}\frac{\partial\psi}{\partial\rho}|\vec{\nabla}\rho|$,

$$\frac{\partial\psi}{\partial\rho}\oint\frac{|\nabla\rho|}{R}\,dl_p = \frac{4\pi}{c} I_T$$

TRANSP numerically integrates the useful geometric factor $\langle|\nabla\rho|^2/R^2\rangle_V = \left(\frac{\partial V}{\partial\rho}\right)^{-1} 2\pi\oint dl_p \cdot \frac{|\nabla\rho|}{R}$. In terms of this we have

$$I_T = \frac{c}{8\pi^2}\left(\frac{\partial V}{\partial\rho}\right)\left(\frac{\partial\psi}{\partial\rho}\right)\langle|\nabla\rho|^2/R^2\rangle_V$$

To relate toroidal current density to gradients in the poloidal field, we take $\partial/\partial\rho$ of both sides:

$$\frac{\partial}{\partial\rho}I_T = \frac{\partial}{\partial\rho}\int_0^\rho d\rho \oint dl_p \cdot \frac{J_T}{|\vec{\nabla}\rho|} = \oint dl_p \cdot \frac{R}{|\nabla\rho|}\cdot\frac{J_T}{R}$$

$$= \frac{1}{2\pi}\left(\frac{\partial V}{\partial\rho}\right)\langle J_T/R\rangle_V$$

So

$$\langle J_T/R\rangle_V = \frac{c}{4\pi}\left(\frac{\partial V}{\partial\rho}\right)^{-1}\cdot\frac{\partial}{\partial\rho}\left[\left(\frac{\partial V}{\partial\rho}\right)\left(\frac{\partial\psi}{\partial\rho}\right)\langle|\vec{\nabla}\rho|^2/R^2\rangle_V\right] \quad (A\phi)$$

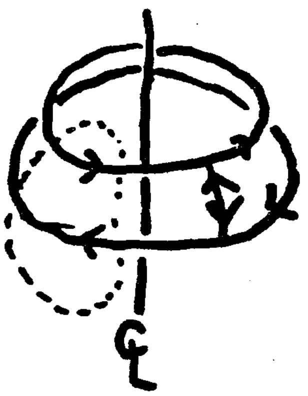
# I. Faraday's Law (Continued)

Thus the expression for Faraday's Law in terms of $\langle \vec{E}\cdot\vec{B}\rangle_V$ is:

$$\frac{\partial}{\partial\rho}\left[\frac{\langle \vec{E}\cdot\vec{B}\rangle_V}{(RB_T)\langle 1/R^2\rangle_V}\right] = \frac{1}{c}\cdot\frac{\partial}{\partial t}\left(\frac{\partial\Psi}{\partial\rho}\right) \qquad (F)$$

# II. Ampere's Law $\oint \vec{B}\cdot d\vec{l} = \frac{4\pi}{c}\int \vec{J}\cdot d\vec{A}$

① for any surface $S$ which is part of a flux surface, $\int \vec{J}\cdot d\vec{A} = 0$ because $\vec{J}$ is tangent to the surface. $\therefore \oint \vec{B}\cdot d\vec{l} = 0$ for any closed contour on $S$.



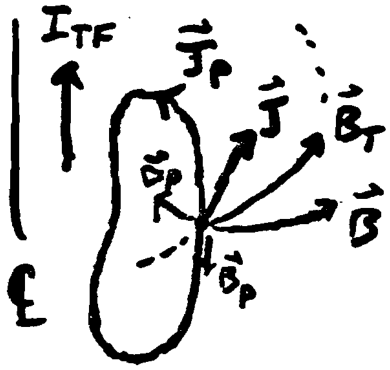

2 circles of radius $R_1$ and $R_2$, on the same flux surface.

Thus for the contour drawn, $R_1 B_{T_1} = R_2 B_{T_2}$ (axisymmetry fixes $B_T$ along the circles).

In fact $(RB_T)$ is constant on a flux surface.

However, $(RB_T)$ can vary from surface due to poloidal para- or diamagnetic plasma currents.

# II. Ampere's Law (continued)




$\vec{\nabla} P = \vec{J} \times \vec{B}$ shown here the direction of $\vec{J}_p$ is such as to reduce $|RB_T|$ when moving to a more interior $\rho$ surface.

③ the poloidal component of Ampere's Law.

The Grad-Shafranov eq.$^n$ $\vec{\nabla} P = \vec{J} \times \vec{B}$ implies the existance of poloidal current in the plasma which will effect the interior toroidal field

$\vec{J}$ tangential to flux surfaces and $\vec{\nabla}\cdot\vec{J} = 0$ imply

$$\frac{\partial I_p}{\partial \rho} = \frac{2\pi R}{|\vec{\nabla}\rho|} J_p$$

is constant on a flux surface.

Then Ampere's Law yields

$$2\pi \frac{\partial (RB_T)}{\partial \rho} = -\frac{4\pi}{c} \frac{\partial I_p}{\partial \rho}$$

When the poloidal field equation is solved and $\langle \vec{J}\cdot\vec{B} \rangle_v$ is known, $\partial I_p/\partial\rho$ can be evaluated using the Grad-Shafronov equation, and $g = (RB_T)/(RB_T^{ext})$ checked. Until then, the equivalent relation

$$|J_p| = -\frac{c}{4\pi} \frac{|\vec{\nabla}\rho|}{R} \frac{\partial}{\partial \rho}(RB_T) \qquad \text{(A1)}$$

is useful.

II. Ampere's Law (continued).

④ "Parallel" Ampere's Law. Ohm's Law, section III., will relate $\langle \vec{E}\cdot\vec{B}\rangle_v$ to $\langle \vec{J}\cdot\vec{B}\rangle_v$ so it is desirable to express Ampere's Law accordingly:

$$\langle \vec{J}\cdot\vec{B}\rangle_v = \langle J_T B_T\rangle_v + \langle J_p B_p\rangle_v$$

$$= (RB_T)\langle J_T/R\rangle_v + \frac{\partial\psi}{\partial\rho}\langle J_p \cdot |\vec{\nabla}\rho|/R\rangle_v$$

use (A0) and (A1) from above

$$\langle \vec{J}\cdot\vec{B}\rangle_v = \frac{c}{4\pi}(RB_T)\left(\frac{\partial V}{\partial\rho}\right)^{-1}\frac{\partial}{\partial\rho}\left[\left(\frac{\partial V}{\partial\rho}\right)\left(\frac{\partial\psi}{\partial\rho}\right)\langle|\vec{\nabla}\rho|^2/R^2\rangle_v\right]$$

$$-\frac{c}{4\pi}\frac{\partial\psi}{\partial\rho}\frac{\partial}{\partial\rho}(RB_T)\langle|\vec{\nabla}\rho|^2/R^2\rangle_v$$

which combines to form

$$\langle \vec{J}\cdot\vec{B}\rangle = \frac{c}{4\pi}\left(\frac{\partial V}{\partial\rho}\right)^{-1}(RB_T)^2\frac{\partial}{\partial\rho}\left[\left(\frac{\partial V}{\partial\rho}\right)\left(\frac{\partial\psi}{\partial\rho}\right)(RB_T)^{-1}\langle|\vec{\nabla}\rho|^2/R^2\rangle_v\right] \qquad (A)$$

## II. Ohm's Law.

$$\langle \vec{E} \cdot \vec{B} \rangle = \eta_{\parallel} \left[ \langle \vec{J} \cdot \vec{B} \rangle_v - \langle \vec{J} \cdot \vec{B} \rangle_v^{ext} \right] \qquad (O)$$

where $\eta_{\parallel}$ is the parallel resistivity

An arbitrary - geometry generalization of Spitzer and/or neoclassical resistivity is needed (not considered here).

$\langle \vec{J} \cdot \vec{B} \rangle_v^{ext}$ represents non-ohmic currents, e.g. bootstrap, beam-driven, and RF driven currents, which must be calculated independently and supplied as input to the poloidal field calculation.

Through Ohm's Law, Faraday's Law (F) and Ampere's Law (A) can be combined to form a single expression relating the time and space derivatives of $\partial \Psi / \partial \rho$.

## IX. The Magnetic Diffusion Equation.

Combining (0), (F) and (A),

$$\frac{1}{c}\frac{\partial}{\partial t}\left(\frac{\partial \psi}{\partial \rho}\right) = \frac{\partial}{\partial \rho}\left\{\frac{c\eta_{\parallel}}{4\pi}\frac{(RB_T)}{\langle R^{-2}\rangle_v (\partial V/\partial \rho)}\frac{\partial}{\partial \rho}\left[\frac{\partial V}{\partial \rho}\frac{\partial \psi}{\partial \rho}\frac{\langle |\nabla\rho|^2/R^2\rangle_v}{(RB_T)}\right]\right\}$$

$$- \frac{\partial}{\partial \rho}\frac{\eta_{\parallel}\langle \vec{J}\cdot\vec{B}\rangle_{ext}}{(RB_T)\langle R^{-2}\rangle_v}$$

In TRANSP we solve the equation on a fixed $\xi = \rho/\rho_{lim} = (\Phi/\Phi_{lim})^{1/2}$ grid, so we transform $\rho \to \rho_{lim}\xi$, $\frac{\partial}{\partial \rho} \to \frac{1}{\rho_{lim}}\frac{\partial}{\partial \xi}$,

and, using $\dot{l} \equiv \frac{1}{\rho_{lim}}\frac{d\rho_{lim}}{dt} = \frac{1}{2\Phi_{lim}}\frac{d\Phi_{lim}}{dt}$,

$$\left.\frac{\partial}{\partial t}\right|_{\rho} \to \left.\frac{\partial}{\partial t}\right|_{\xi} - \xi\dot{l}\left.\frac{\partial}{\partial \xi}\right|_{t}$$

It was also found to be convenient to use

$$\mathring{t} = \frac{1}{q} = \frac{2\pi\,\partial\psi}{\partial\Phi}$$

As the dependent variable.

## ... The Magnetic Diffusion Equation (cont.)

From the definitions of $\xi$ and $\dot{\ell}$,

$$\frac{\partial \psi}{\partial \rho} = \frac{1}{\rho_{lim}} \frac{\partial \psi}{\partial \xi} = \frac{1}{\rho_{lim}} \frac{\dot{\ell}\xi\, \Phi_{lim}}{\pi}$$

Also, from the definition of $\rho$, $\Phi_{lim} = \pi \rho_{lim}^2 B_0$

Substituting for $\partial\psi/\partial\rho$ and applying $\rho \rightarrow \xi$ transformations, the equation becomes

$$\left.\frac{\partial}{\partial t}\right|_{\xi} (\dot{\ell}\xi) + (\dot{\ell}\xi)\dot{\ell} - \xi\dot{\ell}\frac{\partial}{\partial \xi}(\dot{\ell}\xi)$$

$$= \frac{\partial}{\partial \xi}\left\{ \frac{c^2 \eta_{\parallel}}{4\pi} \frac{(RB_T)}{\langle 1/R^2 \rangle_v (\partial V/\partial \xi)} \frac{\partial}{\partial \xi}\left[ \frac{\partial V}{\partial \xi} (\dot{\ell}\xi) \frac{\langle |\nabla\xi|^2/R^2 \rangle_v}{(RB_T)} \right] \right.$$

$$\left. - \frac{\pi c}{\Phi_{lim}} \frac{\partial}{\partial \xi}\left[ \frac{\eta_{\parallel} \langle \vec{J}\cdot\vec{B} \rangle_v^{ext}}{\langle 1/R^2 \rangle (RB_T)} \right] \right\}$$

Note that all occurrances of $B_0$ and $\rho_{lim}$ have cancelled out.

This equation can also be solved for $(\dot{\ell}\xi)$, which in cylindric coordinates is proportional to $B_p$.

## I. Magnetic quantities in terms of $\dot{\ell}$ and $\xi$

Having solved for $\dot{\ell}$ in the Diffusion Equation, all standard magnetic quantities can be defined:

$$q = \frac{1}{\dot{\ell}} \qquad \psi = \int \frac{\partial \psi}{\partial \xi} d\xi = \int \frac{\dot{\ell}\xi\, \Phi_{lim}\, d\xi}{\pi}$$

$$\langle B_p \rangle_v = 2\dot{\ell}\xi\, \Phi_{lim} \left(\frac{\partial V}{\partial \xi}\right)^{-1} \cdot L \qquad (L = \text{poloidal path length})$$

$$\langle \vec{J}\cdot\vec{B} \rangle_v = \frac{c\,\Phi_{lim}}{4\pi^2} \frac{(RB_T)^2}{(\partial V/\partial \xi)} \frac{\partial}{\partial \xi}\left[\frac{\partial V}{\partial \xi} \dot{\ell}\xi \langle |\nabla\xi|^2/R^2 \rangle_v / (RB_T)\right]$$

$$\langle J_T \rangle_A = \frac{c\,\Phi_{lim}}{8\pi^3} \left(\frac{\partial A}{\partial \xi}\right)^{-1} \frac{\partial}{\partial \xi}\left[\frac{\partial V}{\partial \xi} \dot{\ell}\xi \langle |\vec{\nabla}\xi|^2/R^2 \rangle_v\right]^*$$

$$\langle \vec{E}\cdot\vec{B} \rangle_v = \eta_{\parallel}\left(\langle \vec{J}\cdot\vec{B} \rangle_v - \langle \vec{J}\cdot\vec{B} \rangle_v^{ext}\right)$$

$$V_l = 2\pi \langle \vec{E}\cdot\vec{B} \rangle / \left[(RB_T)\langle 1/R^2 \rangle\right]$$

$$\langle E_T \rangle_v = V_l \cdot \langle 1/R \rangle_v \cdot 1/2\pi$$

$$\langle B_p^2/8\pi \rangle_v = (\dot{\ell}\xi\,\Phi_{lim}/\pi)^2 \langle |\nabla\xi|^2/R^2 \rangle / 8\pi$$

$$\langle P_{OH} \rangle_v = \langle \vec{J}\cdot\vec{E} \rangle_v = \langle J_T \rangle_A \cdot \langle E_T \rangle_v$$

$$\langle \vec{\nabla}\cdot(\vec{E}\times\vec{B}) \rangle_v = \left(\frac{\partial V}{\partial \xi}\right)^{-1} \frac{\partial}{\partial \xi}\left[\frac{V_l\,\Phi_{lim}}{2\pi^2}\left(\frac{\partial V}{\partial \xi}\right)\dot{\ell}\xi \langle |\vec{\nabla}\xi|^2/R^2 \rangle_v\right]$$

. . .

---

* $\langle J_T \rangle_A$ area-integrates to toroidal plasma current
$(\partial A/\partial \xi)$ - cross-sectional area element

# II. Magnetic Energy Balance

The Poloidal Field Diffusion Equation implies an energy relation describing the poloidal field energy balance:

$$\underbrace{\left(\frac{\partial V}{\partial s}\right)^{-1}\frac{\partial}{\partial t}\langle B_p^2/8\pi\rangle}_{\frac{d}{dt}(\text{field energy})} + \underbrace{P_{OH}}_{\text{ohmic heating}} = \underbrace{-\langle\vec{\nabla}\cdot(\vec{E}\times\vec{B})\rangle}_{-\text{div (E-M power flux)}} + \underbrace{\frac{\langle B_p^2/8\pi\rangle\left(\frac{\partial V}{\partial s}\right)^{-1}\frac{\partial}{\partial t}\left[\left(\frac{\partial V}{\partial s}\right)\langle|\vec{\nabla}s|^2/R^2\rangle_v\right]}{\langle|\vec{\nabla}s|^2/R^2\rangle_v}}_{\text{compression input to poloidal field}}$$

Which can be evaluated indepently as a check on the solution of the basic poloidal field equation