Rotation Modified Equilibrium Solutions in TRANSP/PTRANSP APS 2008

(updated August 2015 for multi-species rotation and quasineutraity)

PPPL: R. Andre, D. McCune, J. Menard
LLNL: L. D. Pearlstein, L. L. LoDestro

Work performed by LLNL under the auspices of the US DOE, ContractDE-AC52-07NA27344

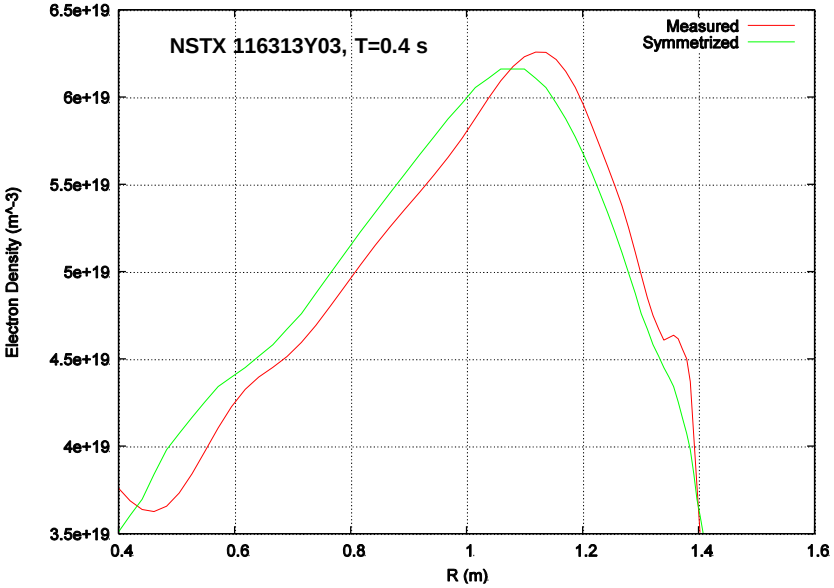
Abstract

Neutral beam injection is used with NSTX to achieve high beta discharges. This can lead to significant toroidal rotation such that the centrifugal pressure becomes a significant fraction of the total plasma pressure. Under these conditions, it has been shown that the particle densities are no longer flux surface functions with a resulting modification of the equilibrium solution. While TRANSP/PTRANSP has traditionally supported an additional contribution to the plasma pressure due to rotation, the total pressure has been assumed to be a flux surface quantity. Accounting for the radial dependence of the pressure caused by rotation requires a modification to the underlying Grad-Shafranov equation. This poster will describe the status of adding the rotation terms to the TEQ and other equilibrium solvers in TRANSP/PTRANSP and the effect they have on the flux surface representation in NSTX. The rotation is assumed to be purely toroidal with the rotation frequency and species temperatures represented as flux surface quantities.

Introduction

- Heating of tokamak plasmas through neutral beam injection can produce significant toroidal rotation.
- Previous work has shown that the centrifugal forces due to rotation will affect the equilibrium solution.
  - Magnetic axis will shift outward.
  - Pressure and densities are no longer flux surface constants.
- Self consistently accounting for rotation in the equilibrium solution requires a modification to the underlying Grad-Shafranov equation.
  - Maschke and Perrin, Plasma Phys., Vol. 22, 1980
  - Hameiri, Phys. Fluids., Vol. 26, 1983

Measured electron density is shifted outward relative to the density after symmetrization with the equilibrium solution



- TRANSP/PTRANSP has traditionally handled rotation by modifying the scalar pressure.
- The current work considers the effect of toroidal rotation on the equilibrium solution in TRANSP/PTRANSP through a rotation modified Grad-Shafranov equation. The implication of poloidally dependent pressure and densities on the transport and other processes in TRANSP/PTRANSP is not considered here but would be a worthwhile pursuit for future study.

Transp/PTransp Traditional Pressure

- Transp/PTransp has traditionally assumed that the pressure P is a flux surface function with the following contributions:

$$P = P_{\text{thermal}} + P_{\text{fast}} + P_{\text{rotation}}$$ Total pressure

$$P_{\text{thermal}} = \sum_{s}^{\text{thermal}} N_s T_s$$ Pressure due to thermal ions

$$P_{\text{fast}} = \sum_{f}^{\text{fast}} \frac{1}{2}\left(\overline{E}_{f,\perp} + 2\overline{E}_{f,\parallel}\right)$$ Pressure due to fast (beam,RF) ions

$$P_{\text{rotation}} = \frac{4}{3}\sum_{s}^{\text{thermal}} \frac{1}{2} m_s N_s \left\langle R^2 \right\rangle \omega^2$$ Pressure due to rotation

Where m_s is the mass of the thermal species and N_s, T_s are the flux surface average density and temperature, $\overline{E}_{f,\perp}, \overline{E}_{f,\parallel}$ are the flux surface average fast ion energy densities in the perpendicular and parallel direction, ω is the mass average thermal rotation frequency and $\left\langle R^2 \right\rangle$ is the flux surface average value of R^2 .

- The rotational energy density defines an isotropic rotational pressure contribution in a manner similar to the thermal and fast ion contribution.
- This ignores the kinematics of the rotation in which the centrifugal force on the plasma is directed in the outward, radial direction.

Rotation Modified Pressure Assumptions

- To self consistently include rotation effects, start from multi-species ideal mhd equations
  - Electrons assumed to have zero mass
  - Assume zero resistivity
- Thermal species will be assumed to have the toroidal velocity

$$\vec{u}_j = \omega_j R \hat{\varphi}$$

- Assume the rotation ω_j and thermal species temperatures T_j are flux surface functions.
- The species densities n_j <u>will not</u> be flux functions but assume the inputs to the equilibrium solver are the flux averaged thermal densities,

$$N_j = \langle n_j \rangle$$

- Assume the fast ions contribute to the scalar pressure p in the traditional way with pressure P_{fast} and electron pressure P_{extra} associated with the fast ion species. Note there is no P_{rotation} term.

<u>Ideal MHD</u>

$$\nabla \cdot \left(n_j \vec{u}_j \right) = 0$$

$$m_j n_j (\vec{u}_j \cdot \nabla) \vec{u}_j = -\nabla p_j + q_j n_j (\vec{E} + \vec{u}_j \times \vec{B})$$

$$\mu_0 \vec{J} = \nabla \times \vec{B}$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} = 0$$

$$\vec{J} = \sum_j^{\text{ions+electrons}} n_j q_j \vec{u}_j$$

$$n_e = \sum_j^{\text{ions}} Z_j n_j$$

$$Z_j = -\frac{q_j}{q_e}$$

Rotation Modified Grad-Shafranov Equation

- The Grad-Shafranov equation determines the equilibrium solution for the poloidal flux $\psi(R,Z)$. With a scalar pressure p and **without rotation**, it has the traditional form,

$$\Delta^*\psi + FF' + \mu_0 R^2 p' = 0$$

$$\Delta^*\psi = R^2 \nabla \cdot \left(\frac{1}{R^2} \nabla \psi \right)$$

with required inputs $p(\psi)$ and $F(\psi)$.

$$p' = \frac{dp}{d\psi}, \quad \text{similar for } F = RB_\varphi$$

- **With rotation** and the assumptions made on the previous page, the rotation modified Grad-Shafranov Equation becomes,

$$\Delta^*\psi + FF' + \mu_0 R^2 \sum_j^{\text{ions+electrons}} \mathrm{e}^{\frac{m_j \omega_j^2 R^2}{2T_j} - \frac{Z_j E}{T_j}} \left[p'_{j0} - p_{j0} \left(\frac{Z_j}{T_j} \right)' E + m_j R^2 p_{j0} \left(\frac{\omega_j^2}{2T_j} \right)' \right] + \mu_0 R^2 (P'_{\text{fast}} + P'_{\text{extra}}) = 0$$

$$p_{j0}(\psi) = \frac{N_j T_j}{\left\langle \mathrm{e}^{\frac{m_j \omega_j^2 R^2}{2T_j} - \frac{Z_j}{T_j} E} \right\rangle} \qquad n_j(\psi, R) = \frac{\mathrm{e}^{\frac{m_j \omega_j^2 R^2}{2T_j} - \frac{Z_j}{T_j} E}}{\left\langle \mathrm{e}^{\frac{m_j \omega_j^2 R^2}{2T_j} - \frac{Z_j}{T_j} E} \right\rangle} N_j \qquad E(\psi, R) = \int_{R_{\min}}^{R_{\max}} R \left[\frac{\sum_j^{\text{ions}} Z_j n_j(\psi, R) \frac{m_j \omega_j^2}{T_j}}{\sum_j^{\text{ions}} Z_j n_j(\psi, R) \left(\frac{1}{T_{\text{electron}}} + \frac{Z_j}{T_j} \right)} \right] dR$$

with required inputs $P_{\text{fast}}(\psi)$ $P_{\text{extra}}(\psi)$ $F(\psi)$ m_j $N_j(\psi)$ $T_j(\psi)$ $\omega_j(\psi)$

Rotation Modified Grad-Shafranov Equation, cont.

- $E(\psi, R)$ enforces quasineutrality and is solved iteratively in the picard loop

- The species specific part of the exponential term can be written as

$$\mathrm{e}^{\frac{m_j \omega_j^2 R^2}{2T_j}} = \mathrm{e}^{\frac{M_j^2(\psi,R)}{2}}, \quad \text{where} \quad M_j(\psi,R) = \frac{\omega_j R}{\sqrt{\frac{T_j}{m_j}}} = \text{mach number}$$

In this context the mach number is the ratio of the rotational velocity to the thermal velocity. It can become large for small temperatures and impurities (large m_j).

- Usually $\left\langle \vec{J} \cdot \vec{B} \right\rangle$ or q is preferred to $F = RB_\varphi$ as an input to the equilibrium solver. The relationship between these flux functions is derived from the flux averaged Grad-Shafranov equation. These relations have the same form using the rotation modified Grad-Shafranov equation except instead of using $\frac{dp}{d\psi}$, the following is used,

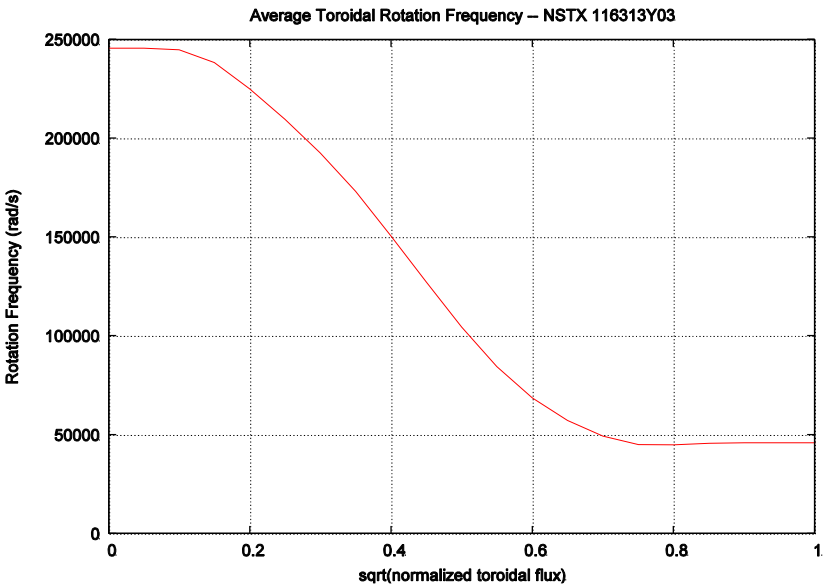
$$\left\langle \left. \frac{\partial p}{\partial \psi} \right|_R \right\rangle \equiv \sum_j^{\text{ions+electrons}} \left[p'_{j0} \left\langle \mathrm{e}^{\frac{m_j \omega_j^2 R^2}{2T_j} - \frac{Z_j}{T_j} E} \right\rangle - p_{j0} \left(\frac{Z_j}{T_j} \right)' \left\langle E \mathrm{e}^{\frac{m_j \omega_j^2 R^2}{2T_j} - \frac{Z_j}{T_j} E} \right\rangle + m_j p_{j0} \left(\frac{\omega_j^2}{2T_j} \right)' \left\langle R^2 \mathrm{e}^{\frac{m_j \omega_j^2 R^2}{2T_j} - \frac{Z_j}{T_j} E} \right\rangle \right] + \frac{dP_{\text{fast}}}{\mathrm{d}\psi} + \frac{dP_{\text{extra}}}{\mathrm{d}\psi}$$

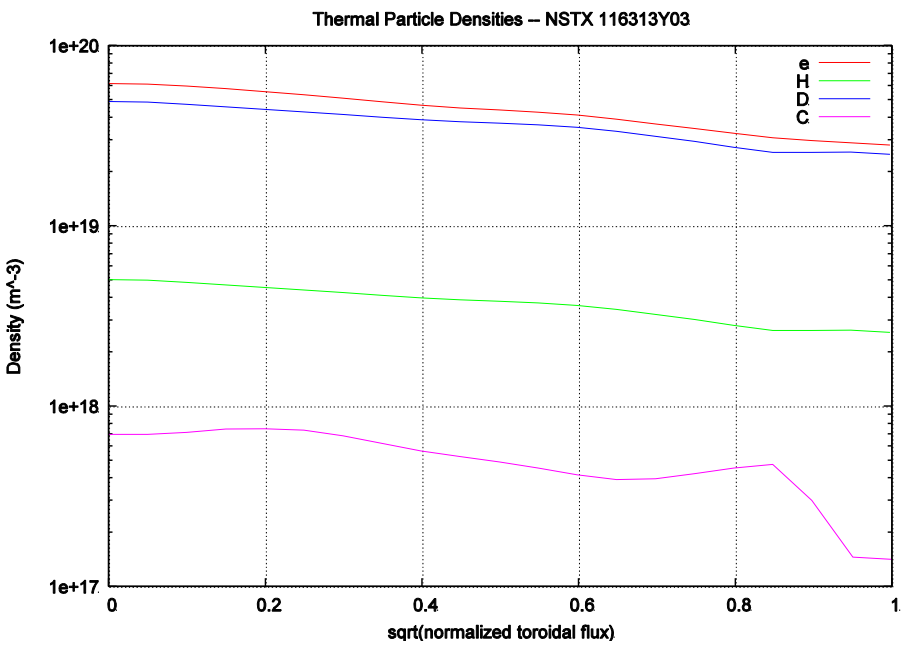
Rotation Modified Equilibrium Status

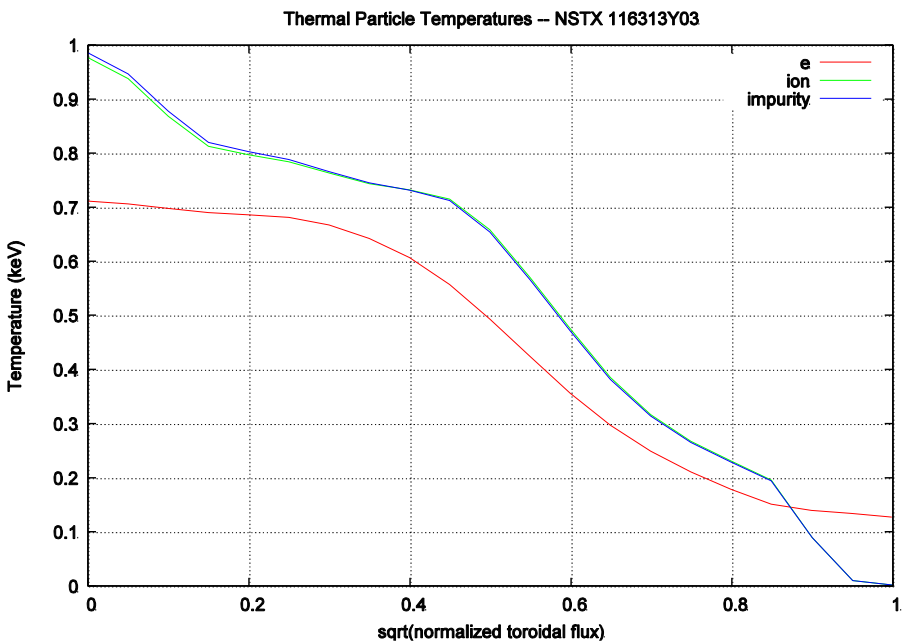
- ISolver
  - ISolver is a free boundary equilibrium code written in IDL by J. Menard and translated to Fortran by R. Andre.
  - Coil and limiter data currently only available for NSTX.
  - Rotation modified equilibrium: **Completed** for time slice analysis.
- TEQ
  - Fixed and free boundary equilibrium solver used in the Lawrence Livermore National Laboratory's Corsica transport code.
  - The inverse solver is the default fixed boundary solver for TRANSP.
- TRANSP/PTRANSP
  - Outside of the equilibrium solver, TRANSP/PTRANSP makes assumptions on the relationship between $\frac{dp}{d\psi}, F, \langle \vec{J} \cdot \vec{B} \rangle$ and q which must be consistent with the solver.
  - Rotation modified equilibrium: **In Progress**

Example: NSTX 116313, Time=0.4 s

- NSTX 116313 shot
  - e,H,D,C thermal species and a D beam fast ion species.
  - NCLASS neoclassical analysis verifies angular rotation of thermal species are comparable except for the electrons which are weakly affected by rotation.
- ISolver time slice equilibrium computed with different pressure models
  - Traditional TRANSP pressure rotation
  - Rotation modified Grad-Shafranov solution
  - No Rotation

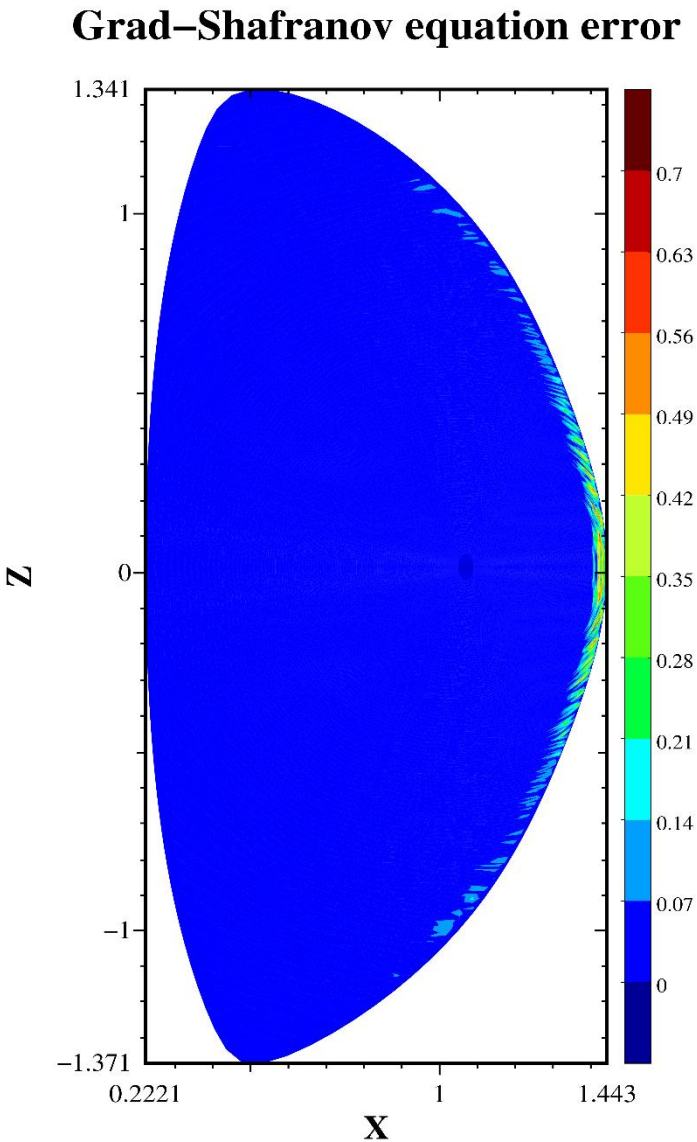






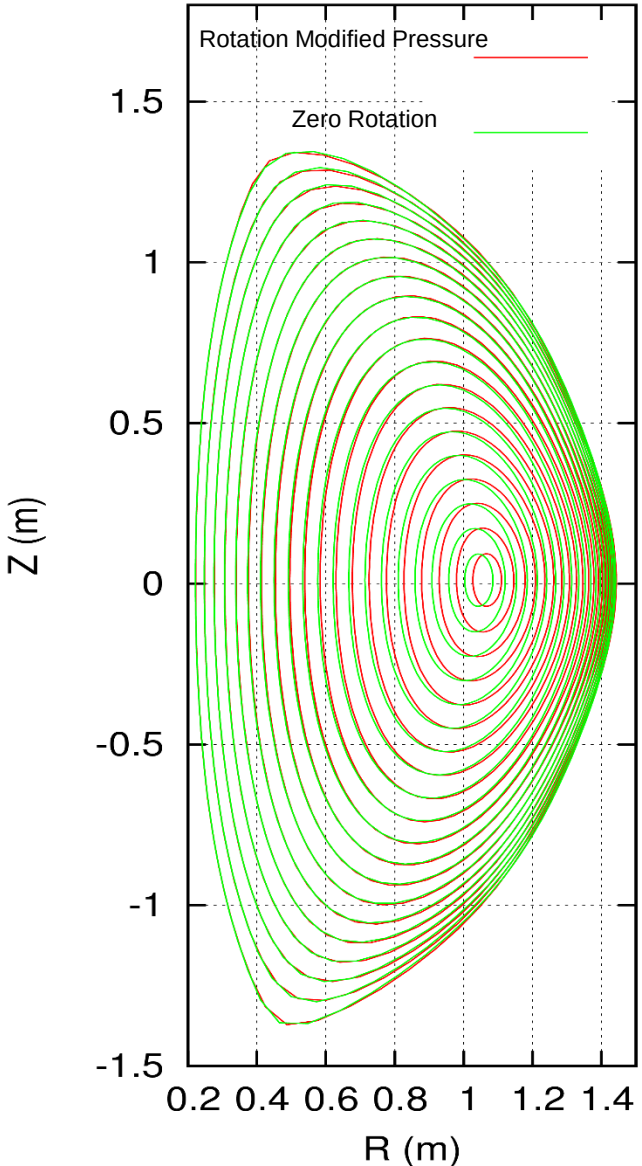
Rotation Modified Grad-Shafranov Equilibrium GS Error

- The error in the rotation modified Grad-Shafranov equation relative to the average value is plotted.
- To limit the mach number, the minimum temperature is bounded below by 50 eV.
- Error on the outer major radius side represents discretization error of the underlying R,Z grid at the edge of the plasma as amplified by the high mach number on the edge.



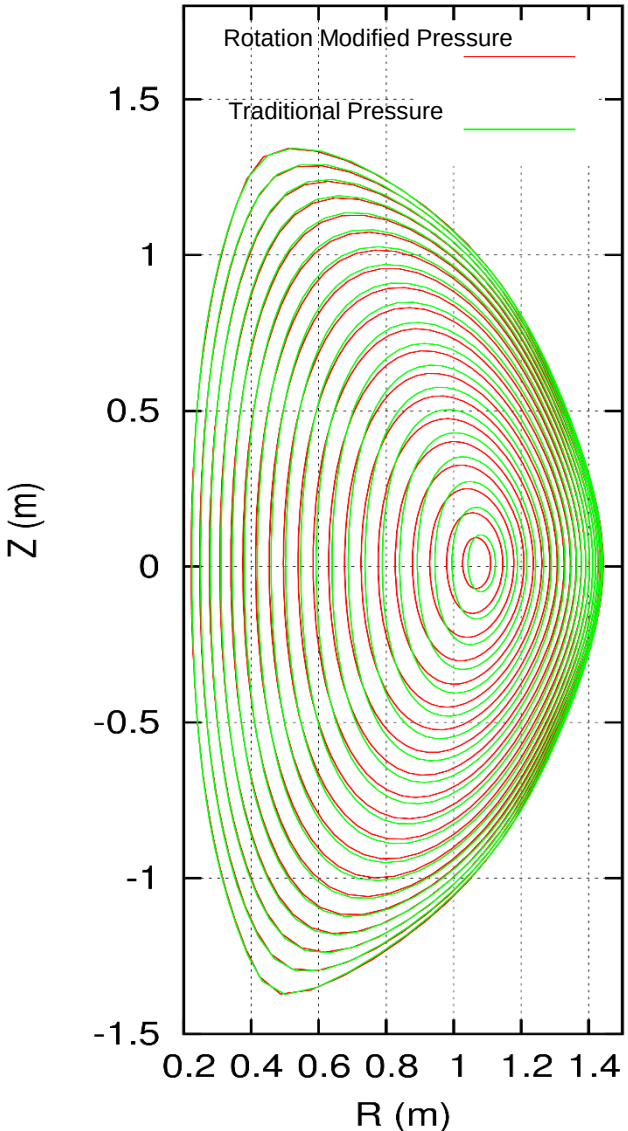
Comparison of Rotation Modified Equilibrium to Zero Rotation

- Adding rotation shifts the center and outside edge of the flux surfaces outward.
- Height of the flux surfaces do not change significantly



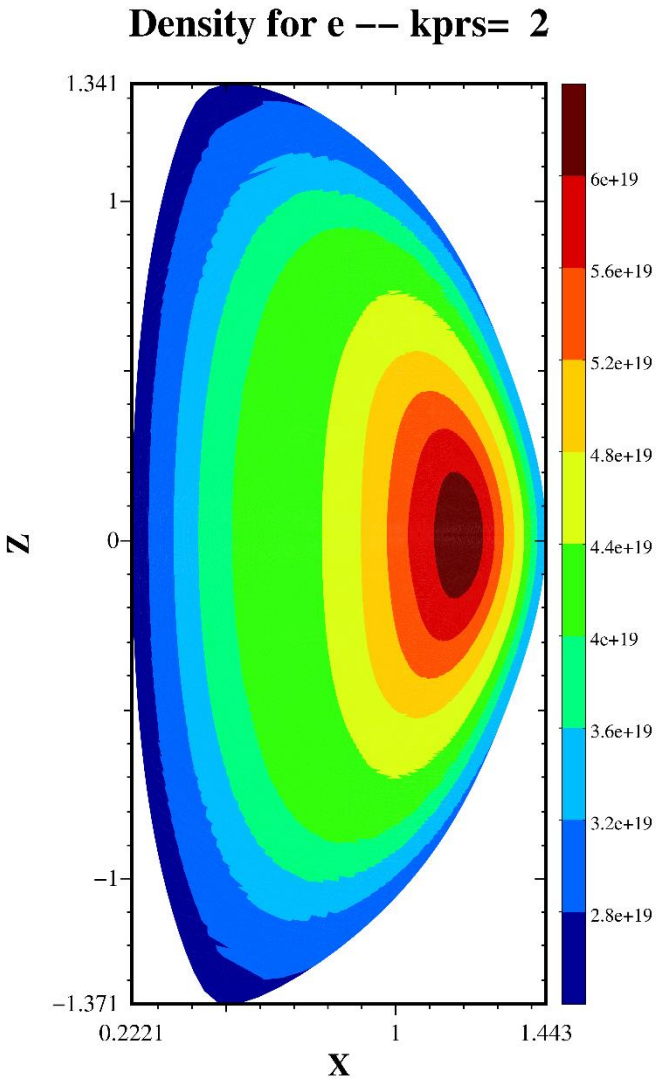
Comparison of Rotation Modified Equilibrium to Traditional TRANSP Rotation

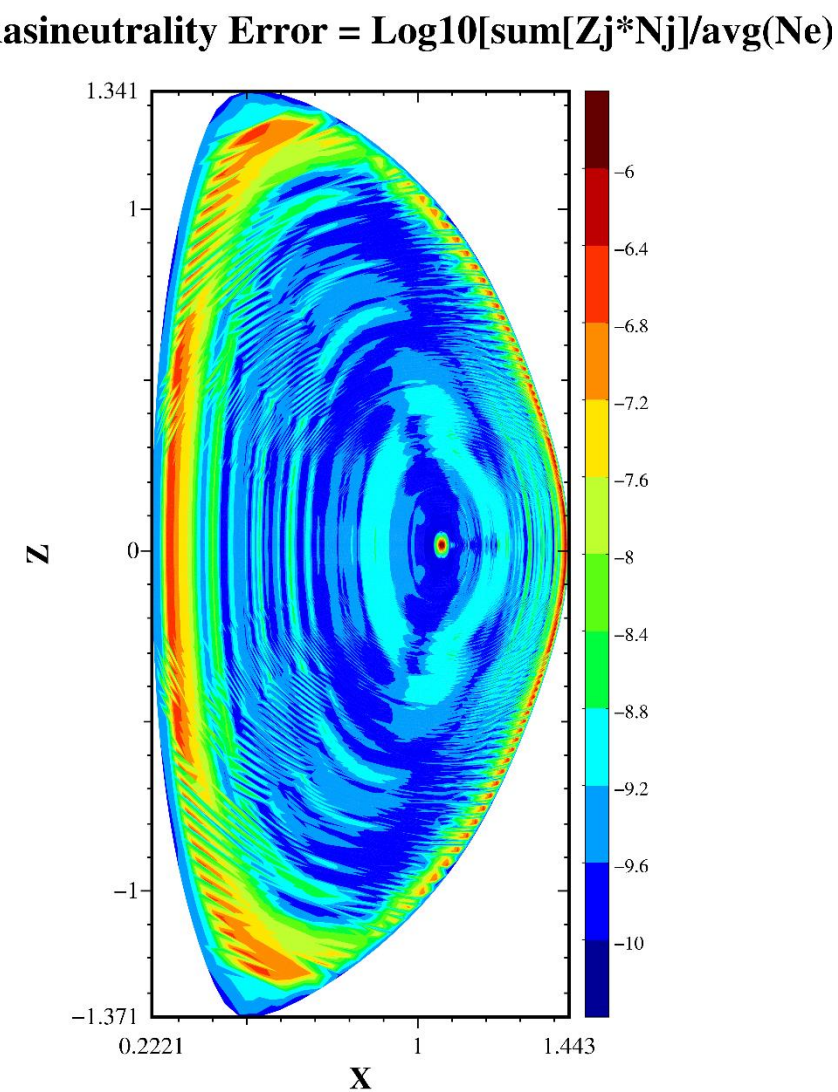
- The traditional TRANSP rotation model, which applies the additional P_{rotation} term to the scalar pressure, does a reasonable job of matching the rotation modified Grad-Shafranov equilibrium.
- The traditional method has a slightly more shifted magnetic axis.
- The elongation of the flux surfaces is also slightly different reflecting that the scalar P_{rotation} pressure approximation applies a force across the entire flux surface.



Electron Density and Quasineutrality

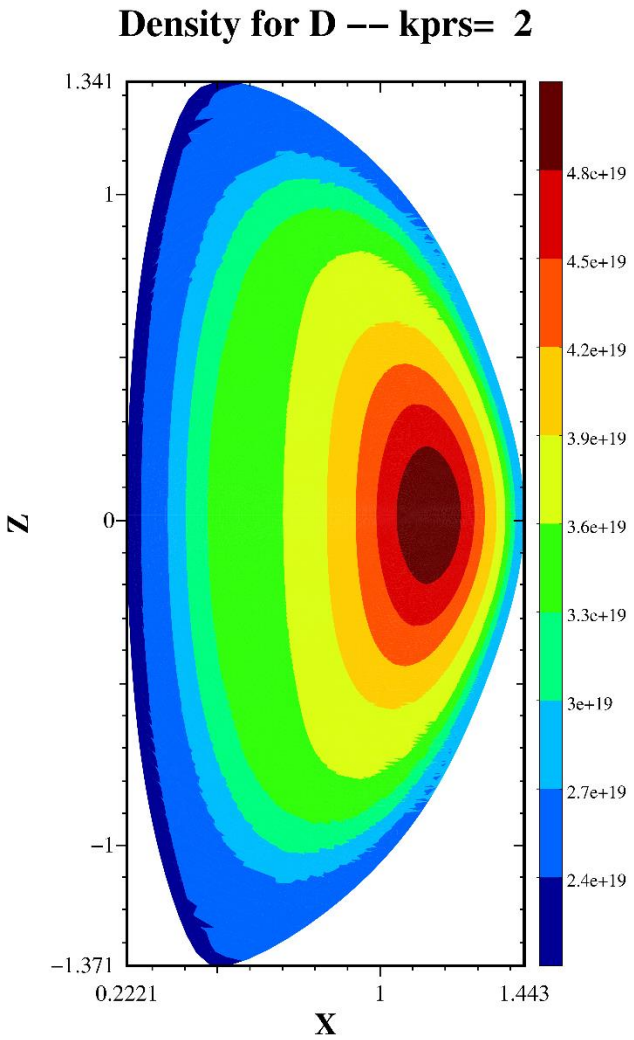
- The electrons are distributed to satisfy quasineutrality with respect to the rotating ion species.

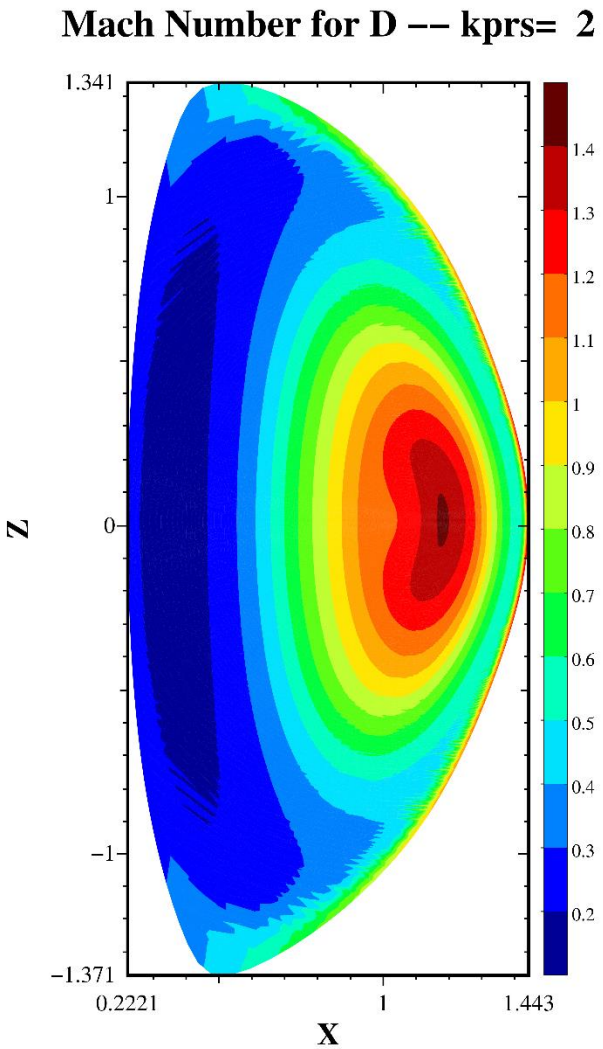




Deuterium Density and Mach Number

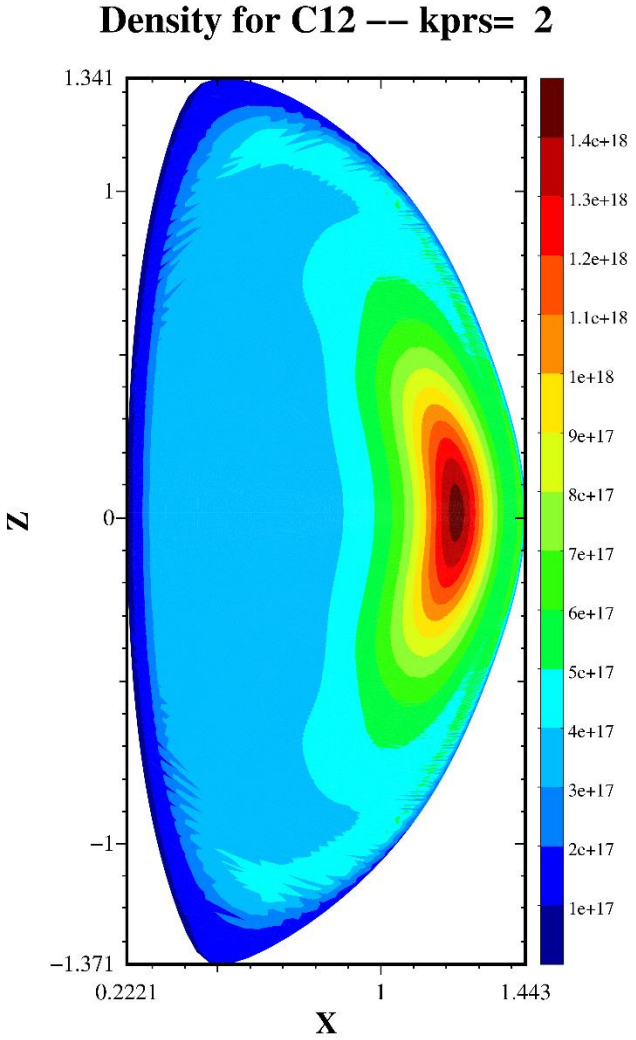
- The main ion species becomes more concentrated on the outer major radius side of the flux surfaces.
- Mach number is of order unity for this shot and time slice.

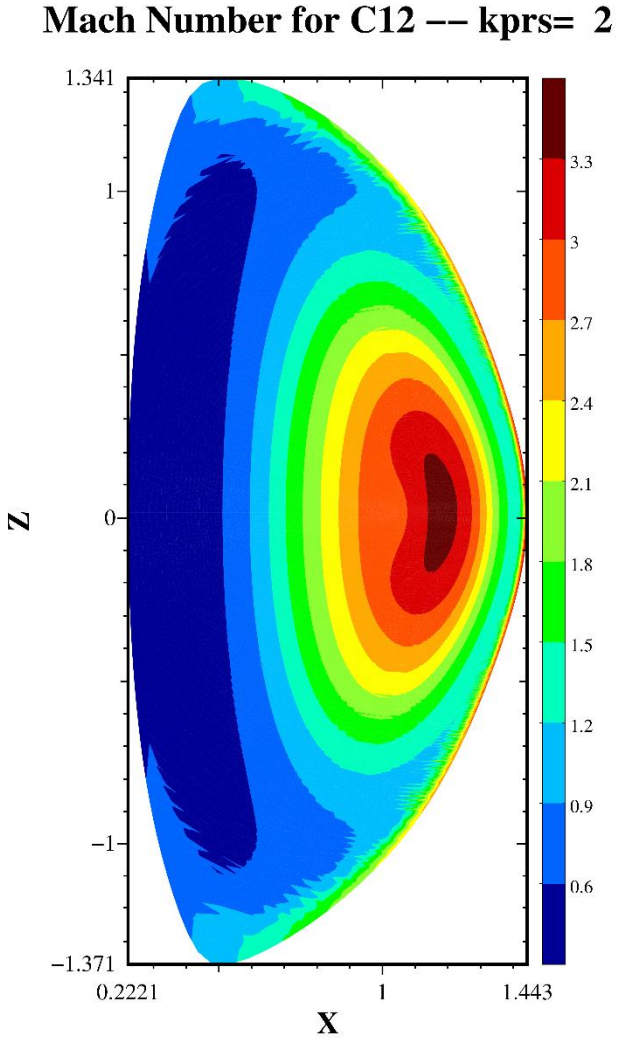




Carbon Density and Mach Number

- The impurity is significantly affected by rotation and is concentrated on the outer edge of the flux surface.





Summary

- The ISolver free boundary equilibrium solver can now produce a solution of the rotation modified Grad-Shafranov equation based on a multi-species ideal mhd approach.
- Additional work is required to move this into TRANSP.
- A time slice solution with and without rotation was demonstrated for NSTX.
- The traditional method of handling rotation in TRANSP through an additional scalar pressure term did a reasonable job of simulating the full rotation model for this time slice.
- The significant effect of rotation on the density distribution suggests that accounting for rotation, not just in the equilibrium solver but in other models in TRANSP, should be considered.