Ondes dans les Plasmas

[Partie I]

Interaction onde / plasma: Bases physiques

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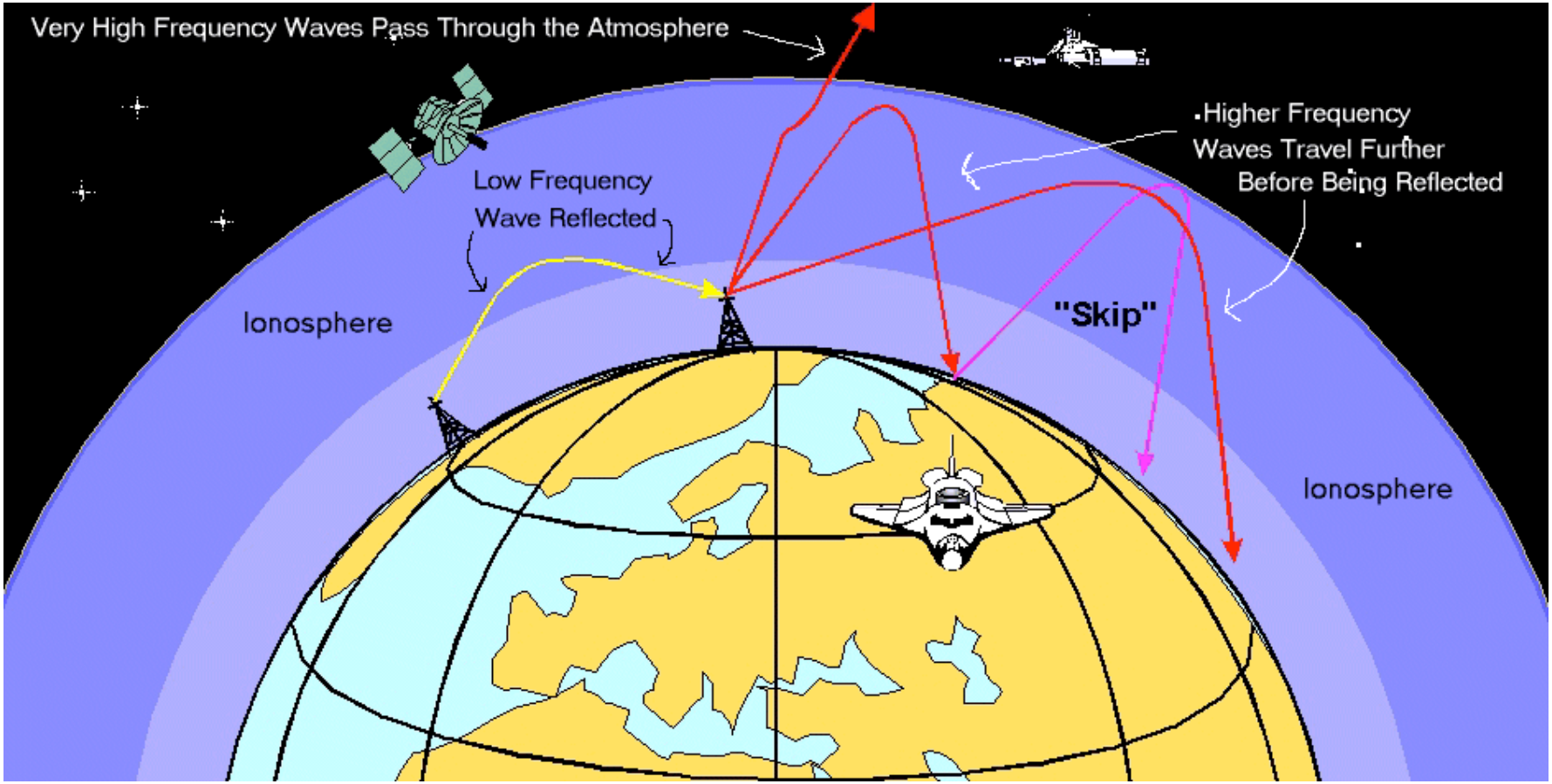
Introduction

Ionosphère / Atmosphère

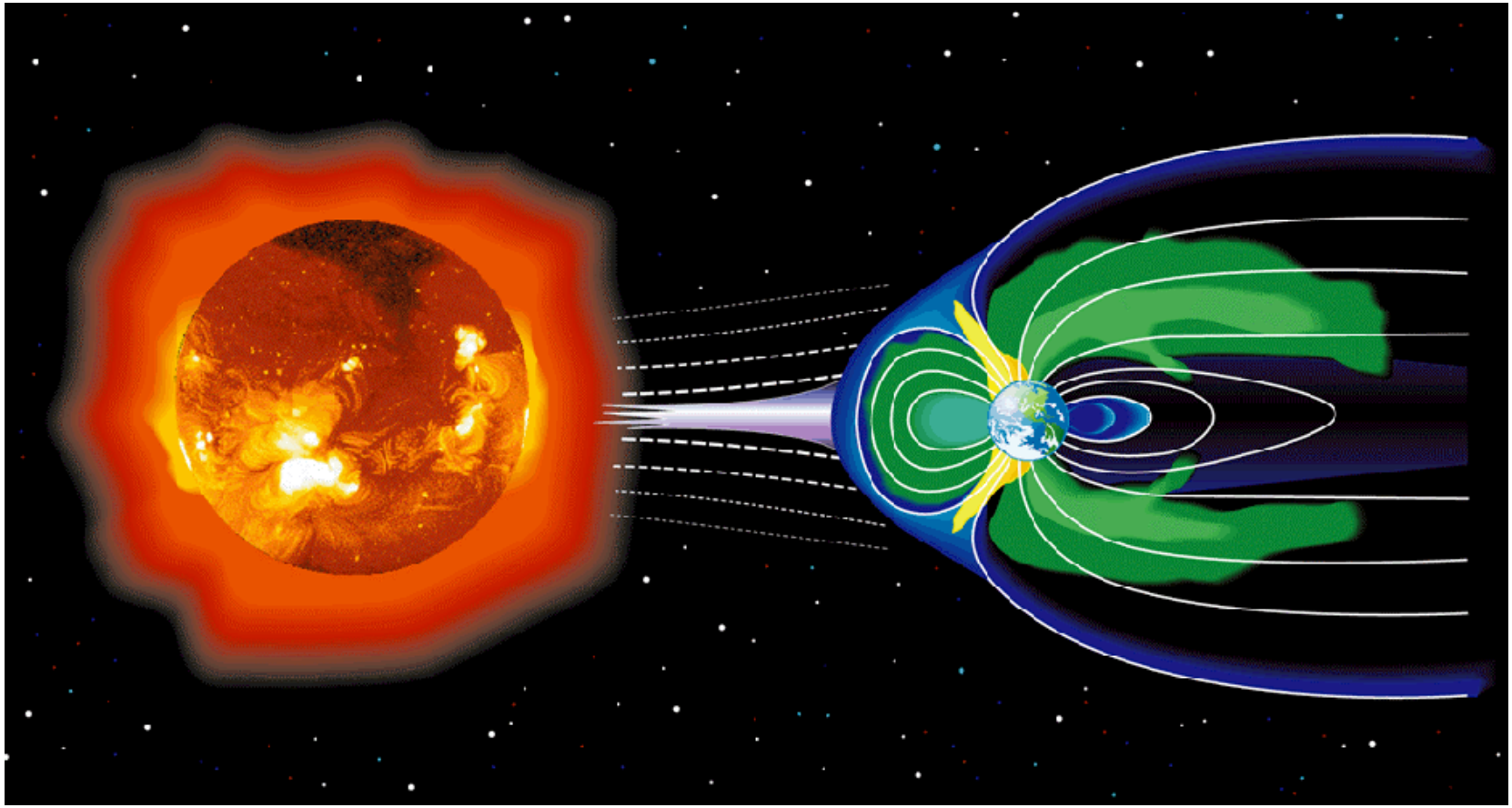


[Source: Nasa - GSFC]

Gradient de densité, coupures :

Réflexions / Transmission

Magnétosphère

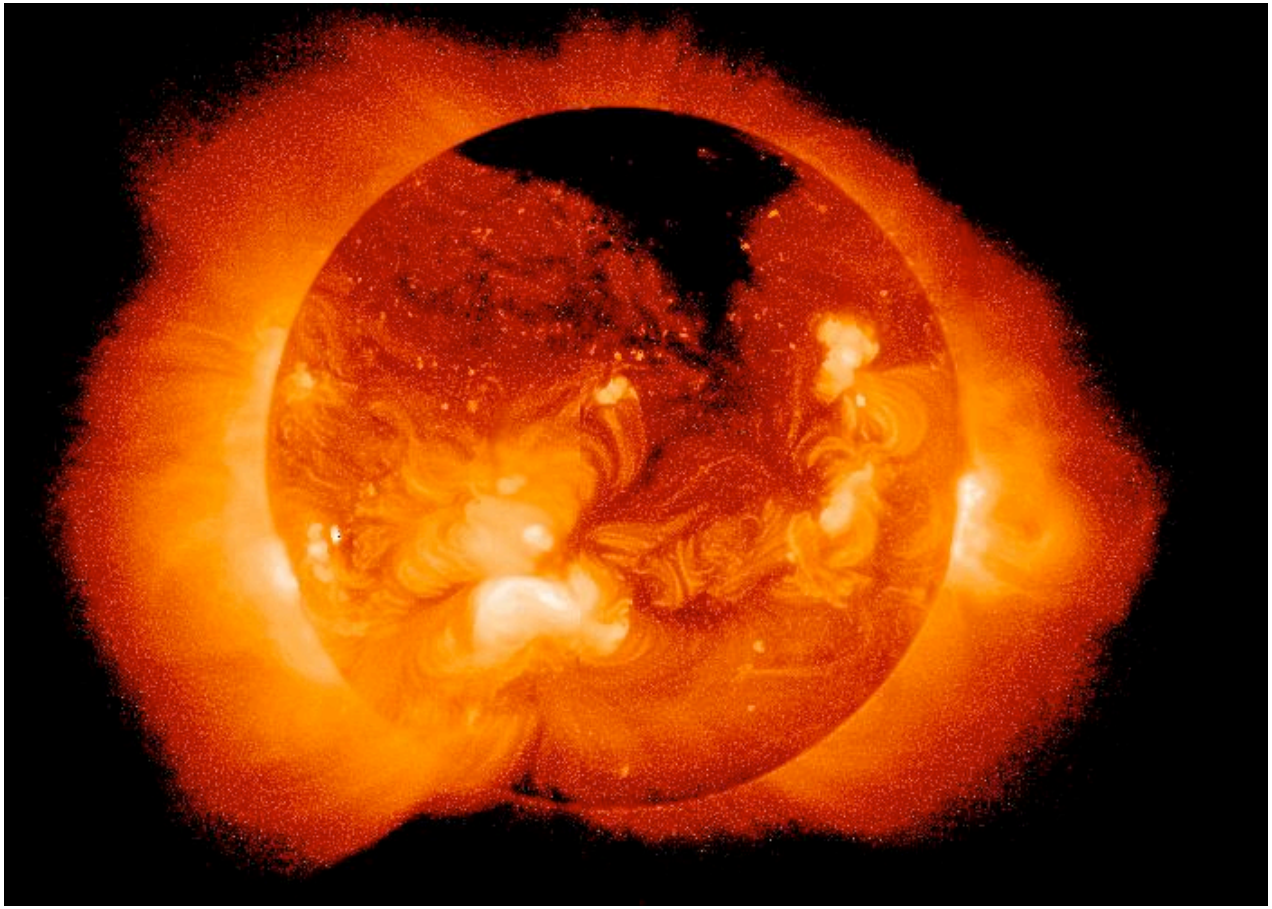


[Source: Nasa - GSFC]

Présence de whistlers:

Précipitation des particules dans les cônes de pertes

Astrophysique solaire

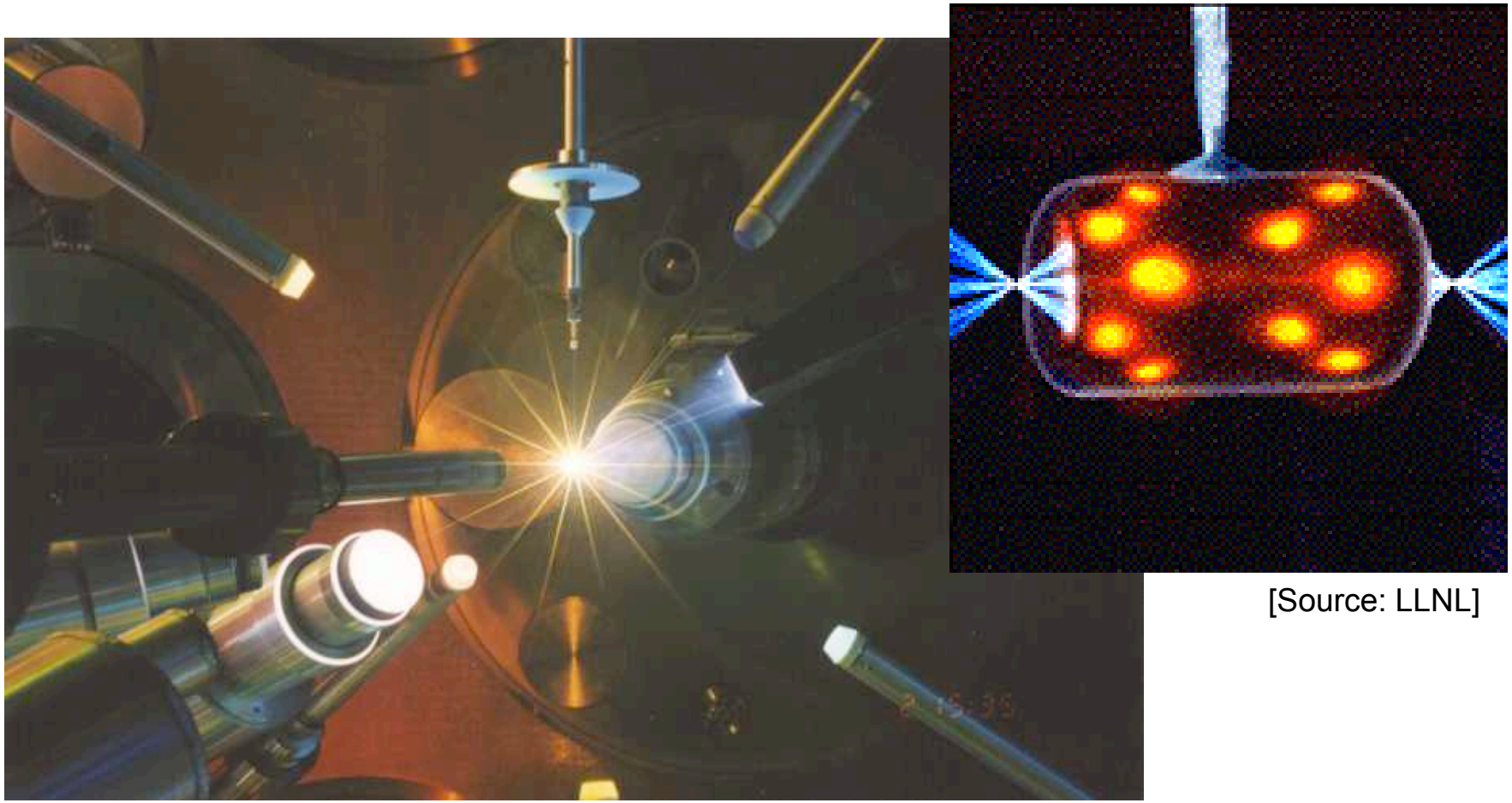


[Source: Nasa - MFSC]

Ondes d'Alfvén:

Chauffage anormal de la couronne solaire (?)

Plasmas confinés par lasers

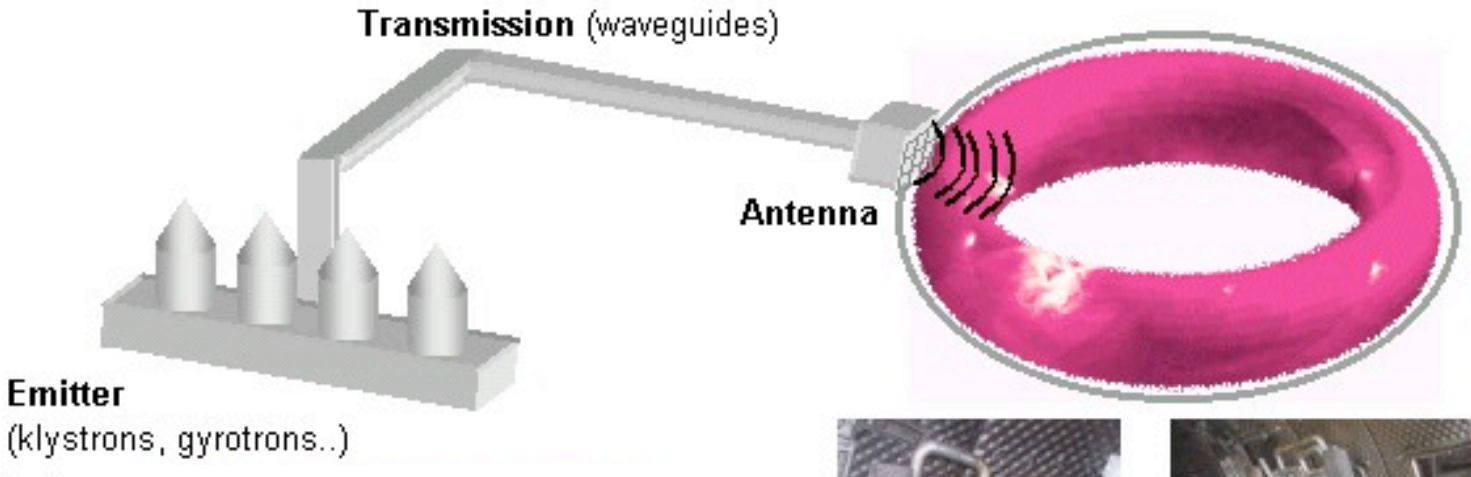


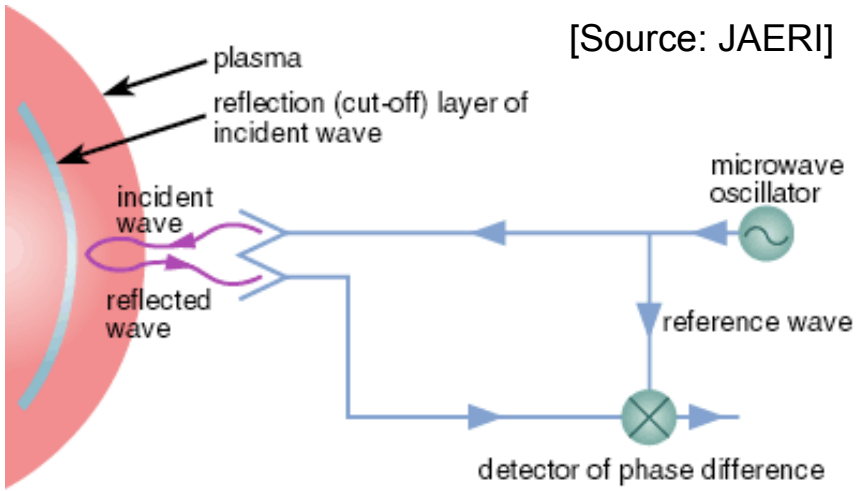
[Source: LLNL]

[Source: U. Rochester]

Interaction non-linéaire laser / plasma

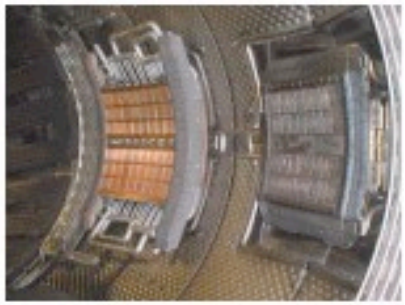
Plasmas de fusion magnétique





Diagnostics

Ion cyclotron heating antenna (Tore Surpa)

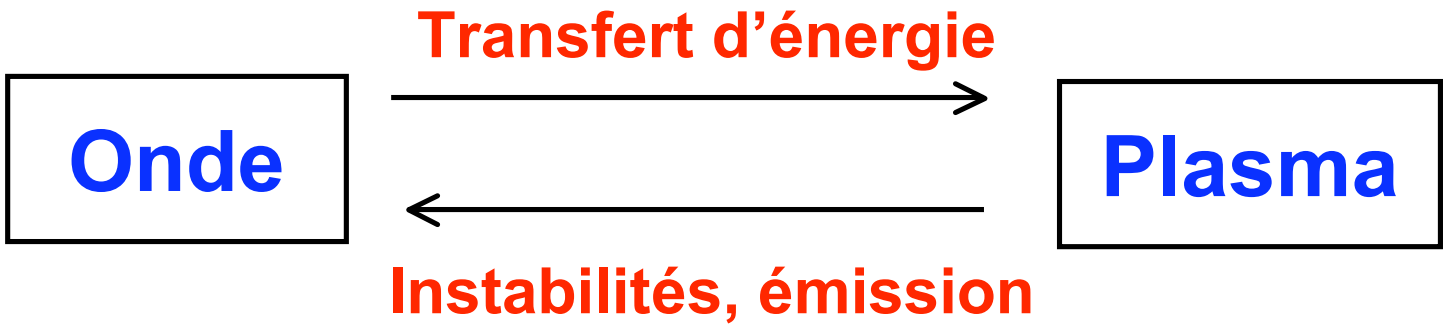


Hybrid heating antenna (Tore Surpa)

Chauffage

Génération de Courant

Le problème



Equations de Maxwell

Description statistique

Equations de base

Equations de Maxwell

- Le coeur du problème

Déplacement électrique

Densité de charges

$$\nabla \cdot \mathbf{D} = \rho$$

Induction magnétique

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

Champ électrique

$$\nabla \times \mathbf{H} = \mathbf{j}_s + \frac{\partial \mathbf{D}}{\partial t}$$

Intensité magnétique

Courant source

- Equation de continuité pour les sources

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \mathbf{j}_s = 0$$

Relation constitutive du milieu

- Milieu isotrope: $\begin{cases} \mathbf{D} = \epsilon \mathbf{E} \\ \mathbf{B} = \mu \mathbf{H} \end{cases}$

- Milieu anisotrope: $\begin{cases} \mathbf{D} = \overline{\overline{\epsilon}} \cdot \mathbf{E} \\ \mathbf{B} = \overline{\overline{\mu}} \cdot \mathbf{H} \end{cases}$

- Plasma: $\begin{cases} \mathbf{D} = \overline{\overline{\epsilon}} \cdot \mathbf{E} \\ \mathbf{B} = \mu_0 \mathbf{H} \end{cases}$

- Lien champ / déplacement électrique: $\mathbf{j}_p + \epsilon_0 \dfrac{\partial \mathbf{E}}{\partial t} = \mathbf{j}_s + \dfrac{\partial \mathbf{D}}{\partial t}$

Courant total **Courant ``réponse''**

- Cohérence du courant:

$$\mathbf{j}_p \equiv \mathbf{j}_s + \mathbf{j} \longrightarrow \mathbf{j} + \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} = \frac{\partial \mathbf{D}}{\partial t}$$

- Champs harmoniques: $(\mathbf{E}, \mathbf{B}) \propto \exp(-i\omega t)$

$$\mathbf{D} = \epsilon_0 \left(\mathbf{E} + \frac{i}{\omega \epsilon_0} \mathbf{j} \right)$$

- Relation constitutive du milieu: $\mathbf{j} \stackrel{?}{=} \mathbf{j}(\mathbf{E})$

Approximation linéaire

- Description statistique du plasma:

$$\rho = \sum_s q_s \int d^3\mathbf{v}\, f_s(\mathbf{r},\mathbf{v},t) \qquad \mathbf{j} = \sum_s q_s \int d^3\mathbf{v}\, \mathbf{v} f_s(\mathbf{r},\mathbf{v},t)$$

- Equation cinétique:

$$\frac{\partial f_s}{\partial t} + \mathbf{v}\cdot\frac{\partial f_s}{\partial \mathbf{r}} + \underbrace{\frac{q_s}{m_s}\Big[(\mathbf{E}_0+\mathbf{E}) + \mathbf{v}\times(\mathbf{B}_0+\mathbf{B})\Big]\cdot\frac{\partial f_s}{\partial \mathbf{v}}}_{\text{Non-linéarité}} = \left(\frac{\partial f_s}{\partial t}\right)_{coll.}$$

- Approximation linéaire:

$$f_s(\mathbf{r},\mathbf{v},t) \equiv F_s(\mathbf{r},\mathbf{v},t) + \tilde{f}_s(\mathbf{r},\mathbf{v},t)$$

Solution d'équilibre

Perturbation

$$\tilde{f}_s \propto \mathbf{E}$$

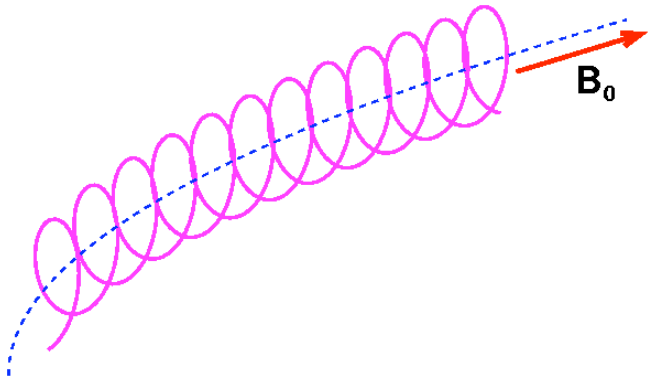
- Réponse linéaire du plasma:

$$(\rho,\mathbf{j}) \propto \mathbf{E}$$

Le milieu plasma

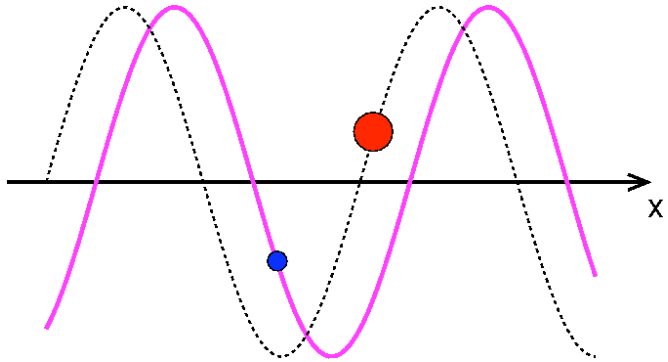
- Anisotropie

$$\mathbf{j}(\mathbf{r},t) = \overline{\overline{\boldsymbol{\sigma}}} \cdot \mathbf{E}(\mathbf{r},t)$$



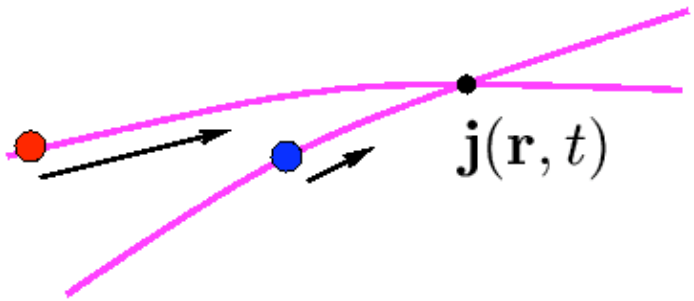
- Dispersion temporelle

$$\mathbf{j}(\mathbf{r},t) = \int_{-\infty}^{t} dt' \, \overline{\overline{\boldsymbol{\sigma}}}(\mathbf{r},t,t') \cdot \mathbf{E}(\mathbf{r},t')$$



- Dispersion spatiale

$$\mathbf{j}(\mathbf{r},t) = \int d^3\mathbf{r}' \, \overline{\overline{\boldsymbol{\sigma}}}(\mathbf{r},\mathbf{r}',t) \cdot \mathbf{E}(\mathbf{r}',t)$$



Tenseur diélectrique

- Courant non local: $\mathbf{j}(\mathbf{r},t) = \int_{-\infty}^{t} dt' \int d^3\mathbf{r}'\, \bar{\bar{\boldsymbol{\sigma}}}(\mathbf{r},\mathbf{r}',t,t') \cdot \mathbf{E}(\mathbf{r}',t')$

- Plasma stationnaire: $\bar{\bar{\boldsymbol{\sigma}}}(\mathbf{r},\mathbf{r}',t,t') = \bar{\bar{\boldsymbol{\sigma}}}(\mathbf{r},\mathbf{r}',t-t')$

- Plasma uniforme: $\bar{\bar{\boldsymbol{\sigma}}}(\mathbf{r},\mathbf{r}',t,t') = \bar{\bar{\boldsymbol{\sigma}}}(\mathbf{r}-\mathbf{r}',t,t')$

- Ondes planes: $(\mathbf{E},\mathbf{B}) \propto \exp\left(i(\mathbf{k}\cdot\mathbf{r}-\omega t)\right)$

$$\longrightarrow \quad \mathbf{j}(\mathbf{k},\omega) = \bar{\bar{\boldsymbol{\sigma}}}(\mathbf{k},\omega)\cdot\mathbf{E}(\mathbf{k},\omega)$$

$$\longrightarrow \quad \mathbf{D} = \epsilon_0\left(\mathbf{E} + \frac{i}{\omega\epsilon_0}\mathbf{j}\right) = \underbrace{\epsilon_0\left(1 + \frac{i}{\omega\epsilon_0}\bar{\bar{\boldsymbol{\sigma}}}\right)}_{\bar{\bar{\boldsymbol{\epsilon}}} \equiv \epsilon_0 \bar{\bar{\mathbf{K}}}} \cdot \mathbf{E}$$

(Tenseur diélectrique)

Relation de dispersion

- Equation d'onde (sans source):

$$\nabla \times \nabla \times \mathbf{E} - \frac{\omega^2}{c^2} \overline{\overline{\epsilon}} \cdot \mathbf{E} = 0$$

- Ondes planes - Plasma homogène:

$$\mathbf{n} \times \mathbf{n} \times \mathbf{E} + \overline{\overline{\epsilon}} \cdot \mathbf{E} = 0, \qquad \mathbf{n} \equiv \frac{c}{\omega}\mathbf{k}$$

$$\longrightarrow \quad \overline{\overline{\mathbf{M}}}_{\mathbf{k},\omega} \cdot \mathbf{E} = \mathbf{0}$$

- Solutions non triviales pour le champ électrique:

$$\det(\overline{\overline{\mathbf{M}}}_{\mathbf{k},\omega}) = 0 \quad \longrightarrow \det\left[\left(n^2 \frac{k_i k_j}{k^2} - \delta_{ij}\right) + \epsilon_{ij}(\mathbf{k},\omega)\right] = 0$$

$\longrightarrow$
- Ondes caractéristiques du plasma uniforme $\quad n = n(\mathbf{k},\omega)$
- Modes propagatifs / Modes évanescents
- Polarisation

Description du milieu plasma

- Enjeu de la modélisation du plasma: $\overline{\overline{\epsilon}}$
- Différents modèles disponibles:
  - Plasma froid
  - Plasma froid + corrections thermiques

    $\longrightarrow$ Description fluide du plasma

  - Plasma chaud - linéaire
  - Plasma chaud - quasi-linéaire

    $\longrightarrow$ Description cinétique du plasma

Ondes dans un plasma fluide

Modèle de plasma froid

- Courant (modèle fluide): $\mathbf{j} = \sum_s \mathbf{j}_s = \sum_s n_s q_s \mathbf{v}_s$

Vitesse macroscopique

- Equation pour l'impulsion:

$$n_s m_s \frac{d\mathbf{v}_s}{dt} = n_s m_s \left(\frac{\partial \mathbf{v}_s}{\partial t} + (\mathbf{v}_s \cdot \boldsymbol{\nabla})\mathbf{v}_s \right) = n_s q_s (\mathbf{E} + \mathbf{v}_s \times \mathbf{B}) - \boldsymbol{\nabla} \cdot \overline{\overline{\boldsymbol{\Phi}}}_s$$

Tenseur de pression

- Plasma froid: $\overline{\overline{\boldsymbol{\Phi}}}_s = \overline{\overline{\mathbf{0}}}$

$$\longrightarrow \quad \frac{d\mathbf{v}_s}{dt} = \frac{q_s}{m_s}(\mathbf{E} + \mathbf{v}_s \times \mathbf{B})$$

- Linéarisation: $\mathbf{B} \equiv \mathbf{B}_0 + \delta\mathbf{B}, \quad \mathbf{E} \equiv \delta\mathbf{E}$

$\longrightarrow$ Au premier ordre et en omettant les "δ"

$$-i\omega m_s \mathbf{v}_s = q_s(\mathbf{E} + \mathbf{v}_s \times \mathbf{B}_0)$$

Modèle de plasma froid (2)

- Champ magnétique statique: $\mathbf{B}_0$ // $\mathbf{e}_z$

$$\begin{cases} -i(v_x + iv_y)\cdot(\omega - \omega_{cs}) &= \dfrac{q_s}{m_s}(E_x + iE_y) \\ -i(v_x - iv_y)\cdot(\omega + \omega_{cs}) &= \dfrac{q_s}{m_s}(E_x - iE_y) \\ -iv_z\omega = \dfrac{q_s}{m_s}E_z \end{cases}$$

$$\omega_{cs} \equiv \frac{q_s B_0}{m_s}$$ **(Fréquence cyclotronique)**

- Champ et vitesse “tournants”:

$$E^{\pm} \equiv \frac{1}{2}(E_x \pm iE_y) \qquad v^{\pm} \equiv \frac{1}{2}(v_x \pm iv_y)$$

$$\longrightarrow \begin{cases} -iv^{\pm}\cdot(\omega \mp \omega_{cs}) &= \dfrac{q_s}{m_s}E^{\pm} \\ -iv_z\omega = \dfrac{q_s}{m_s}E_z \end{cases}$$

Modèle de plasma froid (3)

- Courant:

$$\begin{cases} j^{\pm} = i \sum_s \frac{n_s q_s^2}{m_s} \cdot \frac{E^{\pm}}{\omega \mp \omega_{cs}} \\ j_z = i \sum_s \frac{n_s q_s^2}{m_s} \cdot \frac{E_z}{\omega} \end{cases}$$

- Tenseur conductivité: $\mathbf{j} = \bar{\bar{\boldsymbol{\sigma}}} \cdot \mathbf{E}$ $\longrightarrow$ **Tenseur diélectrique froid**

$$\bar{\bar{\boldsymbol{\epsilon}}} = \epsilon_0 \begin{bmatrix} S & -iD & 0 \\ iD & S & 0 \\ 0 & 0 & P \end{bmatrix}$$

$$\omega_{ps}^2 \equiv \frac{n_s q_s^2}{m_s \epsilon_0}$$

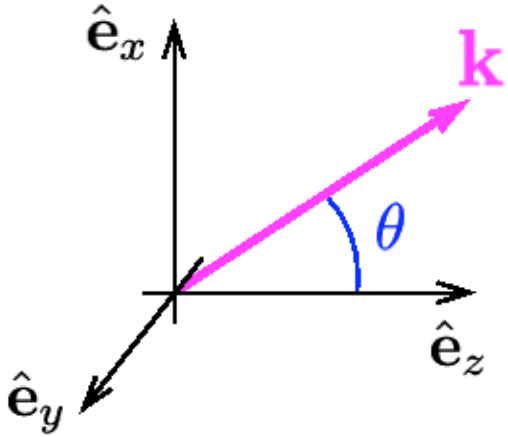
(Fréquence plasma)

$$R \equiv 1 - \sum_s \frac{\omega_{ps}^2}{\omega(\omega + \omega_{cs})} \qquad L \equiv 1 - \sum_s \frac{\omega_{ps}^2}{\omega(\omega - \omega_{cs})} \qquad P \equiv 1 - \sum_s \frac{\omega_{ps}^2}{\omega^2}$$

$$S \equiv \frac{1}{2}(R + L) \qquad D \equiv \frac{1}{2}(R - L)$$

N.B.: Le plasma froid est spatialement non dispersif

Relation de dispersion



$$\mathbf{n} \times \mathbf{n} \times \mathbf{E} + \overline{\overline{\epsilon}} \cdot \mathbf{E} = 0$$

$$\begin{cases} n_x \equiv n\cos(\theta) \\ n_z \equiv n\sin(\theta) \end{cases}$$

$$\begin{pmatrix} S - n^2\cos^2(\theta) & -iD & n^2\cos(\theta)\sin(\theta) \\ iD & S - n^2 & 0 \\ n^2\cos(\theta)\sin(\theta) & 0 & P - n^2\sin^2(\theta) \end{pmatrix} \cdot \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix} = \mathbf{0}$$

- Relation de dispersion:

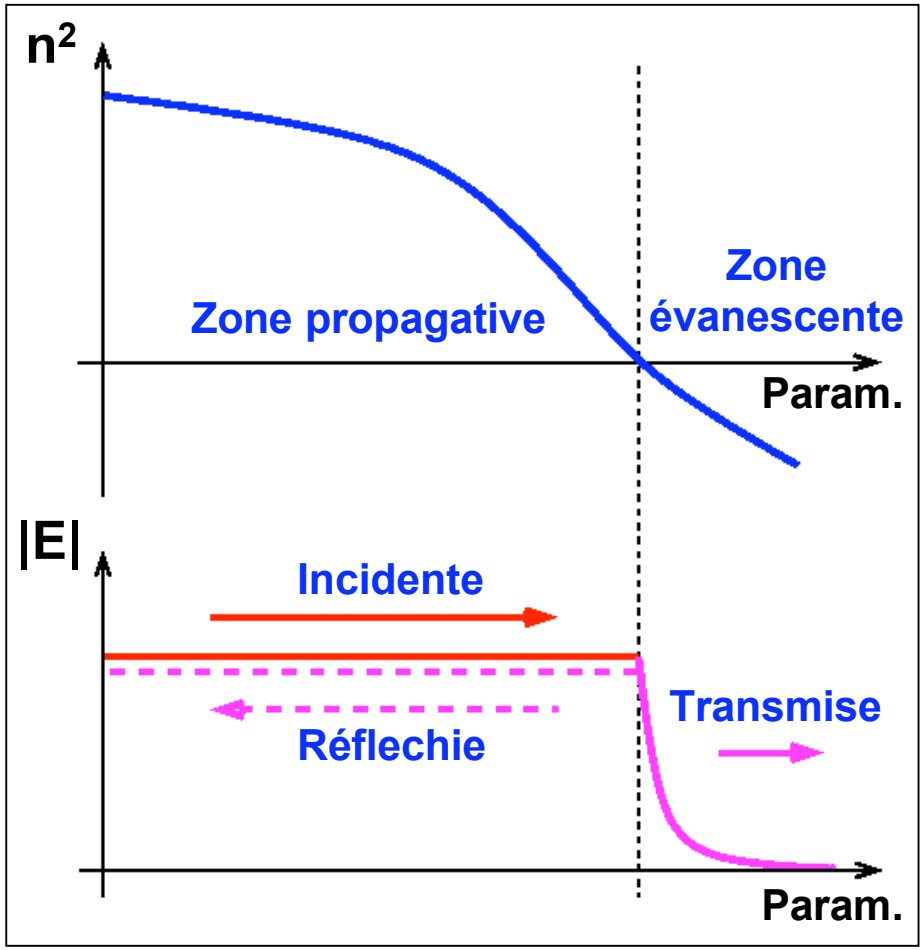
$$An^4 - Bn^2 + C = 0 \quad \longrightarrow$$ **2 modes de propagation**

$$\begin{cases} A = S\sin^2(\theta) + P\cos^2(\theta) \\ B = RL\sin^2(\theta) + PS\big(1 + \cos^2(\theta)\big) \\ C = PRL \end{cases}$$

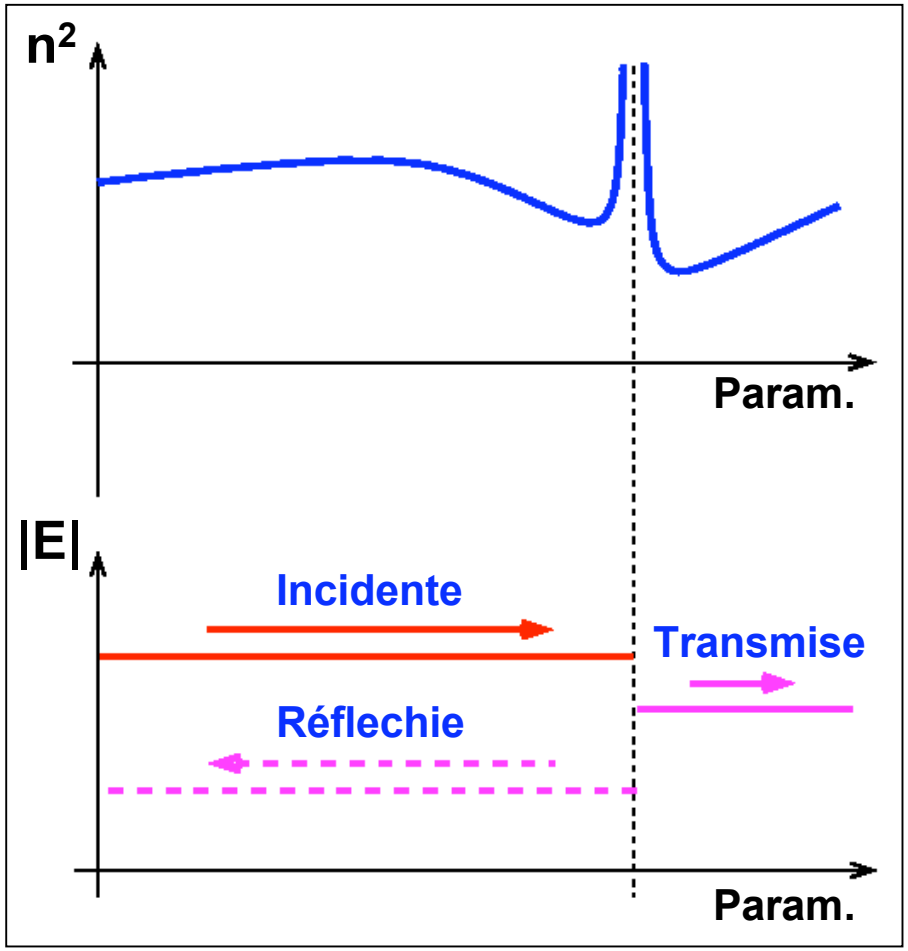
Coupures - Résonances

- Relation de dispersion: $\tan^2(\theta) = -\dfrac{P(n^2-R)(n^2-L)}{(Sn^2-RL)(n^2-P)}$

Coupure: $n^2 = 0$



Résonance: $n^2 \to \infty$



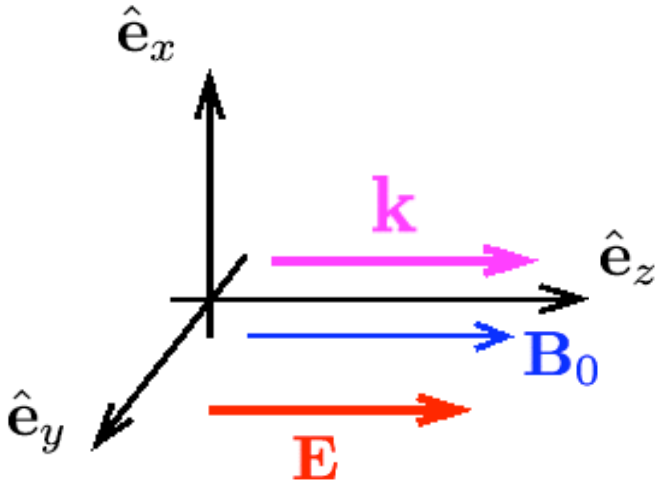
Propagation parallèle

- Equation d'onde (□=0):

$$\left\{ \begin{array}{l} (S-n^2)E_x - iDE_y = 0 \\ iDE_x + (S-n^2)E_y = 0 \\ PE_z = 0 \end{array} \right\} \begin{array}{l} \textbf{(2)} \\ \\ \textbf{(1)} \end{array}$$

1. Onde de Langmuir

- Relation de dispersion: $P = 0 \longrightarrow \omega = \sqrt{\omega_{pe}^2 + \omega_{pi}^2}$
- Vitesses de phase et de groupe non définies (phénomène non propagatif)
- Oscillation parallèle à $\mathbf{B}_0$

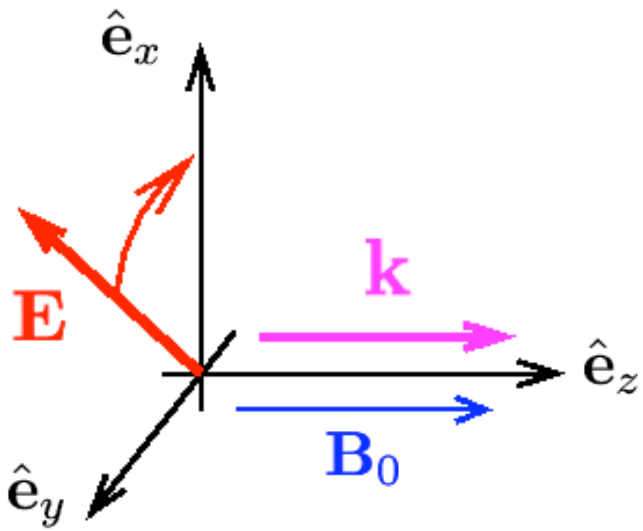


Propagation parallèle (2)

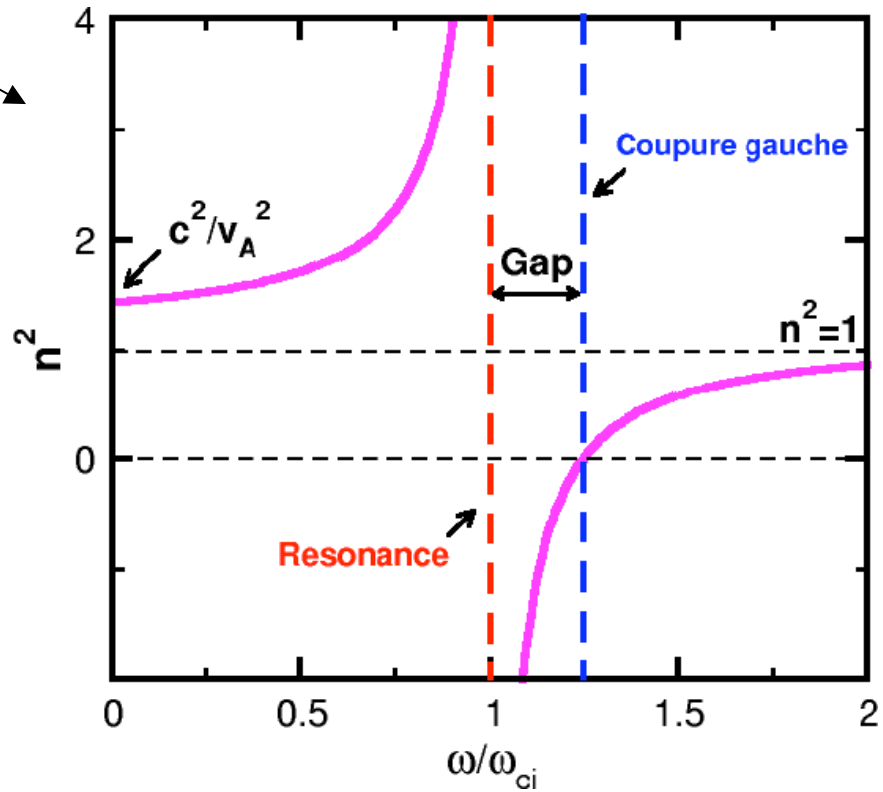
2. Les whistlers

- Equation d'onde: $\begin{cases} (L-n^2)E^+ = 0 & \textbf{(1)} \\ (R-n^2)E^- = 0 & \textbf{(2)} \end{cases}$

- **1) Whistler ionique** *(onde L)*:
  - Relation de dispersion: $n^2 = L$

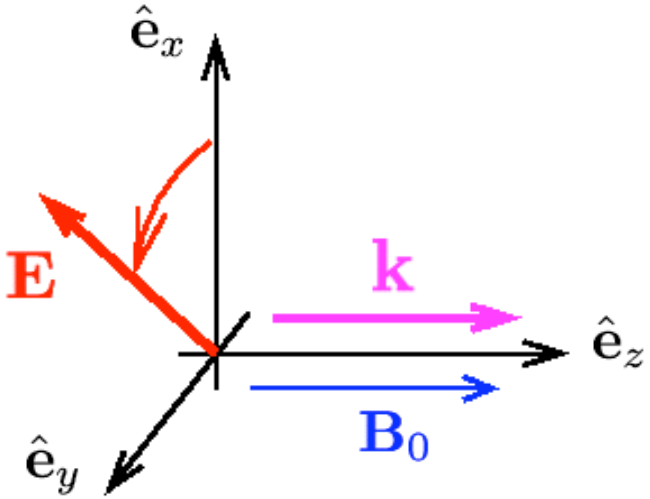


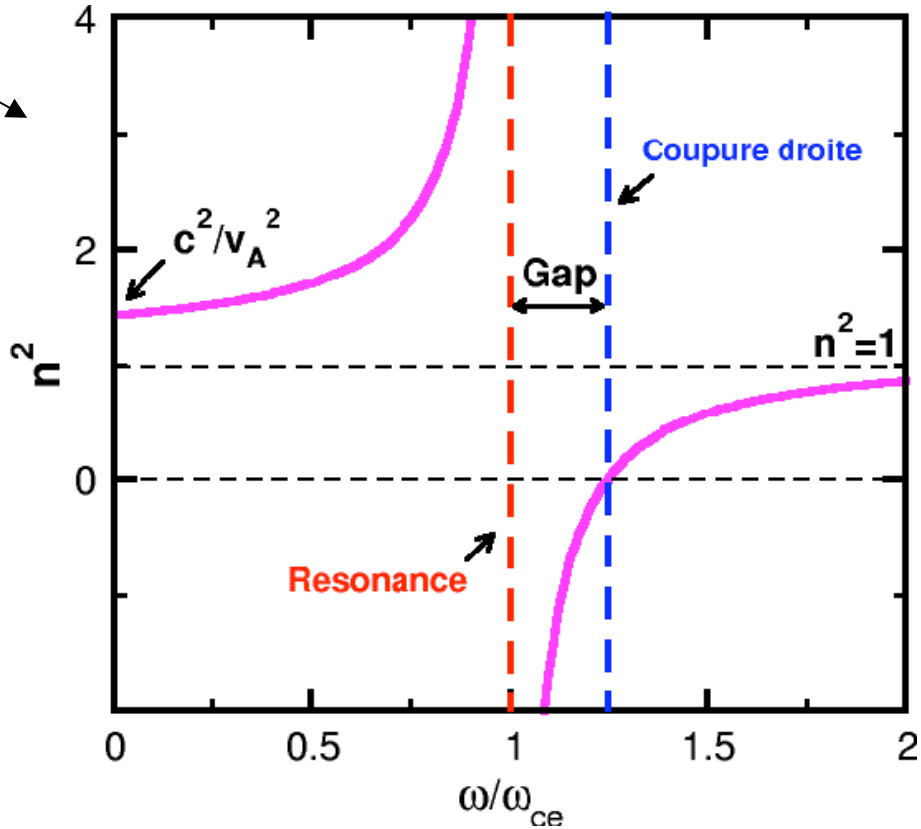
$E^- = 0 \qquad (E_x = iE_y)$



Le whistler électronique

- **2) Whistler électronique** *(onde R)*:
  - Relation de dispersion: $n^2 = R$





$$E^+ = 0 \qquad (E_x = -iE_y)$$

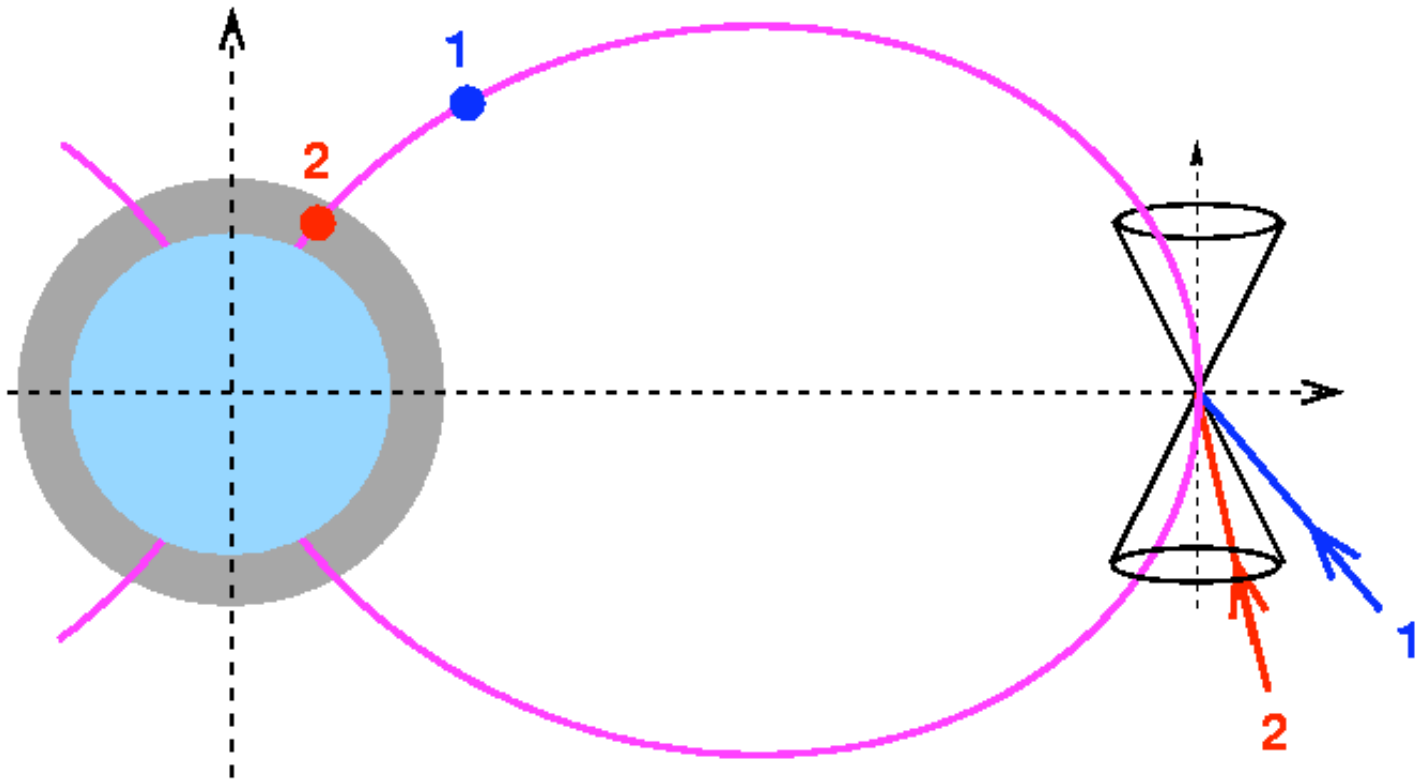
  - Dispersivité:

$$v_\phi = \frac{c}{n} \approx c\sqrt{\frac{\omega\omega_{ce}}{\omega_{pe}^2}} \propto \sqrt{\omega} \qquad v_g = \frac{\partial\omega}{\partial k} \approx 2c\sqrt{\frac{\omega\omega_{ce}}{\omega_{pe}^2}} \propto \sqrt{\omega}$$

$\longrightarrow$ **Forte dispersivité**

Whistlers dans la magnétosphère

Population électronique dans les ceintures de Van Allen

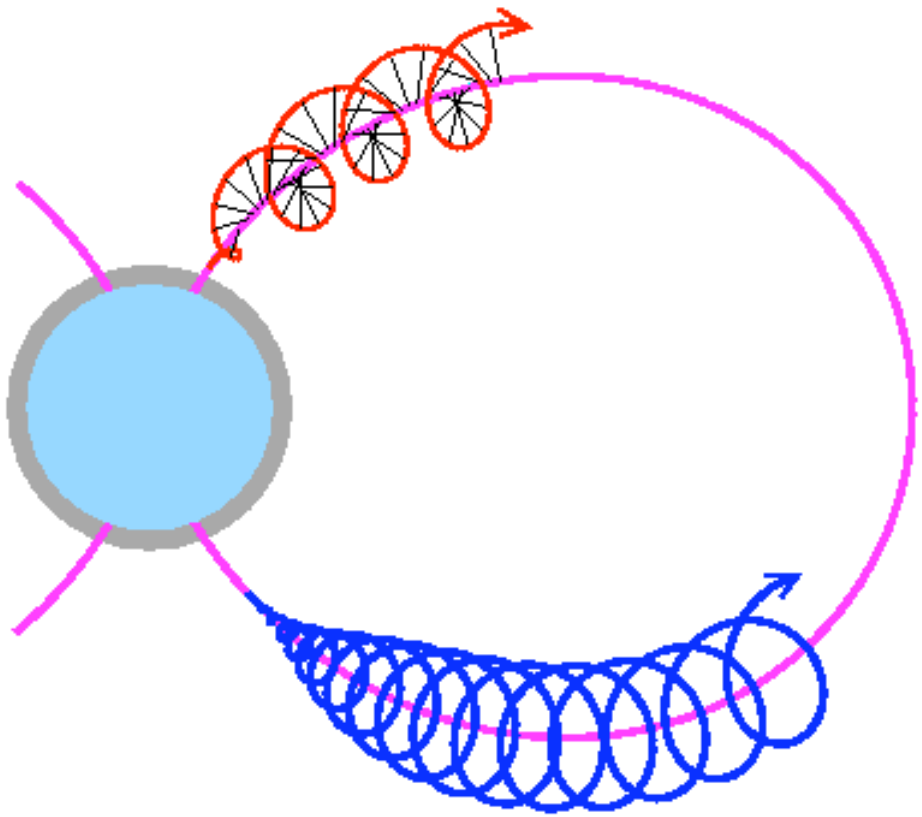


Collisions: diffusion en angle d'attaque: **(1)** □ **(2)**

→ **Pertes très faibles**

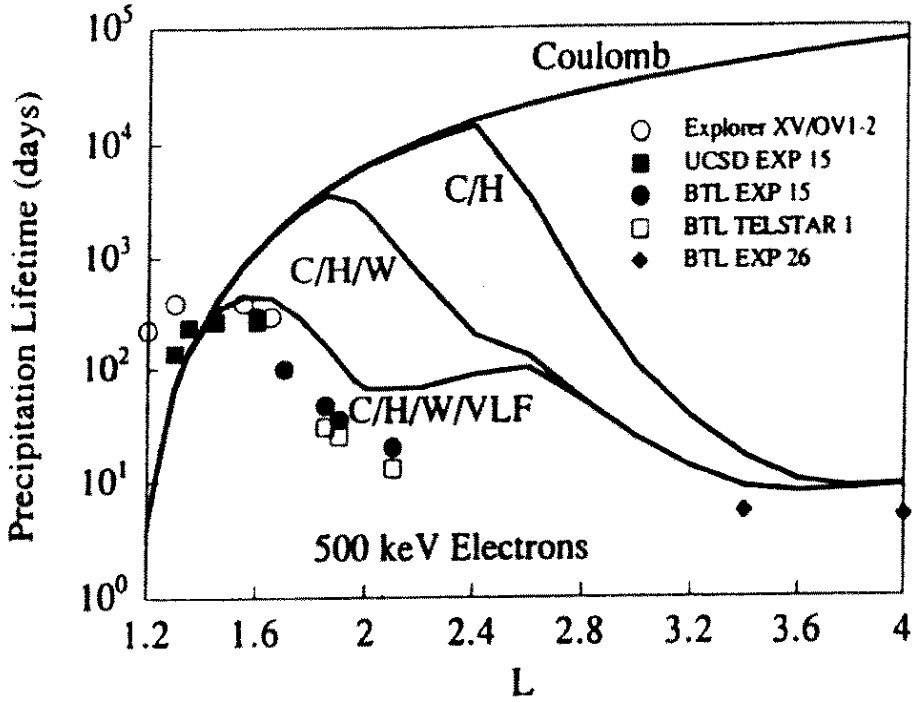
Whistlers dans la magnétosphère (2)

Whistlers guidés



Interaction avec les électrons piégés des ceintures de radiation

Temps de vie électronqiue



[*Source*: R.W. Abel and R.M. Thorne J. Geophys Res. 103 (1998) 2365]

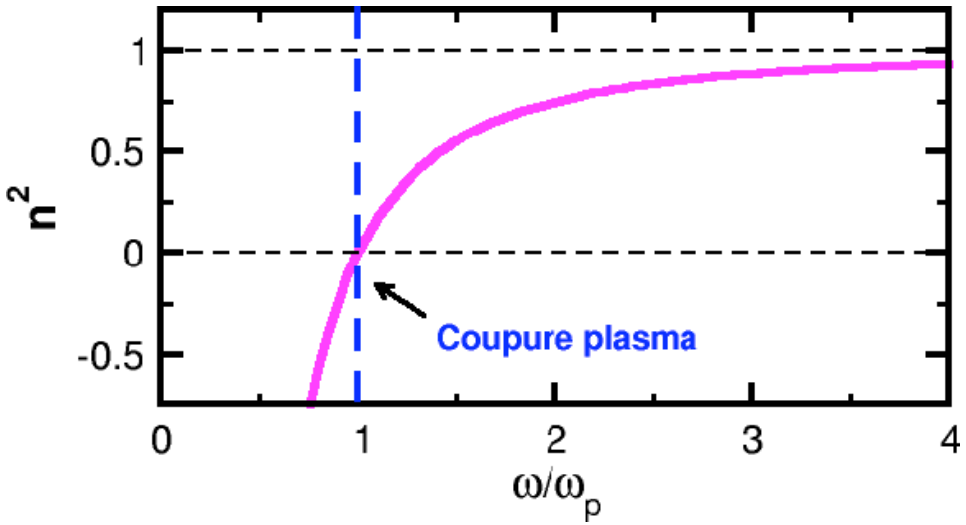
Propagation perpendiculaire

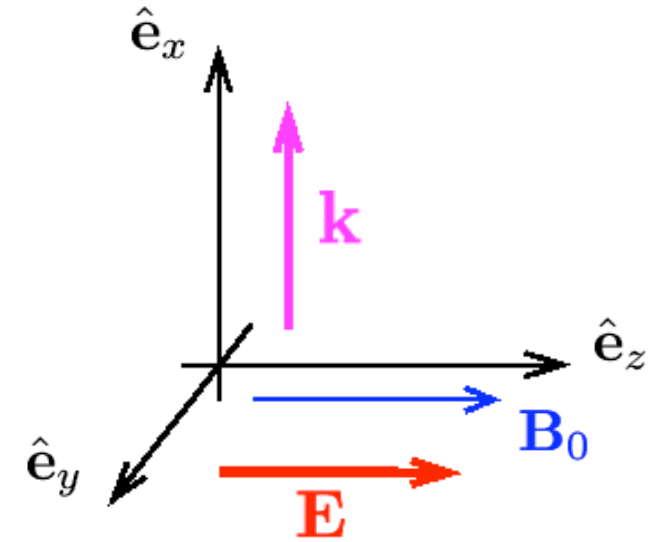
- Equation d'onde (θ=π/2):

$$\left\{\begin{array}{l} SE_x - iDE_y = 0 \\ iDE_x + (S-n^2)E_y = 0 \\ (P-n^2)E_z = 0 \end{array}\right\} \begin{array}{l} \\ \text{(2)} \\ \text{(1)} \end{array}$$

1. Onde ordinaire (O):

- Relation de dispersion: $n^2 = P$
- Propagation pour $\omega^2 > \omega_{pe}^2 + \omega_{pi}^2$



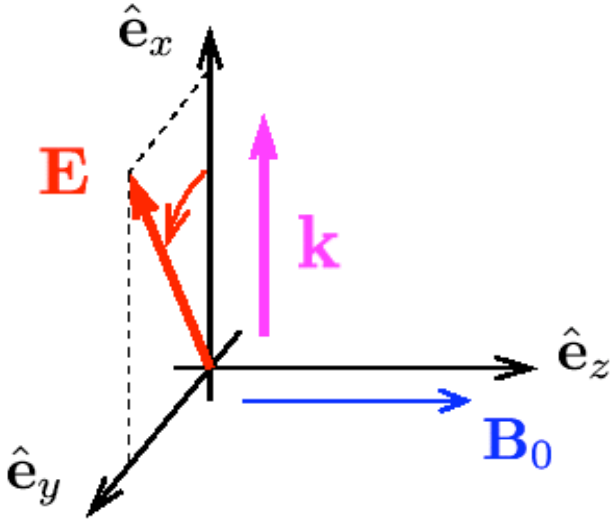


$E_x = E_y = 0 \qquad (\mathbf{E} \parallel \mathbf{B}_0)$

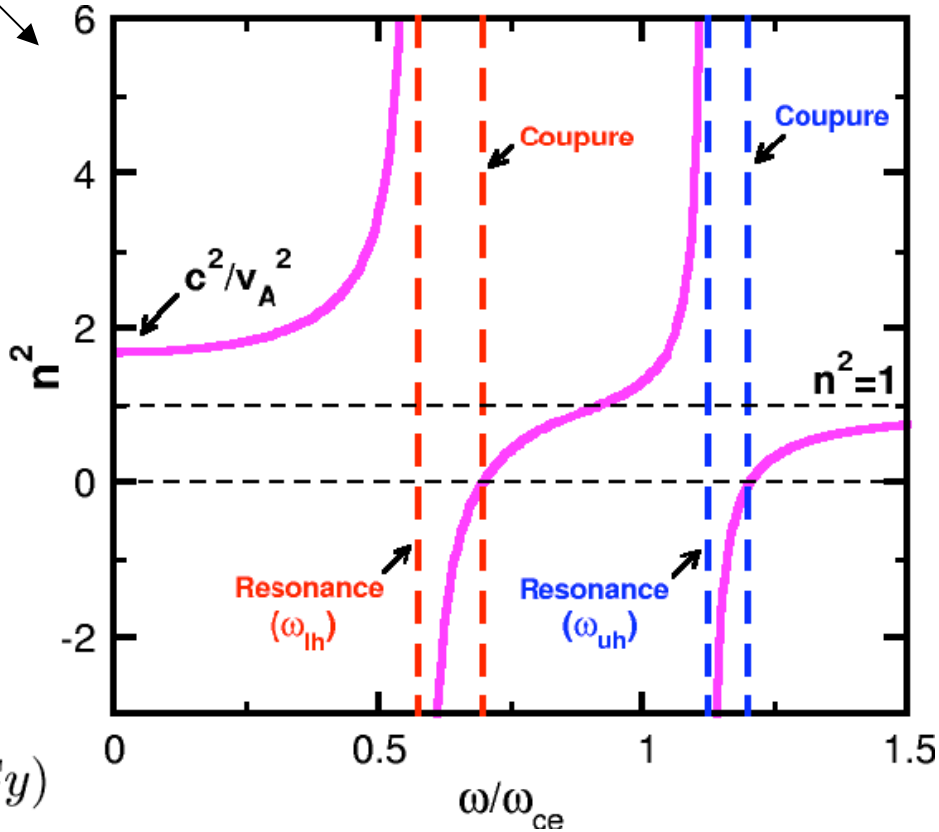
Propagation perpendiculaire (2)

2. Onde extraordinaire (X):

- Relation de dispersion: $n^2 = RL/S$



$$\frac{E_x}{E_y} = i\frac{R-L}{R+L}$$



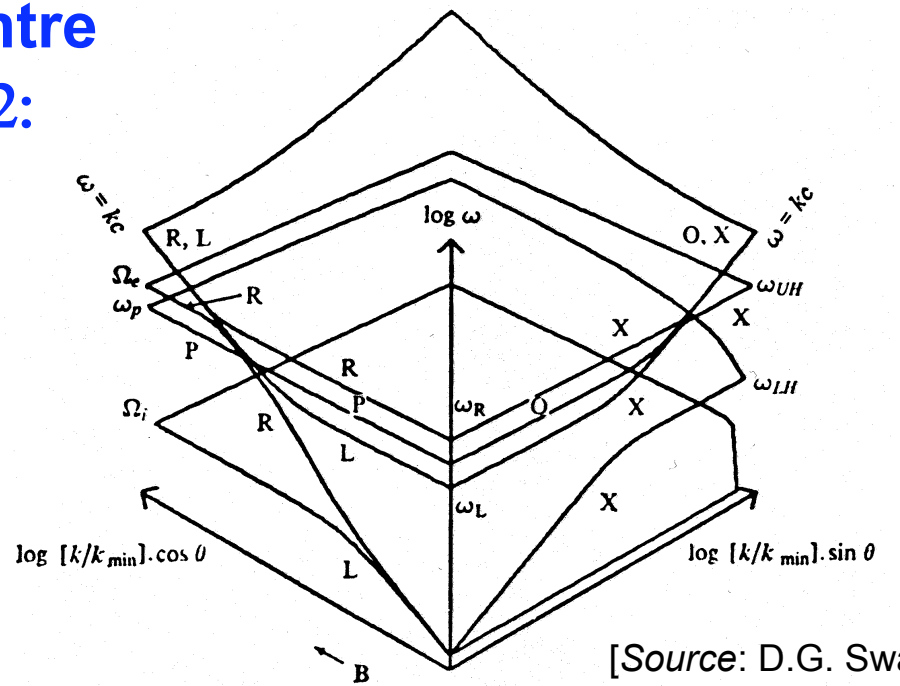
- Phénomène d'écrantage:

$$\omega \to \omega_{ci} : \; E_+ \to 0 \quad (E_x \to -iEy)$$

$$\omega \to \omega_{ce} : \; E_- \to 0 \quad (E_x \to iEy)$$

Propagation à angle quelconque

- **Continuité entre $\theta=0$ et $\theta=\pi/2$:**



[*Source*: D.G. Swanson, Plasma Waves (A.P., San Diego, 1989)]

- **Ex: Ondes MHD: ($\omega \ll \Omega_{ci}$)**

$$\begin{cases} \left(-n^2\cos(\theta)+1+\dfrac{c^2}{v_a^2}\right)\cdot E_x = 0 & \textbf{(Onde d'Alvén torsionnelle)} \\ \left(-n^2+1+\dfrac{c^2}{v_a^2}\right)\cdot E_y = 0 & \textbf{(Onde d'Alvén compressionnelle)} \\ P\cdot E_z = 0 \end{cases}$$

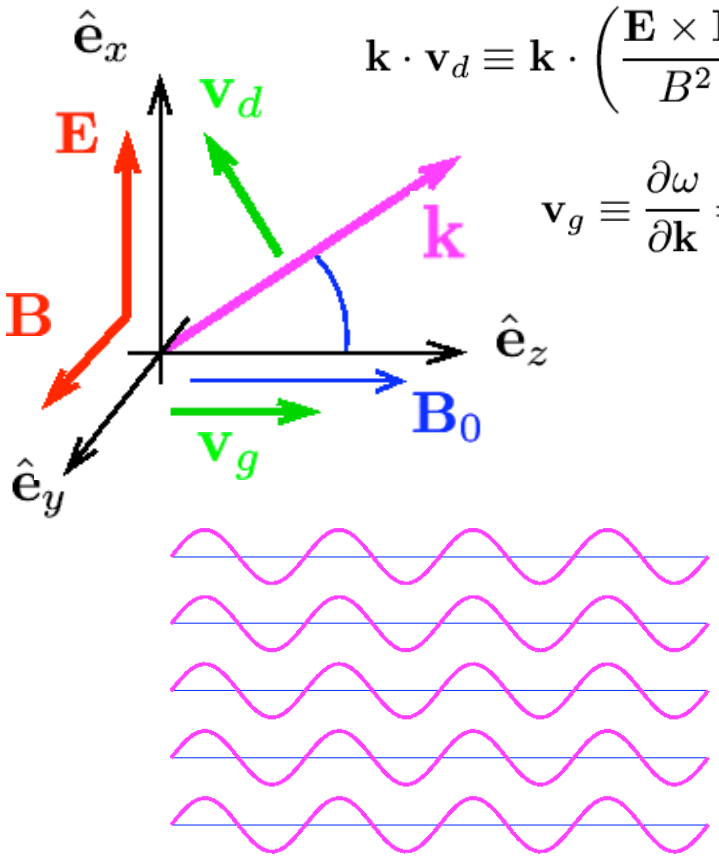
Basses fréquences: ondes d'Alfvén

Onde d'Alvén torsionnelle

$$n^2\cos^2(\theta) = 1 + \frac{c^2}{v_a^2}$$

$$\mathbf{k}\cdot\mathbf{v}_d \equiv \mathbf{k}\cdot\left(\frac{\mathbf{E}\times\mathbf{B}}{B^2}\right) = 0$$

$$\mathbf{v}_g \equiv \frac{\partial\omega}{\partial\mathbf{k}} = v_g\hat{\mathbf{e}}_z$$



Torsion des lignes de champ

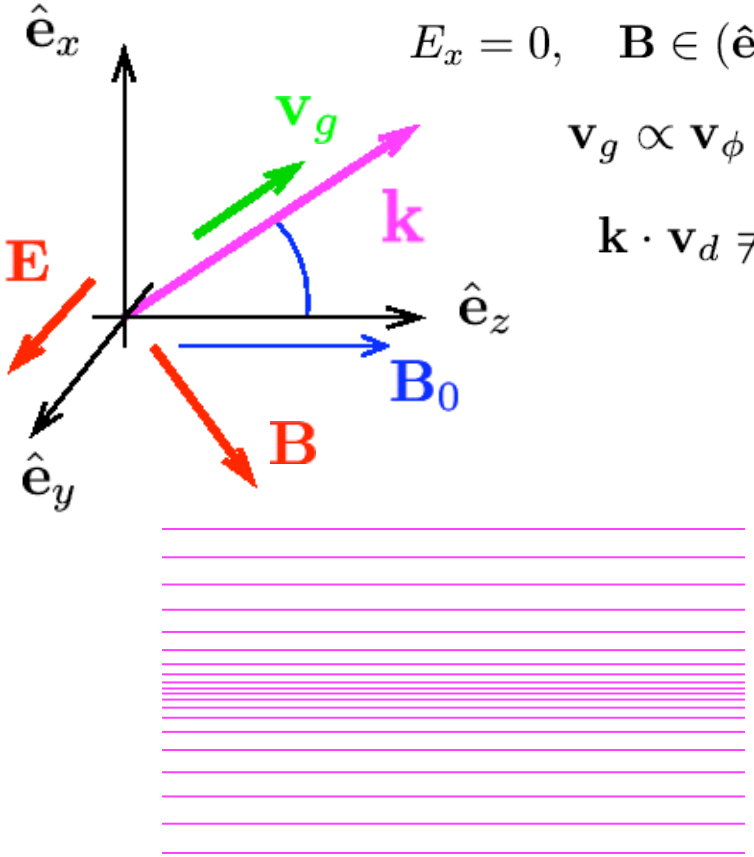
Onde d'Alvén compressionnelle

$$n^2 = 1 + \frac{c^2}{v_a^2}$$

$$E_x = 0, \quad \mathbf{B}\in(\hat{\mathbf{e}}_x,\hat{\mathbf{e}}_z)$$

$$\mathbf{v}_g \propto \mathbf{v}_\phi \propto \mathbf{k}$$

$$\mathbf{k}\cdot\mathbf{v}_d \neq 0$$



Compression des lignes de champ

Plan de vol: le diagramme CMA

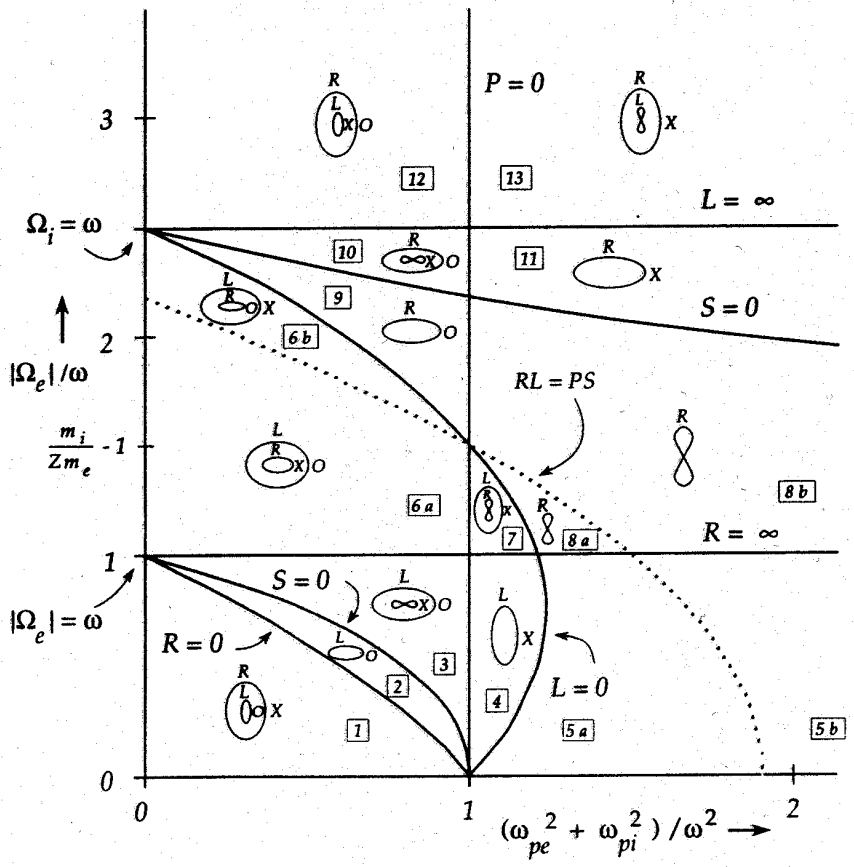
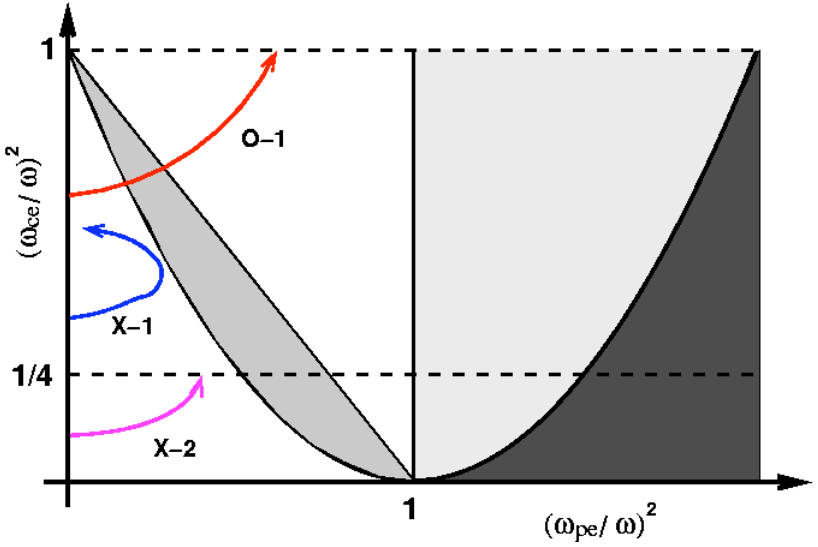


Fig. 2-1. CMA diagram for a two-component plasma. The ion-to-electron mass ratio is chosen to be 2.5. Bounding surfaces appear as lines in this two-dimensional parameter space. Cross sections of wave-normal surfaces are sketched and labeled for each region. For these sketches the direction of the magnetic field is vertical. The small mass ratio can be misleading here: the $L=0$ line intersects $P=0$ at $\Omega_i/\omega_i=1-(Zm_e/m_i)$.

Référence: Stix (1992)

Ex: propagation de rayons dans un tokamak.



L'hypothèse d'homogénéité

- Ondes planes: $(\mathbf{E},\mathbf{B}) \equiv (\mathbf{E}_0,\mathbf{B}_0)\cdot\exp\left(i(\mathbf{k}\cdot\mathbf{r}-\omega t)\right)$

  → Valable en plasma **stationnaire** et **homogène**

  → $\Delta t \gg \dfrac{2\pi}{\omega}$ **[x]**

  → $\Delta r \gg \lambda$ **[o]**

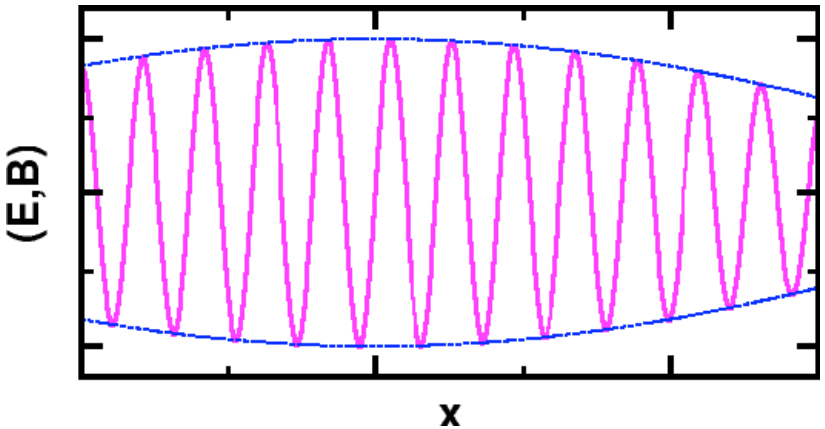
- Description de l'absorption: $\omega \to \omega_r - i\gamma, \quad \mathbf{k} \to \mathbf{k}_r + i\mathbf{k}_i$

  → $e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)} = e^{i(\mathbf{k}_r\cdot\mathbf{r}-\omega_r t)}\cdot\exp(-\mathbf{k}_i\cdot\mathbf{r})\cdot\exp(-\gamma t)$

- Hypothèse WKB: $(\mathbf{E},\mathbf{B}) \equiv \left(\mathbf{E}_0(\mathbf{r},t),\mathbf{B}_0(\mathbf{r},t)\right)\cdot\exp(i\psi)$

  Fonction eikonale

$$\begin{cases} \nabla\psi \equiv \mathbf{k}(\mathbf{r},t) \\ \dfrac{\partial\psi}{\partial t} \equiv -\omega(\mathbf{r},t) \end{cases}$$



→ Séparation des échelles de temps

- Rapide: □
- Lente: $\mathbf{E}_0$, $\mathbf{B}_0$, $\mathbf{k}$, □

→ Applicable en dehors de: coupures, résonances, □#□r

Transferts d'énergie

- **Théorème de Poynting**: $\nabla \cdot (\mathbf{E} \times \mathbf{H}) + \left[\mathbf{B} \cdot \frac{\partial \mathbf{H}}{\partial t} + \mathbf{E} \cdot \frac{\partial \mathbf{D}}{\partial t} \right] = -\mathbf{j}_s \cdot \mathbf{E}$

  (1) **(2)**

- **Milieu ordinaire** (solide…):

  (1) Déplacement atomes, noyaux…: courant de polarisation (réversible)

  (2) Déplacement des charges libres: courant de conduction (irréversible)

  Géneralement: $\square_{mfp} << \square$ ⟶ Equilibre quasi-immédiat (Ex: effet joule)

- **Plasma**:
  - Pas de séparation claire entre courant de conduction / de polarisation
  - Toutes les charges sont libres, mais contribuent à la polarisation ($\square_{mfp} >> \square$)

    ⟶ Pas d'équilibre **sur une période** de l'onde

    ⟶ Pas d'équivalent de la densité d'énergie électromagnétique, du vecteur de Poynting instantané **sur une période**.

    ⟶ Nécessité d'utiliser des **quantités moyennées**

Transferts d'énergie (2)

- **Deux conditions:**
  - Milieu faiblement inhomogène: $\mathbf{Q} \equiv \mathbf{Q}_0 \times \exp(i\psi)$

    $\psi \equiv \psi_r + i\psi_i \longrightarrow \begin{cases} \mathbf{k}_i \equiv \boldsymbol{\nabla}\psi_i \\ \omega_i \equiv -\dfrac{\partial \psi_i}{\partial t} \end{cases}$
  - On travaille sur des quantités moyennées.

- **Théorème de Poynting:**

$$W \equiv \frac{1}{2}\left[\frac{B_0^2}{\mu_0} + \mathbf{E}_0^* \cdot \frac{\partial}{\partial \omega}(\omega \overline{\overline{\epsilon}}^{h}) \cdot \mathbf{E}_0\right]$$

[Energie électromagnétique]

$$\mathbf{T} \equiv -\frac{\omega}{2}\mathbf{E}_0^* \cdot \frac{\partial \overline{\overline{\epsilon}}^{h}}{\partial \mathbf{k}} \cdot \mathbf{E}_0$$

[Flux cinétique]

$$\boxed{\frac{\partial W}{\partial t} + \boldsymbol{\nabla} \cdot (\mathbf{S}_0 + \mathbf{T}) = -P_{abs}}$$

$$\mathbf{S}_0 \equiv \frac{1}{\mu_0}\Re(\mathbf{E}_0^* \times \mathbf{B}_0)$$

[Flux de Poynting]

$$P_{abs} \equiv \omega \mathbf{E}_0^* \cdot \overline{\overline{\epsilon}}^{a} \cdot \mathbf{E}_0$$

[Puissance absorbée]

- **Tenseur diélectrique:** $\overline{\overline{\epsilon}} \equiv \overline{\overline{\epsilon}}^{h} + i\overline{\overline{\epsilon}}^{a}$

Absorption collisionnelle

- **Plasma froid**: $\overline{\overline{\epsilon}}^{a} = 0 \quad \rightarrow \quad P_{abs} = 0$

$$\frac{\partial \overline{\overline{\epsilon}}^{h}}{\partial \mathbf{k}} = \frac{\partial \overline{\overline{\epsilon}}}{\partial \mathbf{k}} = 0 \quad \rightarrow \quad \mathbf{T} = 0$$

Pas d'absorption,
Pas d'échange d'énergie

- **Terme de friction (Crook):** $\dfrac{d\mathbf{v}_s}{dt} = \dfrac{q_s}{m_s}(\mathbf{E} + \mathbf{v}_s \times \mathbf{B}) - \nu_s \mathbf{v}_s$

- **Tenseur diélectrique:**

$$L \equiv 1 - \sum_s \frac{\omega_{ps}^2}{\omega(\omega - \omega_{cs})} \quad \rightarrow \quad 1 - \sum_s \frac{\omega_{ps}^2}{\omega(\omega - \omega_{cs} + i\nu_s)}$$

$$R \equiv 1 - \sum_s \frac{\omega_{ps}^2}{\omega(\omega + \omega_{cs})} \quad \rightarrow \quad 1 - \sum_s \frac{\omega_{ps}^2}{\omega(\omega + \omega_{cs} + i\nu_s)}$$

$$P \equiv 1 - \sum_s \frac{\omega_{ps}^2}{\omega^2} \quad \rightarrow \quad 1 - \sum_s \frac{\omega_{ps}^2}{\omega(\omega + i\nu_s)}$$

$$\overline{\overline{\epsilon}}^{a} \neq \overline{\overline{\mathbf{0}}}$$

- **Puissance absorbée:**

$$P_{abs} = \sum_s \nu_s \frac{\omega_{ps}^2}{\omega^2} \left[A_1 |E^+|^2 + A_2 |E^-|^2 + A_3 |E_z|^2 \right] \neq 0$$

$\longrightarrow$ **Absorption proportionnelle au coefficient de friction**

Corrections thermiques

- **Conservation de l'impulsion:**

$$n_s m_s \frac{d\mathbf{v}_s}{dt} = n_s m_s \left(\frac{\partial \mathbf{v}_s}{\partial t} + (\mathbf{v}_s \cdot \boldsymbol{\nabla})\mathbf{v}_s \right) = n_s q_s (\mathbf{E} + \mathbf{v}_s \times \mathbf{B}) - \boldsymbol{\nabla} \cdot \overline{\overline{\boldsymbol{\Phi}}}_s$$

- **Plasma froid:** $\overline{\overline{\boldsymbol{\Phi}}}_s = \overline{\overline{\mathbf{0}}}$

$$\longrightarrow \quad -i\omega m_s \mathbf{v}_s = q_s (\mathbf{E} + \mathbf{v}_s \times \mathbf{B}_0)$$

- **Effets thermiques:** $\overline{\overline{\boldsymbol{\Phi}}}_s = p_s \times \overline{\overline{\mathbf{1}}}$

$$\longrightarrow \quad -i\omega m_s \mathbf{v}_s = q_s (\mathbf{E} + \mathbf{v}_s \times \mathbf{B}_0) - \frac{1}{n_s} \boldsymbol{\nabla} p_s$$

- **Fermeture:** $p_s = c_s^2 n_s m_s \,, \quad c_s^2 \equiv \gamma_s \dfrac{k_b T_s}{m_s}$

nul pour perturbations transverses

$$\longrightarrow \quad \mathbf{v}_s = i \frac{q_s}{m_s \omega} (\mathbf{E} + \mathbf{v}_s \times \mathbf{B}_0) + \frac{c_s^2}{\omega^2} (\mathbf{k} \cdot \mathbf{v}_s) \cdot \mathbf{k}$$

- **Nouveau tenseur diélectrique:** $\overline{\overline{\boldsymbol{\epsilon}}} = \overline{\overline{\boldsymbol{\epsilon}}}(\mathbf{k}) = \overline{\overline{\boldsymbol{\epsilon}}}_{froid} + i \left(\dfrac{kc}{\omega} \right) \cdot \overline{\overline{\boldsymbol{\epsilon}}}^{(1)} - \left(\dfrac{kc}{\omega} \right)^2 \cdot \overline{\overline{\boldsymbol{\epsilon}}}^{(2)}$

Caractère dispersif

Corrections thermiques: ondes HF

1. Haute fréquence: mode extraordinaire

- Plasma froid (électrons seuls):

$$n_{X,c}^2 = \frac{(\omega^2 - \omega_{pe}^2)\cdot(\omega^2 - \omega_e^2) - \omega_{ce}^2\omega_{pe}^2}{\omega^2\cdot(\omega^2 - \omega_e^2)}$$

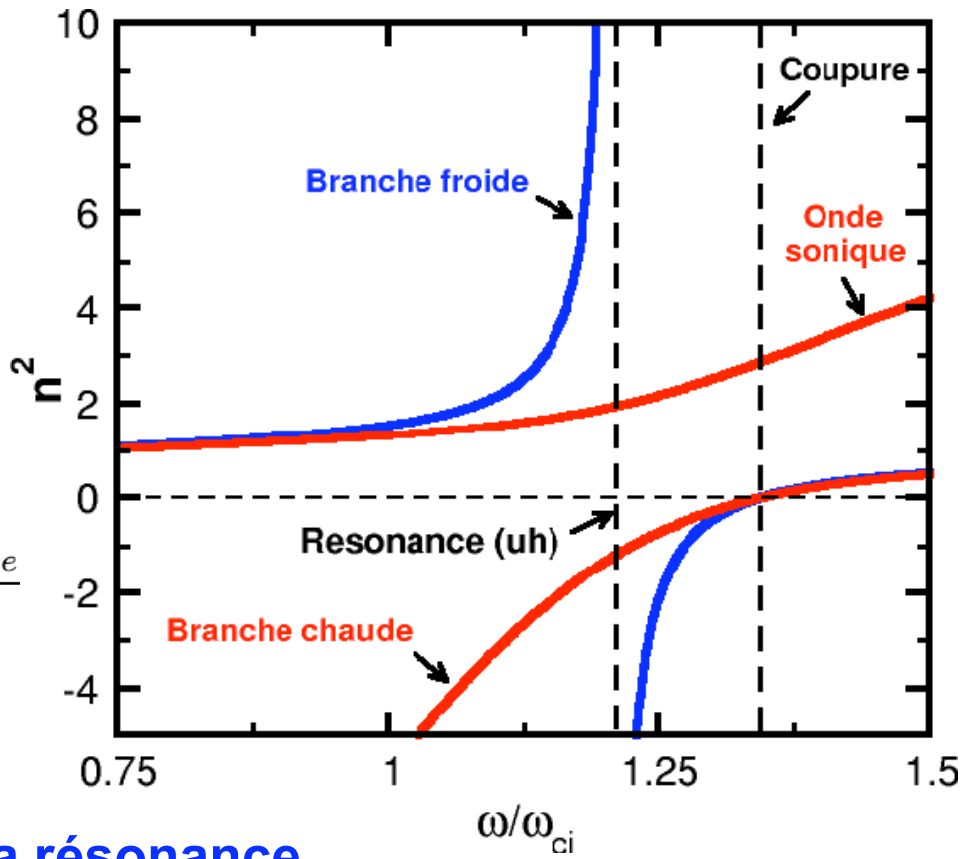
$$\omega_e^2 \equiv \omega_{pe}^2 + \omega_{ce}^2$$

- Avec corrections thermiques:

$$n_{X,h}^2 = \frac{(\omega^2 - \omega_{pe}^2)\cdot(\omega^2 - \omega_e^2 - k^2c_e^2) - \omega_{ce}^2\omega_{pe}^2}{\omega^2\cdot(\omega^2 - \omega_e^2 - k^2c_e^2)}$$

Vitesse thermique

Relation de dispersion



→ **Disparition de la résonance**

→ **Apparition d'une nouvelle branche.**

Corrections thermiques: ondes MHD

2. Basse fréquence: ondes MHD

- **Plasma froid: 2 modes**
  - Onde d'Alfvén torsionnelle:

$$\frac{\omega}{\omega_{ci}} = \left(\frac{kc_s}{\omega_{ci}}\right) \cdot \left(\frac{v_a}{c_s}\right) \cdot \cos(\theta)$$

  - Onde d'Alfvén compressionnelle:

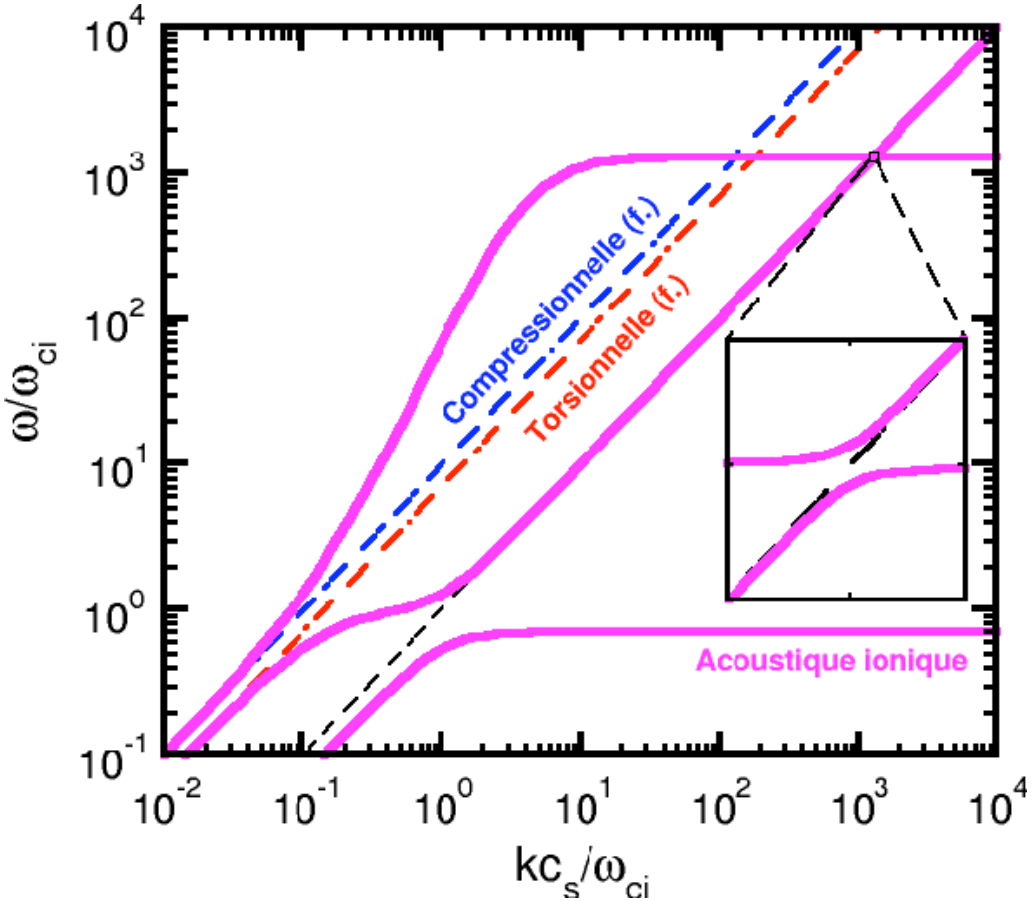
$$\frac{\omega}{\omega_{ci}} = \left(\frac{kc_s}{\omega_{ci}}\right) \cdot \left(\frac{v_a}{c_s}\right)$$

- **Plasma chaud:**

Relation de dispersion
du 3ème ordre en n^2

→ **Nouvelle branche.**

Diagramme de Stringer



Ondes dans un plasma cinétique

Effets cinétiques

- **Ondes: effets micrososcopiques non considérés dans le modèle fluide**
  - Absorption non collisionnelle
  - Effets de rayon de Larmor fini
  - Distributions non-Maxwelliennes

$\longrightarrow$ **Nécessité de décrire les particules individuellement**

- **Traitement général**
  - Equation de Vlasov:

$$\frac{\partial f_s}{\partial t} + \mathbf{v} \cdot \frac{\partial f_s}{\partial \mathbf{r}} + \frac{q_s}{m_s}(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \cdot \frac{\partial f_s}{\partial \mathbf{v}} = 0 \quad \longrightarrow \quad \left.\frac{df_s}{dt}\right|_{\mathbf{R}} = 0$$

Trajectoires non perturbées

  - Trajectoires non perturbées: $\frac{d\mathbf{r}}{dt} = \mathbf{v}, \quad \frac{d\mathbf{v}}{dt} = \frac{q_s}{m_s}\mathbf{v} \times \mathbf{B}_0$

  - Perturbation (1er ordre) de la fonction de distribution:

Distribution d'équilibre

$$\tilde{f}_s(\mathbf{r}, \mathbf{v}, t) = -\frac{q_s}{m_s} \int_{-\infty}^{t} dt' \left[\mathbf{E}(\mathbf{r}', t') + \mathbf{v}' \times \mathbf{B}(\mathbf{r}', t')\right] \cdot \frac{\partial F_s}{\partial \mathbf{v}'}$$

$\longrightarrow$ **Nouveau tenseur diélectrique**

Absorption non collisionnelle

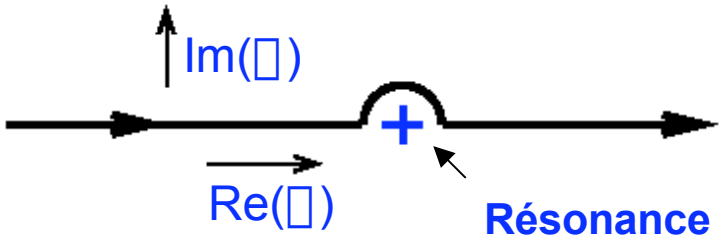
- **Nouveau tenseur diélectrique:** $\bar{\bar{\epsilon}} \equiv \bar{\bar{1}} + \sum_s \bar{\bar{\chi}}_s$

- **Susceptibilité:**

$$\bar{\bar{\chi}}_s \equiv \frac{\omega_{ps}^2}{\omega\omega_{cs}} \sum_{n=-\infty}^{\infty} \int_0^{\infty} dv_\perp 2\pi v_\perp \int_{-\infty}^{\infty} dv_\parallel \left(\frac{\omega_{cs}}{\omega - n\omega_{cs} - k_\parallel v_\parallel} \cdot \bar{\bar{\mathbf{S}}}_n \right)$$

$$\bar{\bar{\mathbf{S}}}_n \equiv \bar{\bar{\mathbf{S}}}_n \left(\frac{\partial F_s}{\partial v_\perp}, \frac{\partial F_s}{\partial v_\parallel}, k_\perp, k_\parallel \right)$$

- **Dénominateur résonant:**



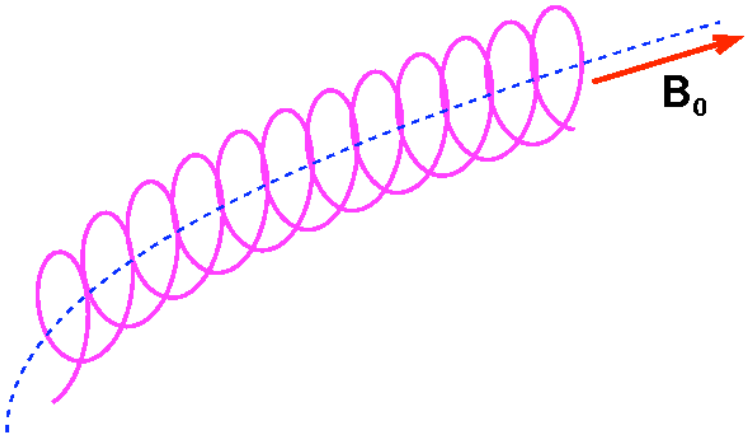
$$\lim_{\nu \to 0} \frac{1}{\omega - n\omega_{cs} - k_\parallel v_\parallel + i\nu} = \mathcal{P}\left(\frac{1}{\omega - n\omega_{cs} - k_\parallel v_\parallel} \right) - i\pi\delta(\omega - n\omega_{cs} - k_\parallel v_\parallel)$$

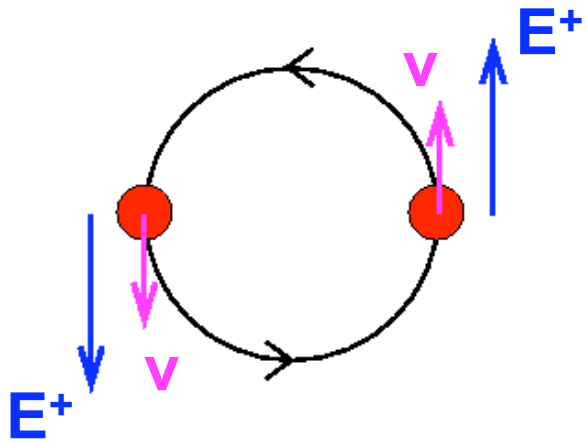
- Effet Döppler ($k_{//}v_{//}$)
- Absorption cyclotronique **(n ≠ 0)**
- Absorption Landau, TTMP **(n = 0)**

Processus d'absorption non collisionnelle

Absorption cyclotronique

- **Interaction avec le mouvement de gyration:** $\omega = n\omega_{cs} + k_{//}v_{//}$

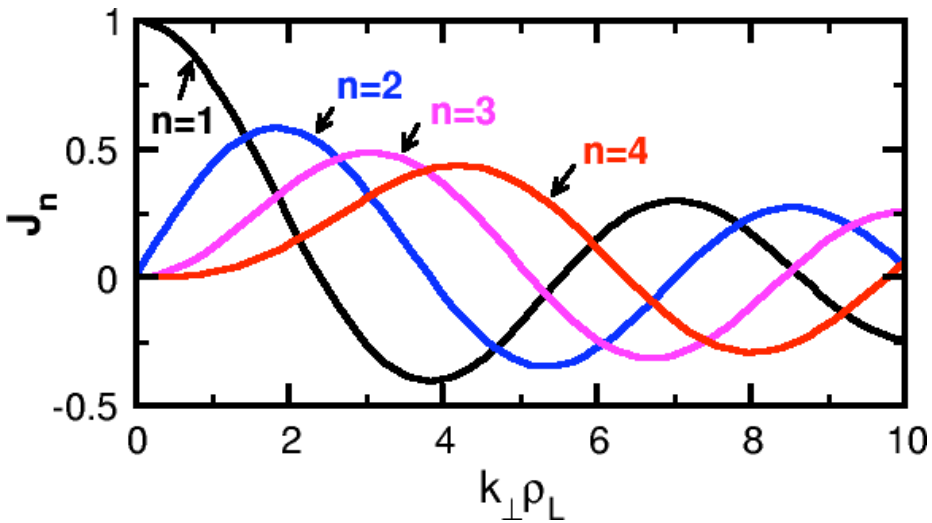




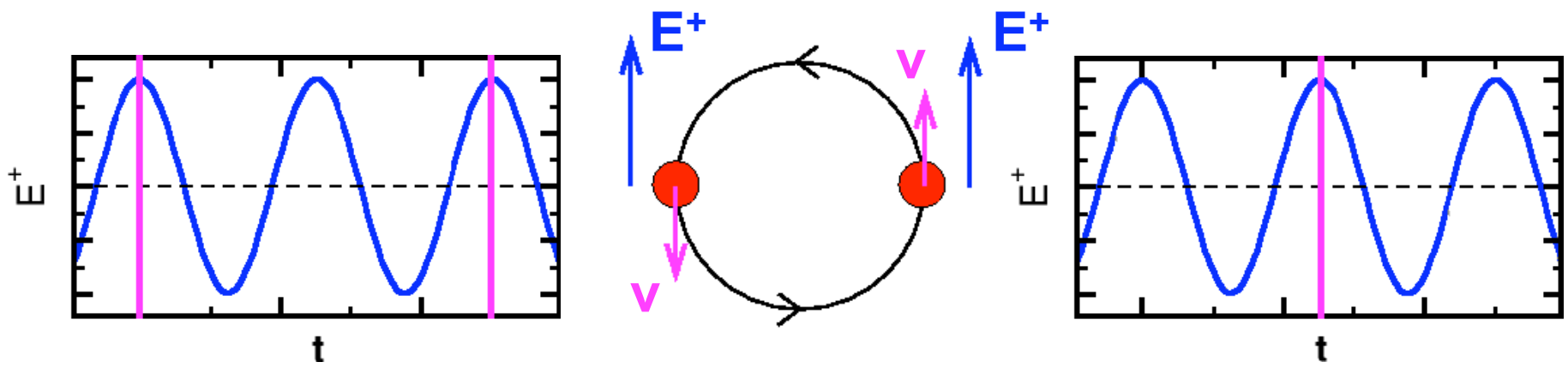
- **Accéleration de la particule:**

$$\frac{dW_\perp}{dt} \equiv \frac{d}{dt}\left(\frac{m_s v_\perp^2}{2}\right) = q_s \mathbf{v}_\perp \cdot \mathbf{E}$$

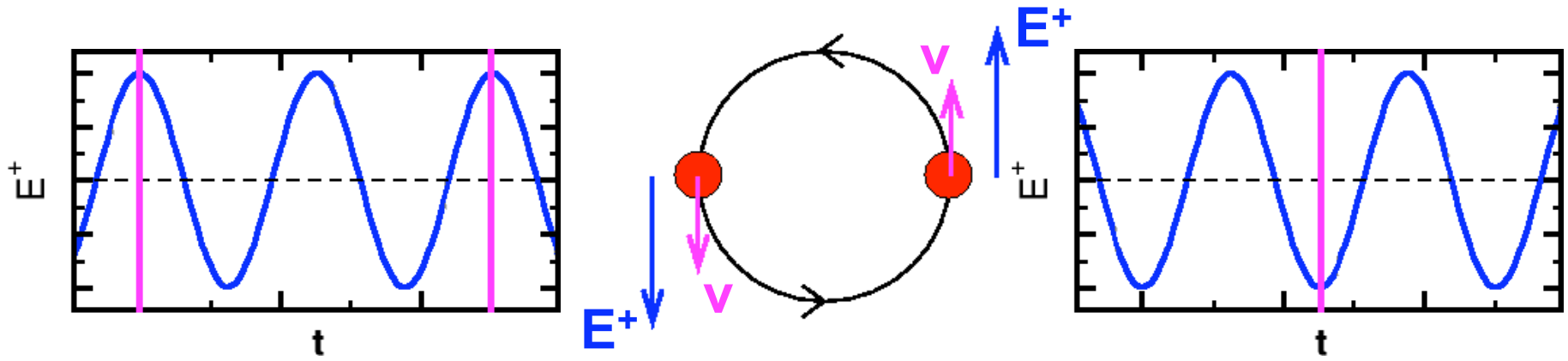
$$\longrightarrow \quad \frac{dW_\perp}{dt} \propto J_n(k_\perp \rho_L)$$



Absorption au premier harmonique



$E=E_0\exp(i\omega t)$ ⟶ **Pas de transfert d'énergie**



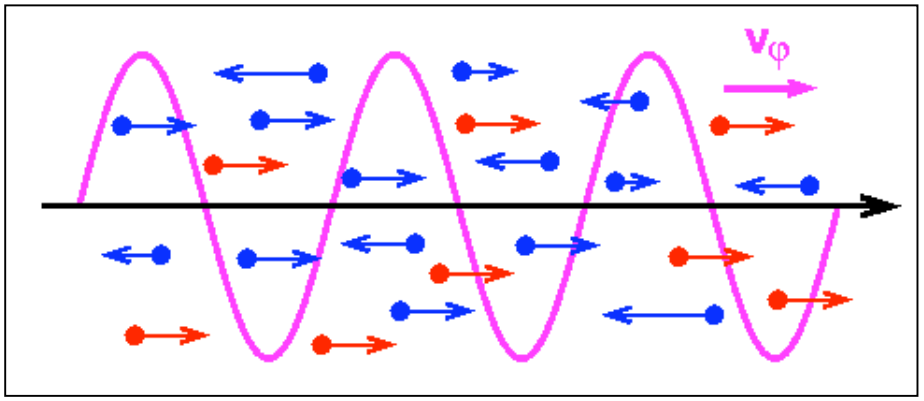
$E=E_0\exp(i(k_x x-\omega t))$, $k_x=\omega/2\pi_L$ ⟶ **Accélération de la particule**

Absorption Landau, TTMP

- **Absorption Landau:** $\omega = k_{//} v_{//}$
  - **Champ oscillant:** $\mathbf{E = E_0 \cos(\omega t - k_{//} z)}$

    ⟶ Interaction avec les particules à $v_{//} = \omega / k_{//}$ (à $\delta v_{//}$ près)

    ⟶ Emission / Absorption

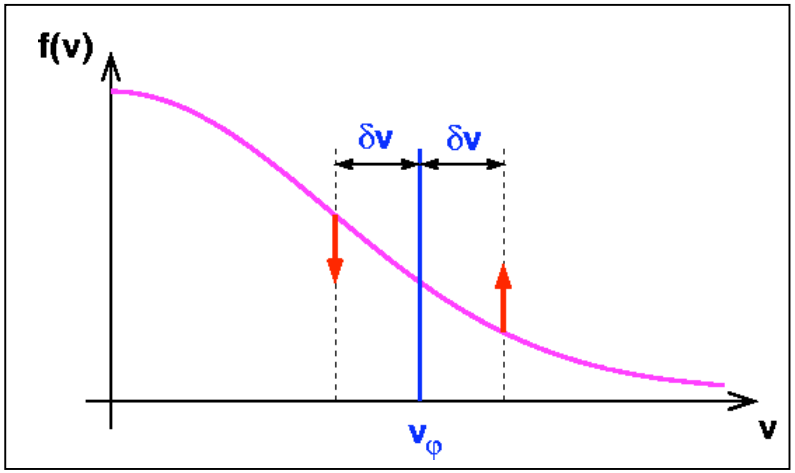


- **Modification de l'énergie parallèle ($W_{//}$):**

$$\frac{dW_{\|}}{dt} = W_{\|} \times \pi \cdot \frac{\omega \omega_p^2}{k_{\|} |k_{\|}|} \cdot \left. \frac{\partial F_s}{\partial v_{\|}} \right|_{v_{\|} = \omega / k_{\|}}$$

⟶ Sensibilité au détails de F_s

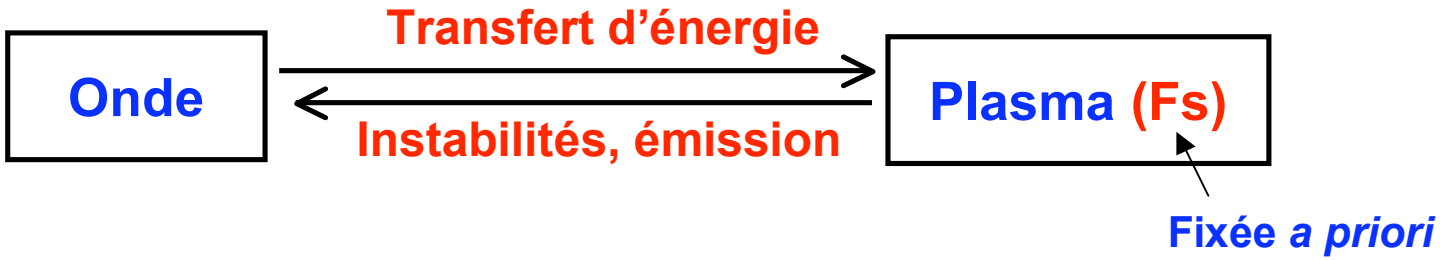
⟶ Cadre de la théorie linéaire mal adapté



- **Transit Time Magnetic Pumping:** $\omega = k_{//} v_{//}$

⟶ Similaire, mais avec $\mathbf{q_s E}$ □ -□□□

Limites d'une description linéaire



Ex: fonction de distribution instable

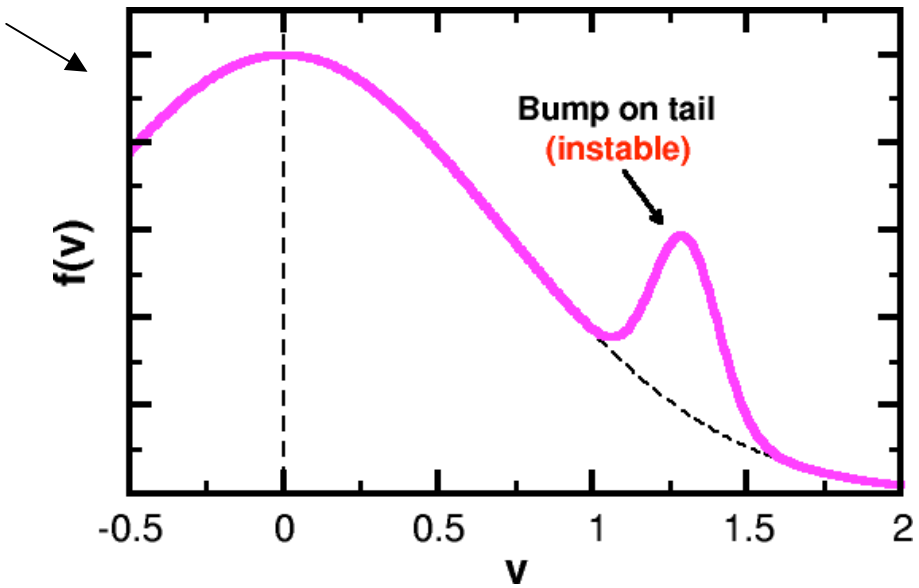
- Taux de croissance linéaire:

$$\gamma = \frac{\pi}{2} \cdot \frac{\omega \omega_p^2}{k|k|} \cdot \frac{\partial F_s}{\partial v}\bigg|_{v=\omega/k}$$

- Amplitude de l'instabilité:

$$A \propto |E|^2 \propto \exp(2\gamma t)$$

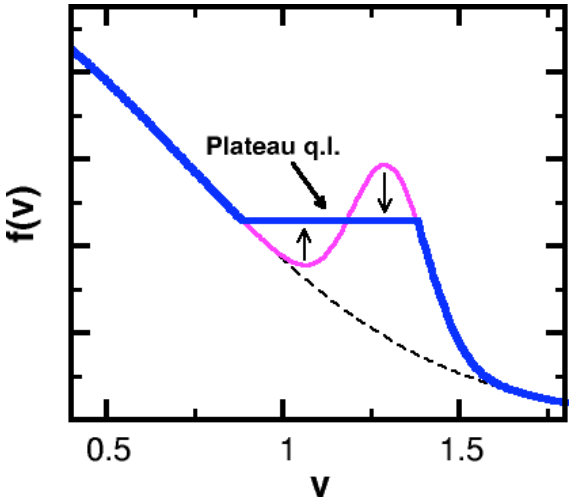
→ Croissance infinie



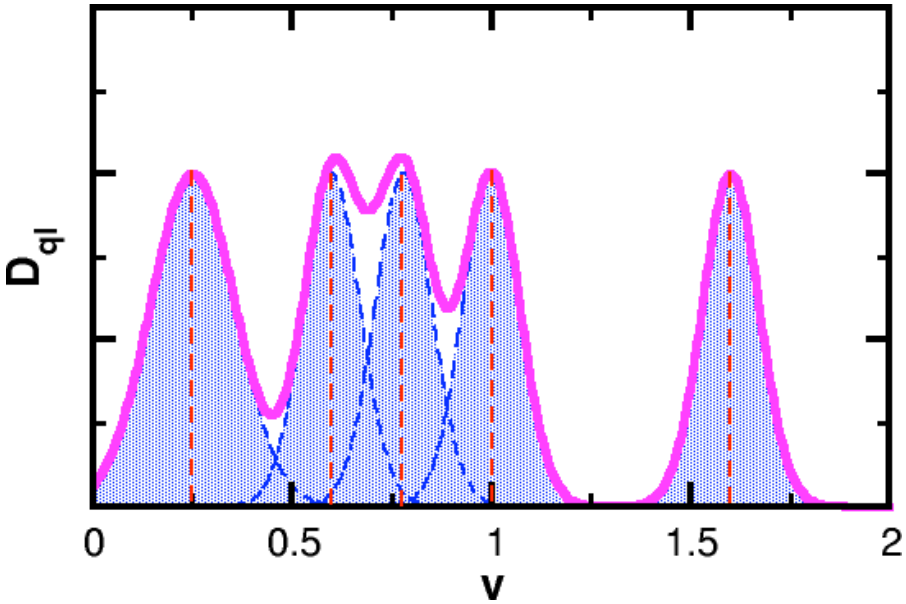
- **Jusqu'où croît réellement l'amplitude ?**
- **Quel est le mécanisme de saturation ?**
- **Quelle est l'action de l'onde sur la fonction d'équilibre ?**

Théorie quasilinéaire

- **Quelques caractéristiques:**
  - **Rétroaction de l'onde sur F_s**
    - ⟶ Diffusion de F_s dans l'espace des vitesses
    - ⟶ Applatissement autour des résonances
  - **Coefficient de diffusion $D_{ql} \square |E_{lin}|^2$**



- **Hypothèses fondamentales:**
  - **Amplitudes modérées**
    - ⟶ Orbites d'ordre 0 respectées
  - **Spectre d'onde dense**
    - ⟶ Pas de phénomène cohérent



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