Equilibrium and Stability of Oblate Free-Boundary FRCs in MRX

S. P. Gerhardt, E. Belova, M. Inomoto*, M. Yamada, H. Ji, Y. Ren

Princeton Plasma Physics Laboratory

** Osaka University*

Outline

- Introduction
  - Brief overview of relevant FRC issues
  - Introduction to the MRX facility & spheromak merging
- Overview of FRC Stability Results in MRX
- Systematic Studies of FRC Stability
  - n=1 tilt/shift instabilities without passive stabilization
  - First experimental observation of co-interchange modes.
  - n≥2 modes often limit lifetime after passive stabilizer is installed
- Modeling of Equilibrium and Stability Properties
  - Equilibrium reconstruction with new Grad-Shafranov code
  - Oblate Plasmas are Tilt Stable from a Rigid-Body Model
  - HYM calculations of Improved Stability Regime at Very Low Elongation

    -E. Belova
- Conclusions

FRC Stability is an Unresolved Issue

Internal Tilt Mode in Prolate FRC (n=1)

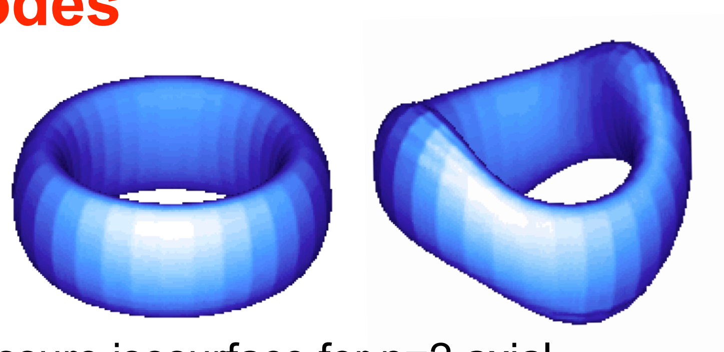
- Plasma current ring tilts to align its magnetic moment with the external field.
- Growth rate is the Alfven transit time.
- Essentially always unstable in MHD.
- Never conclusively identified in experiments
- FLR/non-linear saturation effects almost certainly important. (Belova, 2004)

Oblate FRC: Internal Tilt→External Tilt (n=1)

- For E<1, tilt becomes an external mode
- Can be stabilized by nearby conducting structures, or by very low elongation.
- Radial shifting mode may become destabilized.
- Observed in oblate FRC experiments, avoided with passive stabilizers.

Co-Interchange Modes

- $n \geq 2$ cousins of tilt/shift modes
- For $n \rightarrow \infty$, these are ballooning-like modes
- Low n co-interchange modes ($1<n<9$) computed to be destructive to oblate FRCs (Belova, 2001).
- Never experimentally identified.



Pressure isosurface for n=2 axial co-interchange, calculated by HYM code

MRX is a Flexible Facility for Oblate FRC Studies

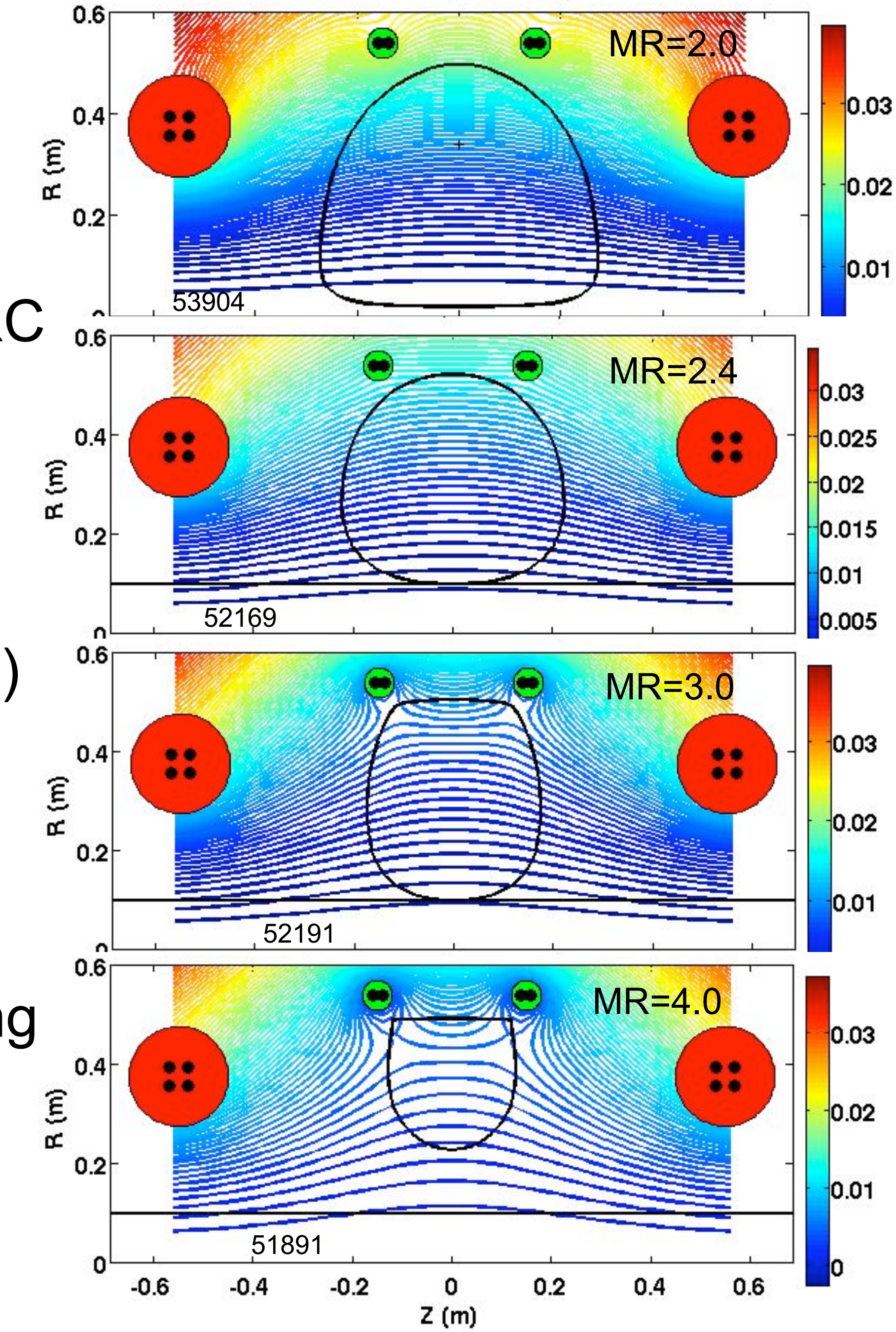
• Spheromak merging scheme for FRC formation.

• FRC shape control via flexible external field (EF) set.
  - Describe EF by Mirror Ratio (MR)

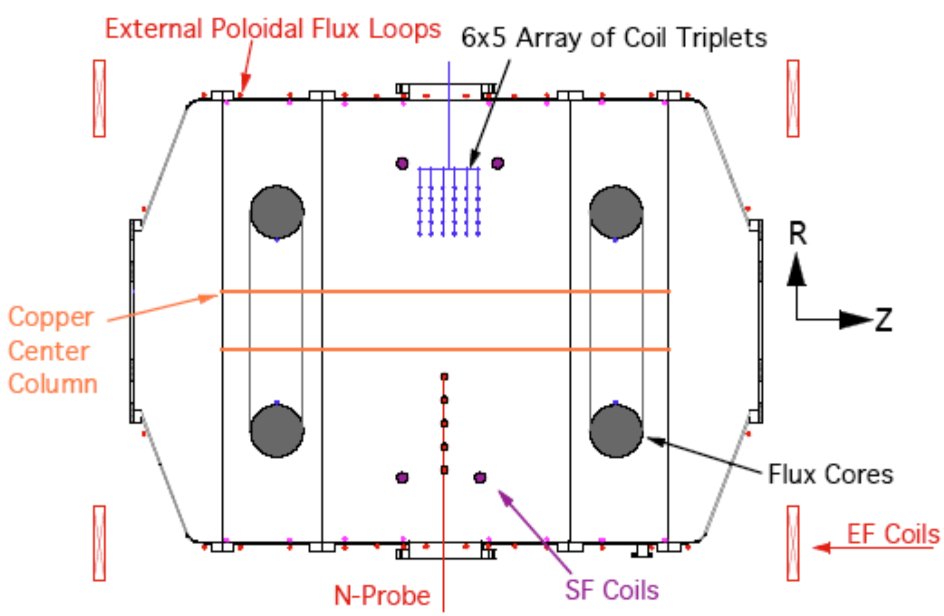
• Extensive internal magnetic diagnostics.

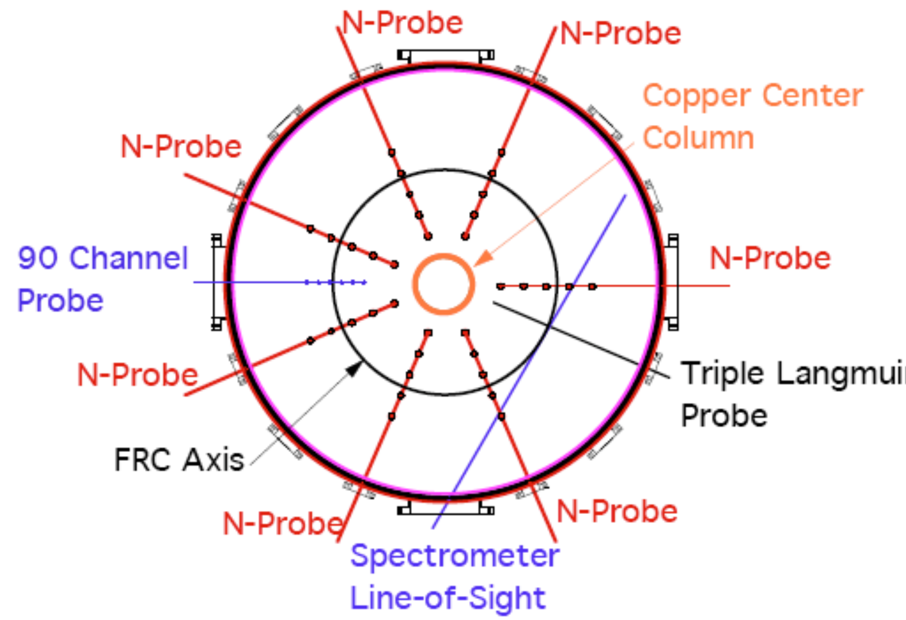
• Passive stabilization via a conducting center column (sometimes).

• First experiments in spring 2005.



Comprehensive Diagnostics For Stability Studies





- 90 Channel Probe: 6x5 Array of Coil Triplets, 4cm Resolution, Scannable
- 105 Channel Toroidal Array: 7 Probes 5 coil triplets
  - ✓Toroidal Mode Number n=0,1,2,3 in B_Z, B_R, B_T
- T_i through Doppler Spectroscopy (He^{+1} @ 468.6nm)
- Copper Center Column for Passive Stabilization
  - •10cm radius, .5 cm thick, axial cut

FRC Formation By Spheromak Merging

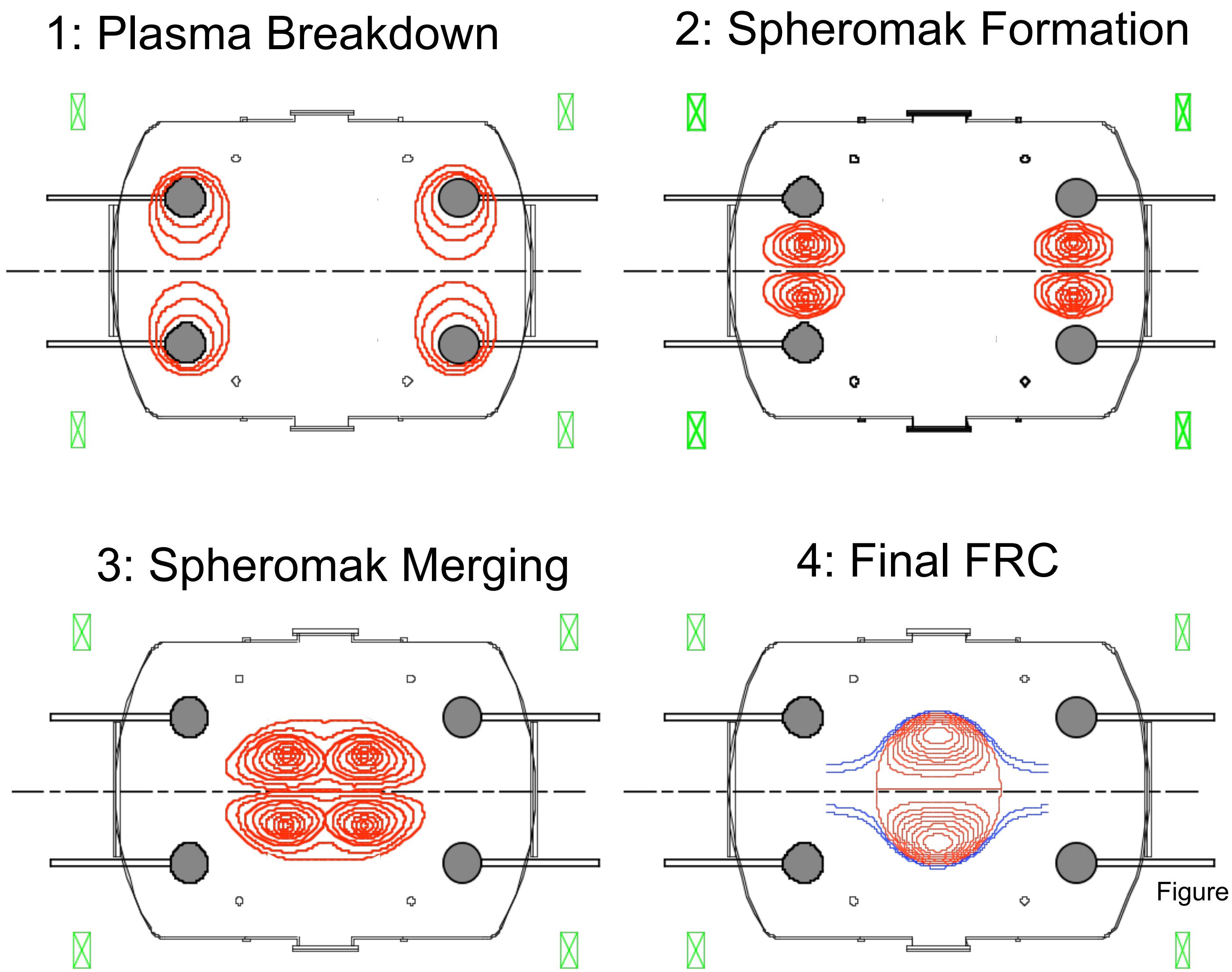
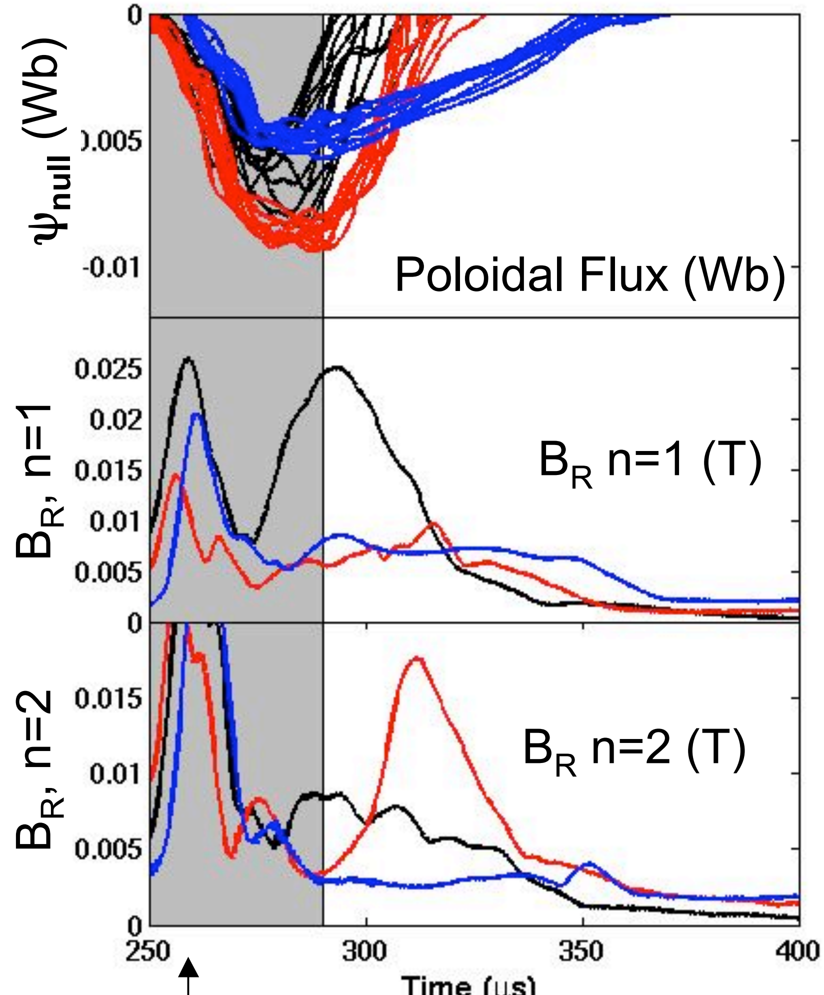


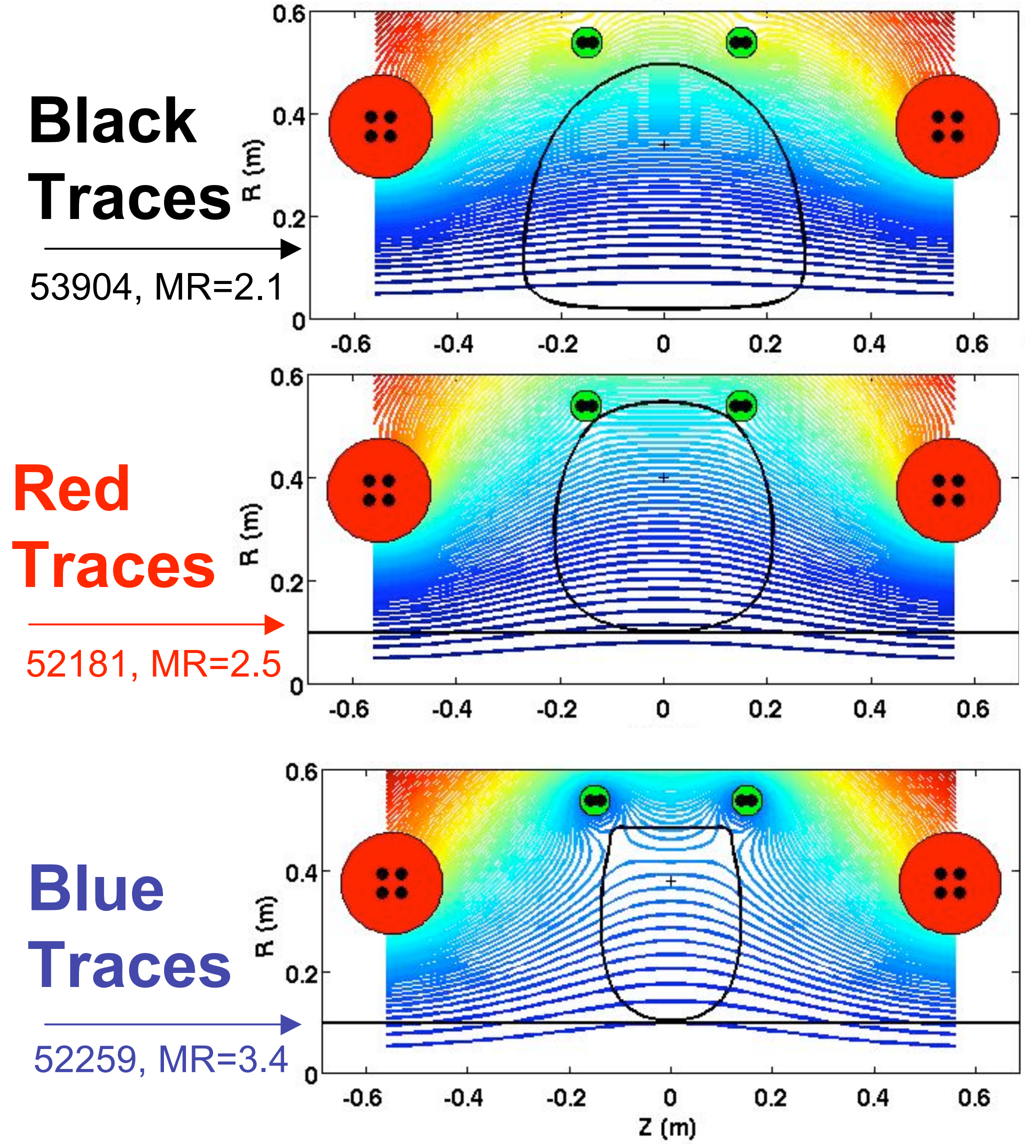
Figure Courtesy of H. Ji.

Technique developed at TS-3, utilized on TS-4 and SSX

Passive Stabilizer and Shape Control Extend the Plasma Lifetime



Large Non-Axisymmetries Before Merging Completion



Systematic Instability Studies Have Been Performed

- The instabilities have the characteristic of tilt/shift and co-interchange modes.

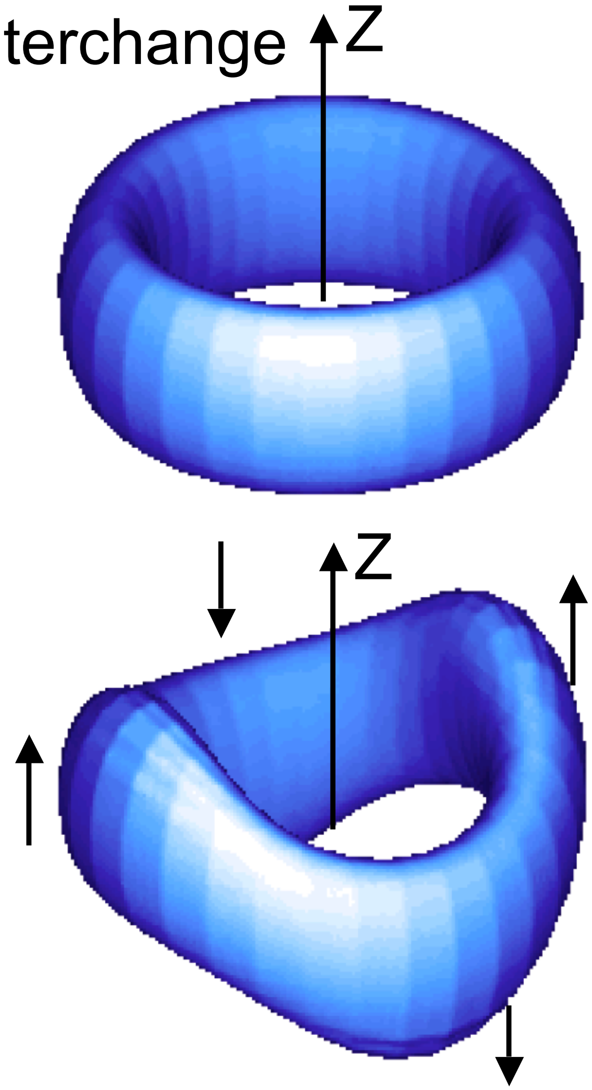
- The center column reduces the n=1 tilt/shift amplitude.

- Co-interchange ($n \geq 2$) modes reduced by shaping.

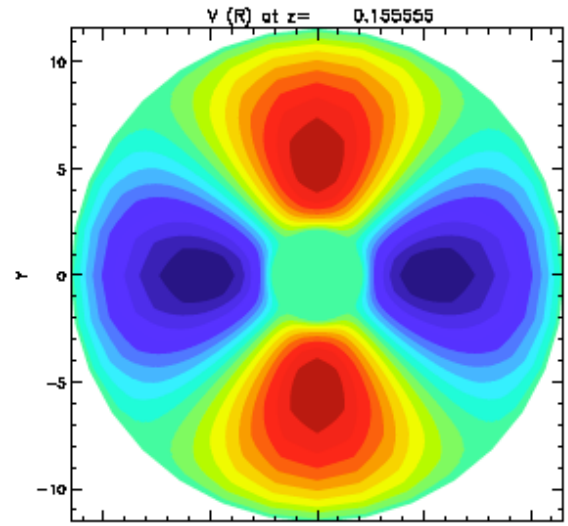
- Co-interchange modes can be as deadly as tilting.

Axial Polarized Mode Appears Strongly in B_R

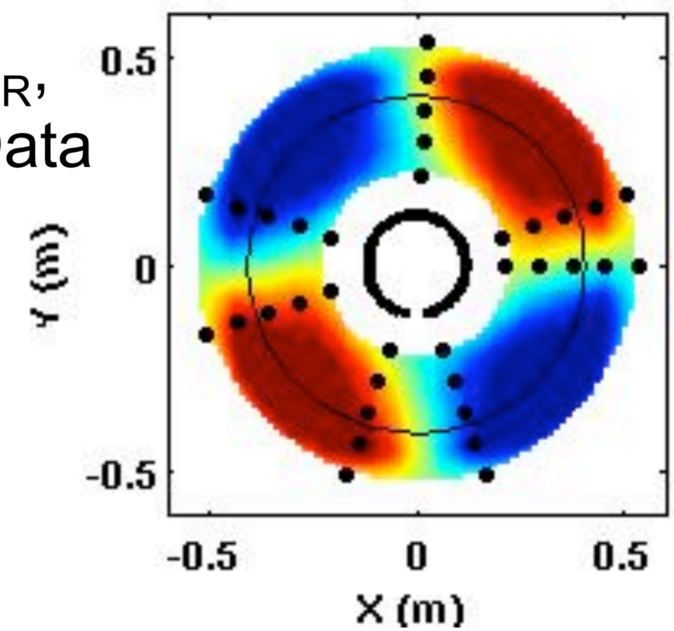
n=2 Axial
Co-Interchange



B_R, HYM



B_R, Data

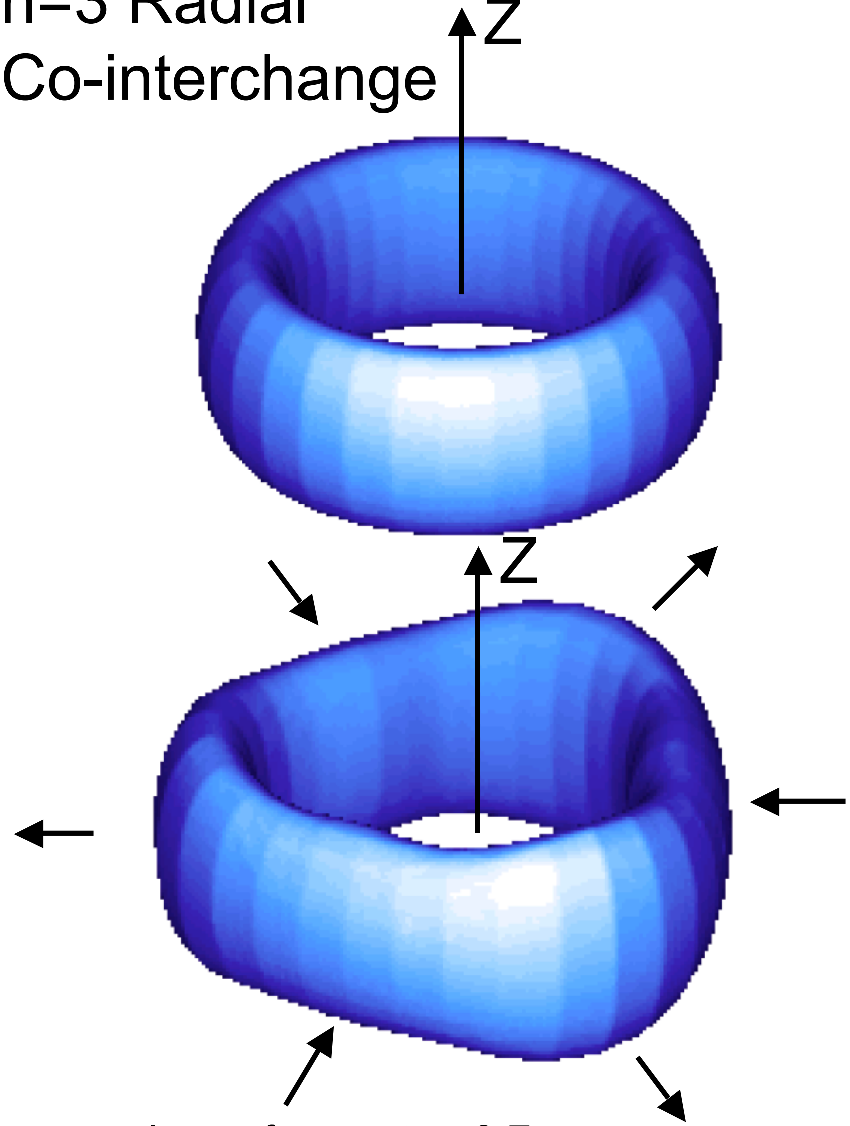


Pressure isosurfaces at $p=0.7p_o$
Calculated for MRX equilibria from HYM code

52182

Radial Polarized Mode Appears Strongly in B_Z

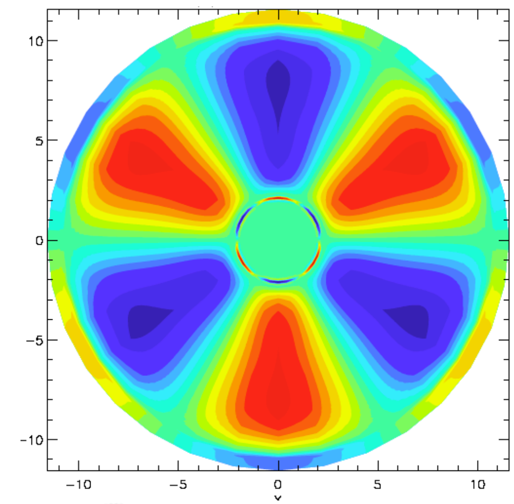
n=3 Radial
Co-interchange



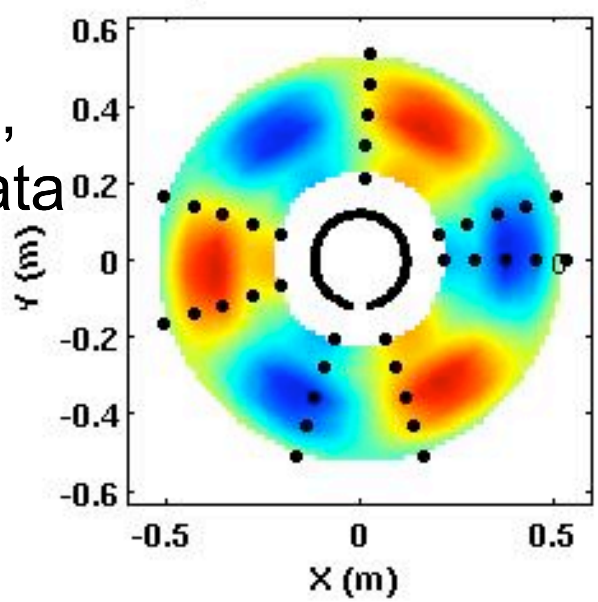
Pressure isosurfaces at $p=0.7p_o$
Calculated for MRX equilibria from HYM code

B_Z Perturbation at Midplane

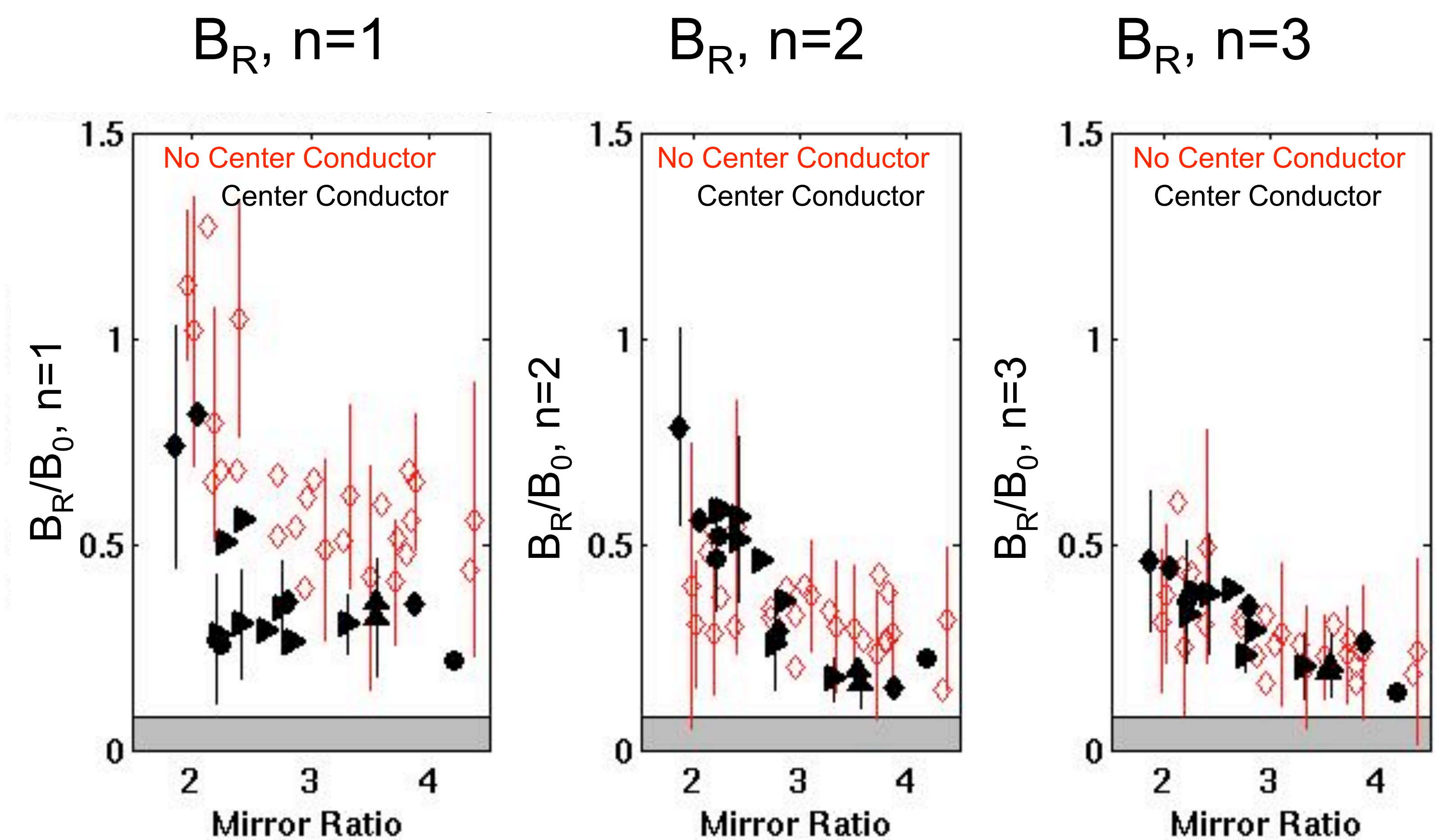
B_Z, HYM



B_Z, Data



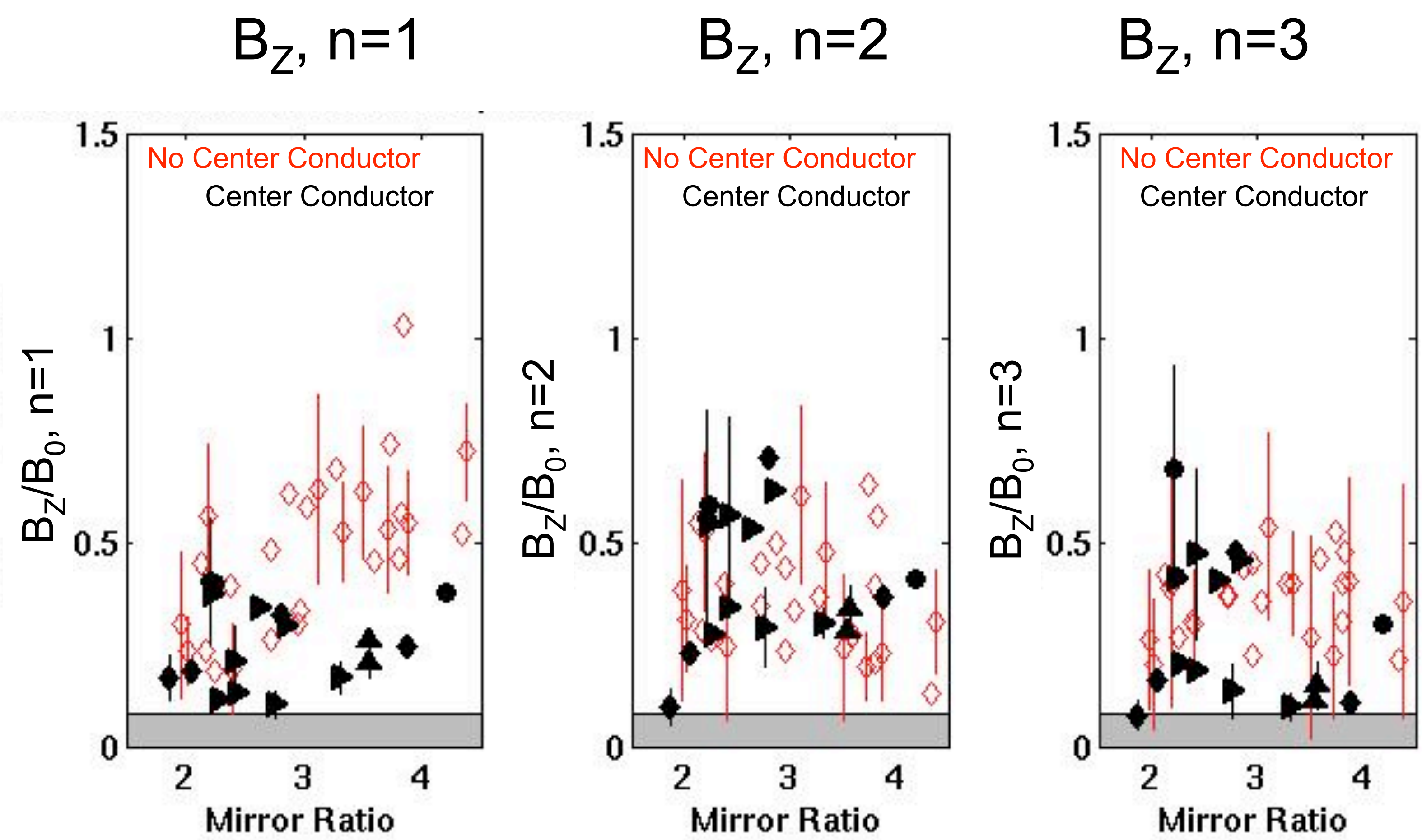
Center Column Reduces Tilt Signature



- n=1 (tilt) reduced with center column
- n=2&3 axial modes reduced at large mirror ratio

Helium

n=1 Shifting Signature Largely Suppressed with Center Column



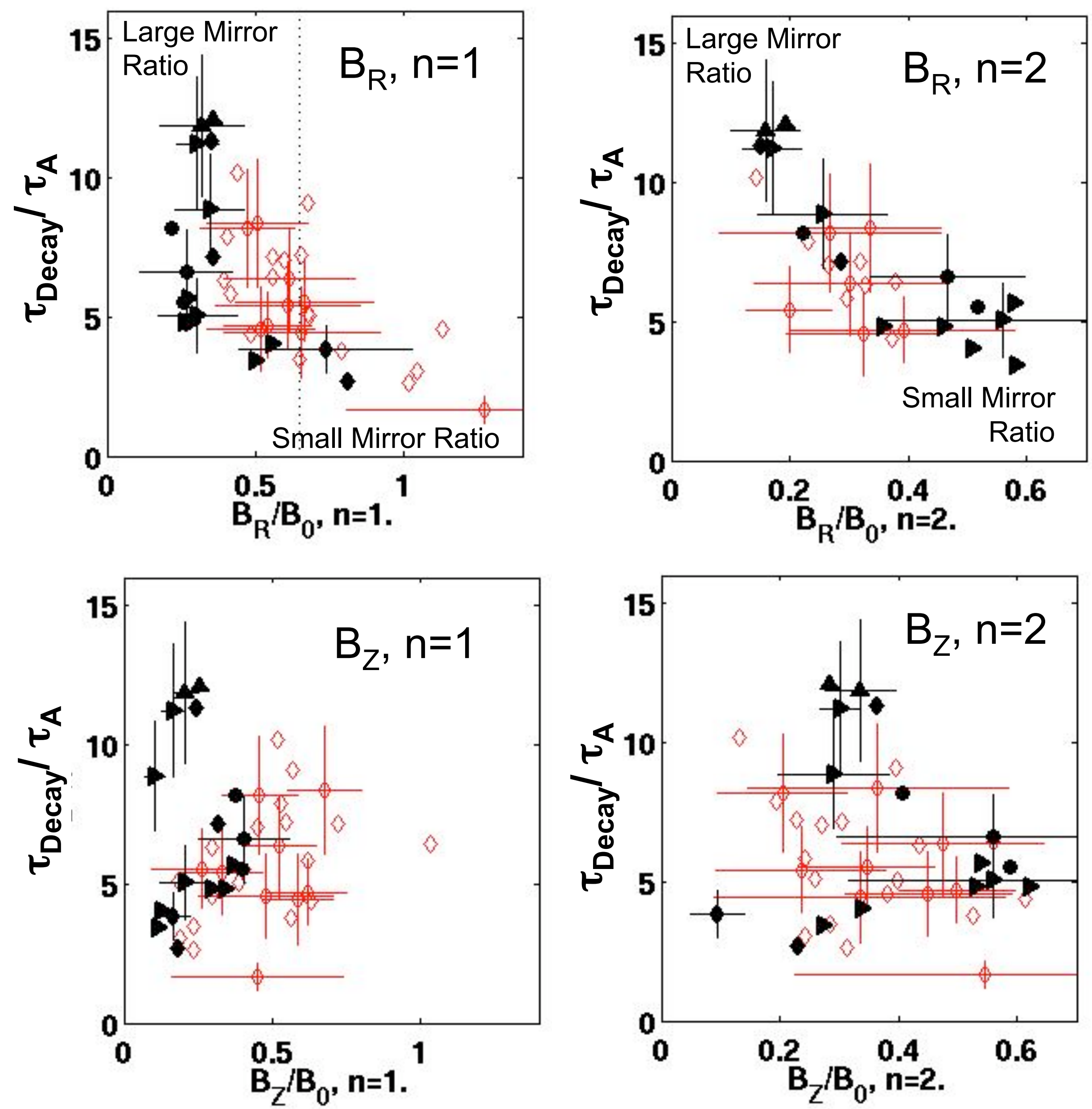
- n=1 reduced by center column
- n=2 & 3 not changed by the passive stabilizer

Helium

Lifetime is Strongly Correlated with B_R Perturbations

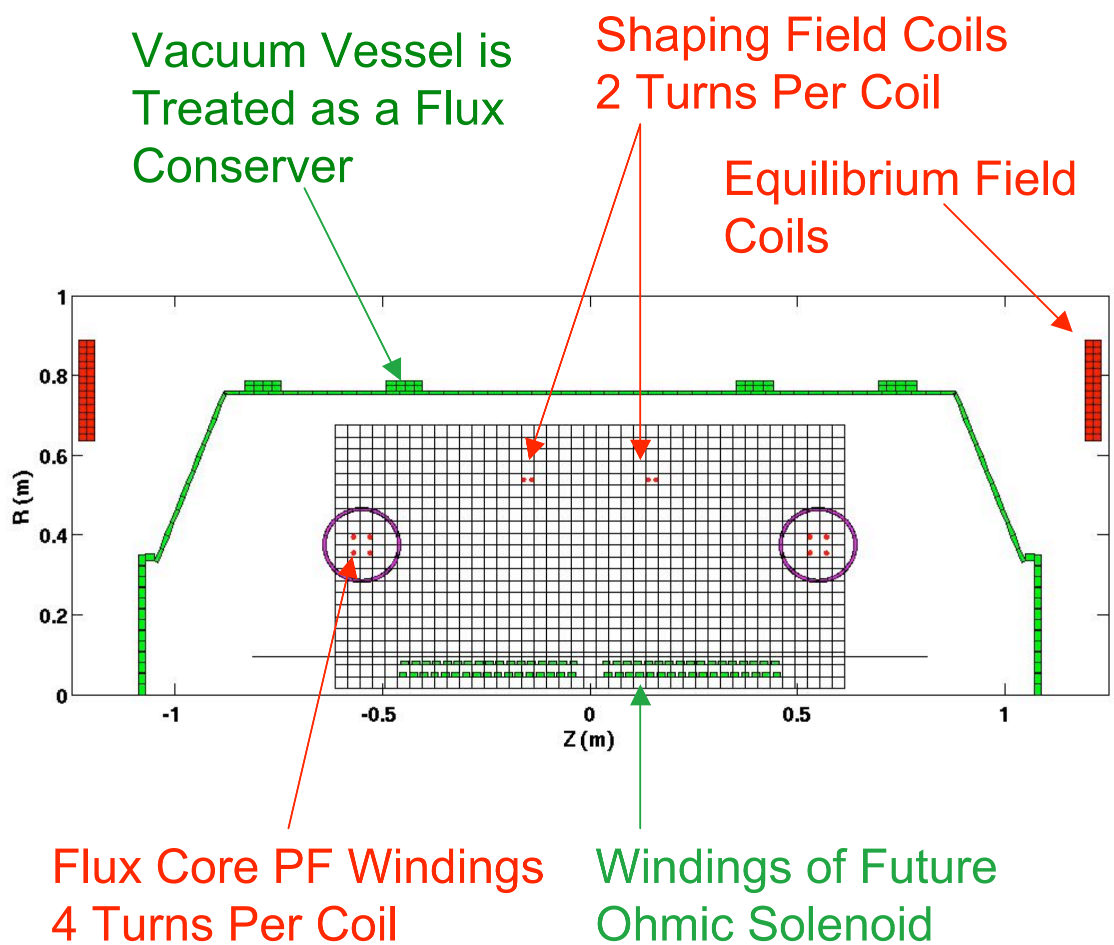
- With & Without Center-Column
- Strong Correlation with the B_R perturbation.

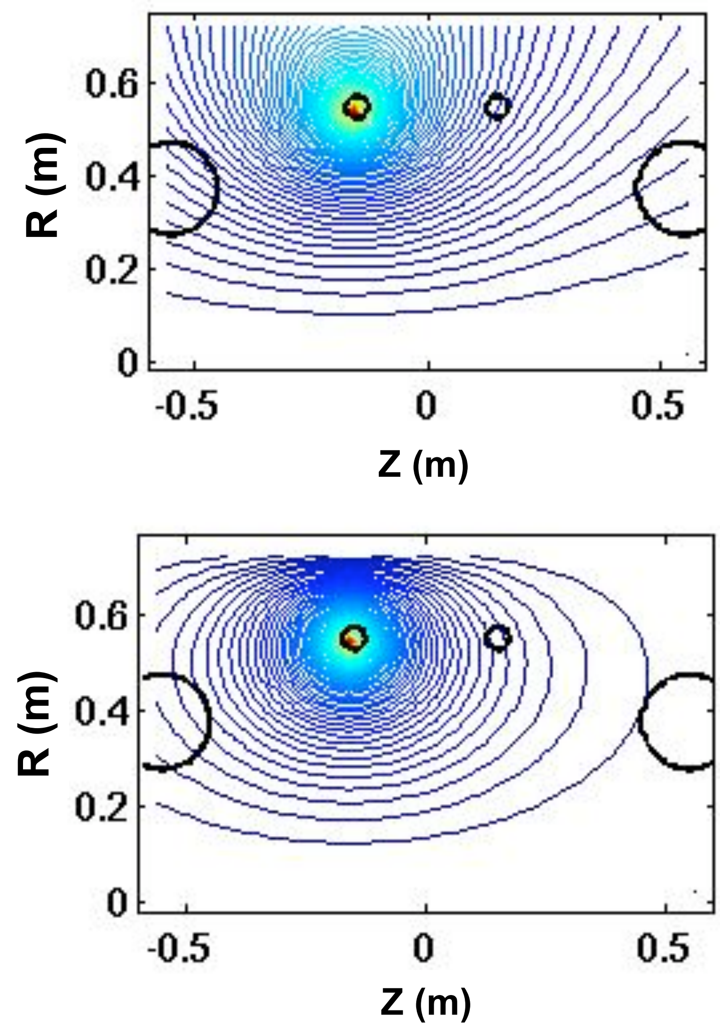
No Correlation with BZ Perturbation.



Helium

Fields Calculated From Axisymmetric Model With Flux Conserving Vessel





*J.K. Anderson et.al. Nuclear Fusion **44**, 162 (2004)

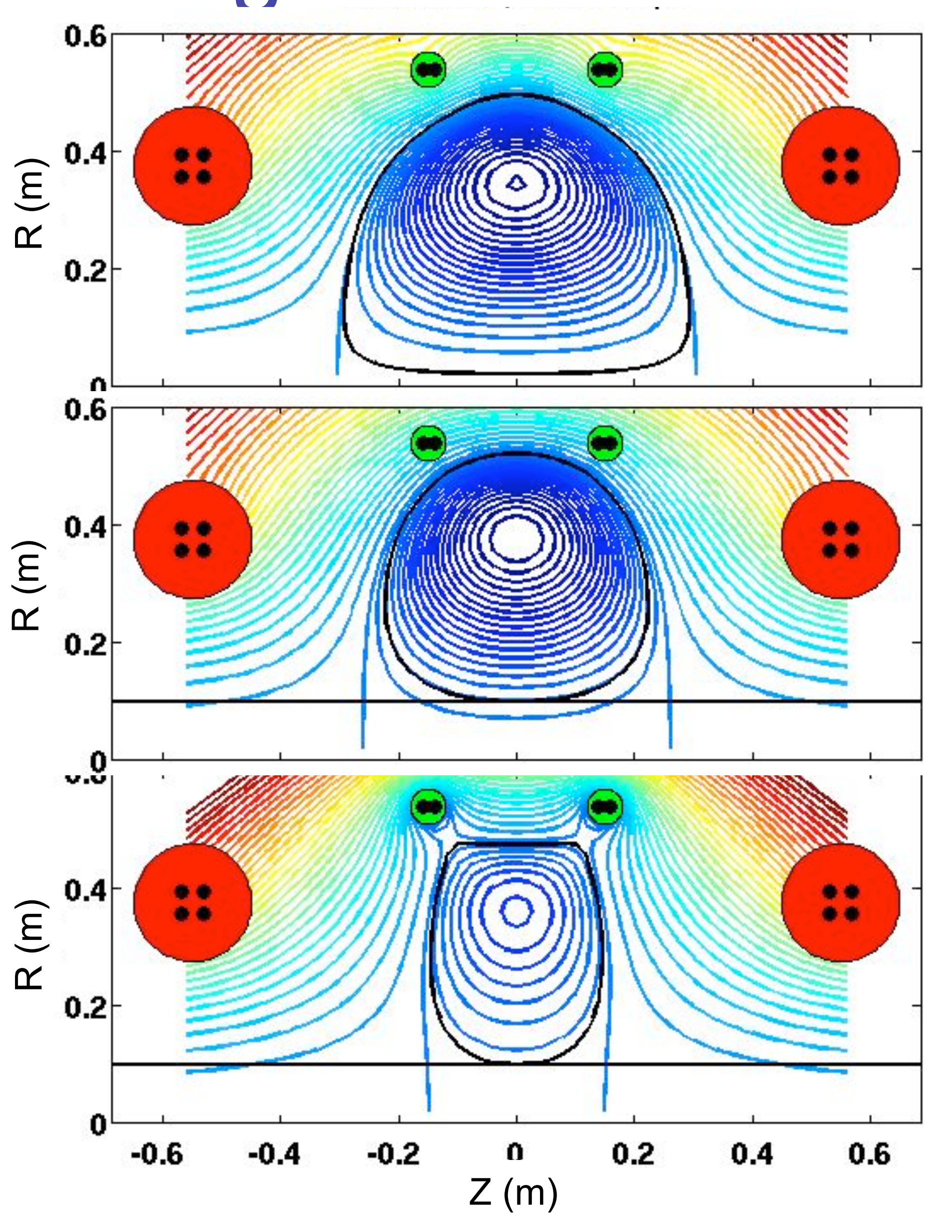
MRXFIT Code Finds MHD Equilibria Consistent with Magnetics Data

- Iterative free-boundary Grad-Shafranov solver.
- Flexible Plasma Boundary
  - Center Column Limited
  - SF Coil Limited
  - X-points
- $P'(\psi)$ & $FF'(\psi)$ optimized for solution matching measured magnetics.

$$p'(\psi) = \sum C_i \hat{\psi}^{\alpha_i}$$

$$FF'(\psi) = C_F \hat{\psi}^{\gamma_F}$$

- Equilibria interfaced to HYM stability code.



Equilibrium Properties Respond to the External Field

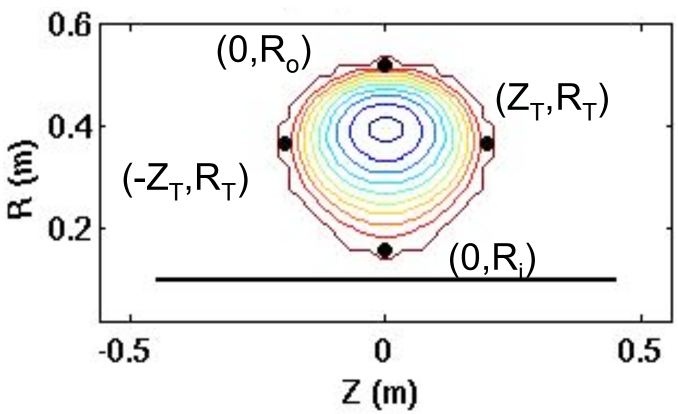
Definitions*:

$$a = {(R_o - R_i)}/{2}$$

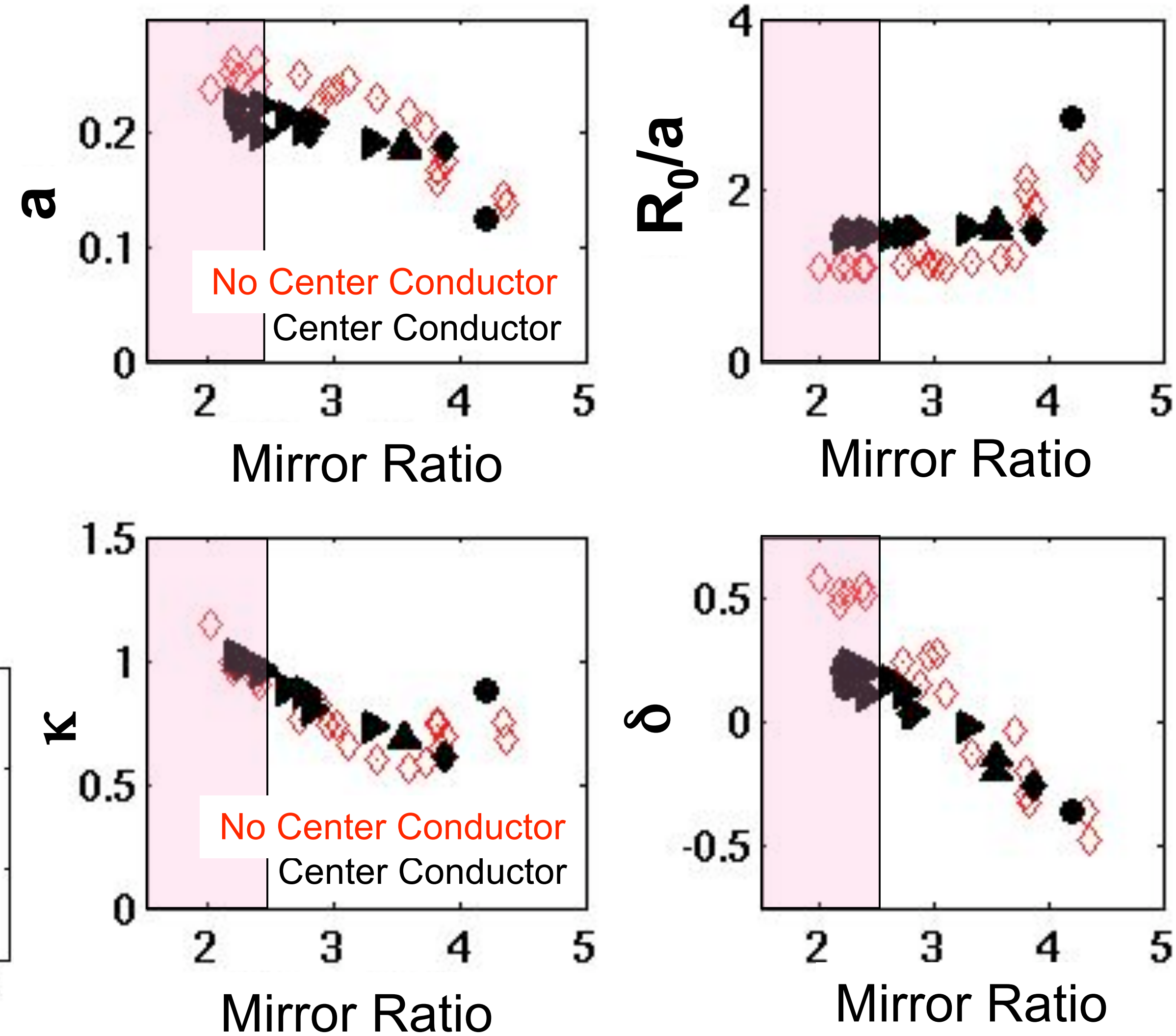
$$R_0 = {(R_o + R_i)}/{2}$$

$$\kappa = {Z_t}/{a} \sim 2E$$

$$\delta = {(R_0 - R_t)}/{a}$$



*S.B. Zheng, A.J. Wooten, & E. R. Solano, Phys. Plasmas 3,1176 (1996)



Pink region: Caution! Large Non-axisymmetries!

Rigid-Body Model Used to Estimate Tilt/Shift Stability

$$\text{Current Profile: } J_\phi = J_\phi(R,Z)$$

$$\text{Equilibrium Field: } \begin{cases} B_Z = B_Z(R,Z) \\ B_R = B_R(R,Z) \end{cases}$$

Tilting

$$n_{decay}(R,Z) = -\frac{R}{B_Z}\left[\frac{\partial B_Z}{\partial R} - \frac{Z}{R^2}\frac{\partial}{\partial R}\left(2RB_R + ZB_Z\right)\right]$$

$$N_X = \theta_X \underbrace{\left[\iint \pi R^2 J_\phi B_Z \left(1 - n_{decay}\right) dA\right]}_{\text{Torque}}$$

Shifting

$$n_{shift}(R,Z) = -\frac{R}{B_Z}\frac{\partial B_Z}{\partial R}$$

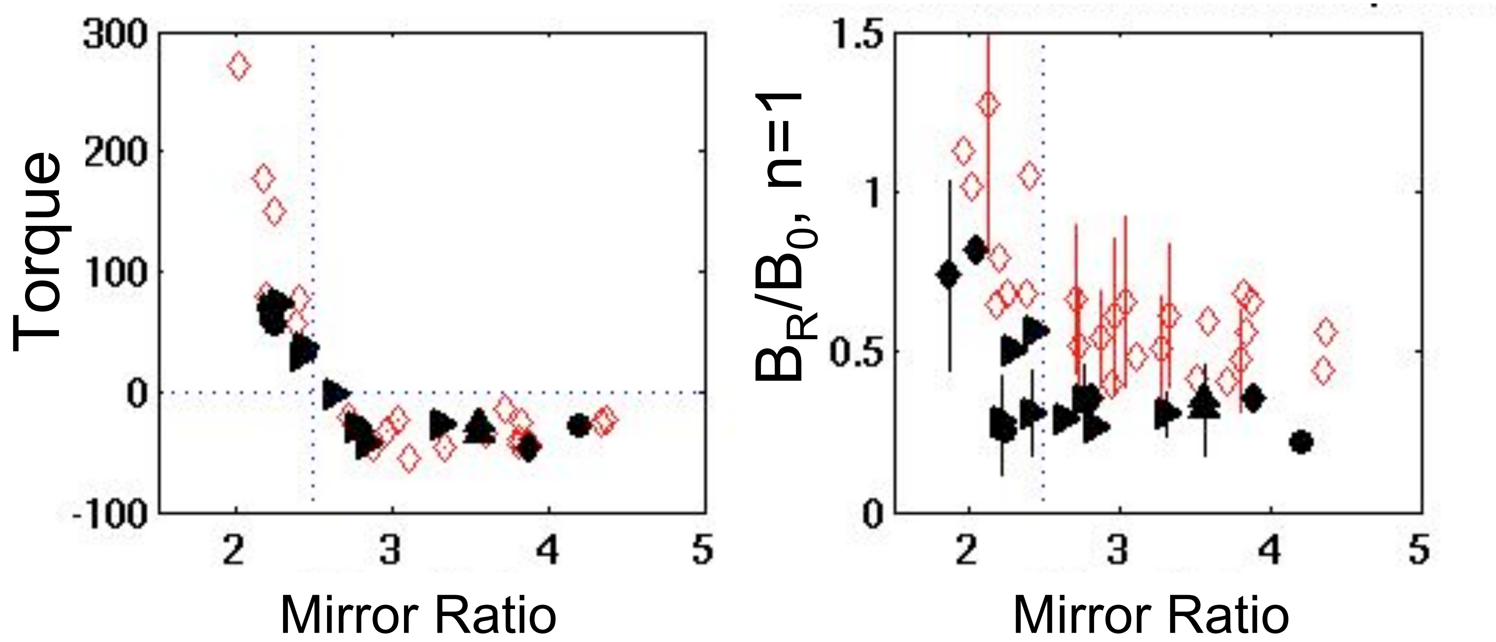
$$F_X = -\xi_X \underbrace{\left[\iint \pi R^2 J_\phi B_Z n_{shift} dA\right]}_{\text{Force}}$$

Tilting Stable: $n>1$

Radial Shifting Stable: $n<0$

H. Ji, et al, Phys. Plasmas **5**, 3685 (1998)

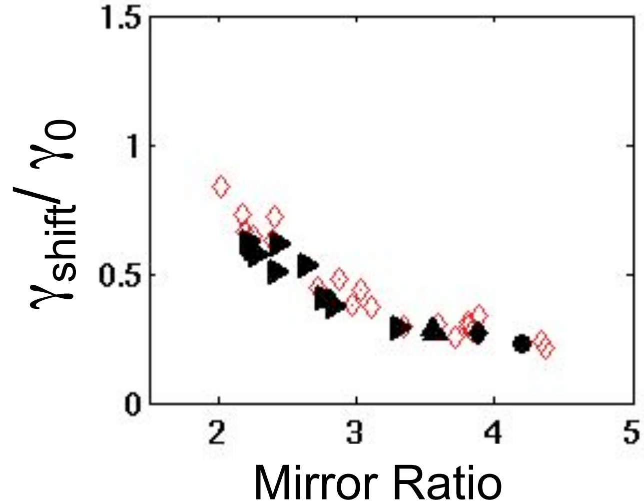
MRX Plasmas Transition to the Tilt Stable Regime



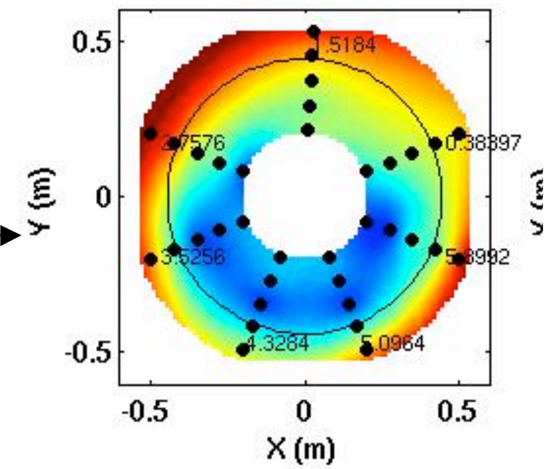
$$T = \iint \pi R^2 J_\phi B_Z \left(1 - n_{decay}\right) dA$$

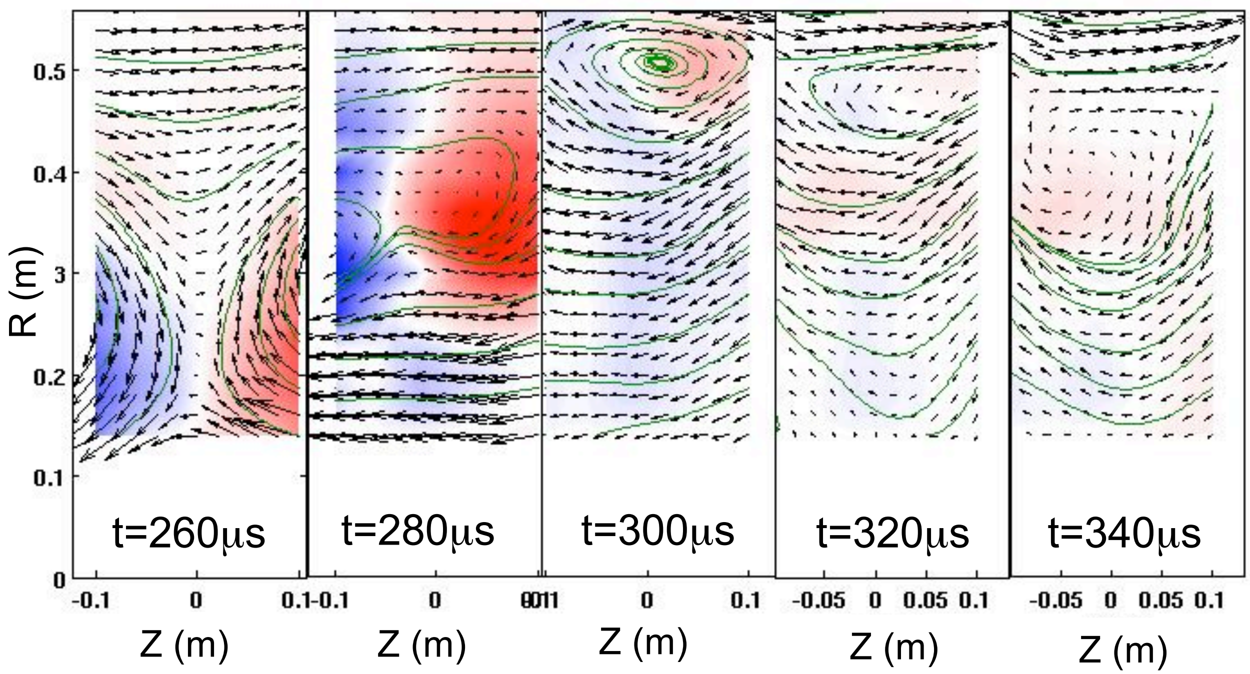
- Plasmas with MR>2.5 predicted to be in the tilt-stable regime.
- Simple model for center-column m=1 eddy currents used.
- Marginal comparison: Tilt often develops during merging phase.

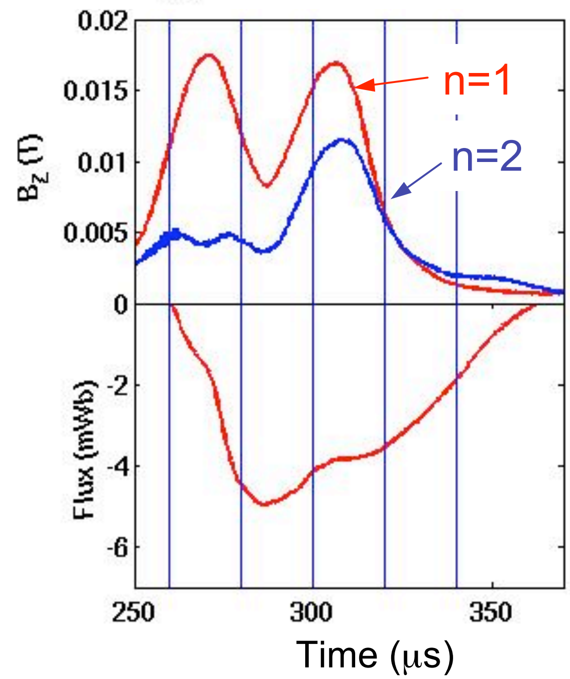
Rigid Body Shift Often Present, But May Be Benign



Midplane B_Z contours at time of large shift

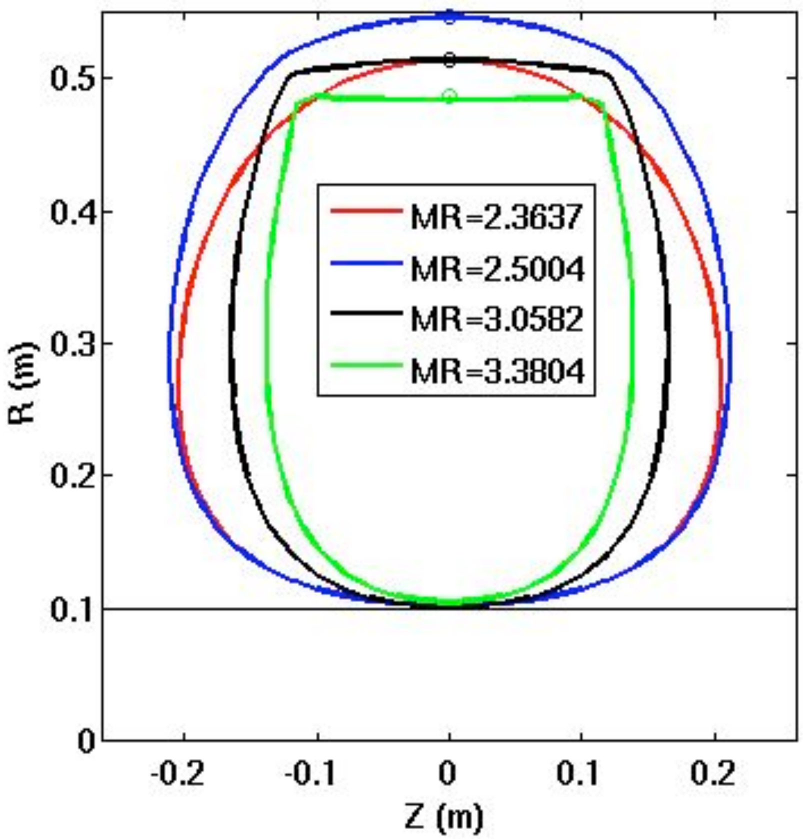


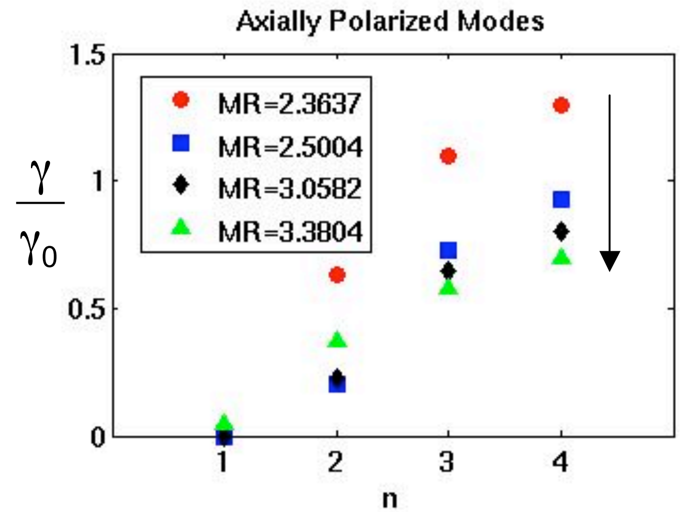




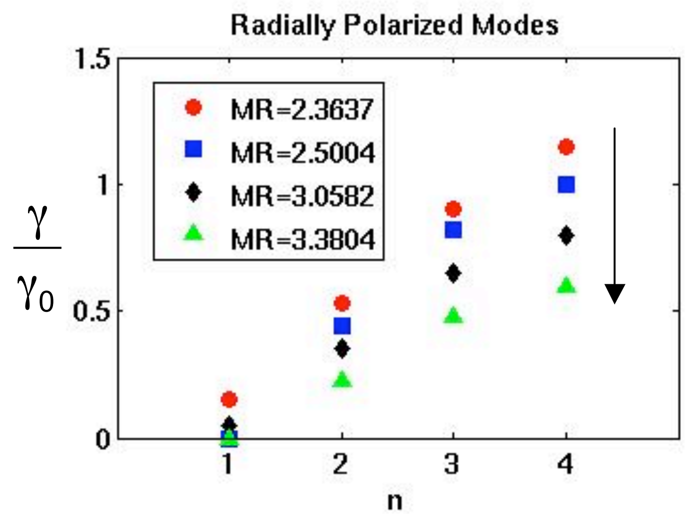
HYM Calculations Indicate Reduced Growth Rates at Larger Mirror Ratio

4 Configurations Considered So Far





Increasing mirror ratio



Increasing mirror ratio

Calculations By E. Belova

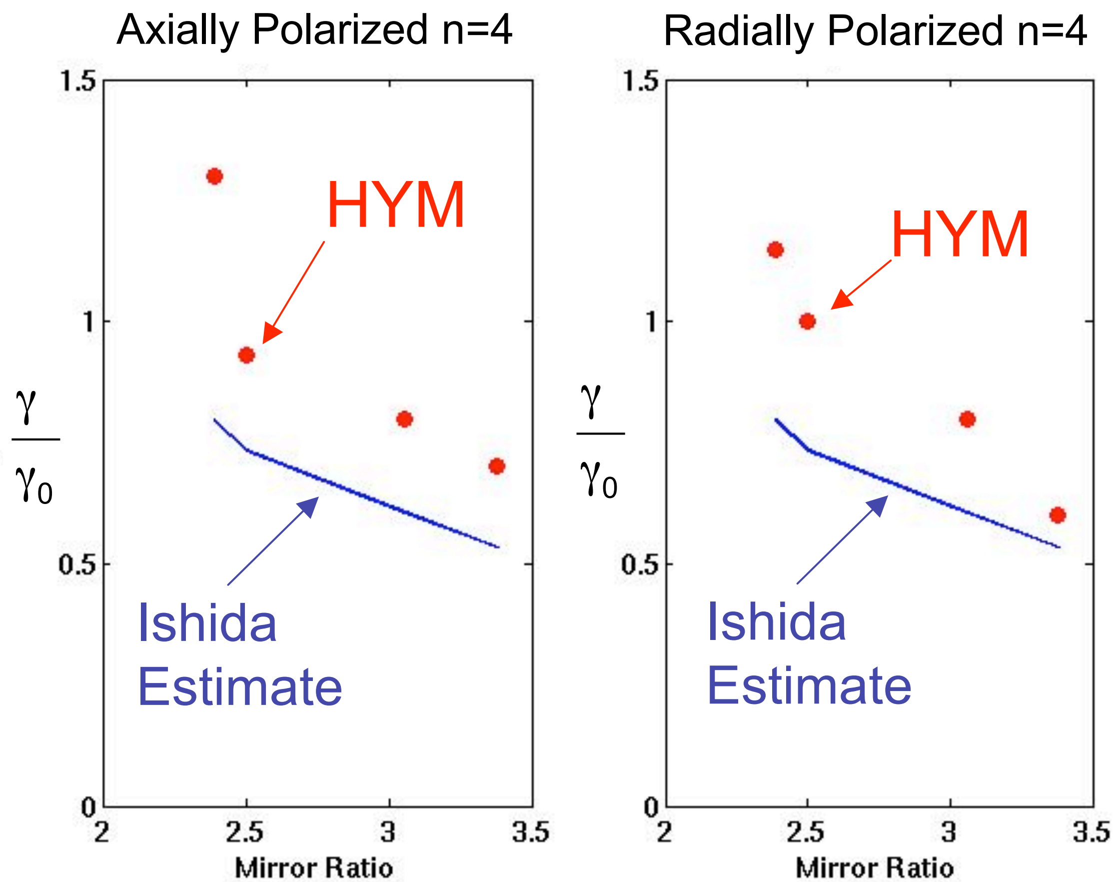
Local Mode Stability Improves At High Mirror Ratio

$$\omega^2 = \frac{\delta W}{T}$$

Ishida, Shibata, & Steinhauer
"Pure" Displacement, n>>1 limit

$$\omega^2 = \frac{-\frac{1}{2}\frac{\partial}{\partial\psi}\oint dl\kappa R B^2}{2\pi\int \rho \frac{dl}{B}}$$

Growth Rates for Local Modes Reduced at Higher Mirror Ratio.



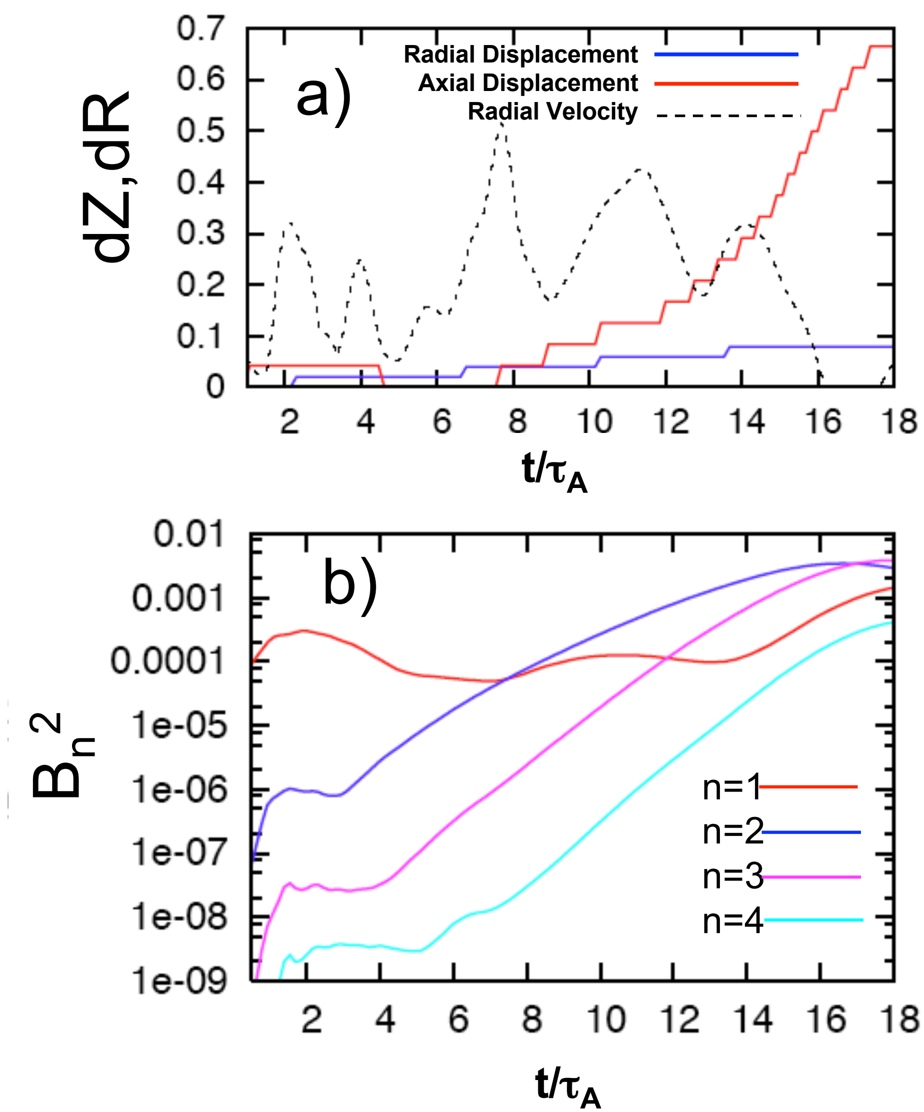
J.R. Cary, Phys. Fluids **24**, 2239 (1981)
A. Ishida et al, Phys. Plasmas **3**, 4278 (1996)

Similar Shift Saturation Observed in Simulation

- Simulation without center-column.

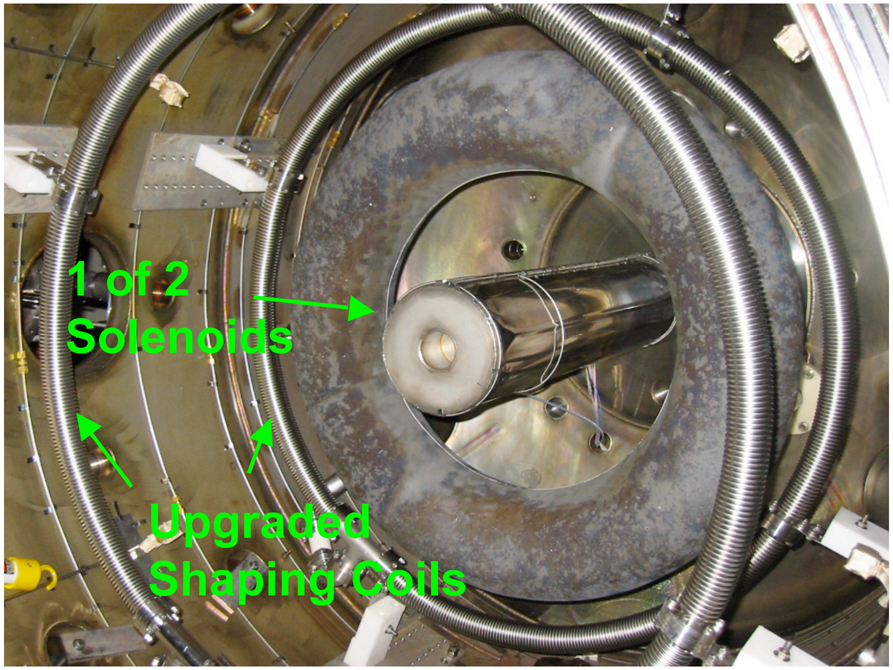
- Radial velocity oscillates, preventing fast growth of the shift mode.

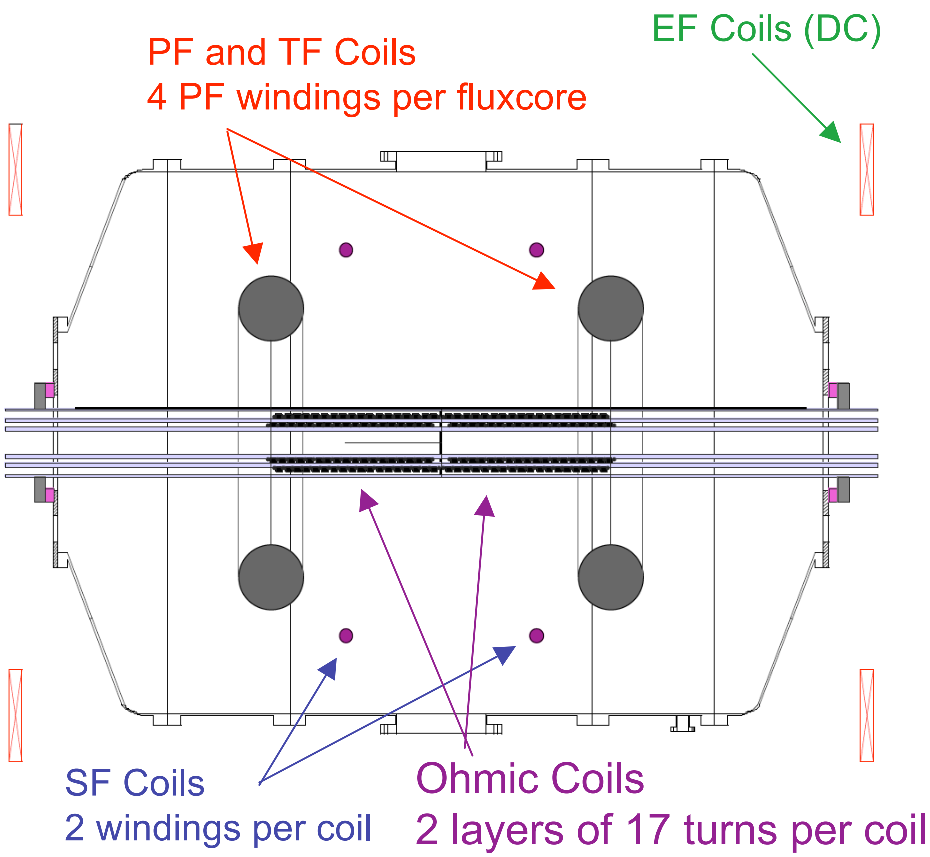
- Compression of Strong B_Z field prevents growth of the radial shift mode.



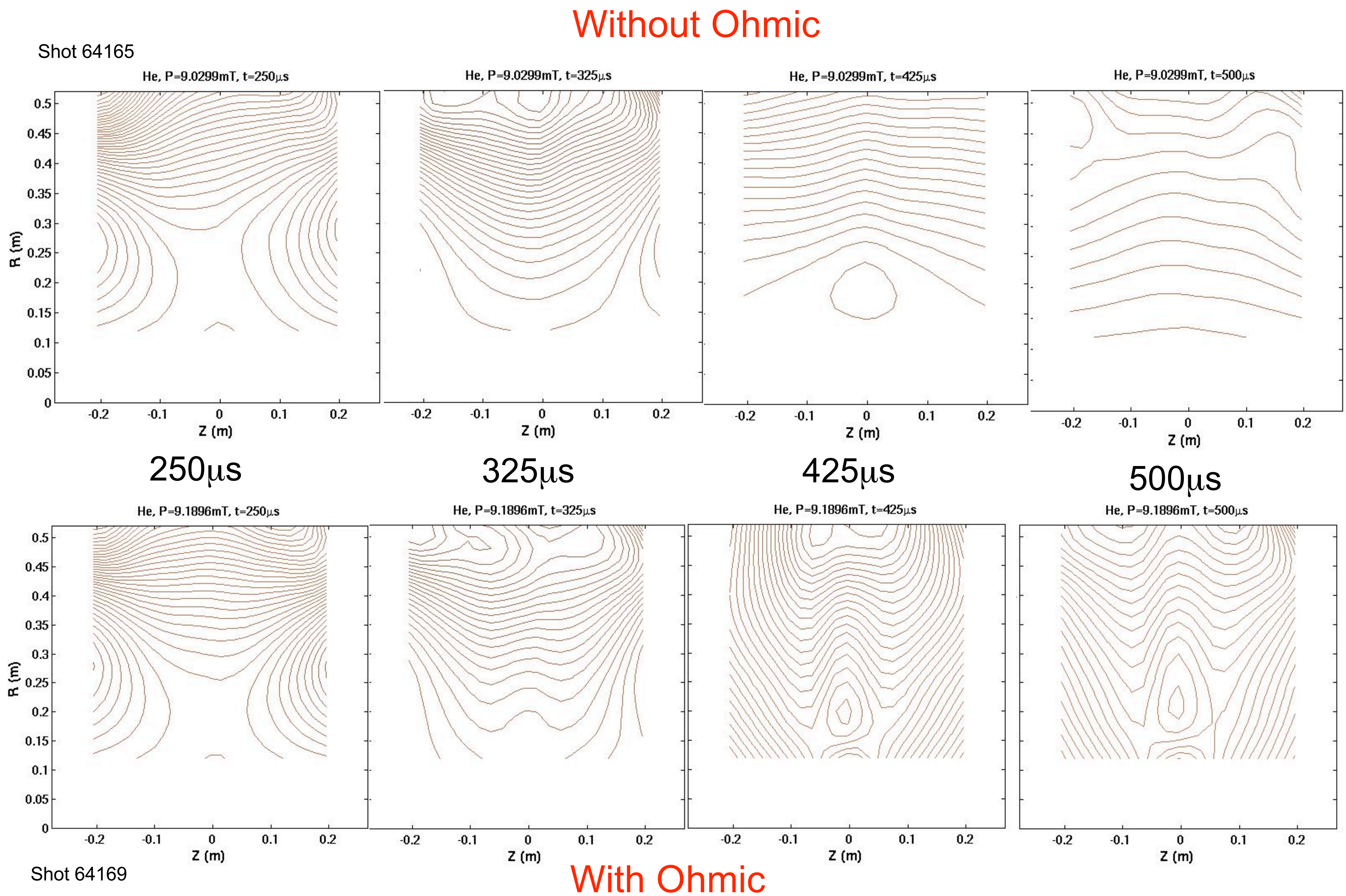
FRC Capabilities Recently Upgraded, Including Ohmic Solenoid

- Thin Inconel Liner allows Ohmic Flux To Escape
- New shaping coils encased in .007" thick formed bellows eliminates previous ceramic breaks, with two independent turns per coil.
- Newly expanded 2D probe array extends coverage by a factor of six.
- Three capacitor banks to share between 4 coils (TF, PF, SF, Ohmic).
- Ohmic return flux trapped by vessel…decreases effective EF
- First plasma during week of 10/3/2006

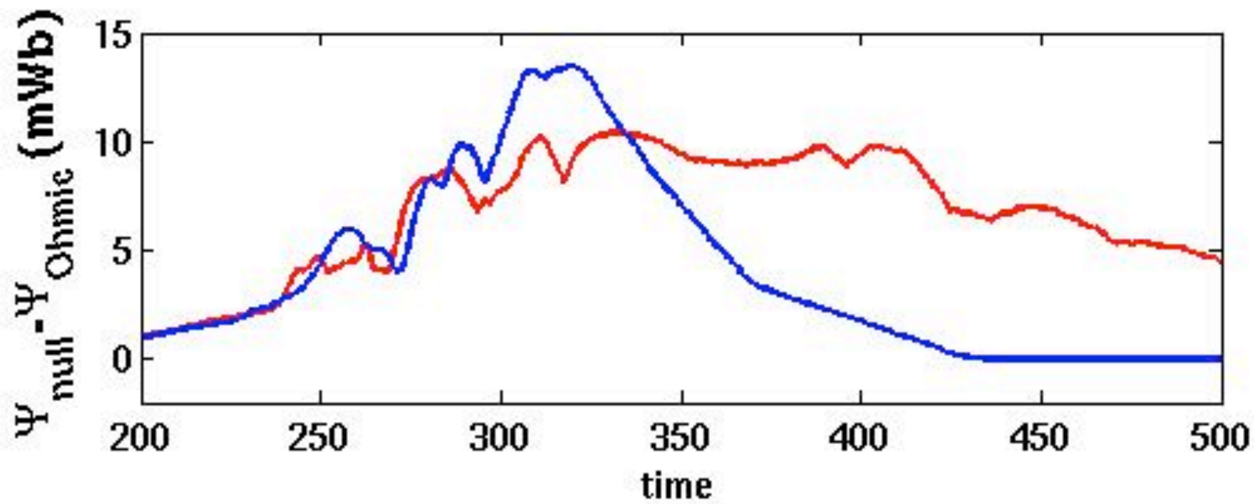


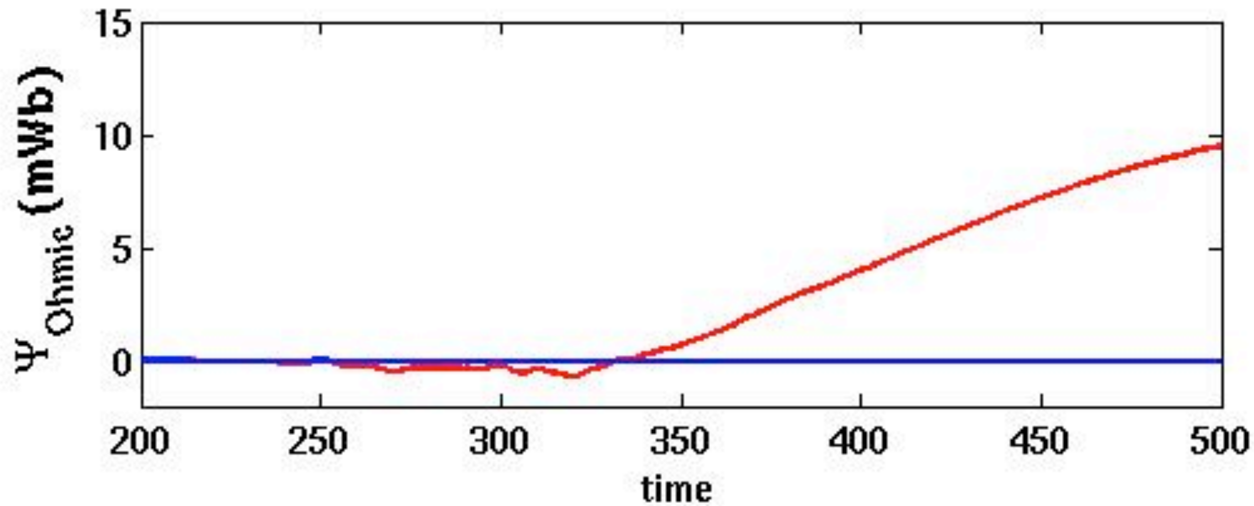


Ohmic Sustainment Demonstrated



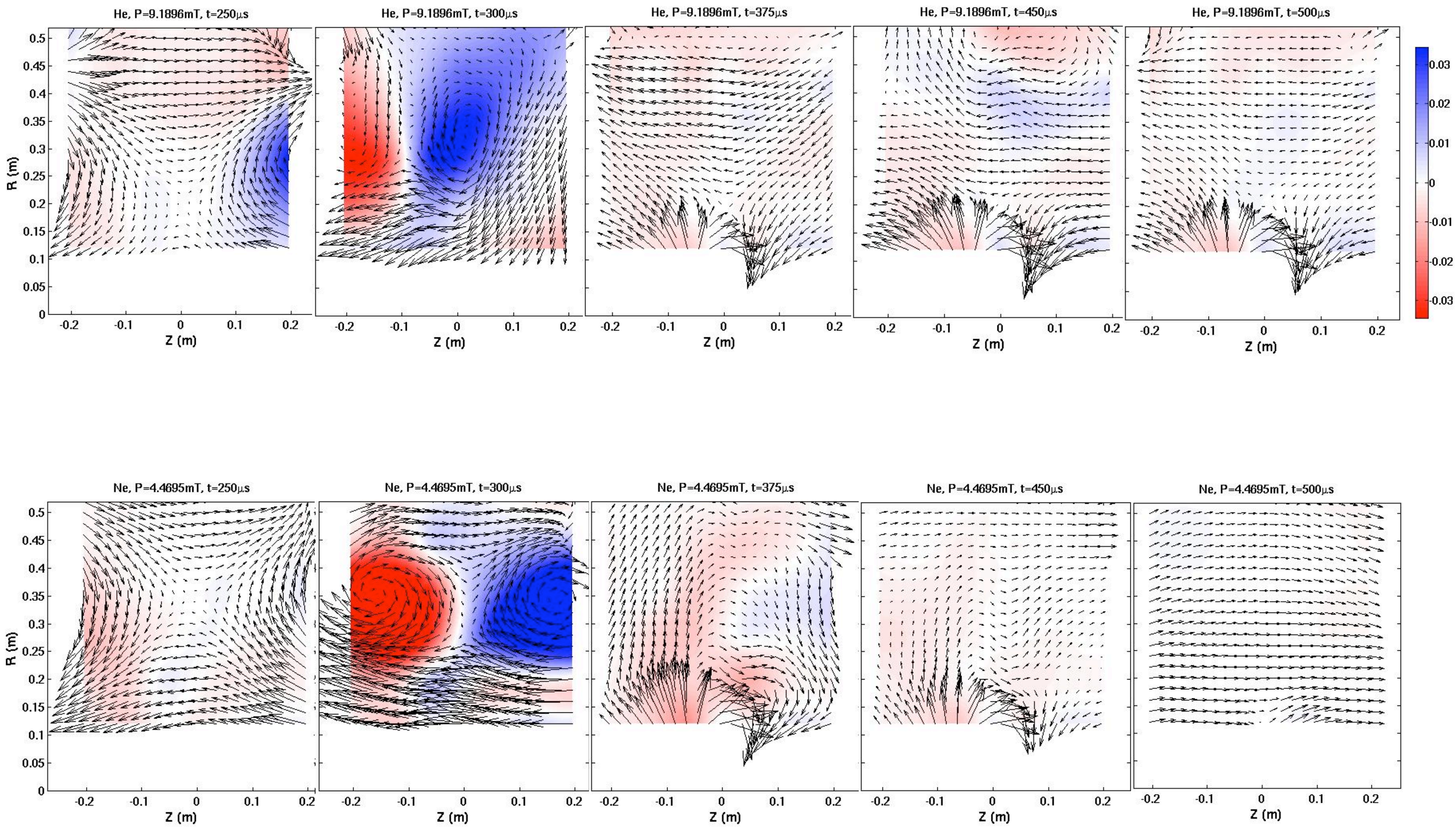
Flux Sustained for Substantially Longer With Ohmic





Ohmic Successful Only In Plasmas with Good Shaping

Shot 64169

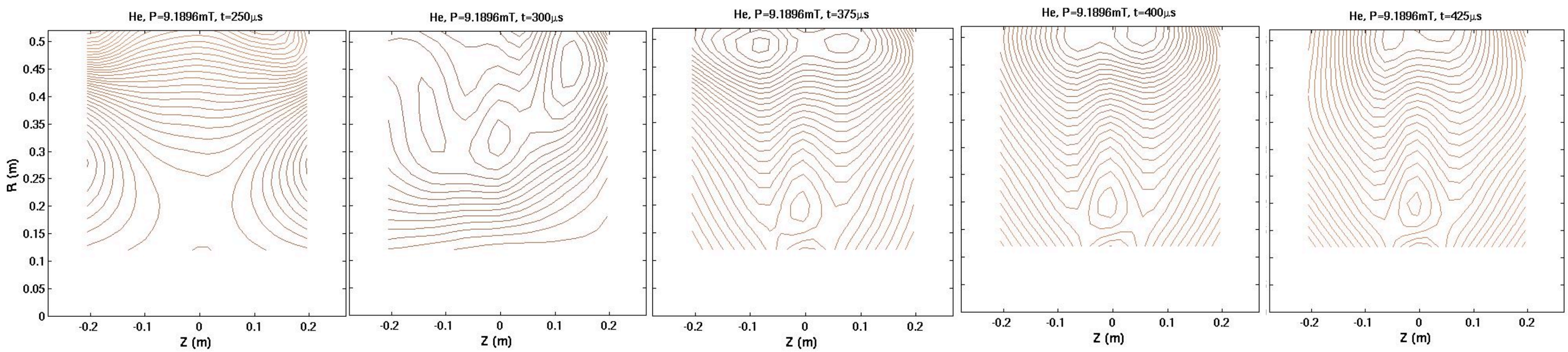


Shot 63988

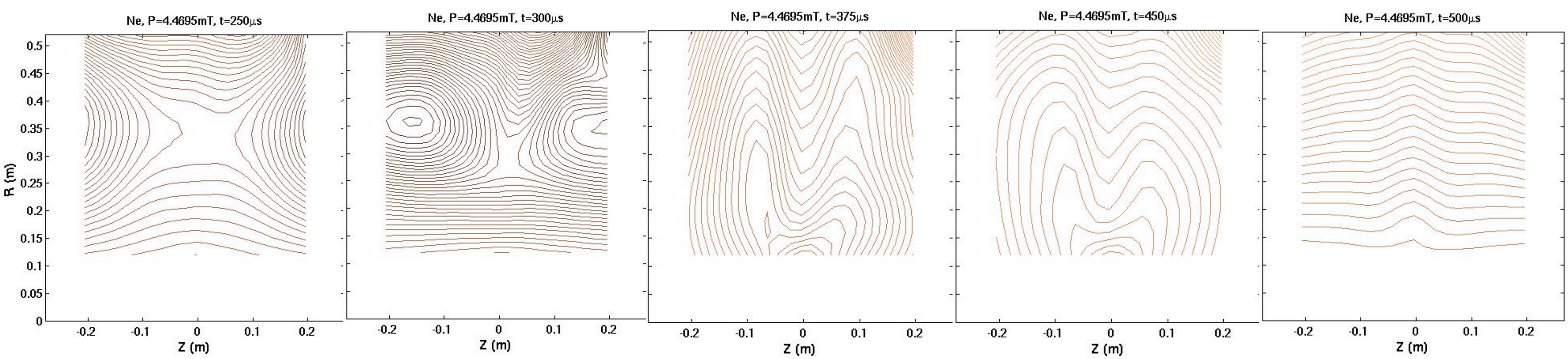
Ohmic Successful Only In Plasmas with Good Shaping

Shot 64169

Shaping Field in Series With PF Coils



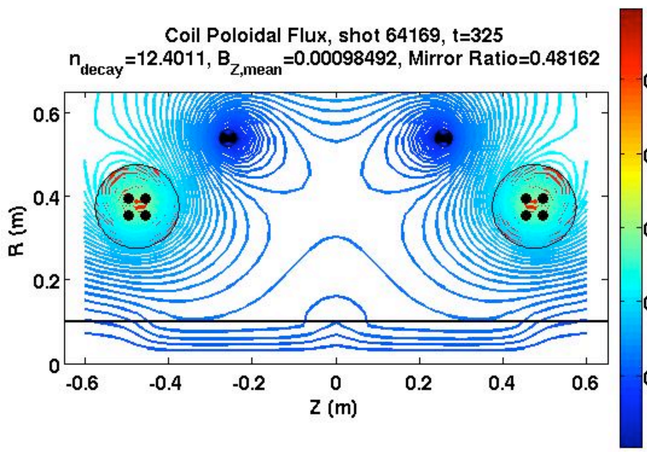
250μs   300μs   375μs   400μs   425μs

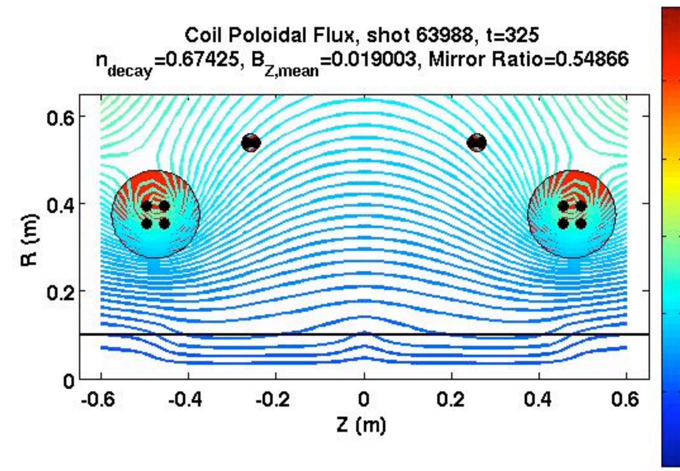


No Shaping Field With PF Coils

Shot 63988

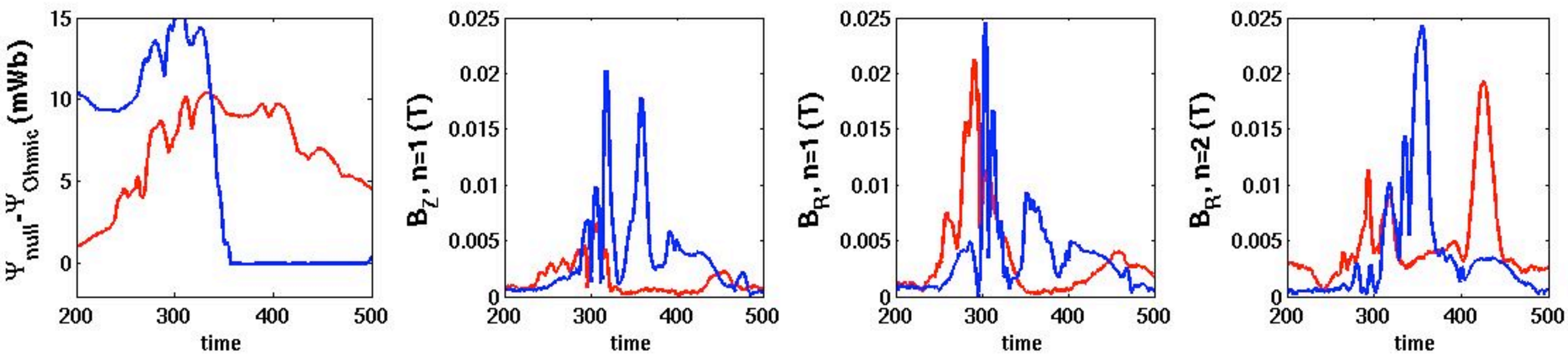
Equilibrium field shaping Eliminates Instabilities, Allowing Flux Ramp-Up





PF With SF

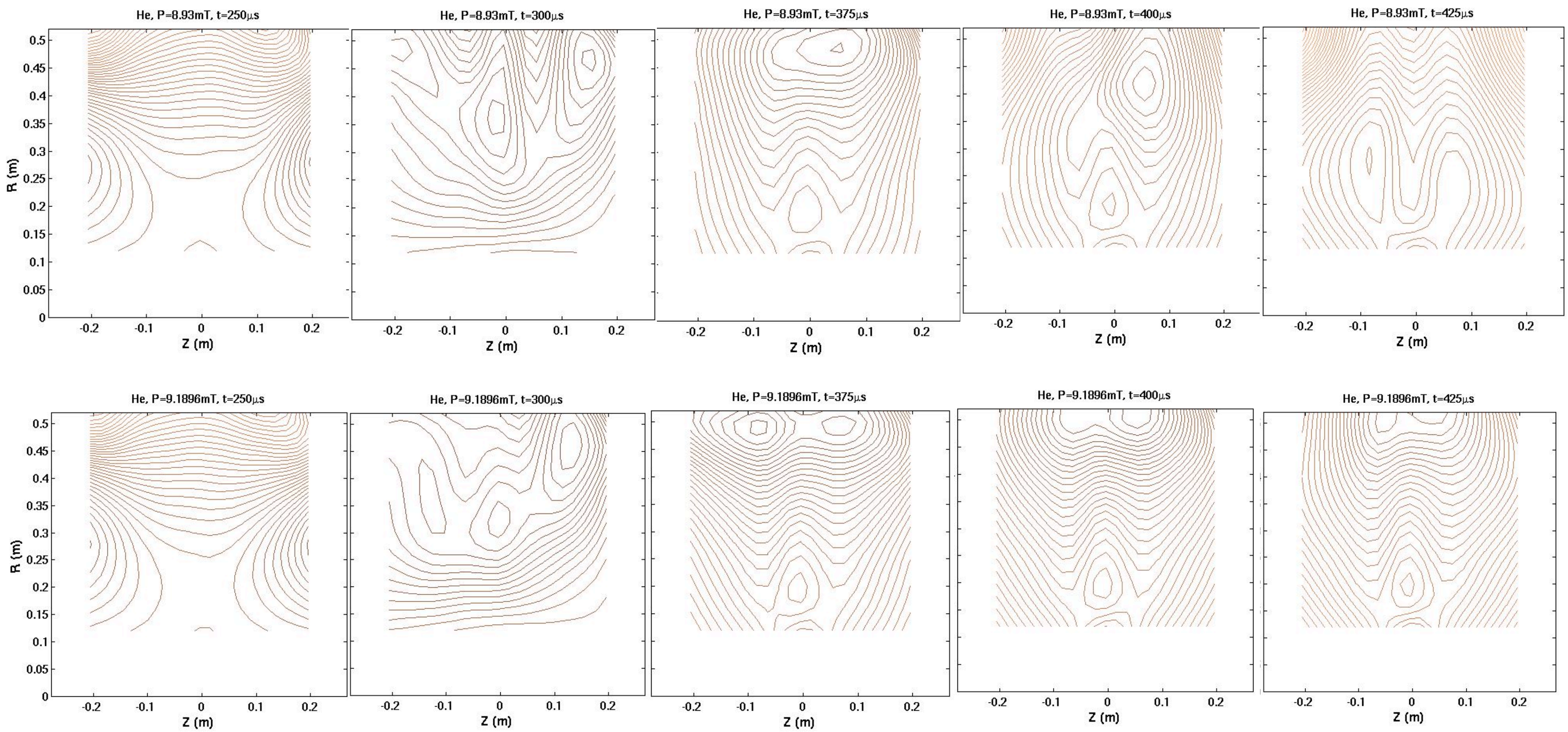
PF Alone



Outward Drift Partially Compensated by SF in Series with Ohmic

SF Coil in Series With Ohmic

Shot 64152

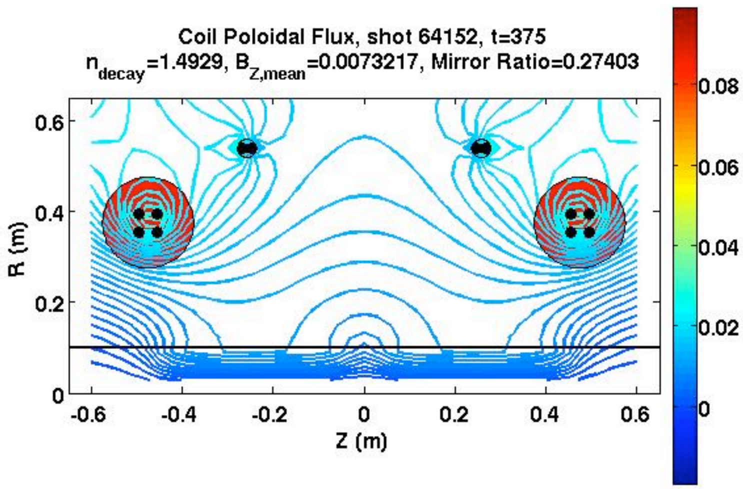


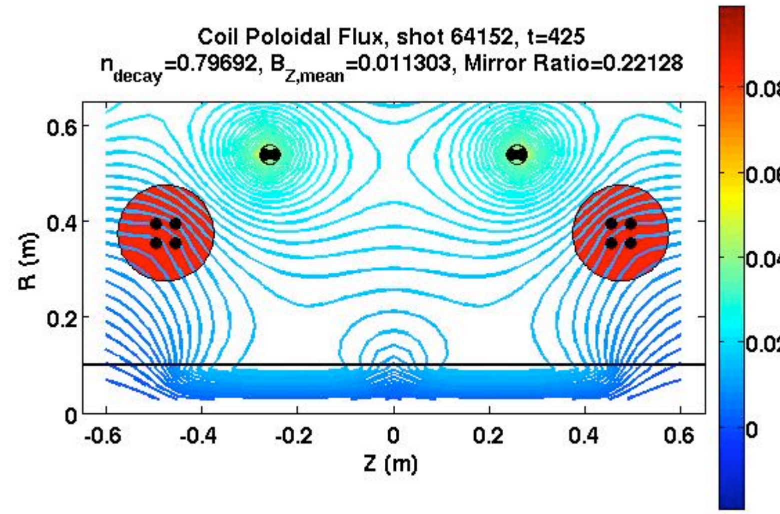
Shot 64169

No SF Coil in Series With Ohmic

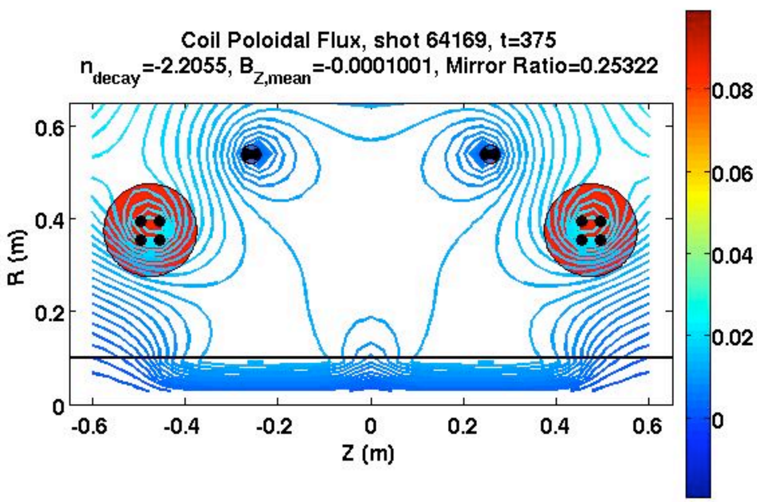
Equilibrium Field Differences With Vertical Field Cancellation

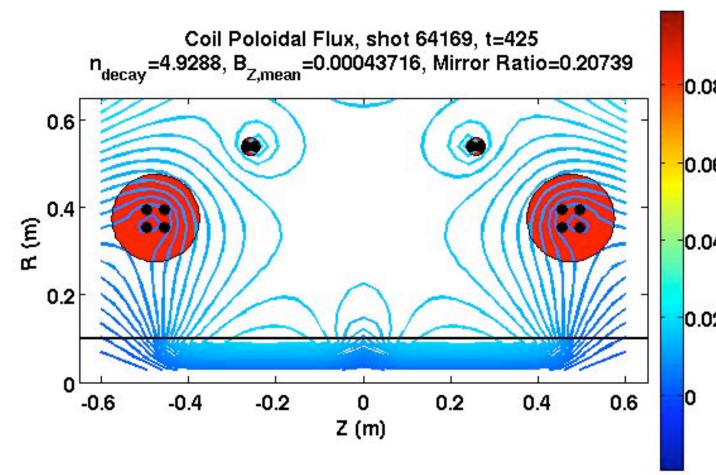
With SF in Series with Ohmic





Without SF in Series with Ohmic





Results Supportive of Proposed SPIRIT* Program

(*Self-organized Plasma with Induction, Reconnection, and Injection Techniques)

- Merging spheromaks for formation of oblate FRC.
  - ✓ Process has been demonstrated in MRX.
- Shaping and passive conductors to stabilize n=1 modes.
  - ✓ Demonstrated to work with a center column.
  - SPIRIT program calls for conducting shells.
- Transformer to increase B and heat the plasma.
  - ✓ Initial results illustrate current sustainment
  - Significant optimization yet to be done
- Neutral beam to stabilize dangerous n≥2 modes.
  - Need for beam is clearly demonstrated, especially at larger elongation.
  - Well on the way to a suitable target plasma.

Conclusions

- FRCs formed in MRX under a variety of conditions, including the unique E<0.5 regime.
- Large n=1 tilt/shift instabilities observed in MRX plasmas without passive stabilization.
- Co-interchange mode has been identified for the first time, and show to be as deadly as tilting.
- A regime with small elongation demonstrates improved stability to $n \geq 2$ axial modes and extended lifetime.
- Equilibrium reconstruction technique has been demonstrated, illustrating FRC boundary control.
- Initial experiments illustrate Ohmic sustainment.

The End

Derivation of Formula For Local Mode Growth Rate

$$\vec{\xi} = \xi_\psi \vec{e}_\psi + \xi_\phi \vec{e}_\phi + \xi_\chi \vec{e}_\chi \xrightarrow{\text{P1}} \begin{aligned} X(\psi,\chi) &= RB\xi_\psi \\ Y(\psi,\chi) &= \frac{in}{R}\xi_\phi \\ Z(\psi,\chi) &= \frac{\xi_\chi}{B} \end{aligned} \xrightarrow{\text{P2}} \begin{aligned} X &= rB\cos(\theta-\theta_0) \\ Z &= \frac{1}{B}\sin(\theta-\theta_0) \end{aligned}$$

$$T = \iiint Mn|\xi|^2 d^3x = 2\pi \int d\psi \int Jd\chi Mn\left(\frac{X^2}{R^2B^2} + \frac{Y^2R^2}{n^2} + B^2Z^2\right) \xrightarrow{\text{P2 \& P3}} T' = 2\pi \oint nM\frac{dl}{B}$$

$$\partial W = \iiint d^3x\left\{\left|\vec{Q}\right|^2 - \vec{J}\cdot\vec{Q}\times\vec{\xi} - \gamma p\left(\nabla\cdot\vec{\xi}\right)^2 + \left(\nabla\cdot\vec{\xi}\right)\left(\vec{\xi}\cdot\nabla p\right)\right\} \xrightarrow{\text{P2}} W' < -\frac{1}{2}\frac{\partial}{\partial\psi}\oint \kappa RB^2 dl$$

$$\omega^2 = \frac{W'}{T'} = \frac{-\frac{1}{2}\frac{\partial}{\partial\psi}\oint \kappa RB^2 dl}{2\pi\oint nM\frac{dl}{B}}$$

- First Written Explicitly in Ishida, Shibata, and Steinhauer, Phys. Plasmas **3**, 4278 (1996)
- Approximate agreement with Variational Analysis for Prolate FRCs in P4

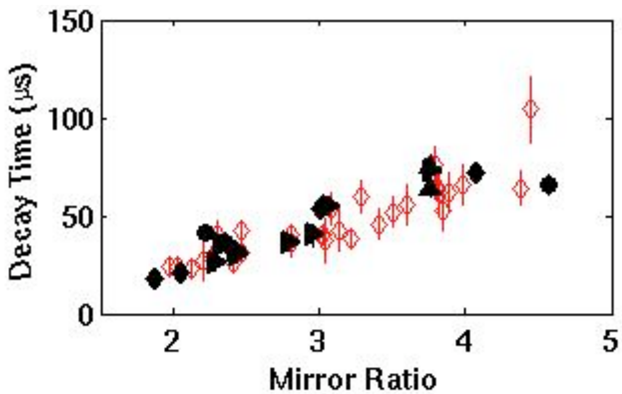
P1: I. B. Bernstein, E.A. Frieman, M.D. Kruskal, and R.M. Kulsrud, Proc. Royal Society A **244**, 17 (1958)
P2: J.R. Cary, Phys. Fluids **24**, 2239 (1981)
P3: A. Ishida, N. Shibata, and L.C. Steinhauer, Phys. Plasmas **3**, 4278 (1996)
P4: A. Ishida, N. Shibata, and L.C. Steinhauer, Phys. Plasmas **1**, 4022 (1994)

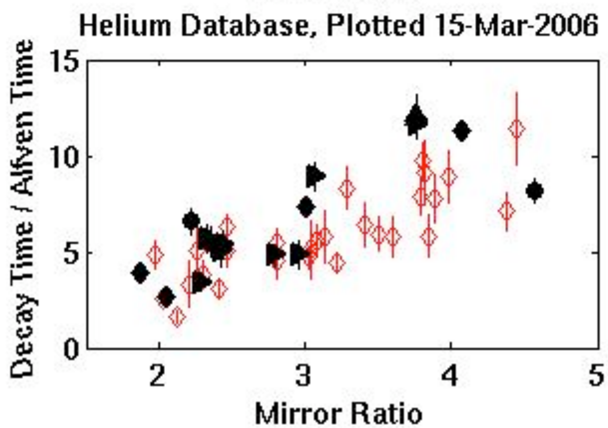
Plasma Parameters

	D_2	Helium	Neon
Fill Pressure (mTorr, molecules/cm^{-3})	8-10, 3×10^{14}	7-9.5, 2×10^{14}	3.5-5, 1.3×10^{14}
n_e, T_e	1×10^{14}, 10	$(1-2) \times 10^{14}$, 10-14	$(2-3) \times 10^{14}$, 10
$B_{Z,Sep}$ (Gauss)	.03-.02	.03-.02	.03-.02
V_A (m/s)	$3-2 \times 10^4$	$2-3 \times 10^4$	1×10^4
Z_S (m)			
τ_A (μs)	3-7	5-10	10-30
$\lambda_{i,mfp}$ (cm)	4	3	2
$\omega_{ci}\tau_i$	1	1	0.5
$\bar{s}$		3-1	1.5-1
E		.6-.3	.6-.3
$\bar{s}/E$		7-4	2-3

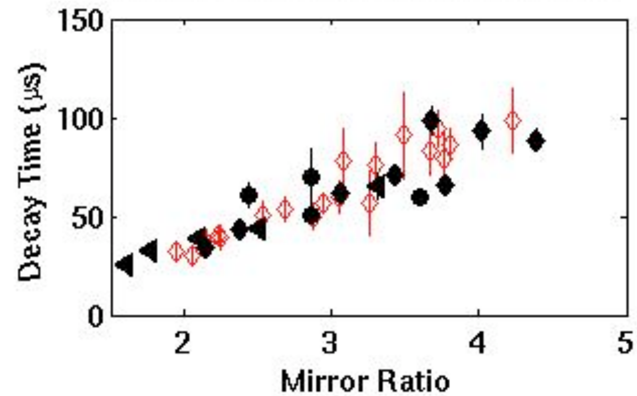
Plasma Lifetime Longest At Large Mirror Ratio

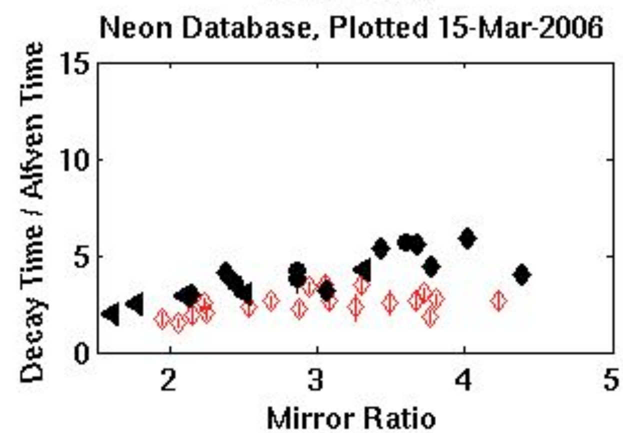
Helium





Neon





2 Trends

- Lifetime increases with larger mirror ratio.
- Center column does not substantially increase the lifetime.

Condition For Kinetic Effects

Kinetic effects matter when: $\gamma = C\frac{V_A}{Z_S} = C\frac{V_A}{ER_S} < \omega^*$

The Diamagnetic drift Frequency is: $\omega^* = \vec{k}\cdot\vec{V}_D = \frac{T}{eBL_p}k = \frac{kv_{th}^2}{2L_p\omega_{ci}}$

Note that for β~1: $v_{th} \sim V_A$

The wavenumber is related to the Major radius as: $k = \frac{n}{R_{null}}$

The separatrix radius is related to the null radius by: $R_0 = 1.4R_{null}$

Combine these as:

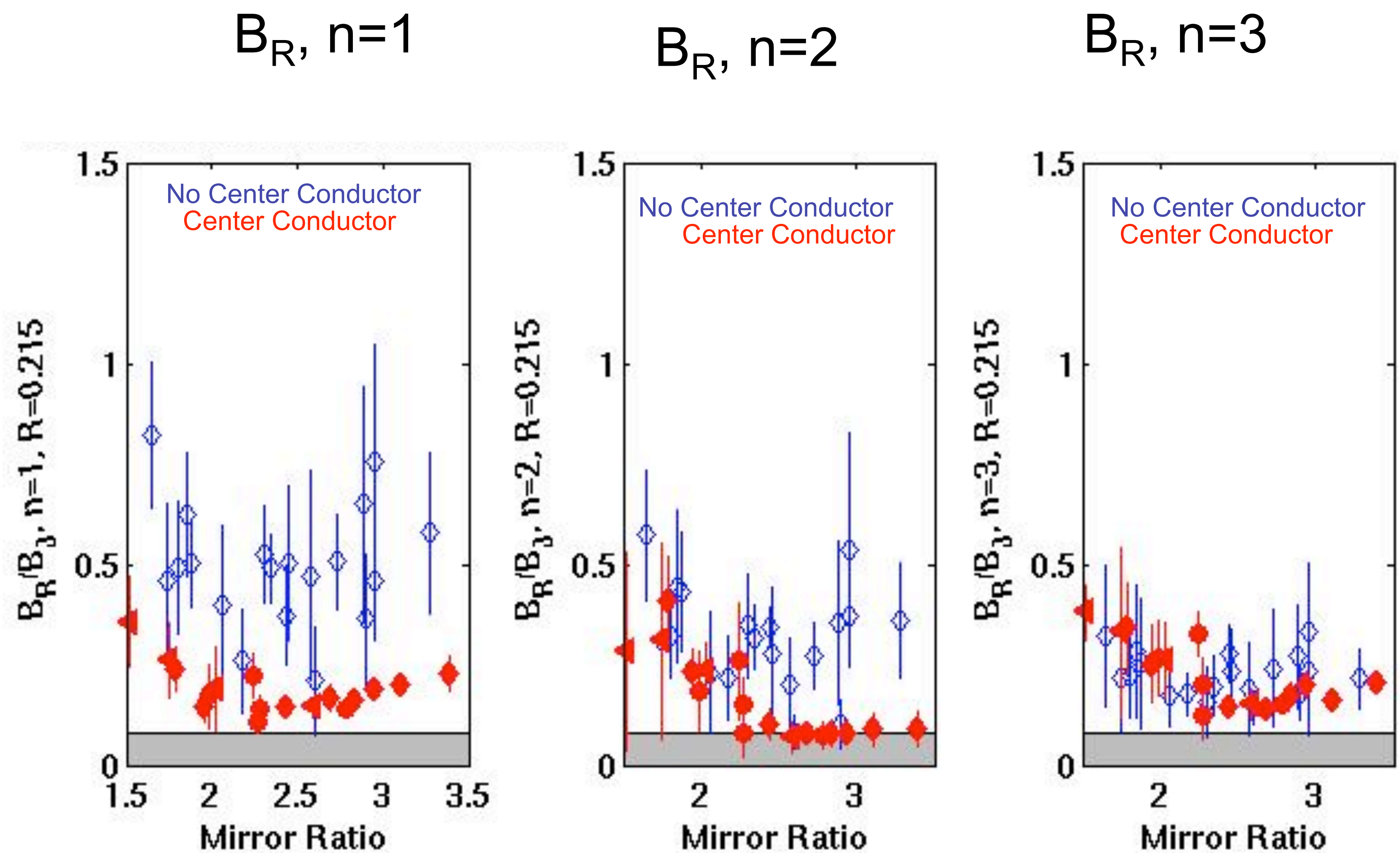
$$\gamma < \omega^*$$

$$C\frac{V_A}{ER_S} < \frac{kv_{th}^2}{2L_p\omega_{ci}}$$

$$\frac{C}{1.4E} < \frac{n\rho_i}{2L_p}$$

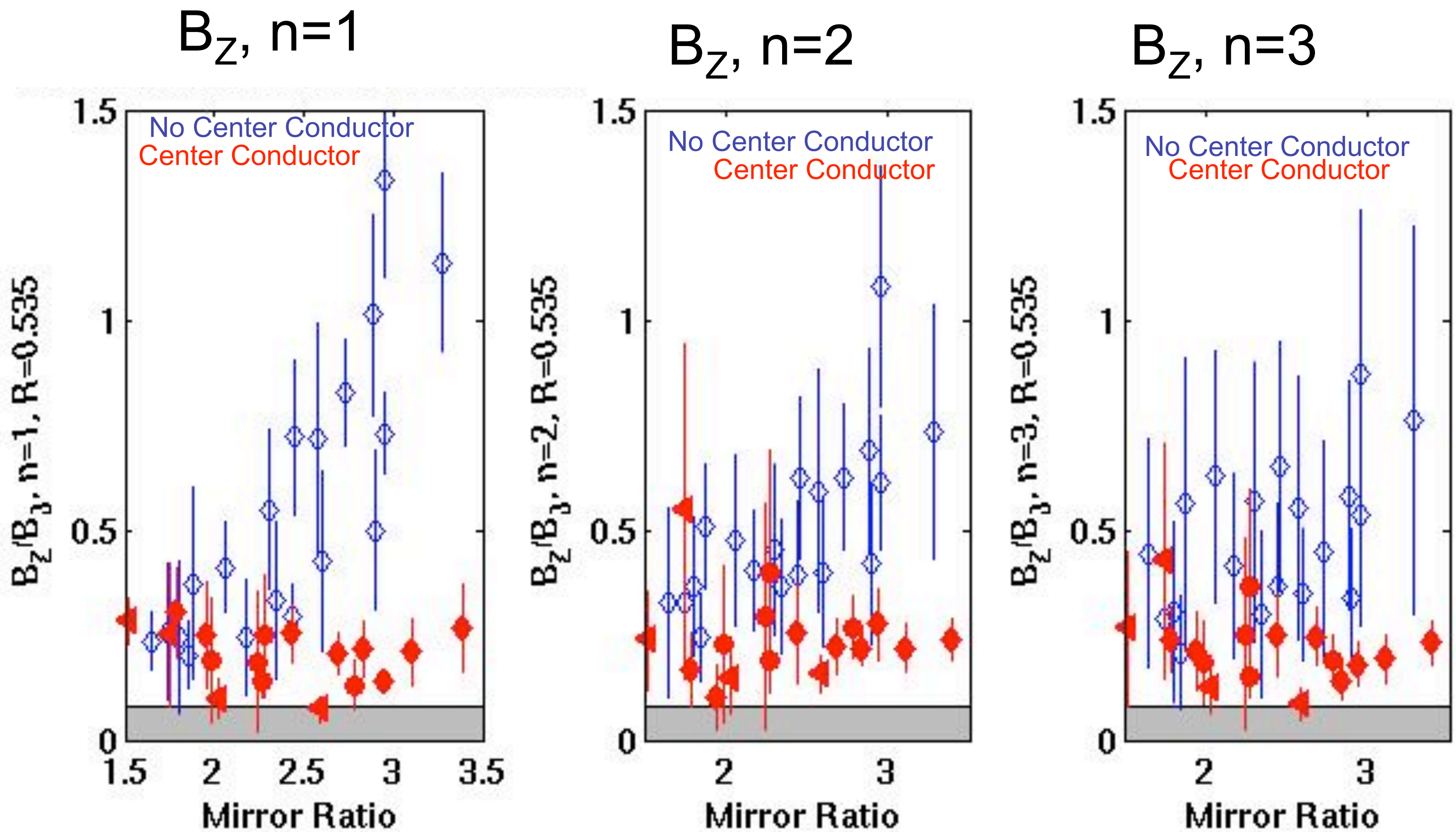
$$\frac{\bar{s}}{E} < \left(\frac{1.4}{2}\right)n$$

Neon Tilting Suppressed With Center Column



Neon

Center Column Reduces Rigid Body Shift Signature



Neon

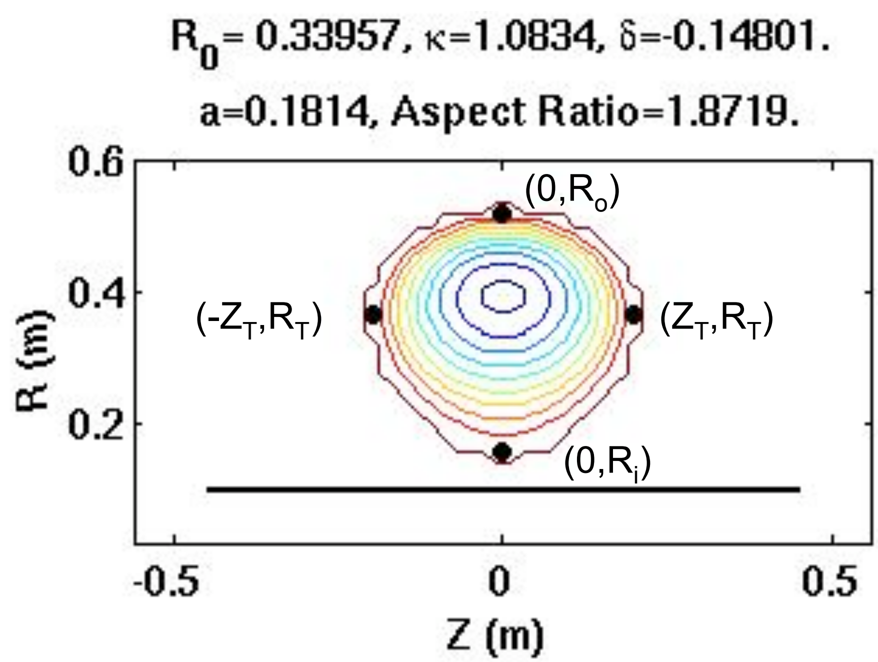
Analytic Equilibrium Model by Zheng Provides Approximation to Current Profile

6 Fit parameters in Model:

- 4 Parameters determine the Plasma shape
- 2 Parameters determine Pressure and Toroidal field:

$$-(2\pi)^2 \mu_0 \frac{dp}{d\psi} = A_1$$

$$(2\pi\mu_0)^2 F \frac{dF}{d\psi} = A_2$$



Poloidal flux specified as:

$$\Psi = c_1 + c_2 R^2 + c_3(R^4 - 4R^2Z^2) + c_4\left[R^2 \ln(R) - Z^2\right] + \frac{A_1}{8} R^4 - \frac{A_2}{2} Z^2$$

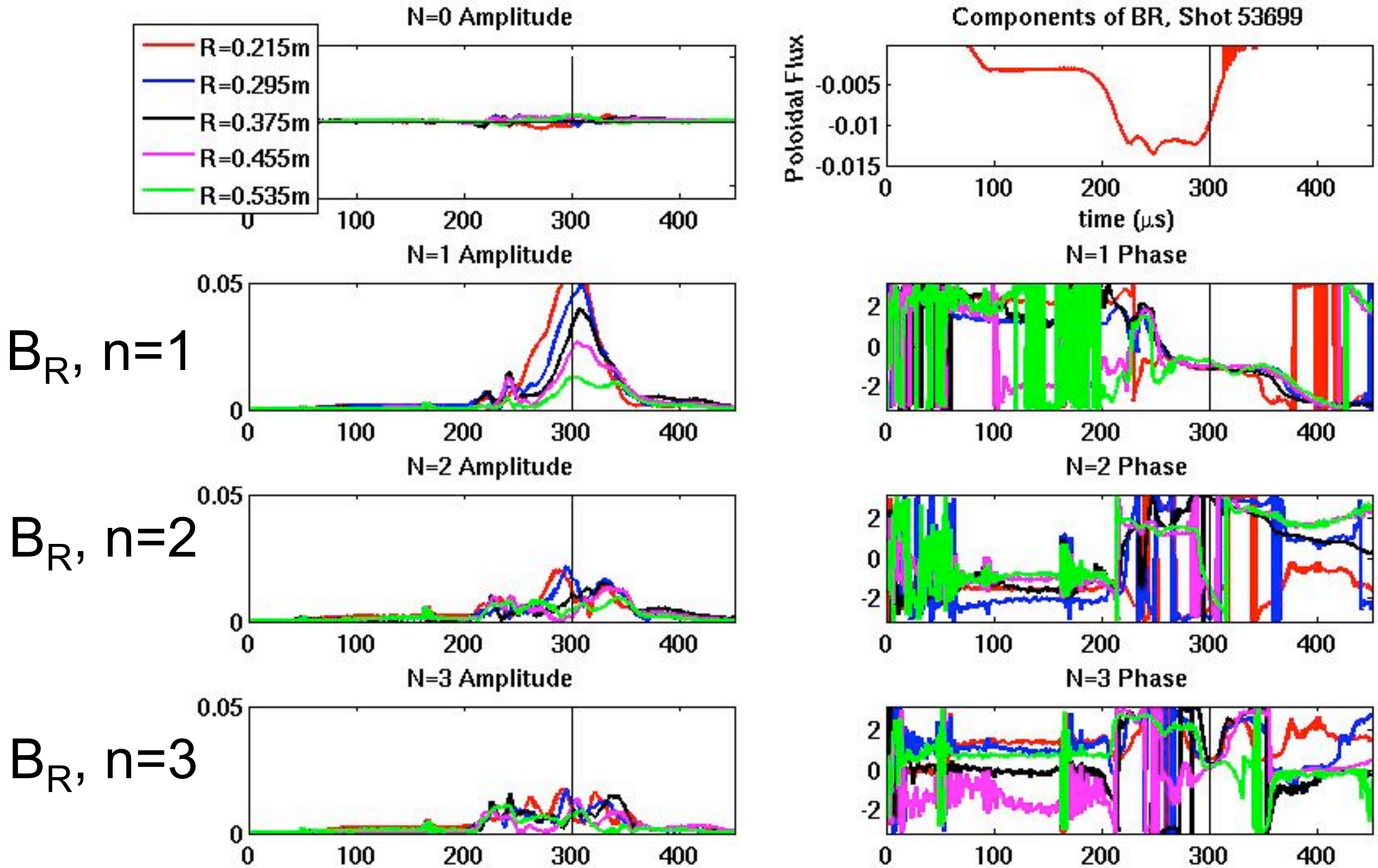
Magnetic Field:

$$B = F\nabla\phi + \frac{1}{2\pi}\nabla\psi \times \nabla\phi$$

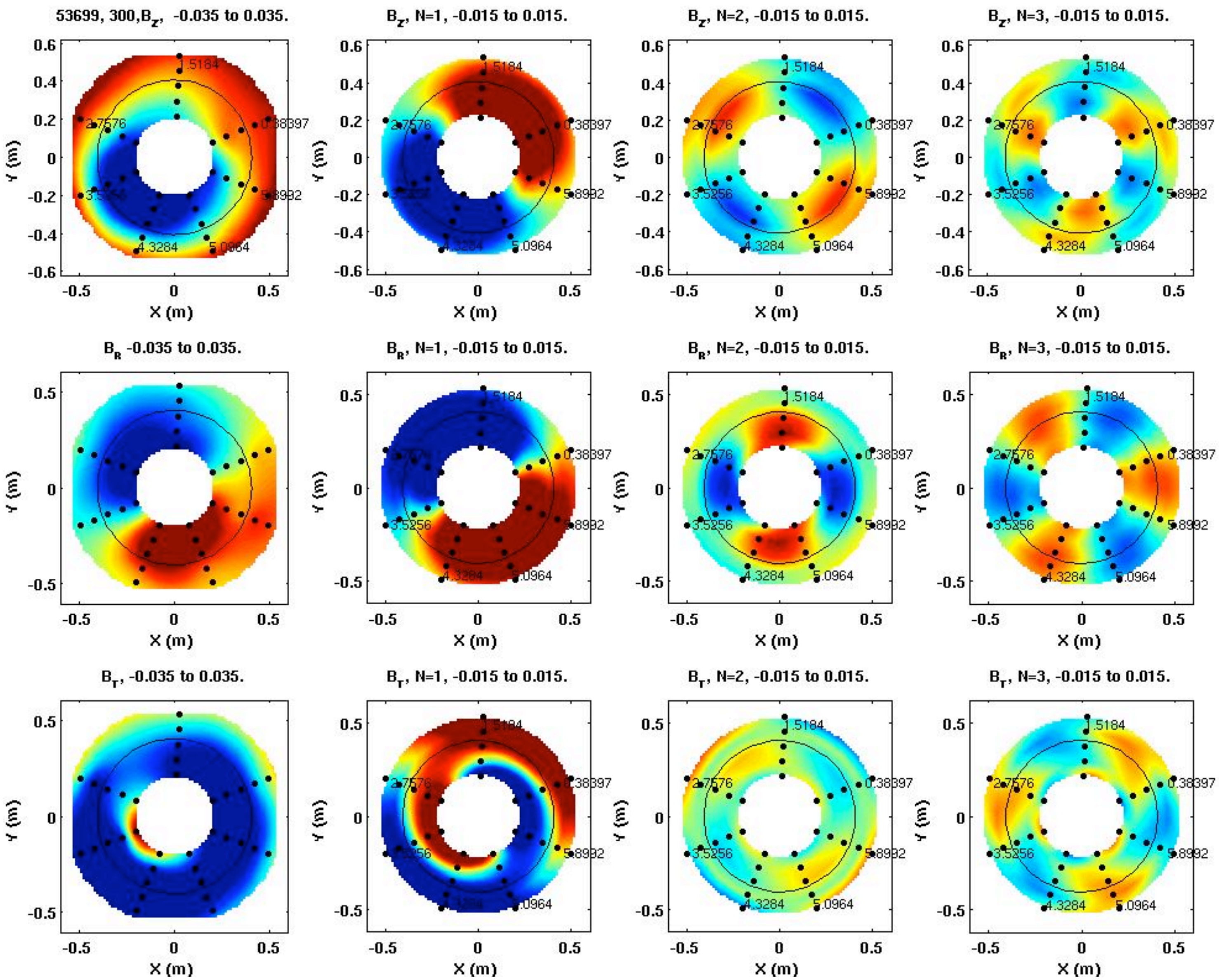
Used to Generate Initial Equilibrium for MRXFIT

S.B. Zheng, A.J. Wooten, & E. R. Solano, Phys. Plasmas **3**,1176 (1996)

Spheromak Tilt is Dominated by n=1

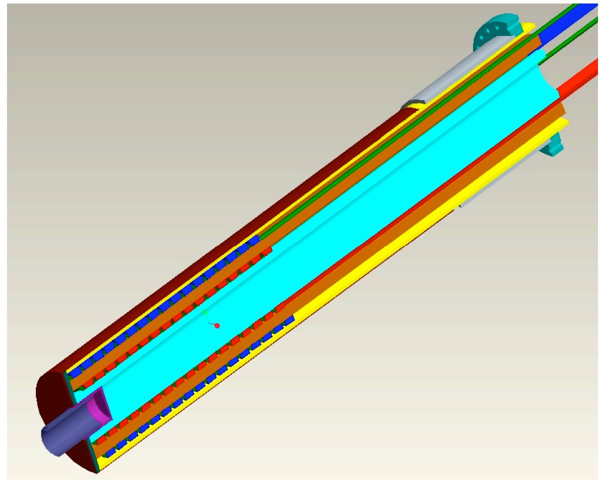


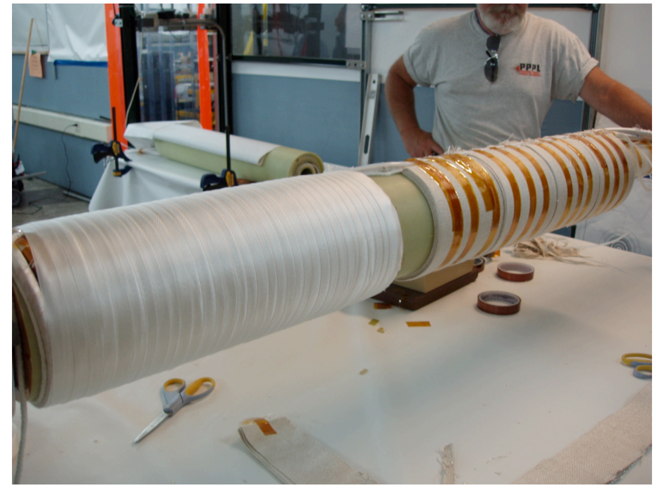
Strong n=1 during Tilting Spheromak



Transformers Used to Sustain Future FRC Plasmas

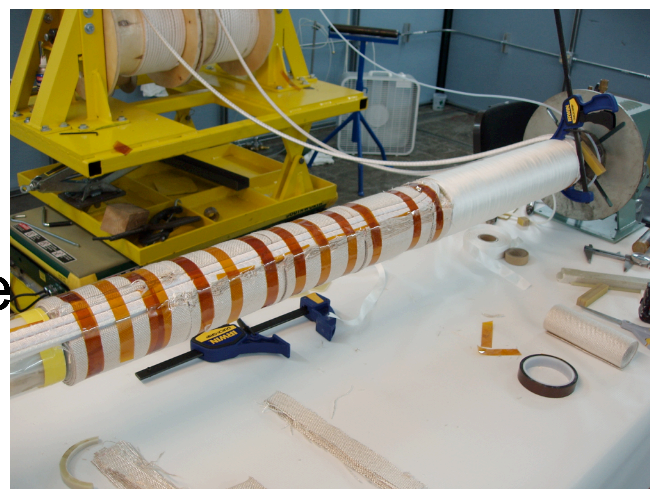
- Two transformers, one inserted from each end of MRX.

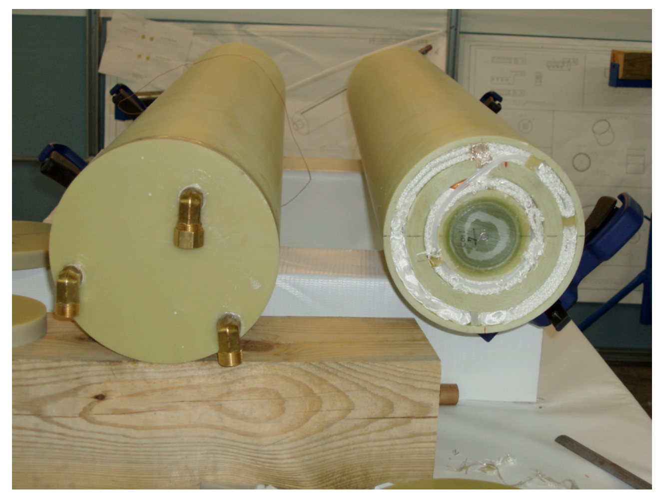




- Total flux of 100mWb at 100kA.

- 10T on axis at 100kA.



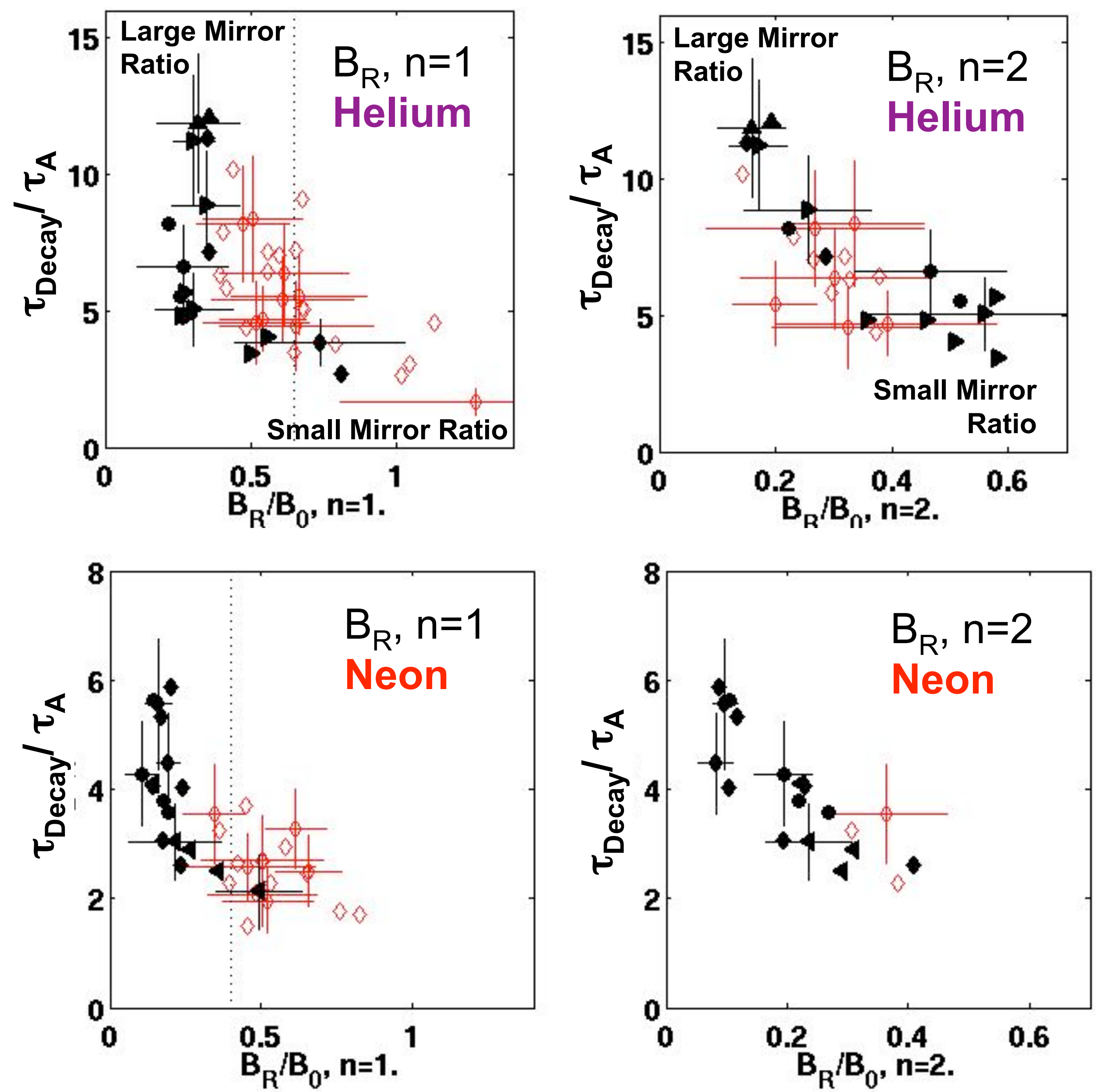


- Only vacuum jacke remains to be completed

Lifetime is Strongly Correlated with B_R Perturbations

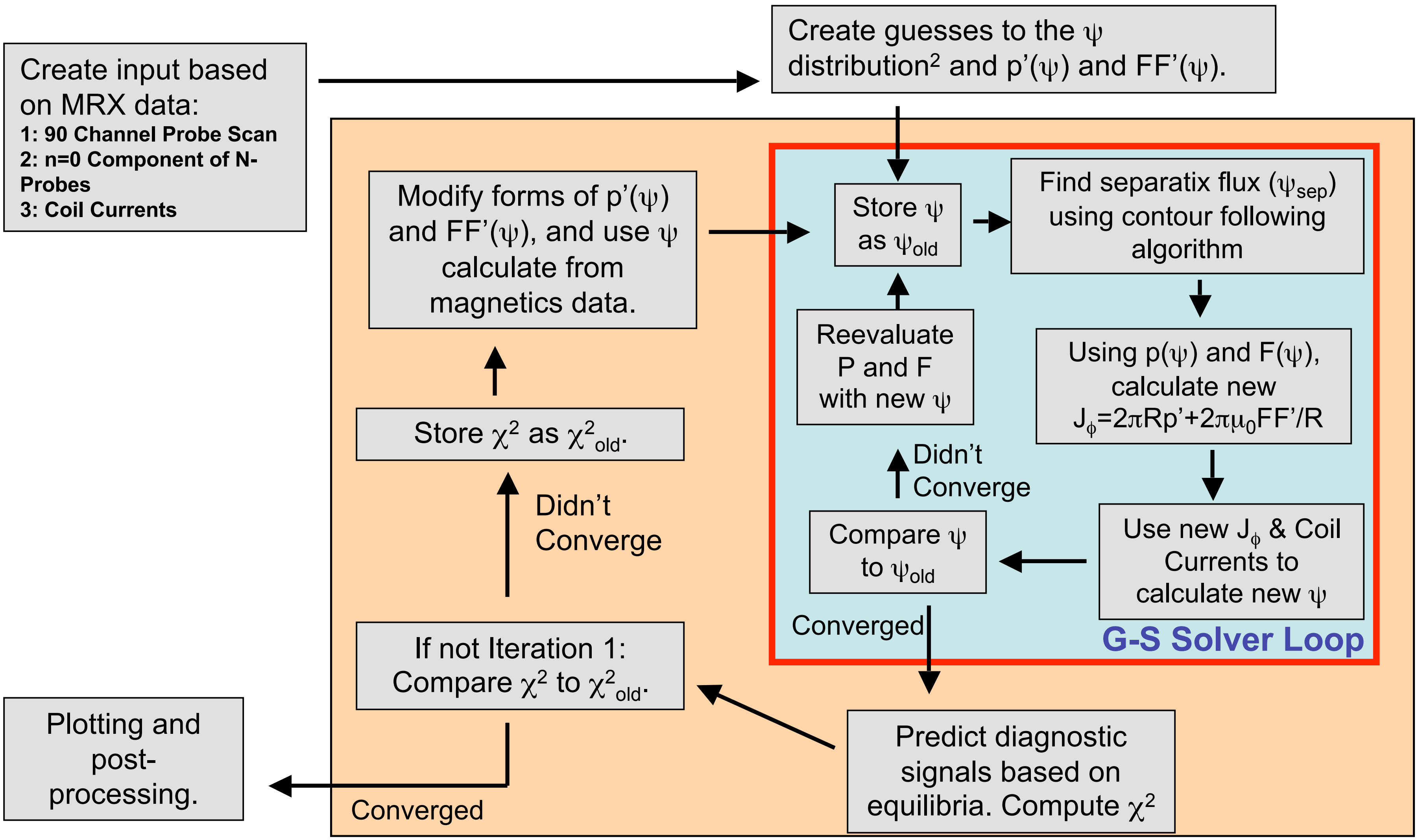
With & Without Center-Column **Helium**

With & Without Center-Column **Neon**



Helium & Neon

MRXFIT[1] Solves G-S Eqn. Subject to Magnetic Constraints



1) J.K. Anderson et.al. Nuclear Fusion **44**, 162 (2004)
2) S.B. Zheng, A.J. Wooten, & E. R. Solano, Phys. Plasmas **3**,1176 (1996)

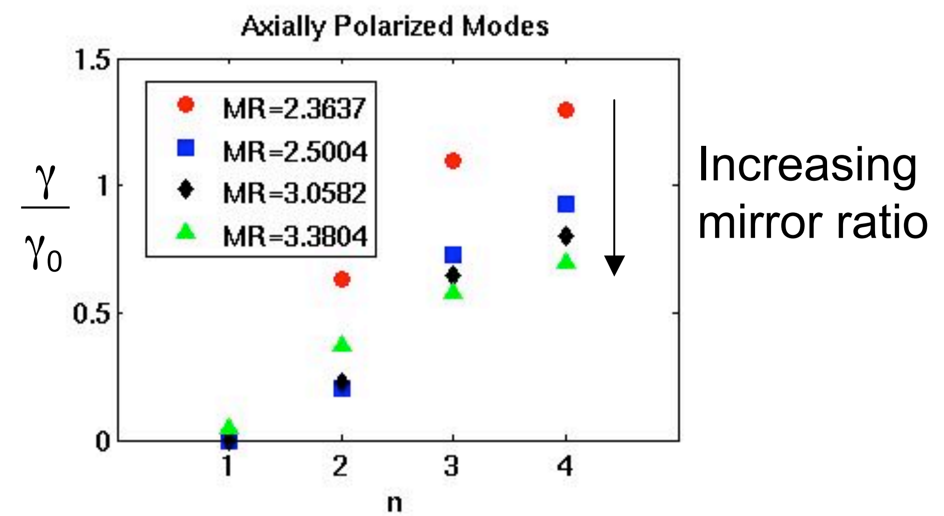
HYM Calculations Indicate Reduced Growth Rates at Larger Mirror Ratio

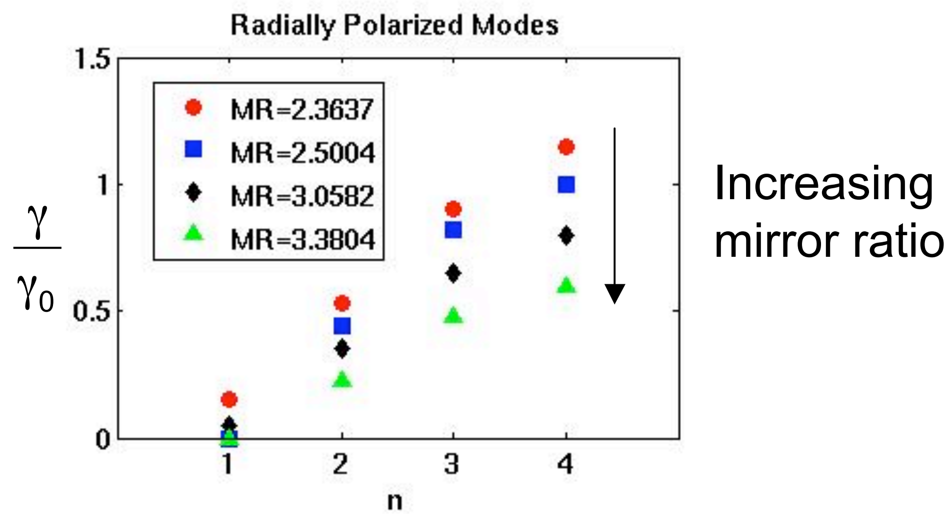
Consider MR=2.5 case

N	γ/γ_o, Measured	γ/γ_o, HYM
2, axial	1	0.3
3, axial	1.4	0.7

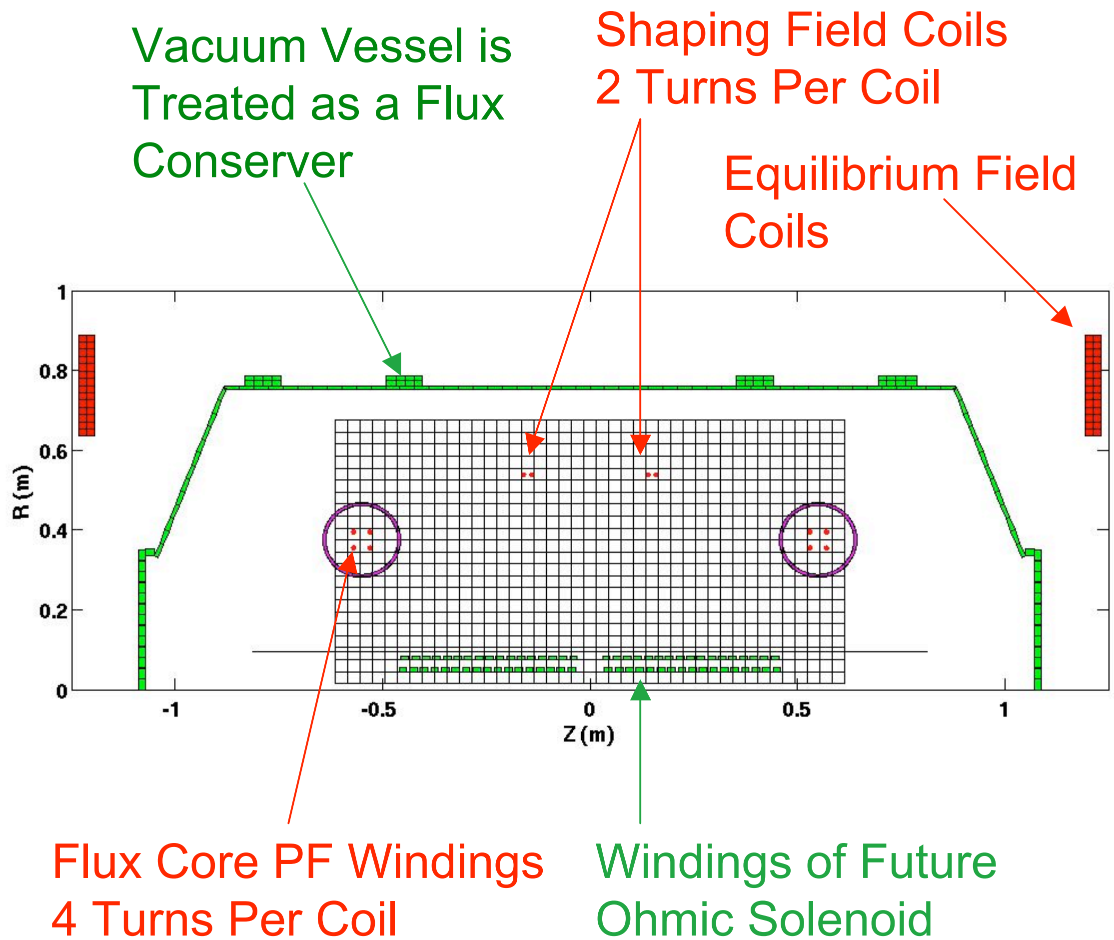
Consider MR=3.0 case

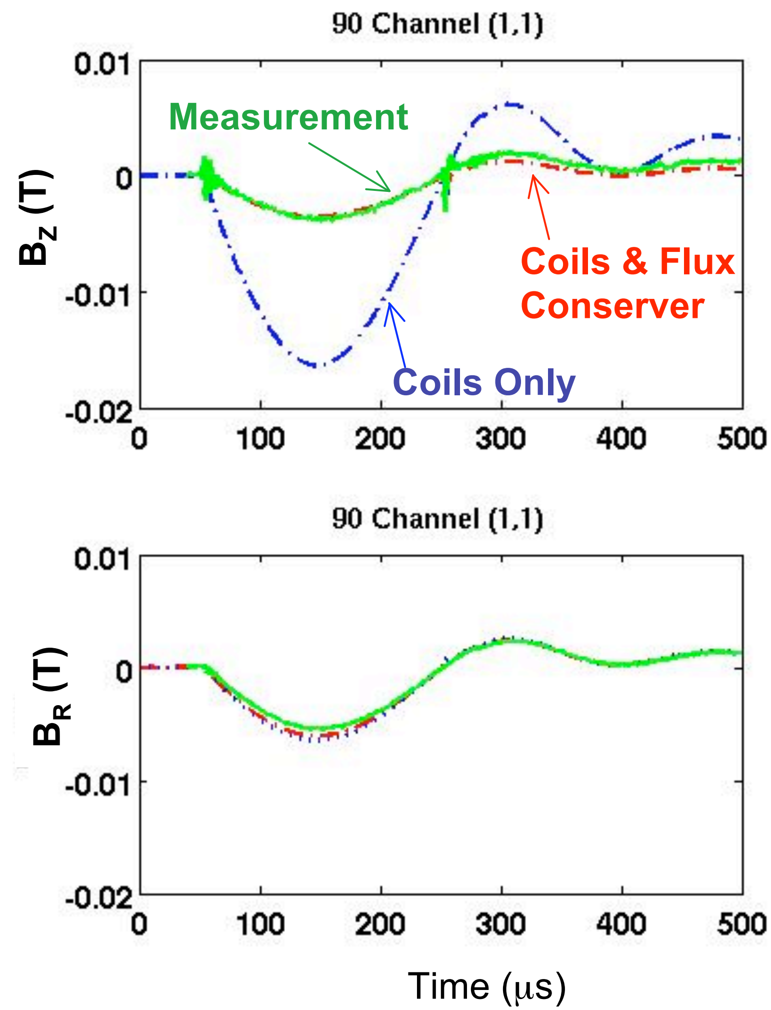
N	γ/γ_o, Measured	γ/γ_o, HYM
2, radial	1.2	0.65
3, radial	2	0.8





Fields Calculated From Axisymmetric Model With Flux Conserving Vessel

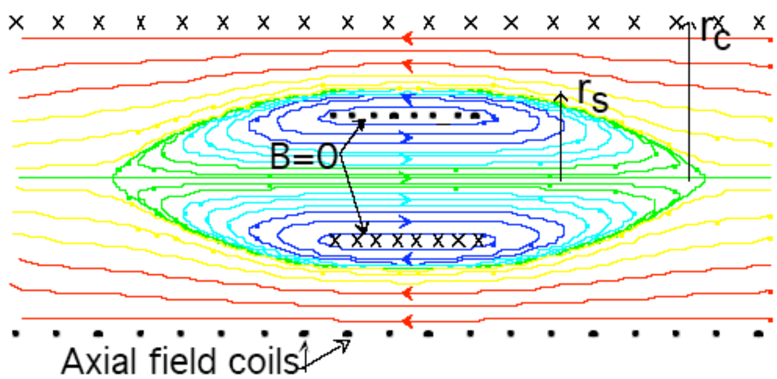


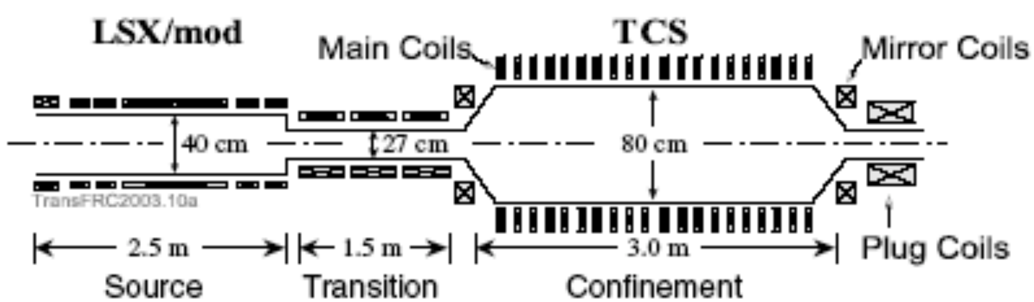


*J.K. Anderson et.al. Nuclear Fusion **44**, 162 (2004)

FRCs have Potential Advantages as Fusion Reactors

FRC→toroidal plasma configuration, with toroidal current, but minimal toroidal field.



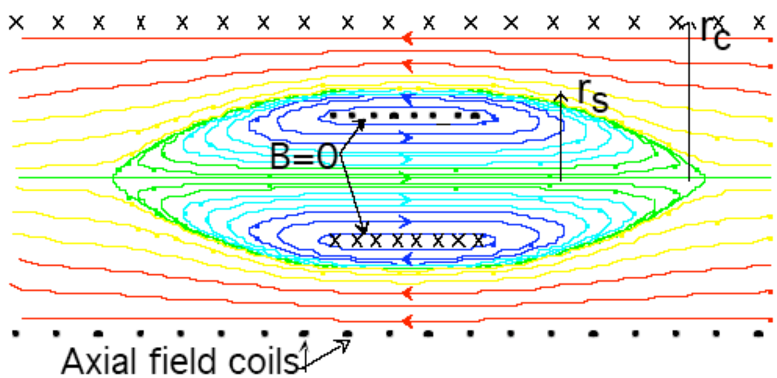


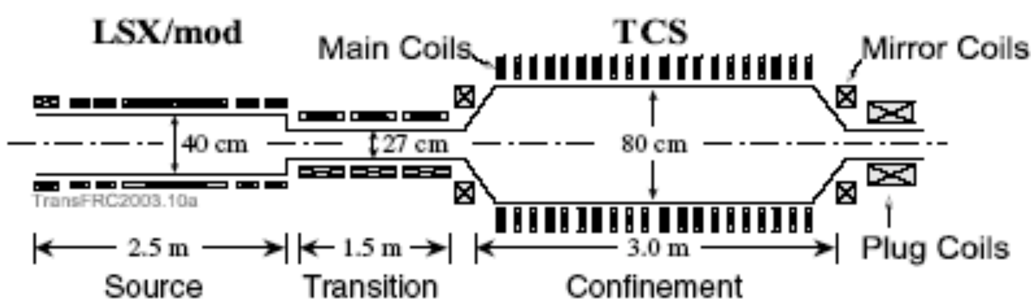
H. Guo, Phys. Rev. Lett. **92**, 245001

- Intrinsically high β (β~1)
- Natural divertor structure
- Only circular axisymmetric coils
- No material objects linking plasma column (ideally)
- Translatable (formation and fusion in different places)

FRCs have Potential Advantages as Fusion Reactors

FRC→toroidal plasma configuration, with toroidal current, but minimal toroidal field.

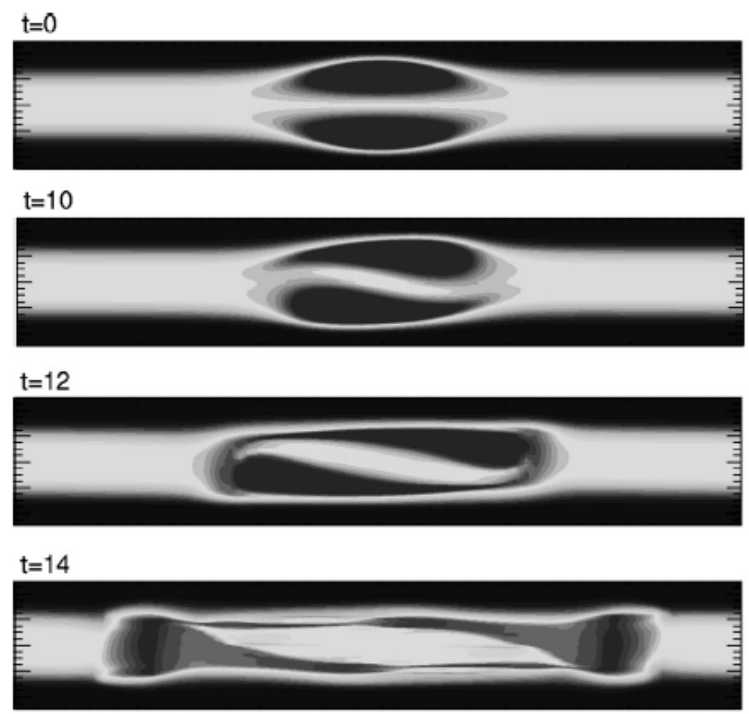




H. Guo, Phys. Rev. Lett. **92**, 245001

Problem:

Predicted to Be MHD Unstable



Pressure Contours, Disruptive Internal Tilt
Belova et al, Phys. Plasmas 2000