BIFURCATED 3D EQUILIBRIA IN THE LHD AND ITER

P.R. Garabedian, N.Y.U.

The NSTAB computer code applies the MHD variational principle to calculate equilibrium and stability of toroidal plasmas in three dimensions. Differential equations are solved in a conservation form that describes force balance correctly across islands that are treated as discontinuities. The method has been applied to the LHD stellarator, the DIII-D tokamak, and ITER. Sometimes the solution of the equations turns out not to be unique, and there may exist bifurcated equilibria that are nonlinearly stable when other theories predict linear instability. Resulting estimates of stability for the DIII-D turn out to be well correlated with observations in the experiment, and the calculations are consistent with measurements of β as high as 4% in the LHD.

The Maxwell stress tensor

$$\mathbf{T} \;=\; \mathbf{B}\,\mathbf{B} \;-\; (B^2/2 \;+\; p)\,\mathbf{I}$$

enables one to put the partial differential equations describing MHD force balance in the conservation form

$$\nabla\cdot\mathbf{T} \;=\; 0\,, \quad \nabla\cdot\mathbf{B} \;= 0\;.$$

Comparable finite difference equations have the advantage that when they are summed over a test volume they telescope down to a correct statement over the boundary of force balance

$$\int\!\!\int\; \mathbf{T}\cdot\mathbf{N}\,dS \;=\; 0\;.$$

We apply this method to calculate discontinuous solutions

$$\mathbf{B} = \nabla s\times\nabla\theta = \nabla\phi + \zeta\nabla s$$

of the MHD variational principle

$$\delta\int\!\!\int\!\!\int (B^2/2 - p(s))\,dV = \; 0\;.$$

Islands occur where the spectrum B_{mn} defined by

$$\frac{1}{B^2} = \sum B_{mn}(s)\cos\left[m\theta - (n-\iota m)\phi\right]$$

activates small denominators of the parallel current

$$\frac{\mathbf{J}\cdot\mathbf{B}}{B^2} = p'\sum\frac{mB_{mn}(s)}{n-\iota m}\;\cos\left[m\theta-(n-\iota m)\phi\right]\;.$$

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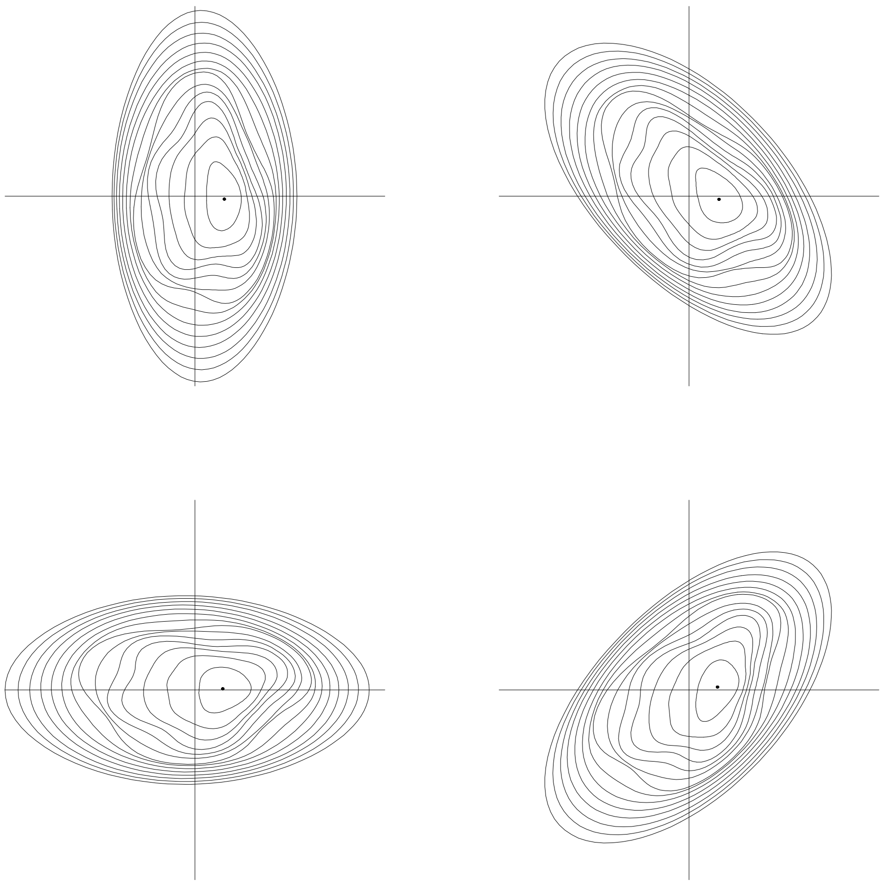
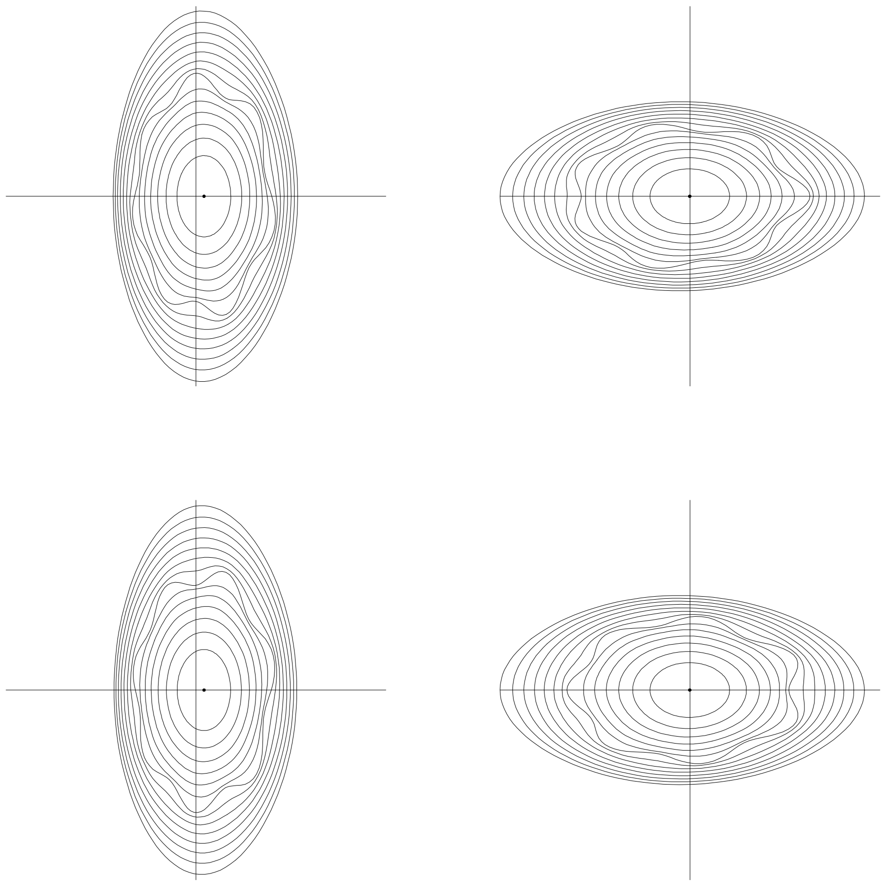
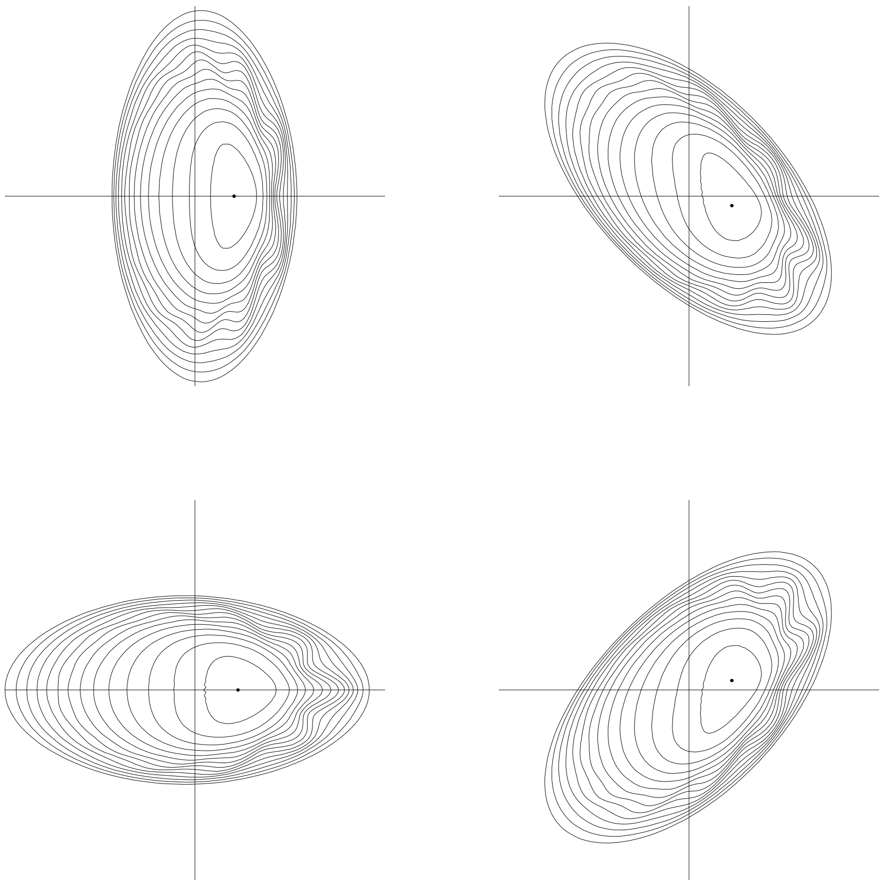


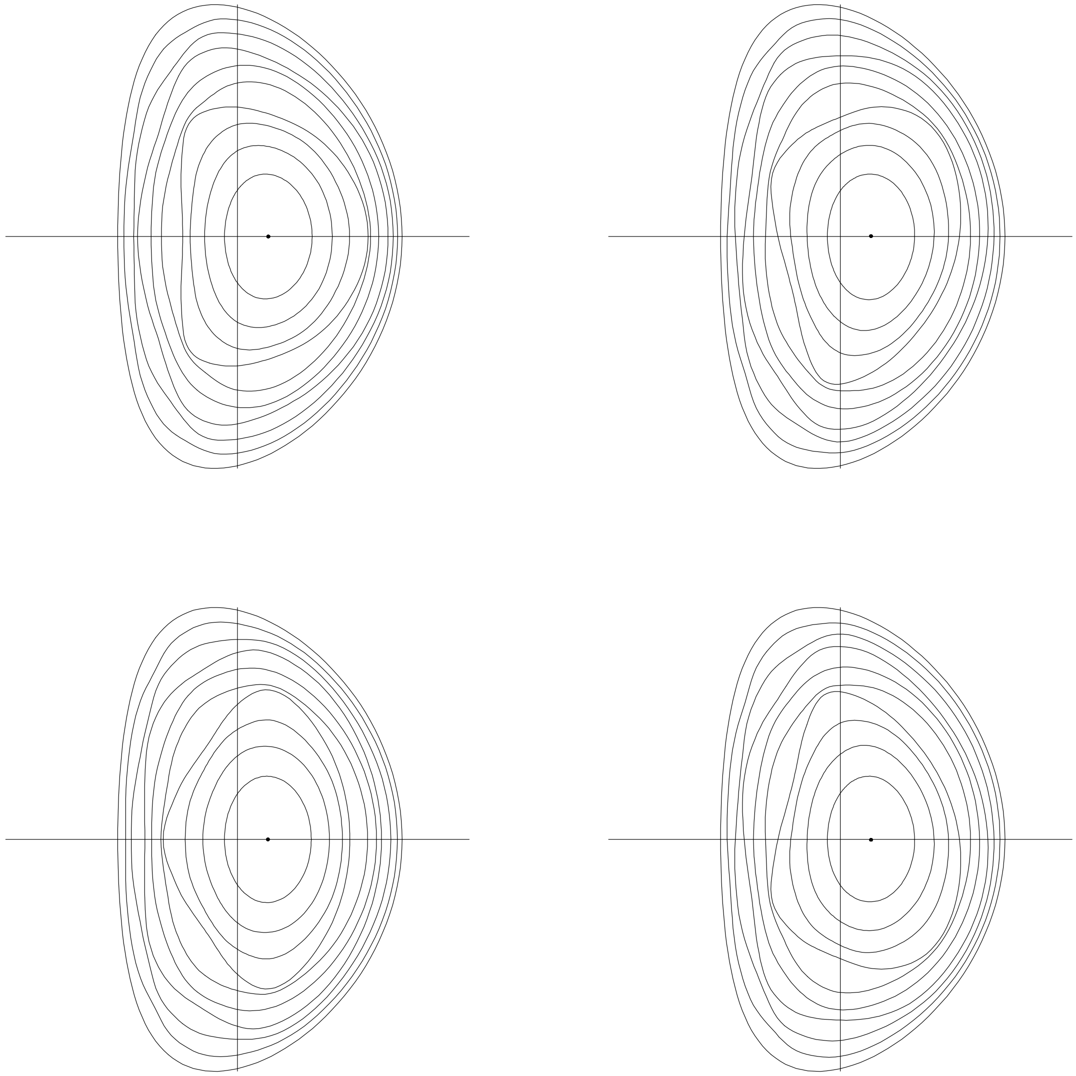
Flux surfaces at four cross sections over five field periods of a bifurcated LHD equilibrium at $\beta = 0.02$ with the magnetic axis located at major radius $R = 3.6$ m. For a triangular pressure profile $p = p_0(1 - s^{0.7})$ like observed values of the electron temperature, the solution is linearly unstable but nonlinearly stable. The existence of several solutions implies linear instability because there are saddle points in the energy landscape.



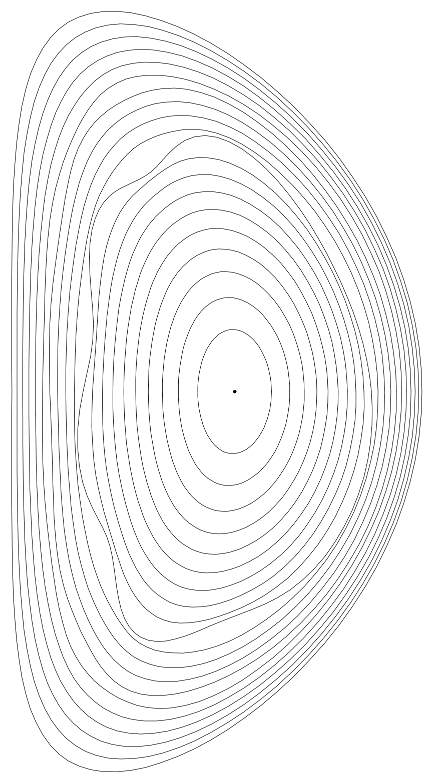
Poincaré sections of the magnetic surfaces of a bifurcated LHD equilibrium at a β of 2.5% in the range observed by the experiment. The islands at a resonance where $\iota = 5/7$ represent a nonlinearly saturated unstable mode that does not limit the solution significantly.



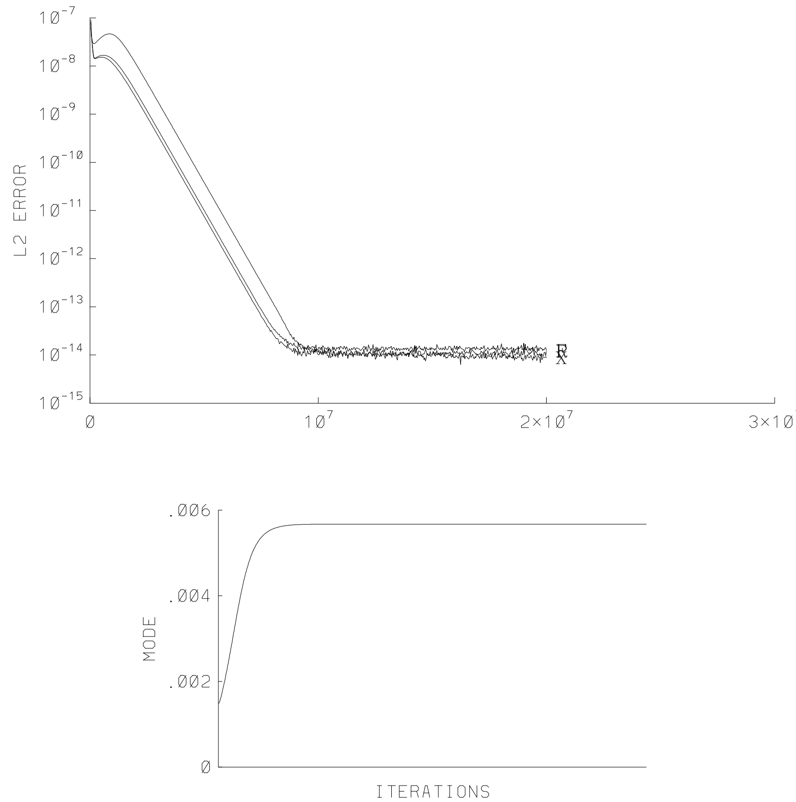
Poincaré map of the flux surfaces at four cross sections over one field period of a bifurcated LHD equilibrium at $\beta = 0.04$ with the magnetic axis shifted inward to a position with major radius $R = 3.6$ m. For a broad pressure profile $p = p_0(1 - s^2)$, this exceptionally accurate solution has magnetic surfaces with ripple suggesting that it is only marginally stable. At these conditions a similar plasma was observed in the experiment.



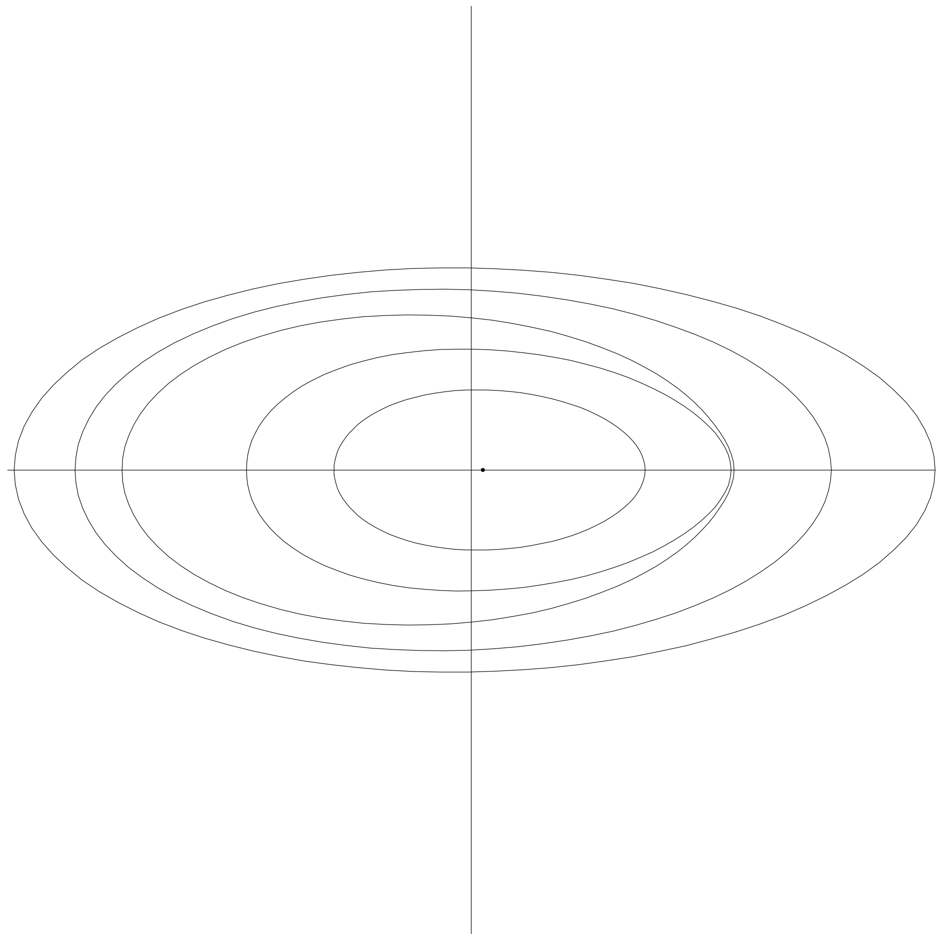
Four cross sections of the flux surfaces over half the torus of a bifurcated DIII-D equilibrium at $\beta = 0.02$ with $p = p_0(1 - s^{1.1})^{1.1}$ and with net current bringing the rotational transform into the interval $0.9 > \iota > 0.3$. There is a large $m = 3,\ n = 2$ magnetic island at $\iota = 2/3$ in the solution. The diagram displays what are presumably neoclassical tearing modes.



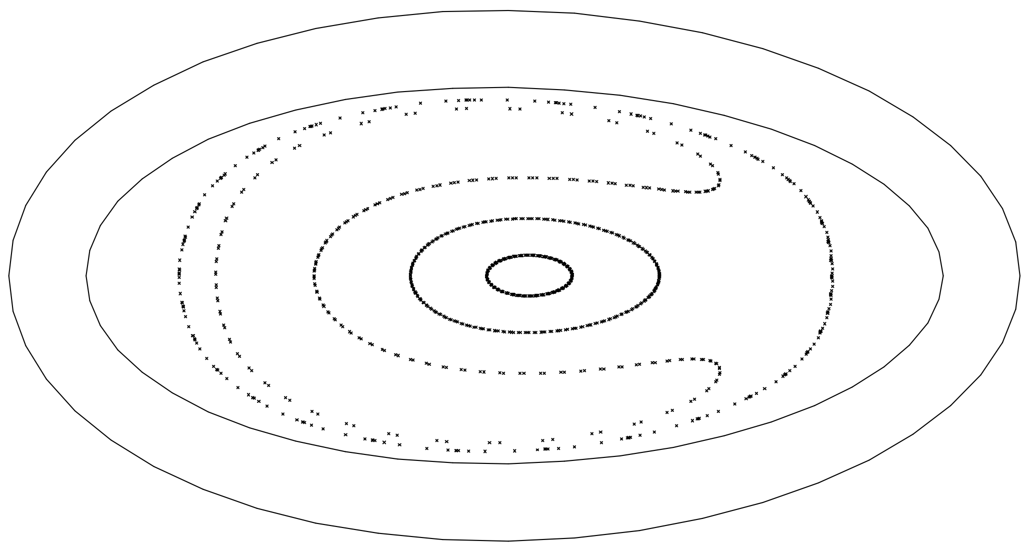
Poincaré section of the flux surfaces of a bifurcated DIII-D equilibrium at $\beta = 0.095$ with $p = p_0(1 - s^{1.1})^{1.1}$ and with net current bringing the rotational transform into the interval $0.9 > \iota > 0.4$. There is a helical island in this fully converged three-dimensional solution of an axially symmetric MHD problem. Shaping coils have been applied to reduce the aspect ratio.



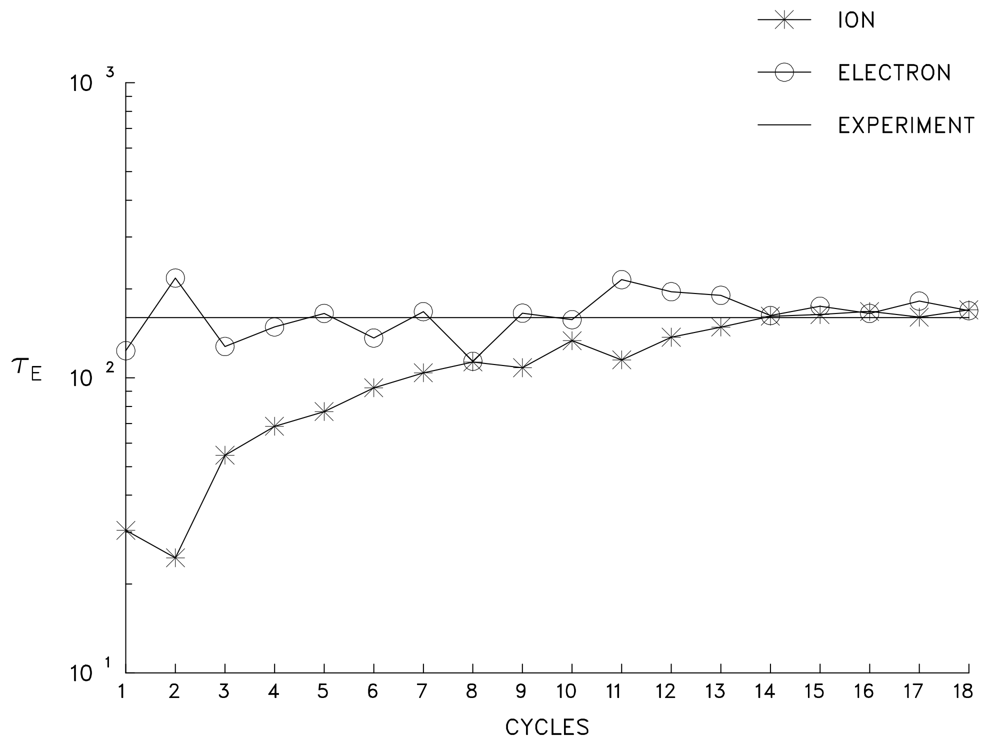
NSTAB iterations to a bifurcated equilibrium for a tokamak problem with axially symmetric boundary conditions. A forcing term was used early in the run to trigger a mode that appears as a discontinuity in the three-dimensional solution. This double precision calculation converges to the level of round off error, showing that the discrete problem has been solved.



Zero β calculation of a Poincaré section of flux surfaces for a Shohet stellarator with reversed poloidal field. Despite the nested surface hypothesis made in the NSTAB code that was used, a large magnetic island can be discerned where the rotational transform ι changes sign.



Tracing of magnetic lines through a Poincaré section of a Shohet stellarator with reversed poloidal field. A large magnetic island is seen where the rotational transform ι changes sign. The plasma surface is plotted together with a control surface used for Biot-Savart computation of the vacuum magnetic field.



Cycles of a calculation of the energy confinement time τ_E in milliseconds for an NBI shot of the LHD experiment using a quasineutrality algorithm to adjust the electric potential Φ. Oscillations of Φ along the magnetic lines model anomalous transport, so there is agreement with the measured value at low collision frequency.

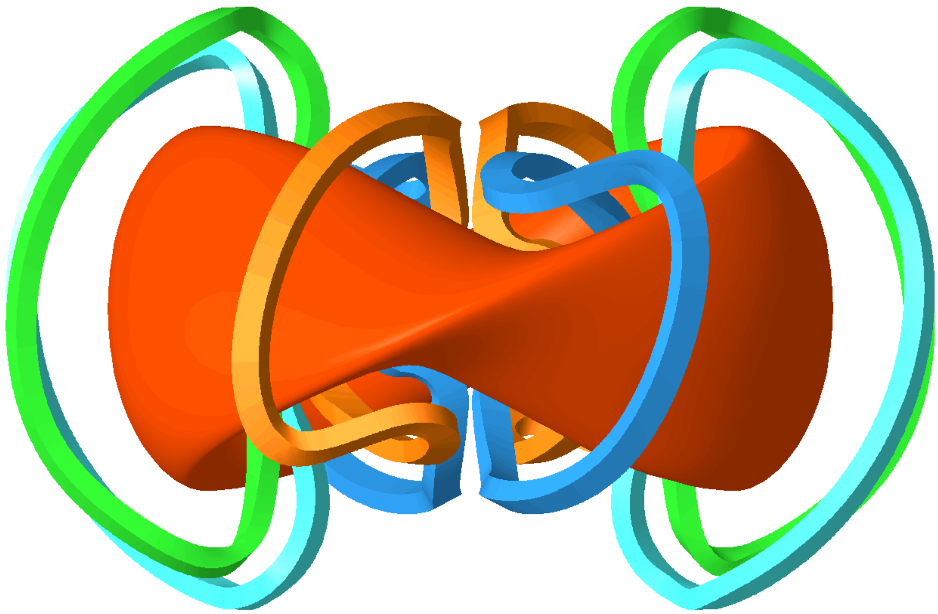


Diagram of a two field period reactor in a magnetic field given by the Biot-Savart law. Eight only moderately twisted modular coils produce robust flux surfaces that do not deteriorate when changes are made in the vertical and toroidal fields. There is good access between the coils for maintenance. (Courtesy of Tak-Kuen Mau and Tsueren Wang.)