Calculations of helically symmetric equilibria with PIES

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Outline:

1) PIES algorithm

2) PIES convergence

   a) Temporal convergence

   b) m, n, k convergence

3) Effects of a model bootstrap current

4) Future plans

PIES "Princeton Iterative Equilibrium Solver" - solves the MHD equilibrium equations with no assumptions about nested surfaces

PIES solves the equation:

$$\vec{J} \times \vec{B} = \nabla P$$

by Picard iteration by solving this equation for $\vec{J}$ and then solving for $\vec{B}$ from

$$\nabla \times \vec{B} = \vec{J}$$
$$\nabla \cdot \vec{B} = 0$$

When solving for J, $J_{\perp}$ is found from the equilibrium equation and $J_{\parallel}$ is found by demanding that $\nabla \cdot \vec{J} = 0$.

Thus, given $\vec{B}^n$ one finds for $\vec{J}^n$, and then $\vec{B}_{temp}^{n+1}$ and then the Picard blend step is made:

$$\vec{B}^{n+1} = \vec{B}^n \alpha + \vec{B}_{temp}^{n+1}(1-\alpha)$$

Where α is the blend or acceleration parameter.

PIES uses two grids. One is the background coordinates, on which $\vec{B}_{temp}^{n+1}$ is found. The background coordinates are usually generated by the VMEC code. The other grid is the magnetic coordinate grid, on which $J_{\parallel}$ is found.

PIES temporal convergence - convergence with iteration number

We have been able to reduce the maximum correction from iteration to iteration to 10^{-8} to 10^{-9}. Previously, we had been able to get 10^{-3} to 10^{-5}, sometimes leading to bistable states.

What was done to get this improvement?

1) PIES now uses background coordinates directly to map $J_{\parallel}$ from magnetic coordinates to the background coordinates.

Previously, we had used

$\vec{X}(r_b, \theta_b)$ and $\vec{X}(r_m, \theta_m)$

to find

$r_m = r_m(r_b, \theta_b)$ and $\theta_m = \theta_m(r_b, \theta_b)$

by an iteration method. The iteration often failed to converge to a low tolerance and this spoiled the overall convergence of the PIES code.

PIES now use

$r_b = r_b(r_m, \theta_m)$ and $\theta_b = \theta_b(r_m, \theta_m)$

to find

$r_m = r_m(r_b, \theta_b)$ and $\theta_m = \theta_m(r_b, \theta_b)$

This lead to significant improvement in the convergence of the iterative algorithm. It also lead to a more faithful representation of the flux surfaces on the background coordinates.

2) PIES now uses splines to invert $r_b = r_b(r_m, \theta_m)$ and $\theta_b = \theta_b(r_m, \theta_m)$ directly.

First, for each magnetic coordinate r_{m_j} we evaluate r_b and θ_b for a grid of θ_{m_i}

$$r_{b_{i,j}} = r_b(r_{m_j}, \theta_{m_i})$$

$$\theta_{b_{i,j}} = \theta_b(r_{m_j}, \theta_{m_i})$$

This then allows us to calculate the spline $r_b = r_b(r_{m_j}, \theta_b)$.

We then evaluate r_b on an r_{m_j}, θ_{b_i} grid. We can now construct a spline for

$$r_m = r_m(r_b, \theta_b)$$

We do a similar calculation to find

$$\theta_m = \theta_m(r_b, \theta_b)$$

3) PIES now uses an irregular grid for the radial magnetic coordinate. Previously, PIES used a grid regularly spaced in the background coordinates. This caused a problem whenever the edge of the island crossed a grid point. The irregular grid is constructed to have grid points (guard points) just outside the island and one grid point inside the island to define the quasi-magnetic coordinates. Currently, the magnetic grid is equally spaced between islands, but we are now in the process of choosing the grid points to be at the most irrational surfaces.

PIES convergence - m, n, k convergence. Here m and n are the poloidal and toroidal mode numbers and k is the radial grid number.

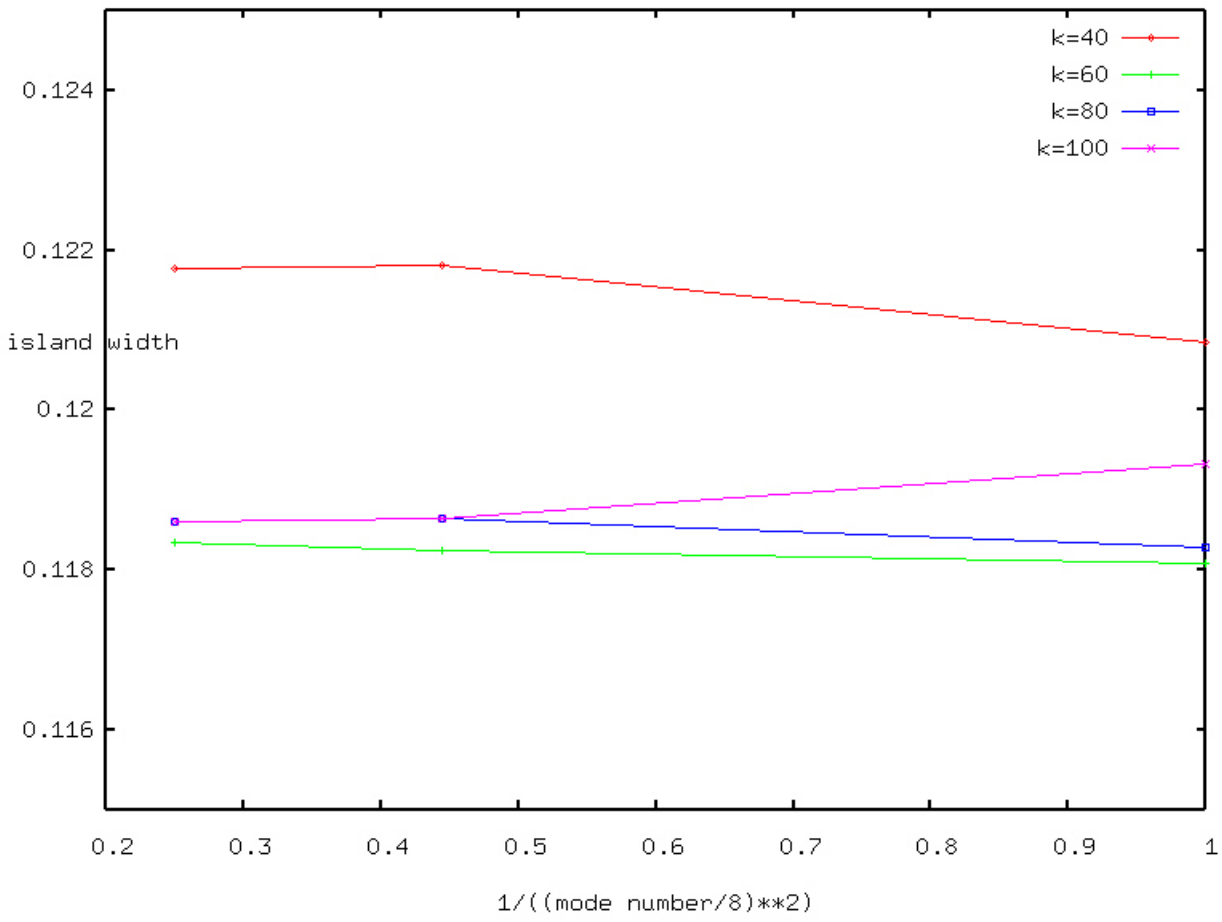
We test the convergence of the improve algorithm on a Furth, Rutherford, Selberg like profile

$$q = 1.5(1 + (r/.635)^3)^{\frac{2}{3}}$$

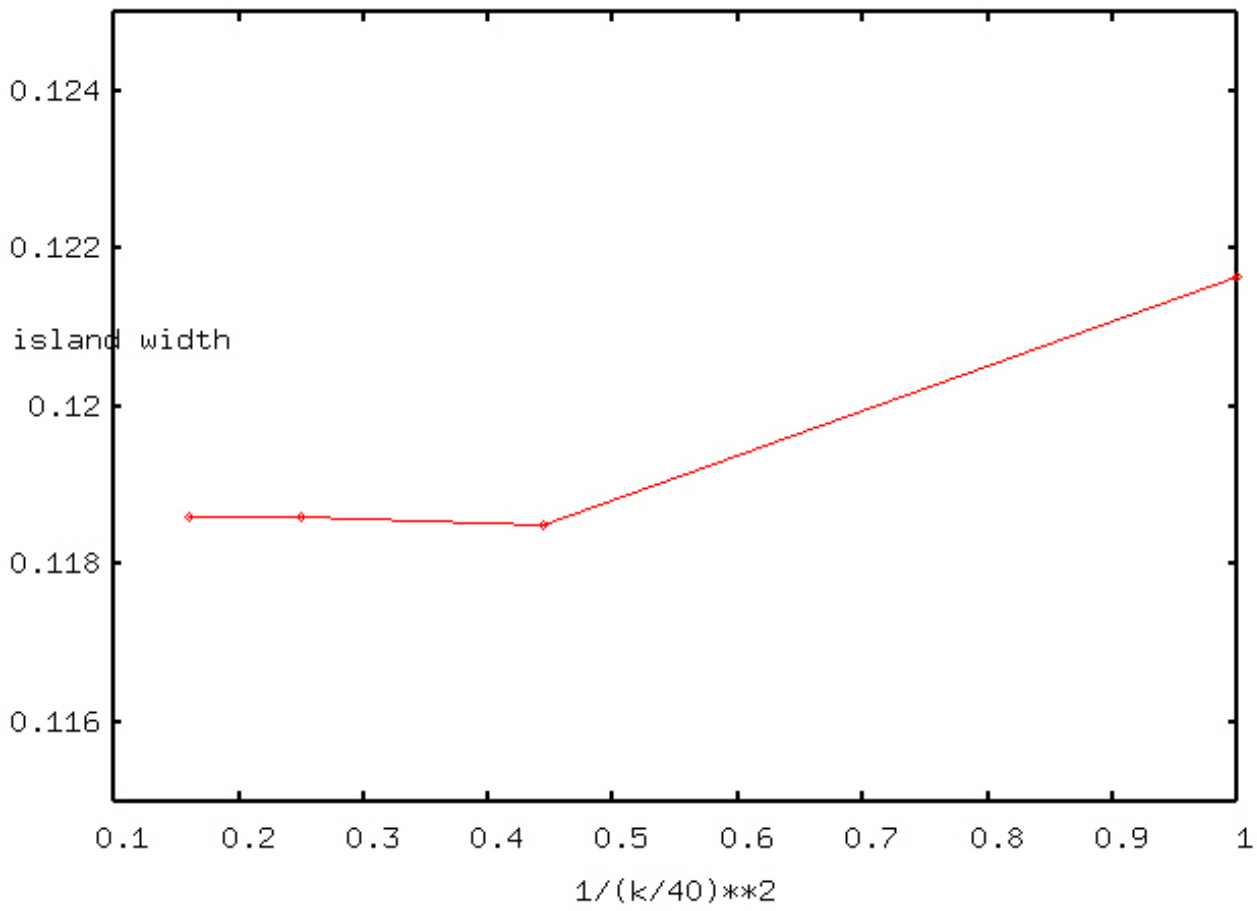
This is intermediate between a peaked and rounded profile having a q=2 surface near r=.5 and a q at the edge of 4.3. The saturated island width is around 10%. Beta is zero.

This was run with the new algorithm for $m = 8,12,16$ and $k = 40,60,80,100$

Resulting in the following plot of convergence in mode number:



The plot of the results converged in mode number verses number of radial zones shows good convergence.



Effects of a model bootstrap current.

A simple model to simulate the effect of a bootstrap current by increasing or decreasing the current in side the island has been implemented . The island width is very sensitive to the bootstrap effect for the equilibrium described above.

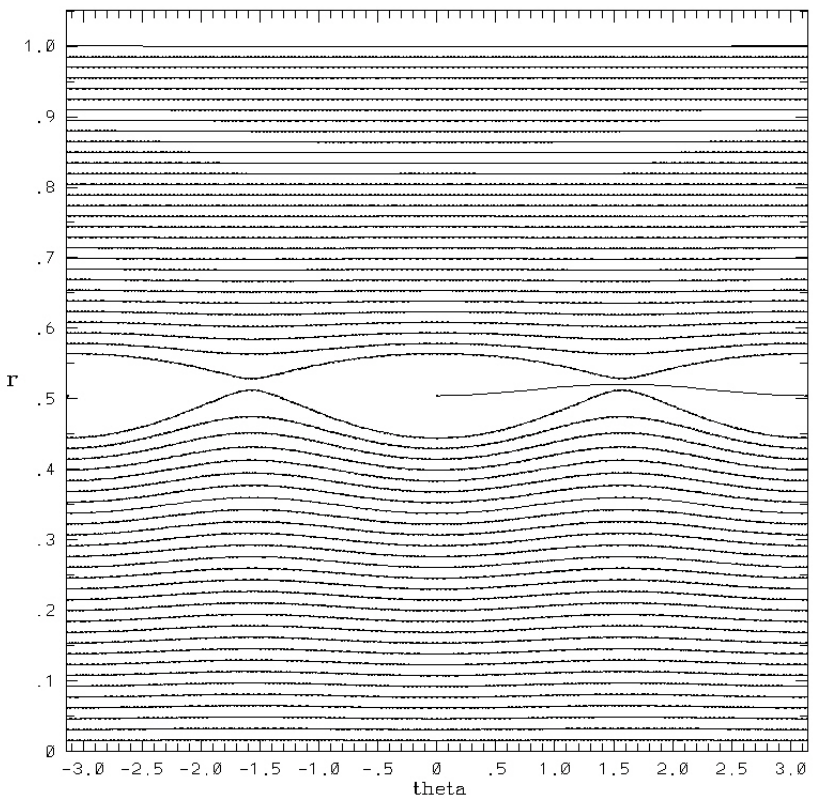
A 10% decrease in the current inside the island causes the island to increase by more that a factor of 2.

An increase in the island size is expected since $q' > 0$.

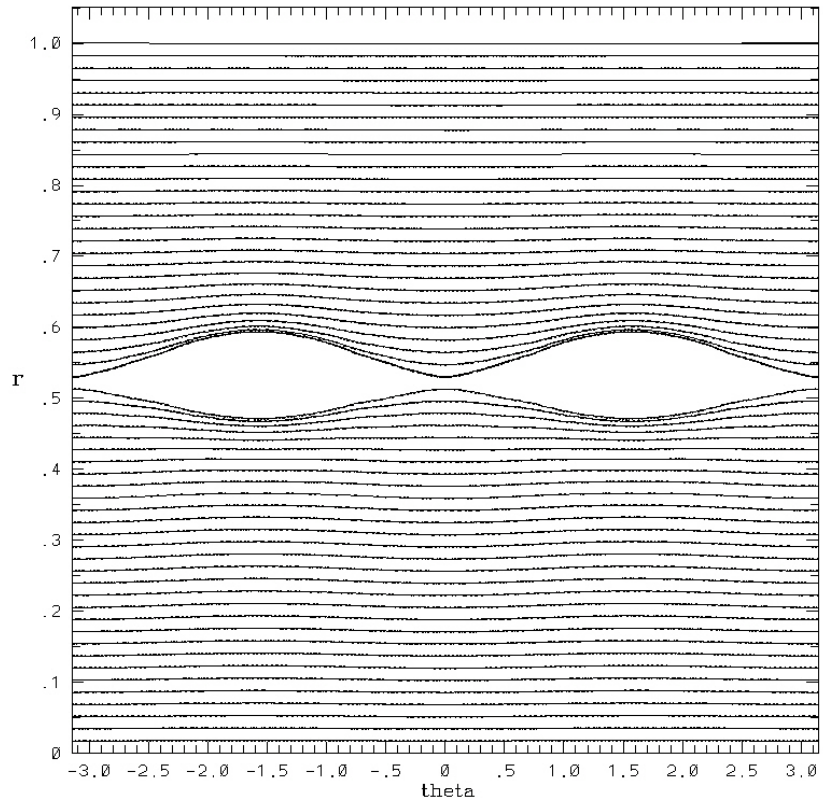
A 2% increase in the current inside the island causes the island to decrease by a factor of 2. A 3% increase makes the equilibrium oscillate between a tiny island and a zero width island.

When the current in the island is increase by 10% the perturbed current effect is so large it causes the island to change phase as seen in the following plots.

Flux contours on background coordinates when current is flattened in island and island width has reached saturation



Flux contours after one iteration when current is the island is increased by 10% starting from the saturated island case at the left

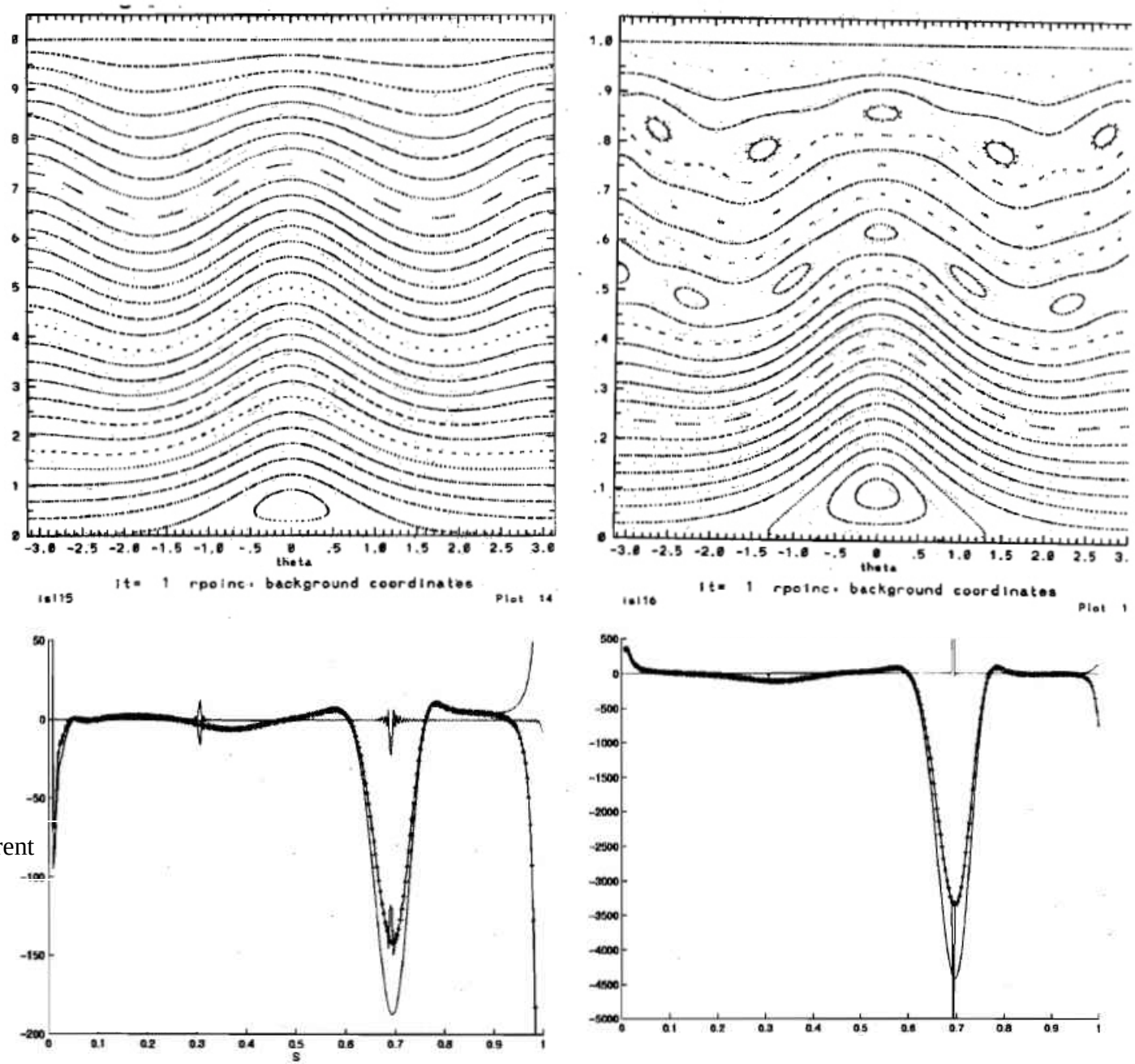


Plans

1) Compare with PIES results with a code that solves the helical Grad-Shafranov equation.

2) Do finite beta - probably need to use iota profile for selection of magnetic coordinates

3) Improve PIES temporal convergence by using Jacobian-free Newton-Krylov methods.

4) Explore using model bootstrap current to stabilize m=1 in tokamaks .

5) Explore using model bootstrap current to find delta-function currents for equilibria without islands.