Calculation of Ion Flows in Stellarators: Configurational Variations

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Acknowledgements: Jeff Harris, Hideo Sugama, Shin Nishimura, Andrew Ware, Steve Hirshman, Wayne Houlberg, Jim Lyon

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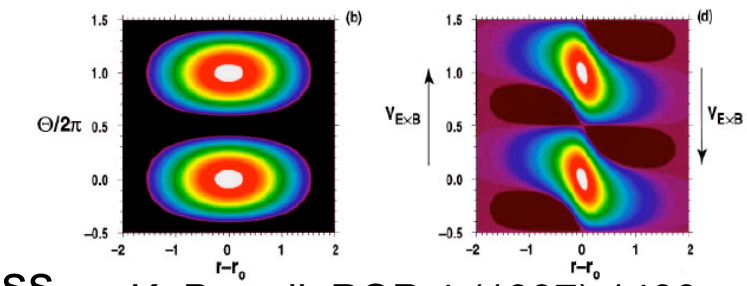
3D shaping offers a large design space for stellarator configurations

- Recent stellarator optimizations have reduced cross-field neoclassical losses << anomalous losses
  - Ripple reduction, quasi-symmetry, isodynamicity, omnigeneity
  - Hybrid (with plasma current) devices, aspect ratios down to ~ 2
- Within these devices a variety of parallel momentum transport characteristics are present
  - Minimize parallel viscous stress: $B \bullet (\nabla \bullet \Pi) = \mu_{\parallel}^{n} u_{\parallel} + \mu_{\parallel}^{E} q_{\parallel} + \ldots$
  - poloidal/toroidal velocity shearing
  - two-dimensional flow structure within magnetic surfaces
- Parallel transport: a new dimension for stellarator optimizations ?

Motivation for parallel transport/flow analysis:

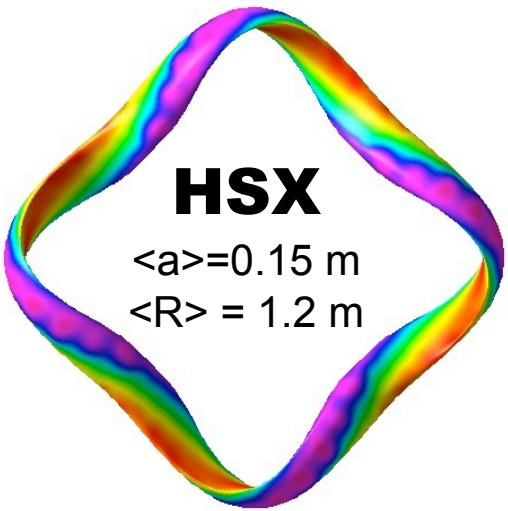
- Universal nature of E x B shearing for shredding eddies over a wide range of configurations/turbulence mechanisms
  - Tokamak specific drives: grad p
  - Stellarator specific drives: Γ_i - Γ_e
  - Drives shared in common:
    - momentum, Reynold's stress, orbit loss

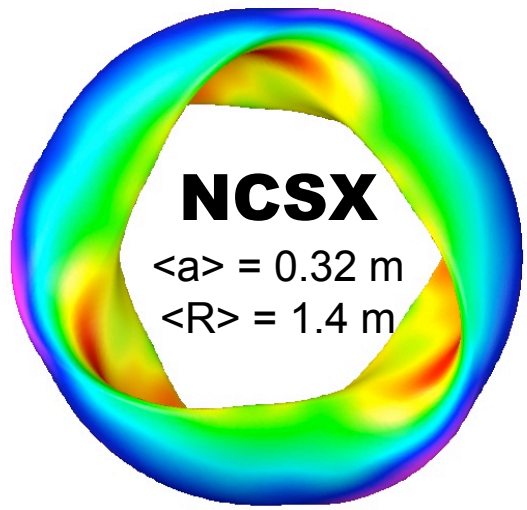


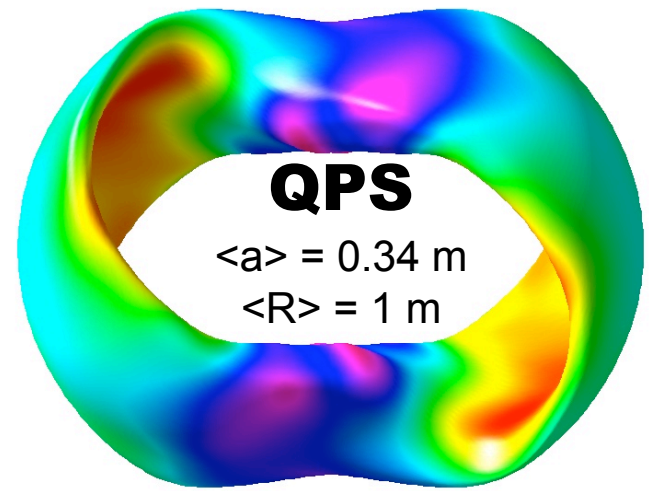
K. Burrell, POP 4 (1997) 1499

- ITER/DEMO H-mode power thresholds
  - 50 to 200 MW (1999 ITER Basis document)
  - Current design: 40 to 70 MW
  - DEMO: up to 300 MW
  - Will alpha heating power (isotropic) count the same way as power (anisotropic) on which these scaling estimates are based?
- Could a stellarator lower these power thresholds by optimizing around E x B shearing rates ?

Advances in stellarator optimization have allowed the design of 3D configurations with magnetic structures that approximate: straight helix/tokamak/connected mirrors:

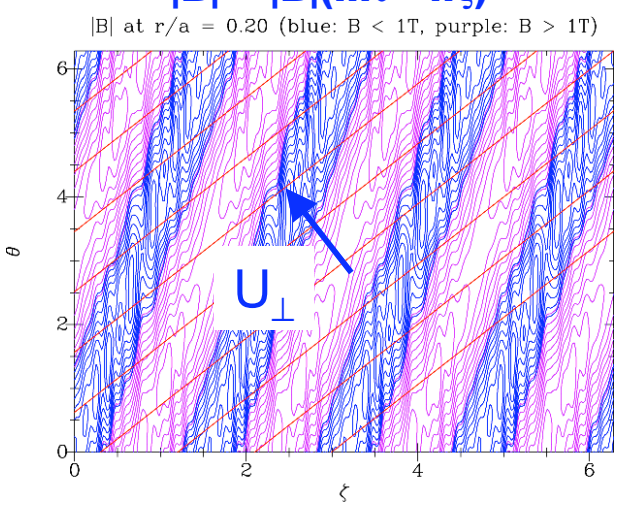






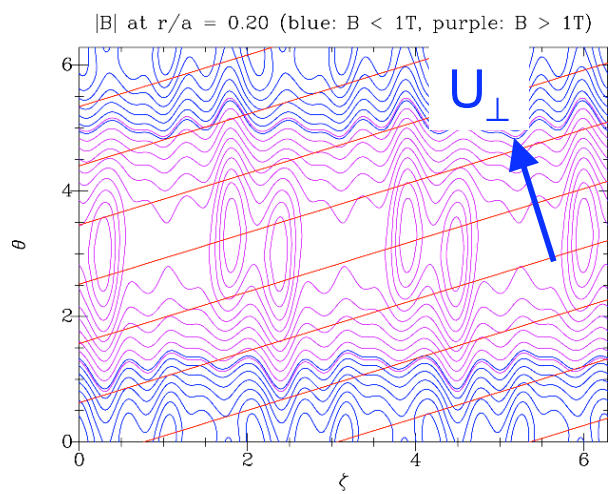
Quasi-helical symmetry

$|B| \sim |B|(m\theta - n\zeta)$



Quasi-toroidal symmetry

$|B| \sim |B|(\theta)$



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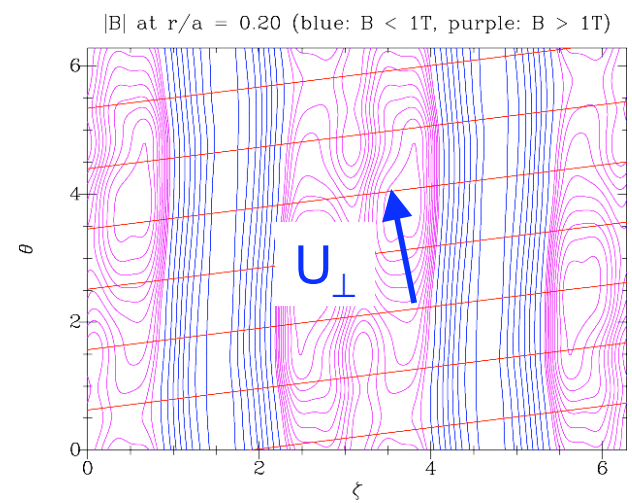

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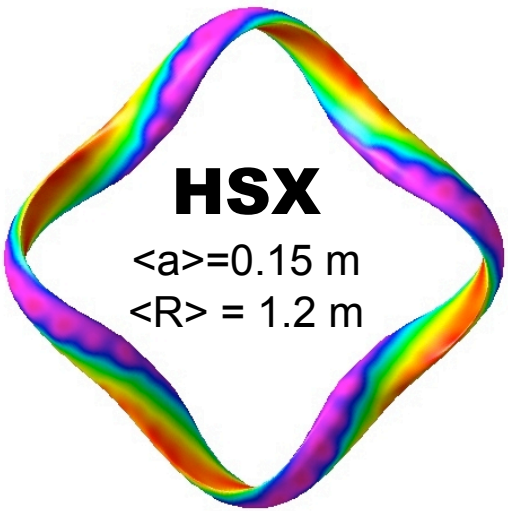

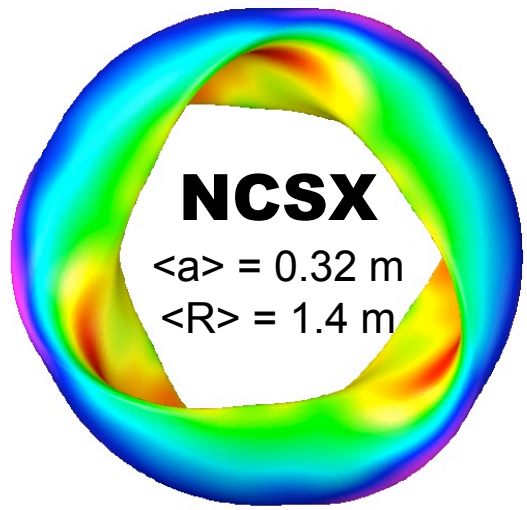

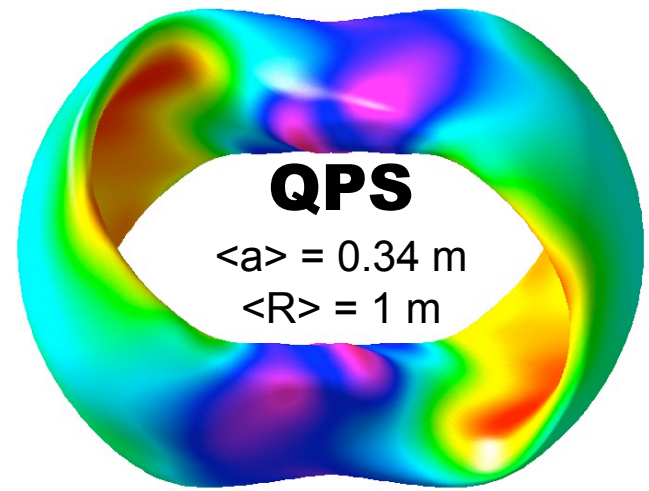

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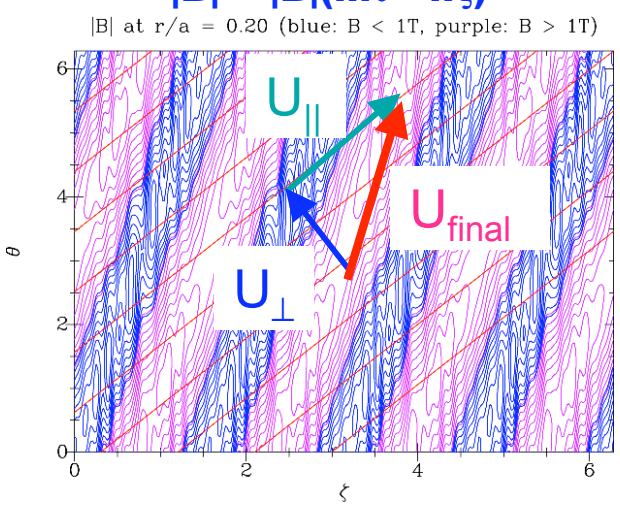

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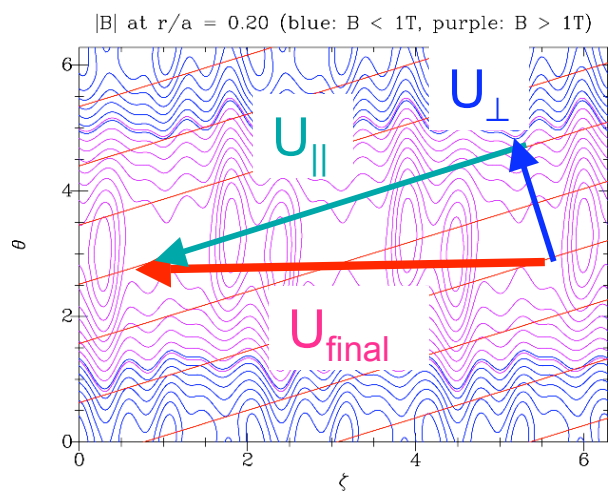

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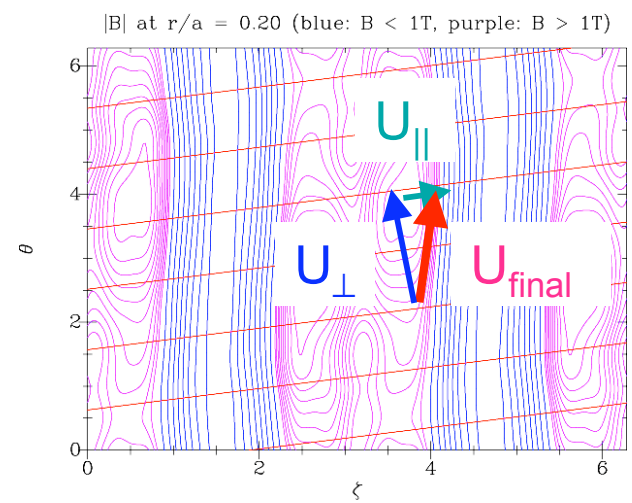
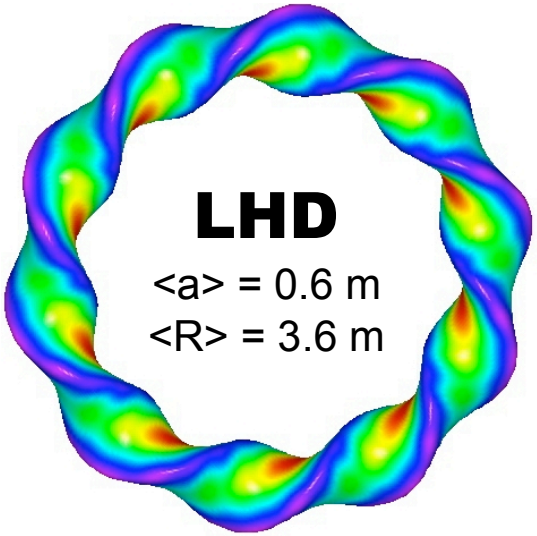
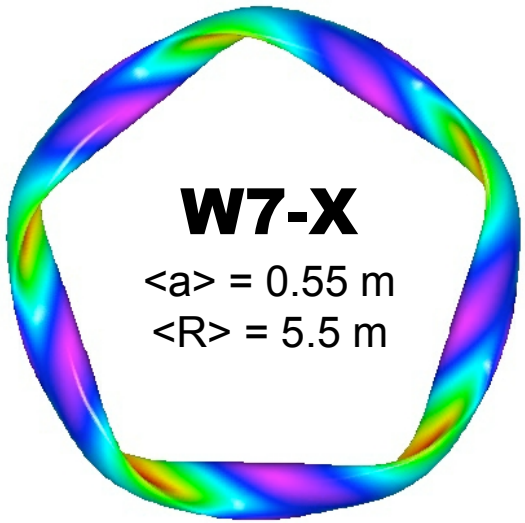
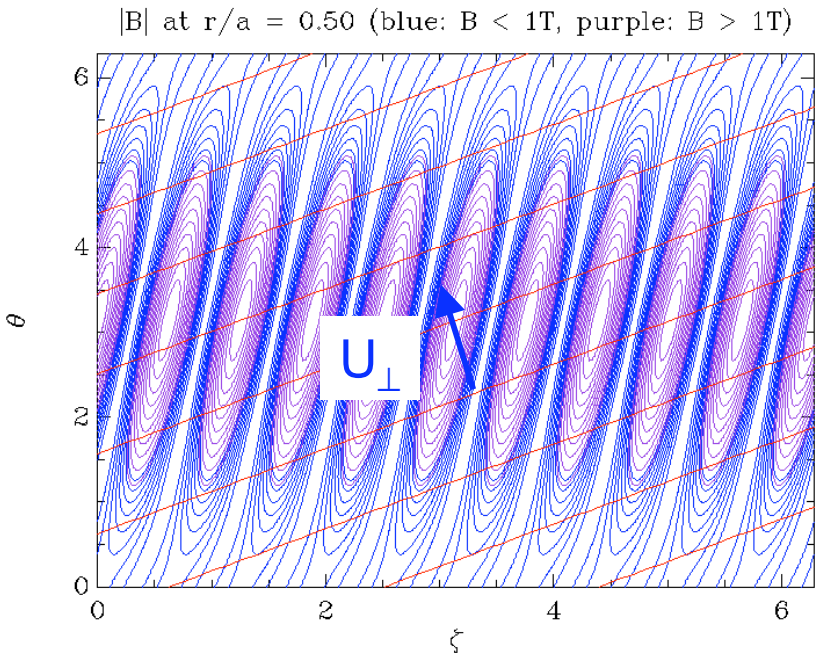
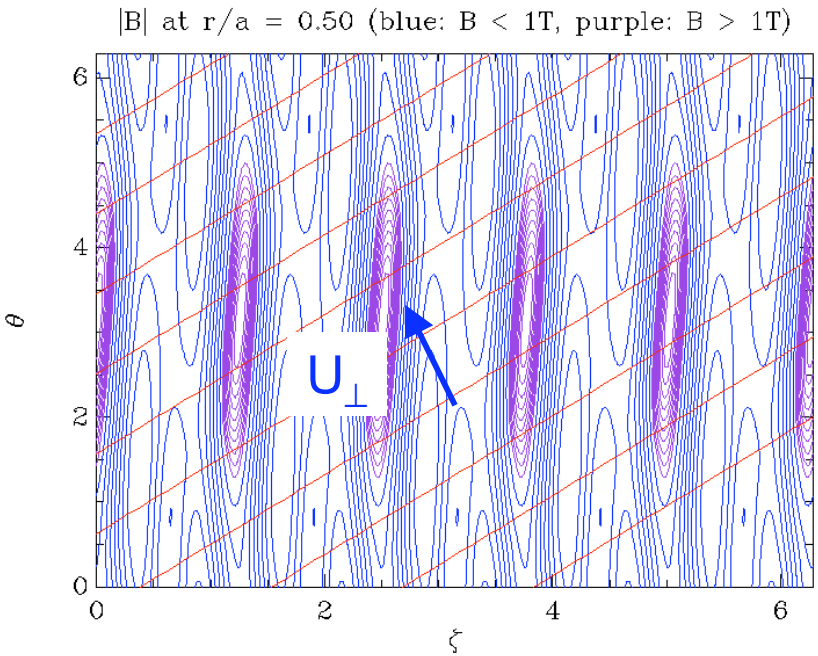


In addition, LHD and W7-X achieve closed drift surfaces by inward shifts (LHD) and finite plasma β effects (W7-X)









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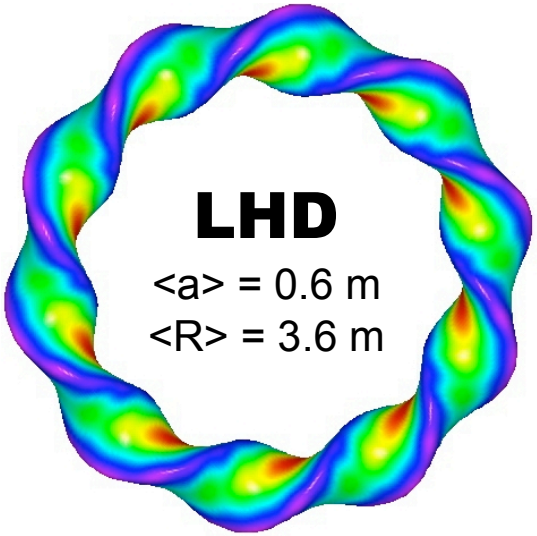
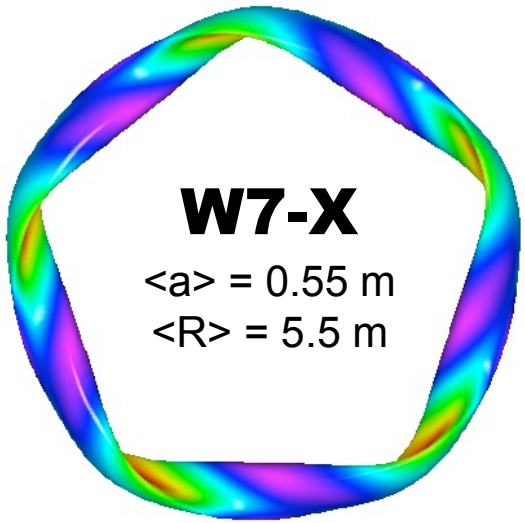
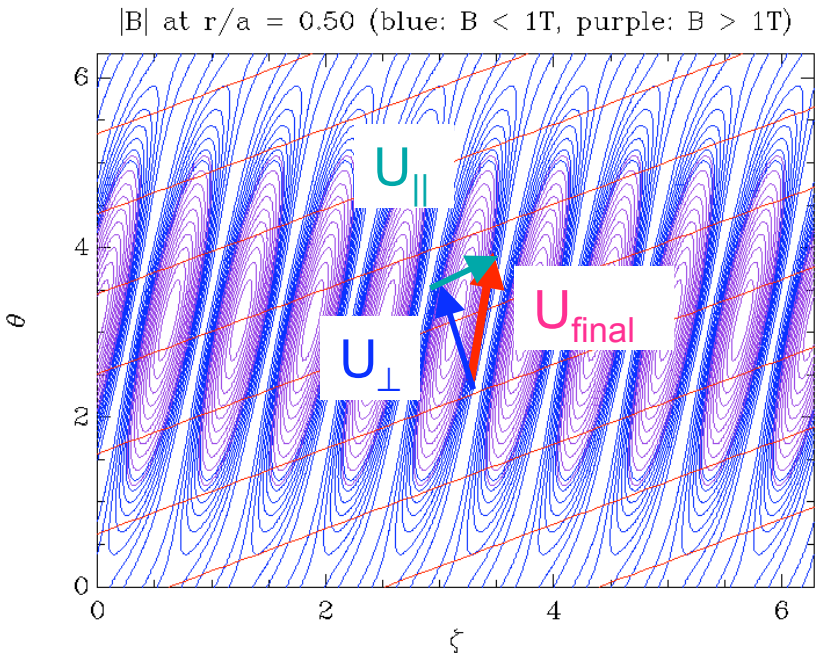
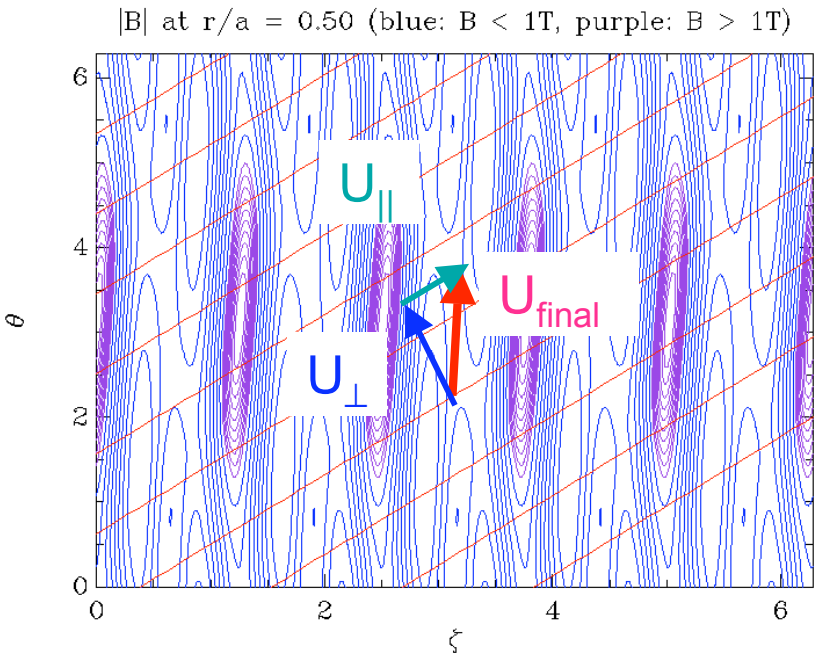








Development of stellarator moments methods

- Viscosities incorporate all needed kinetic information
  - Momentum balance invoked at macroscopic level rather than kinetic level
  - Multiple species can be more readily decoupled
- Recent work
  - Moments method, viscosities related to D_{11} D_{31}, D_{33}

    M. Taguchi, Phys. Fluids B4 (1992) 3638

    H. Sugama, S. Nishimura, Phys. of Plasmas **9** (2002) 4637 - has related viscosities to Drift Kinetic Equation Solver transport coefficients
  - DKES: D_{11} (diffusion of n,T), D_{13} (bootstrap current), D_{33} (resistivity enhancement)

    W. I.Van Rij and S. P. Hirshman, *Phys. Fluids B*, **1**, 563 (1989)
- Implemented into a suite of codes that generate the transport coefficient database, perform velocity convolutions, find ambipolar roots, and calculate flow components
  - D. A. Spong, Phys. Plasmas **12** (2005) 056114
  - D. A. Spong, S. P., Hirshman, et al., Nuclear Fusion 45 (2005) 918

Moments Method Closures for Stellarators

The parallel viscous stresses, particle and heat flows are treated as fluxes conjugate to the forces of parallel momentum, parallel heat flow, and gradients of density, temperature and potential:

$$\begin{bmatrix} \left\langle \vec{B}\cdot\left(\vec{\nabla}\cdot\overleftrightarrow{\Pi}_a\right)\right\rangle \\ \left\langle \vec{B}\cdot\left(\vec{\nabla}\cdot\overleftrightarrow{\Theta}_a\right)\right\rangle \\ \Gamma_a \\ Q_a/T_a \end{bmatrix} = \begin{bmatrix} M_{a1} & M_{a2} & N_{a1} & N_{a2} \\ M_{a2} & M_{a3} & N_{a2} & N_{a3} \\ N_{a1} & N_{a2} & L_{a1} & L_{a2} \\ N_{a2} & N_{a3} & L_{a2} & L_{a3} \end{bmatrix} \begin{bmatrix} \langle u_{\parallel a}B\rangle / \langle B^2\rangle \\ \frac{2}{5p_a}\langle q_{\parallel a}B\rangle / \langle B^2\rangle \\ -\frac{1}{n_a}\frac{\partial p_a}{\partial s} - e_a\frac{\partial \Phi}{\partial s} \\ -\frac{\partial T_a}{\partial s} \end{bmatrix}$$

- Analysis of Sugama and Nishimura related monoenergetic forms of the M, N, L viscosity coefficients to DKES transport coefficients
- Combining the above relation with the parallel momentum balances and friction-flow relations

$$\left\langle \vec{B}\cdot\left(\vec{\nabla}\cdot\overleftrightarrow{\Pi}_a\right)\right\rangle - n_a e_a \langle BE_\parallel\rangle = \langle BF_{\parallel a1}\rangle$$

$$\left\langle \vec{B}\cdot\left(\vec{\nabla}\cdot\overleftrightarrow{\Theta}_a\right)\right\rangle = \langle BF_{\parallel a2}\rangle$$

$$\begin{bmatrix} \langle BF_{\parallel a1}\rangle \\ \langle BF_{\parallel a2}\rangle \end{bmatrix} = \sum_b \begin{bmatrix} l_{11}^{ab} & -l_{12}^{ab} \\ -l_{12}^{ab} & l_{22}^{ab} \end{bmatrix} \begin{bmatrix} \langle Bu_{\parallel b}\rangle \\ \frac{2}{5p_b}\langle Bq_{\parallel b}\rangle \end{bmatrix}$$

Leads to coupled equations that can be solved for $\langle u_{\parallel a}B\rangle$, $\langle q_{\parallel a}B\rangle$, Γ_a, Q_a

Using solutions for an electron/ion plasma, the self-consistent electric fields, bootstrap currents and parallel flows can be obtained

(Appendix C - H. Sugama, S. Nishimura, Phys. Plasmas **9** (2002) 4637)

Radial particle flows required for ambipolar condition ➡ self-consistent energy fluxes and bootstrap currents

$$\begin{bmatrix} \Gamma_e \\ q_e / T_e \\ \Gamma_i \\ q_i / T_i \\ J_{BS}^E \end{bmatrix} = \begin{bmatrix} L_{11}^{ee} & L_{12}^{ee} & L_{11}^{ei} & L_{12}^{ei} & L_{1E}^{e} \\ L_{21}^{ee} & L_{22}^{ee} & L_{21}^{ei} & L_{22}^{ei} & L_{2E}^{e} \\ L_{11}^{ie} & L_{12}^{ie} & L_{11}^{ii} & L_{12}^{ii} & L_{1E}^{i} \\ L_{21}^{ie} & L_{22}^{ie} & L_{21}^{ii} & L_{22}^{ii} & L_{2E}^{i} \\ L_{E1}^{e} & L_{E2}^{e} & L_{E1}^{i} & L_{E2}^{i} & L_{EE} \end{bmatrix} \begin{bmatrix} X_{e1} \\ X_{e2} \\ X_{i1} \\ X_{i2} \\ X_E \end{bmatrix}$$

where $X_{a1} \equiv -\frac{1}{n_a}\frac{\partial p_a}{\partial s} - e_a \frac{\partial \Phi}{\partial s}$, $X_{a2} \equiv -\frac{\partial T_a}{\partial s}$, $X_E = \langle BE_\parallel \rangle / \langle B^2 \rangle^{1/2}$

Parallel mass and energy flows:

$$\begin{bmatrix} \langle Bu_{\parallel i} \rangle \\ \frac{2}{5p_i}\langle Bq_{\parallel i} \rangle \end{bmatrix} = \left[\frac{1}{\langle B^2 \rangle} \begin{bmatrix} M_{i1} & M_{i2} \\ M_{i2} & M_{i3} \end{bmatrix} - \begin{bmatrix} l_{11}^{ii} & -l_{12}^{ii} \\ -l_{12}^{ii} & l_{22}^{ii} \end{bmatrix} \right]^{-1} \begin{bmatrix} N_{i1} & N_{i2} \\ N_{i2} & N_{i3} \end{bmatrix} \begin{bmatrix} X_{i1} \\ X_{i2} \end{bmatrix}$$

The neoclassical theory provides $\langle u_{\|}B\rangle$ and the ambipolar electric field E_r (from solving $\Gamma_i = \Gamma_e$). The final term needed is U, the Pfirsch-Schlüter flow.

$$\vec{v} = \frac{1}{eB}\left(-\frac{1}{n}\frac{\partial p}{\partial s} + E_r\right)\vec{\nabla}s \times \frac{\vec{B}}{B} + \left[\frac{\langle u_{\|}B\rangle}{\langle B^2\rangle} + \frac{1}{eB}\left(-\frac{1}{n}\frac{\partial p}{\partial s} + E_r\right)U\right]\frac{\vec{B}}{B}$$

Integrating $\vec{\nabla} \bullet \vec{v} = 0$ leads to an equation for U:

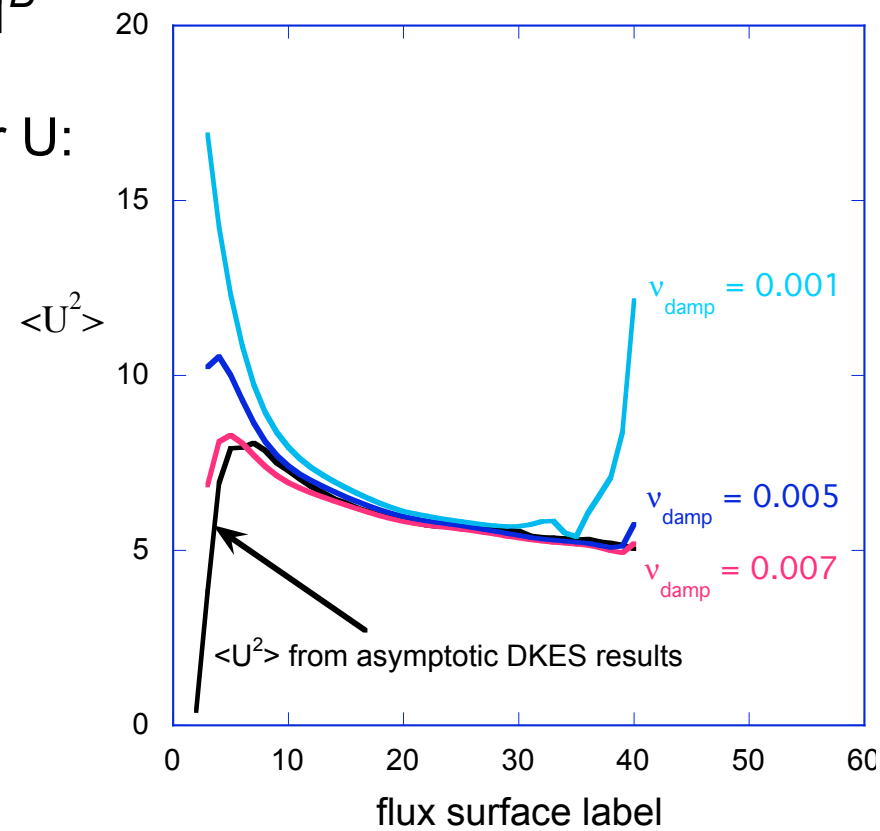
$$\vec{B} \bullet \vec{\nabla}\left(\frac{U}{B}\right) = \vec{B} \times \vec{\nabla}\psi \bullet \vec{\nabla}\left(\frac{1}{B^2}\right) \quad (stellarator)$$

$$U = I(\psi)\left[\frac{B^2}{\langle B^2\rangle} - 1\right] \quad (tokamak)$$

$\langle U^2\rangle$ can be obtained by:

-solving this equation directly (with damping to resolve singularities at rational surfaces)

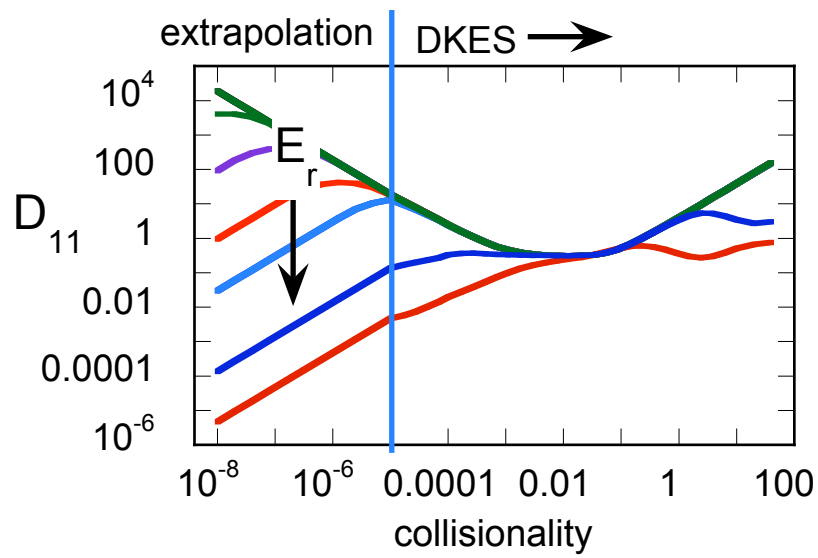
- matching to high collisionality DKES coefficient: $\langle U^2\rangle = 1.5 D_{11} v/\nu$ (for large ν)

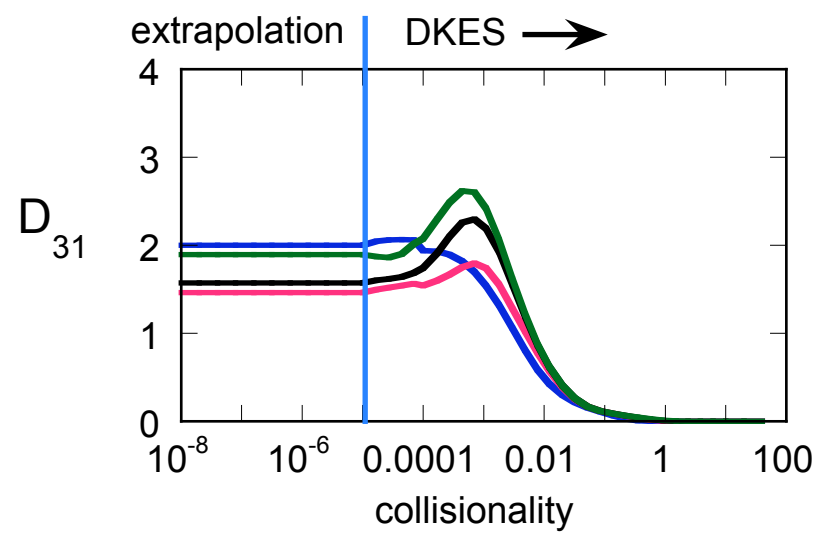


We will give both flux surface averages of component of the total flow velocity as well as showing its two-dimensional variation within a flux surface.

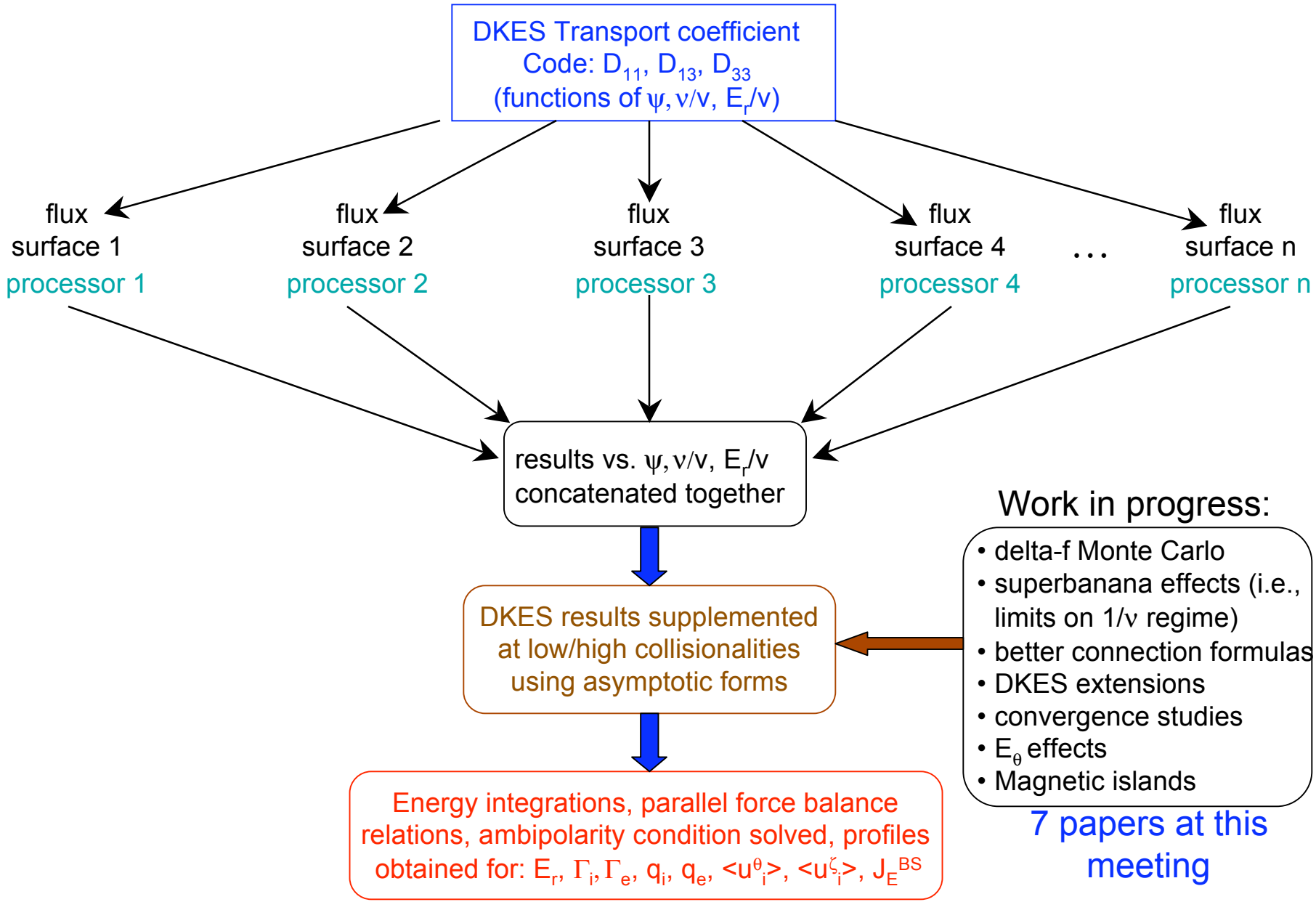
Current approach: DKES data base + analysis/physics based extensions

- Approach
  - Apply to regimes insensitive to extrapolations
  - Improve as better methods become available
- Connection formulas currently used:
  - Low collisionality diffusion coefficient D_{11}
    - $D_{11} \propto 1/\nu$ for plateau > ν > E_r
    - $D_{11} \propto \nu$ for $\nu < E_r$
    - $D_{11} \propto \nu^{1/2}$ for transition region
  - Low collisionality bootstrap coefficient D_{31}
    - Match lowest D_{31} from DKES to constant
  - High collisionality, high electric field
    - Incompressibility assumption breaks down
    - Viscosities can go negative or develop unphysical scalings with ν
    - Merge into $E_r = 0$ result for $\nu > \nu_{critical}$



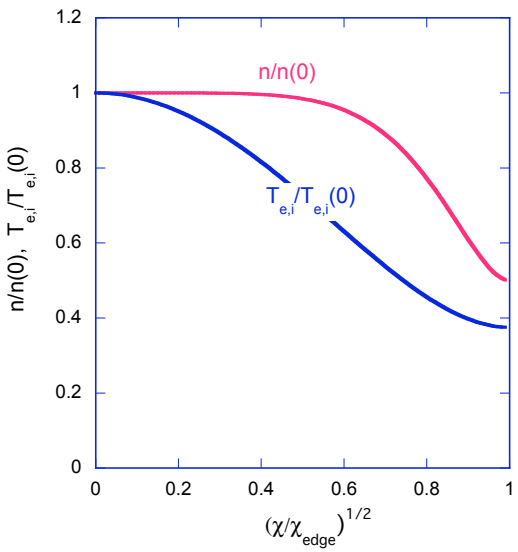


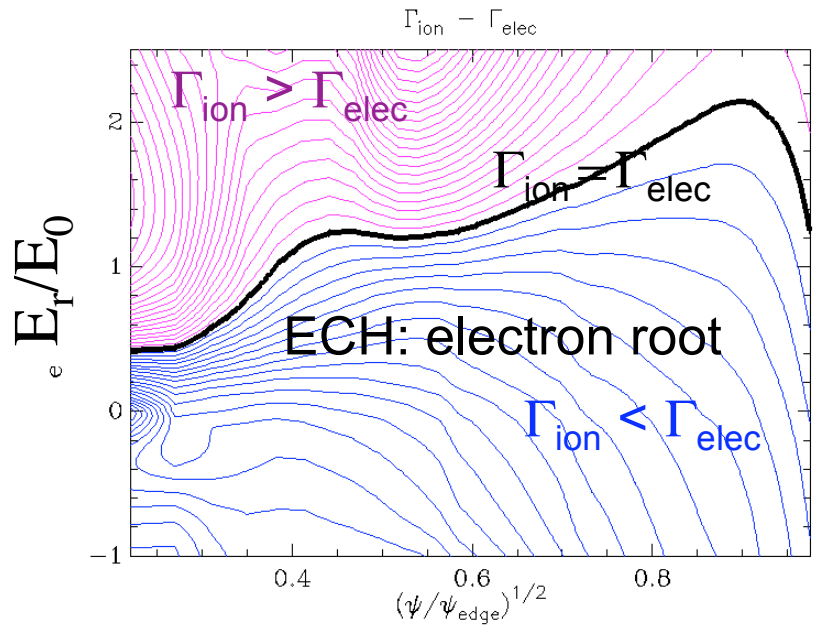
Parallel Environment for Neoclassical Transport Analysis (PENTA)

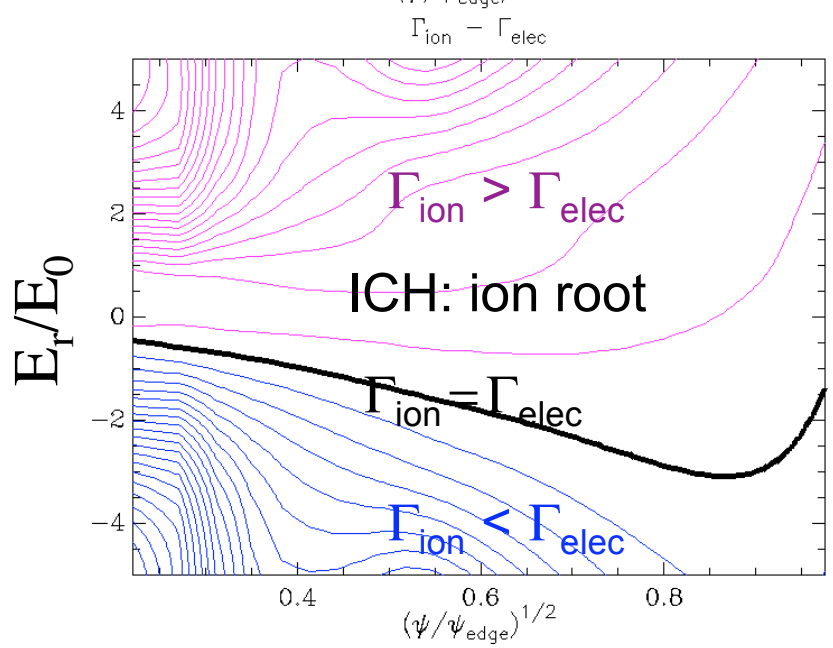


The flow model will be applied to two parameter ranges with radially continuous/stable electric field roots

- ECH regime:
  - $n(0) = 2.5 \times 10^{19}$ m^{-3}, $T_e(0) = 1.5$ keV, $T_i(0) = 0.2$ keV
- ICH regime
  - $n(0) = 8 \times 10^{19}$ m^{-3}, Te(0) = 0.5 keV, $T_i(0) = 0.3$ keV



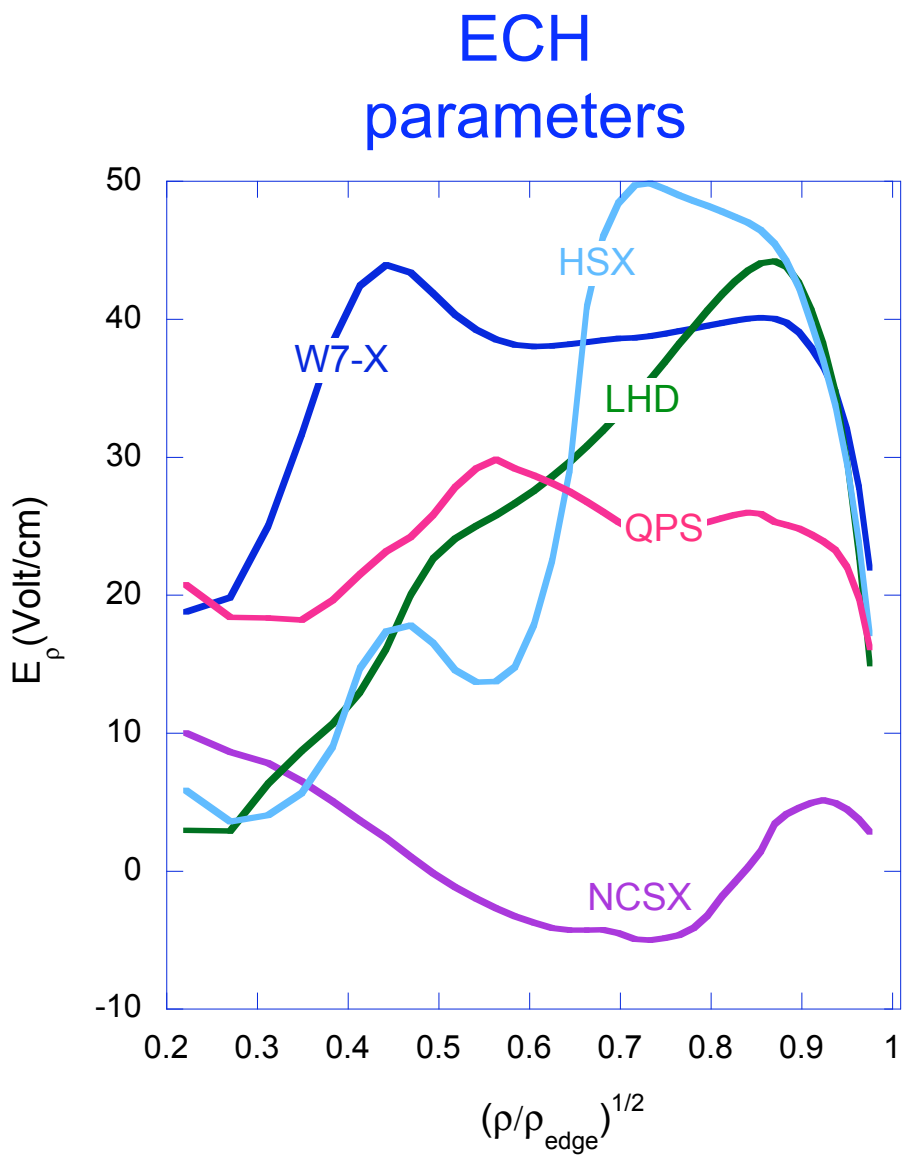


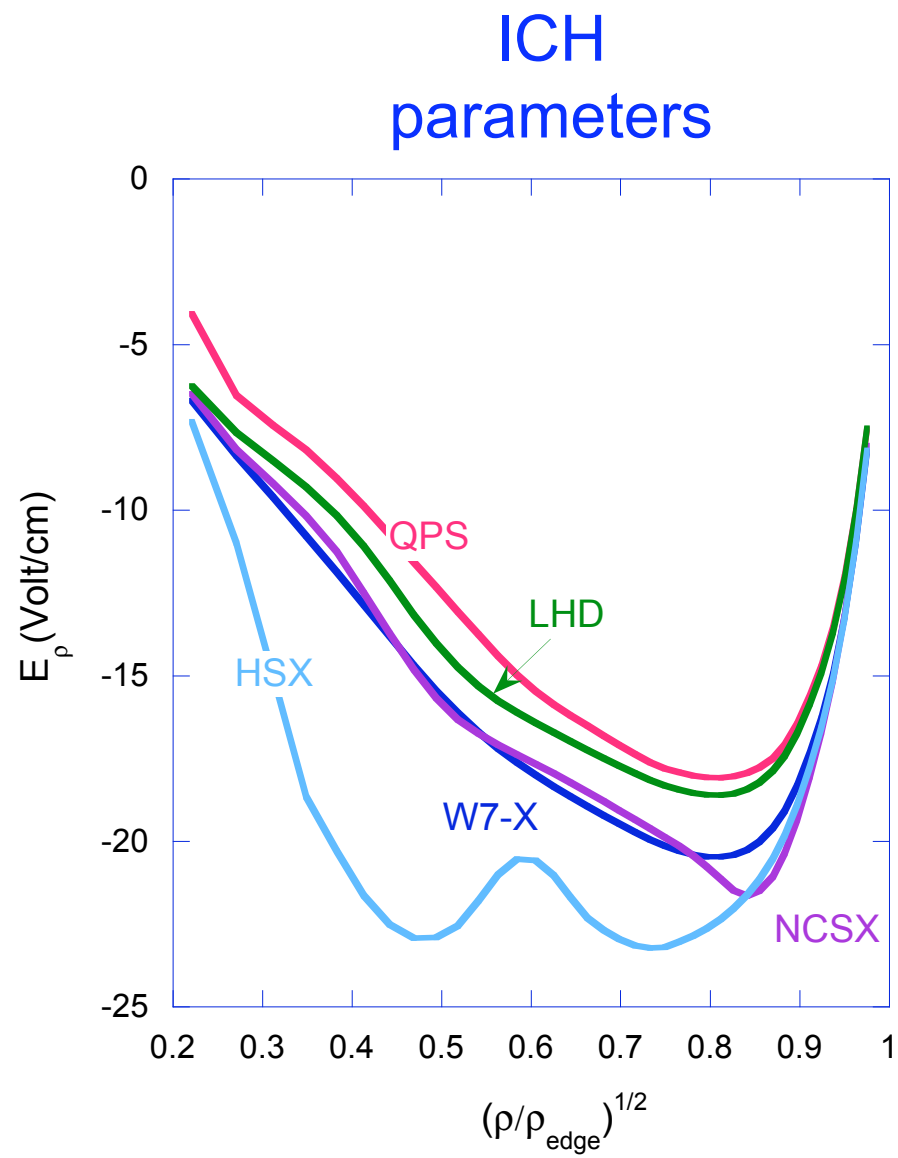


Roots chosen to give stable restoring force for electric field perturbations through:

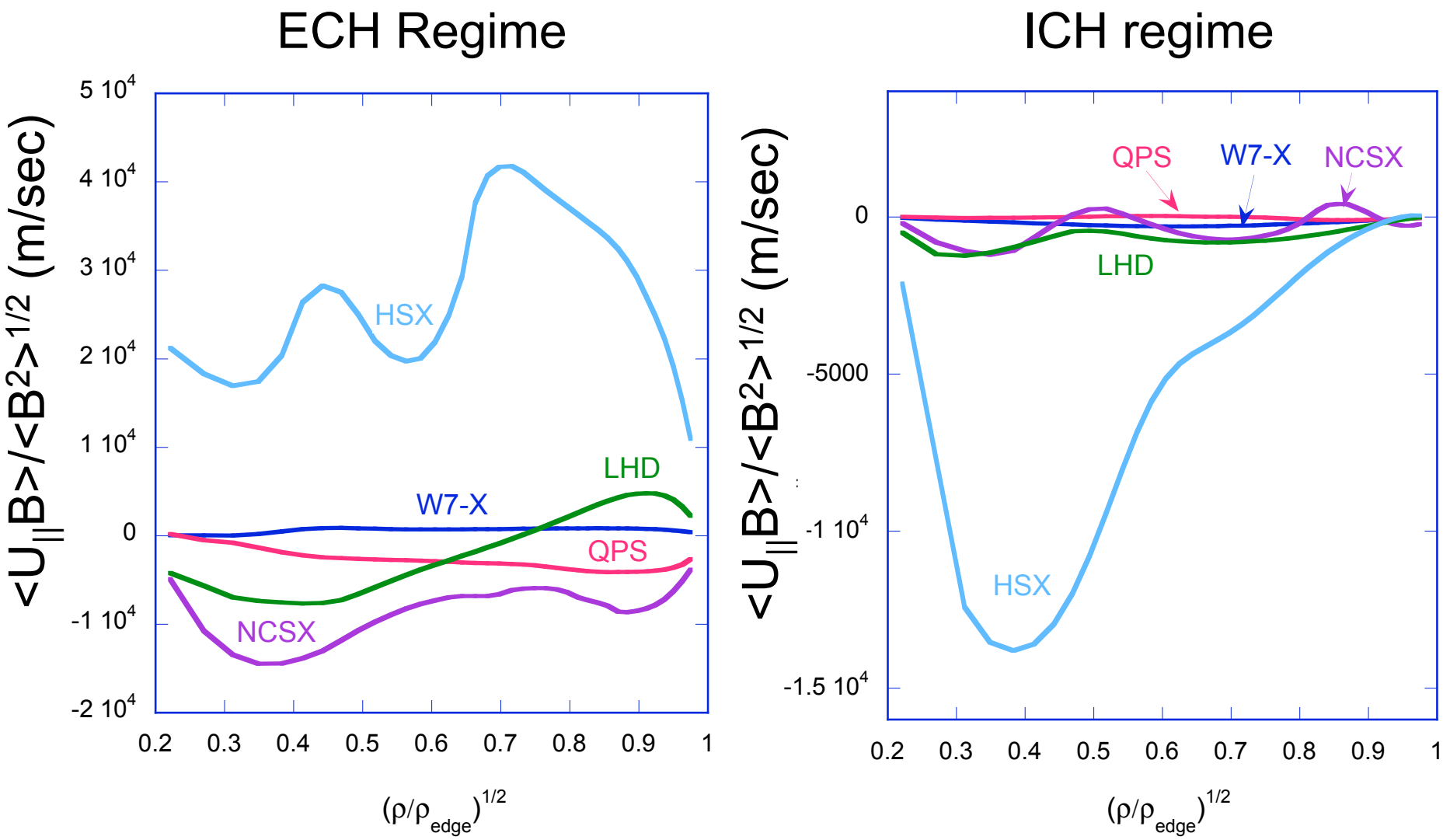
$$\frac{\varepsilon_\perp}{q}\frac{\partial E_r}{\partial t} = \Gamma_e - \Gamma_i$$

Electric field profiles for ICH (ion root) and ECH (electron root) cases





The neoclassical parallel flow velocity can vary significantly among configurations. The lowest levels are present in W7-X. Higher $u_{||}$ flows characterize HSX/NCSX



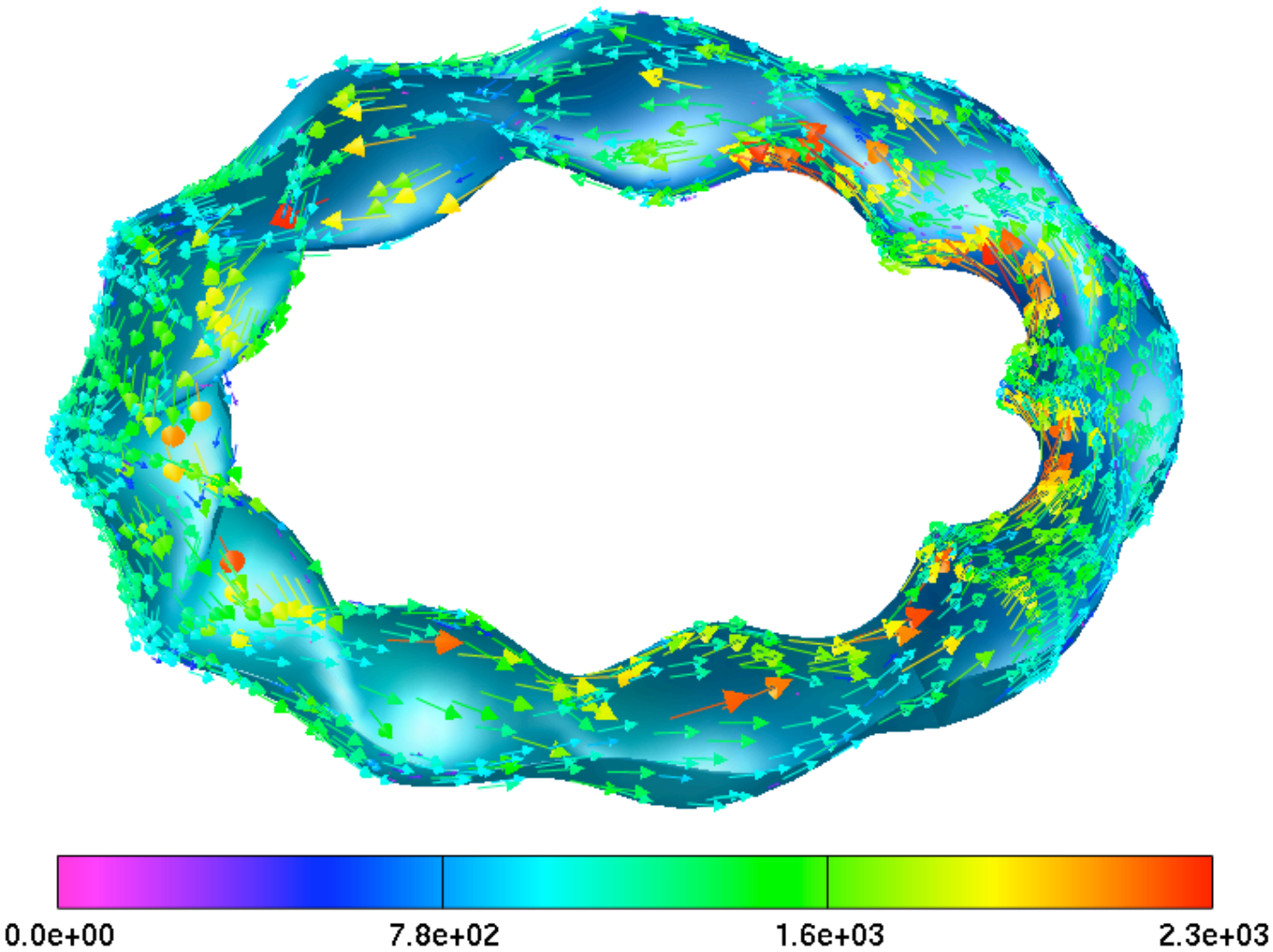
2D flow variation within a flux surface - reveals features not present in flux surface averages

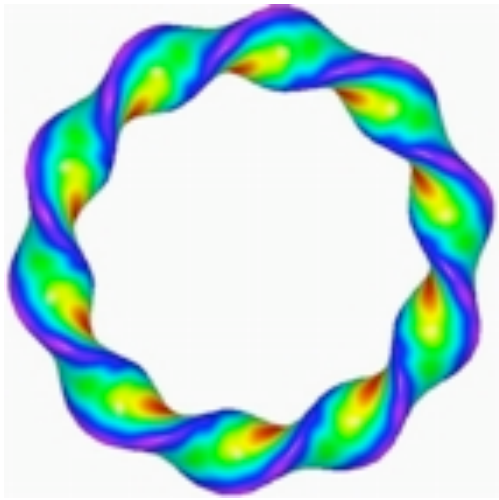
$$\vec{V} = \frac{X_1}{eB^2}\vec{\nabla}s \times \vec{B} + \frac{\langle u_{\parallel} B \rangle}{\langle B^2 \rangle}\vec{B} + \frac{X_1}{e}\tilde{U}\vec{B} \quad \text{where } X_1 \equiv -\frac{1}{n}\frac{\partial p}{\partial s} - e\frac{\partial \Phi}{\partial s}$$

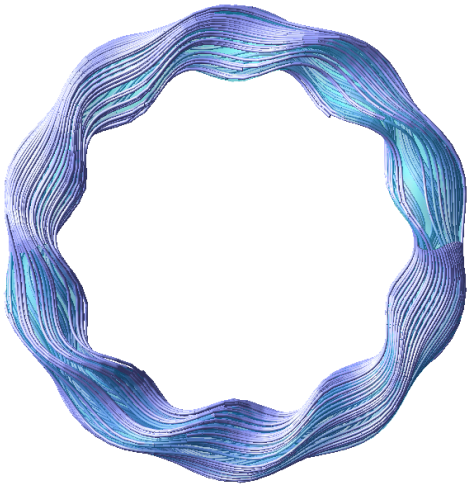
diamagnetic and E x B	neoclassical parallel flow	Pfirsch-Schlüter - for $\vec{\nabla} \bullet \left(\rho \vec{V}\right) = 0$	
$\propto$ 1/B	B	(…)B	(variation within a flux surface)

- Visualization of flows in real (Cartesian space) at 75% flux surface
- Indicates flow shearing (geodesic) within a flux surface over shorter connection lengths than for a tokamak
  - Could impact ballooning, interchange stability, microturbulence
- Implies need for multipoint experimental measurements and/or theoretical modeling support

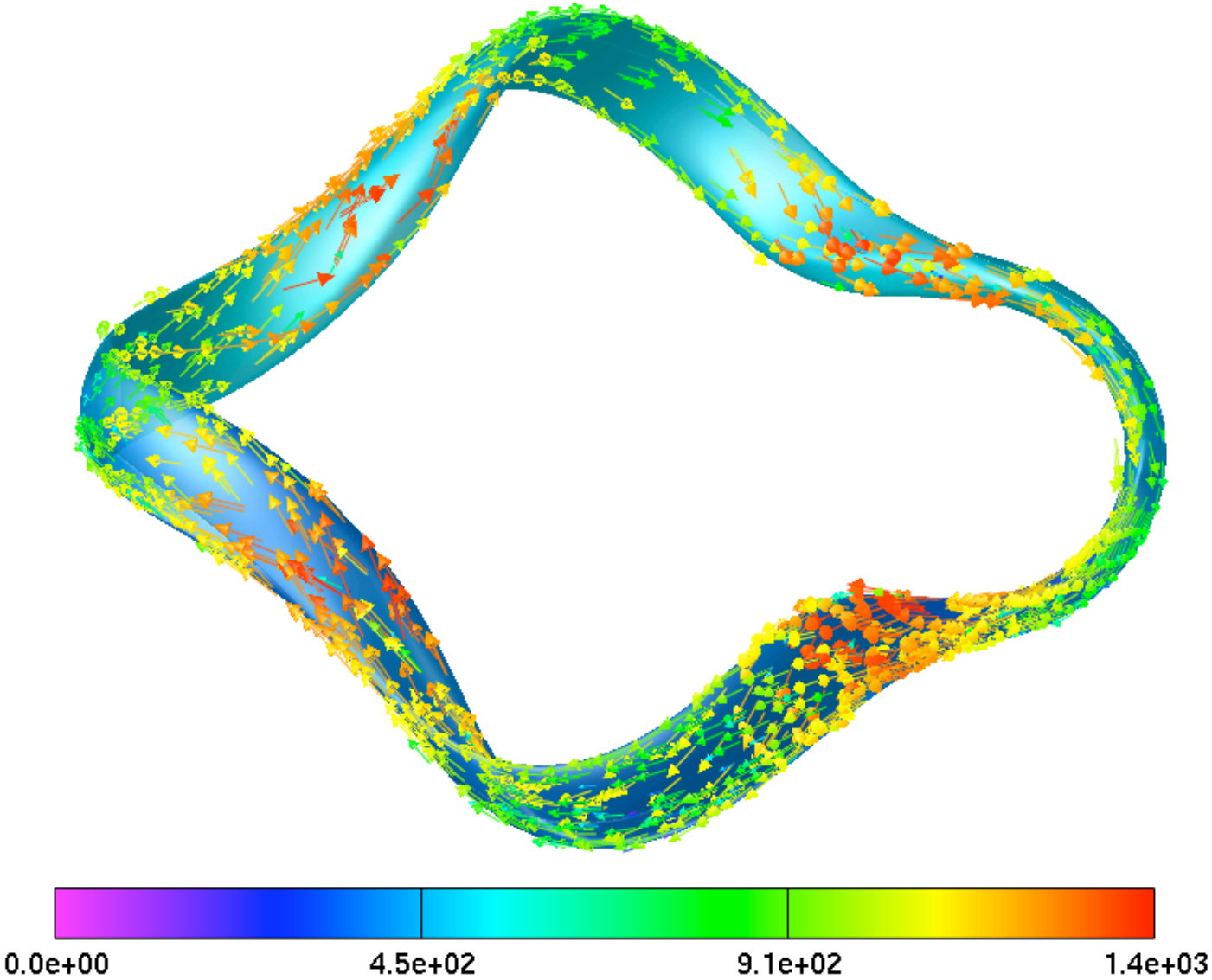
LHD 2D-flow variation

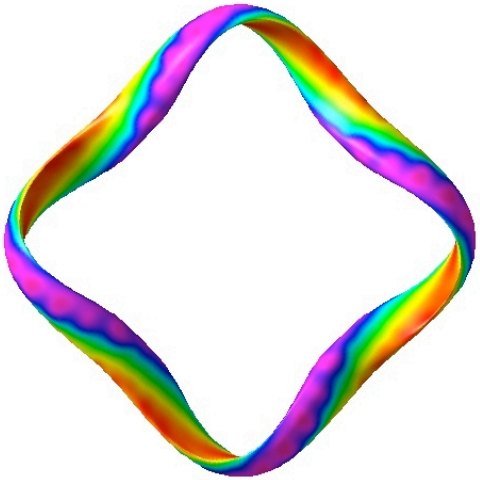


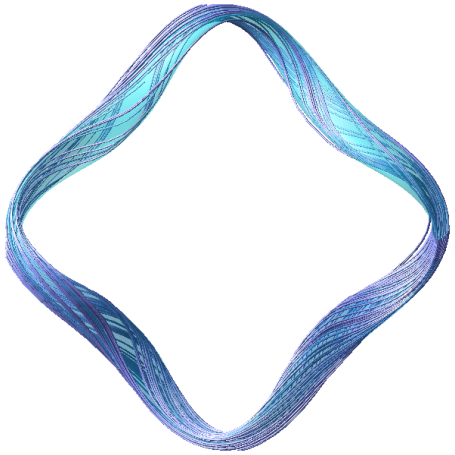




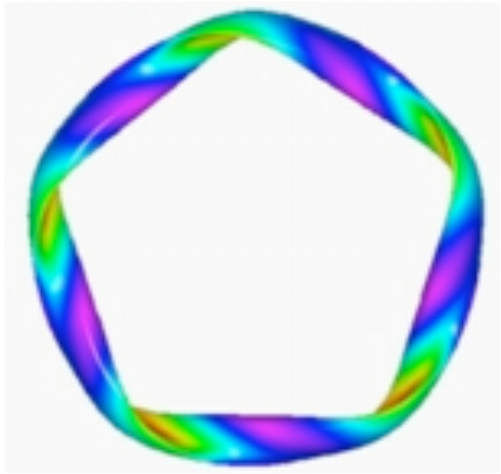
HSX 2D-flow variation

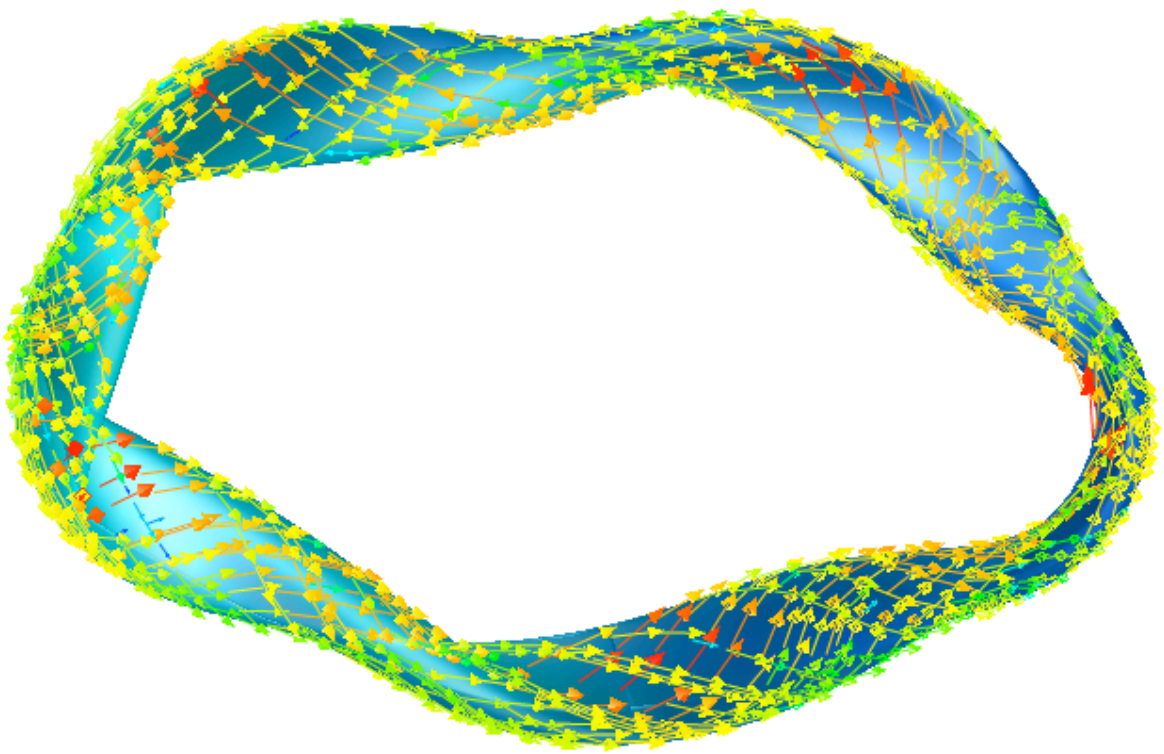


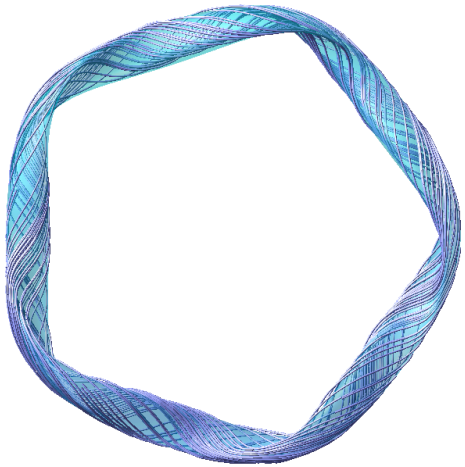




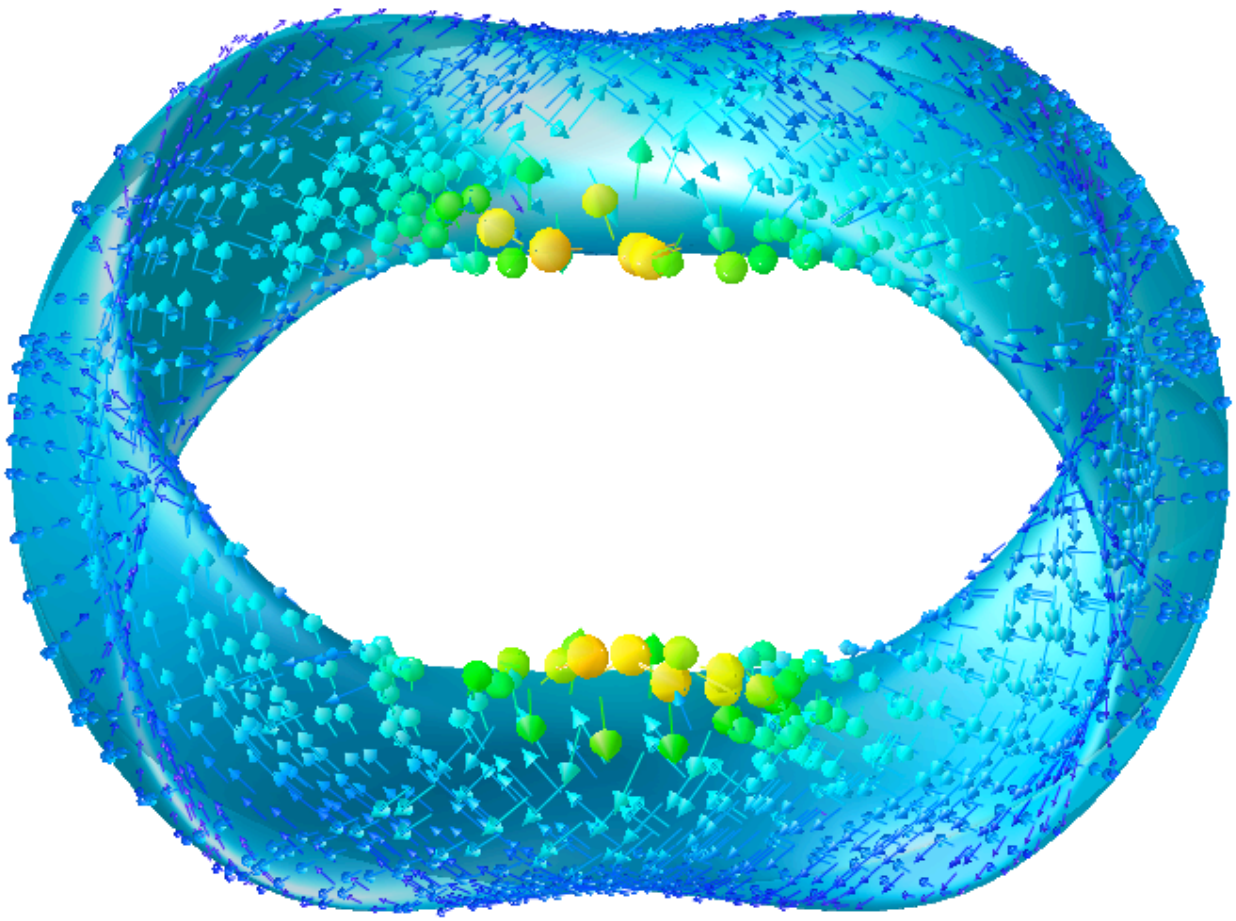
W7-X 2D-flow variation

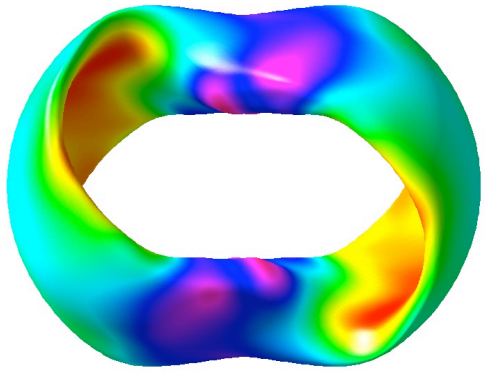




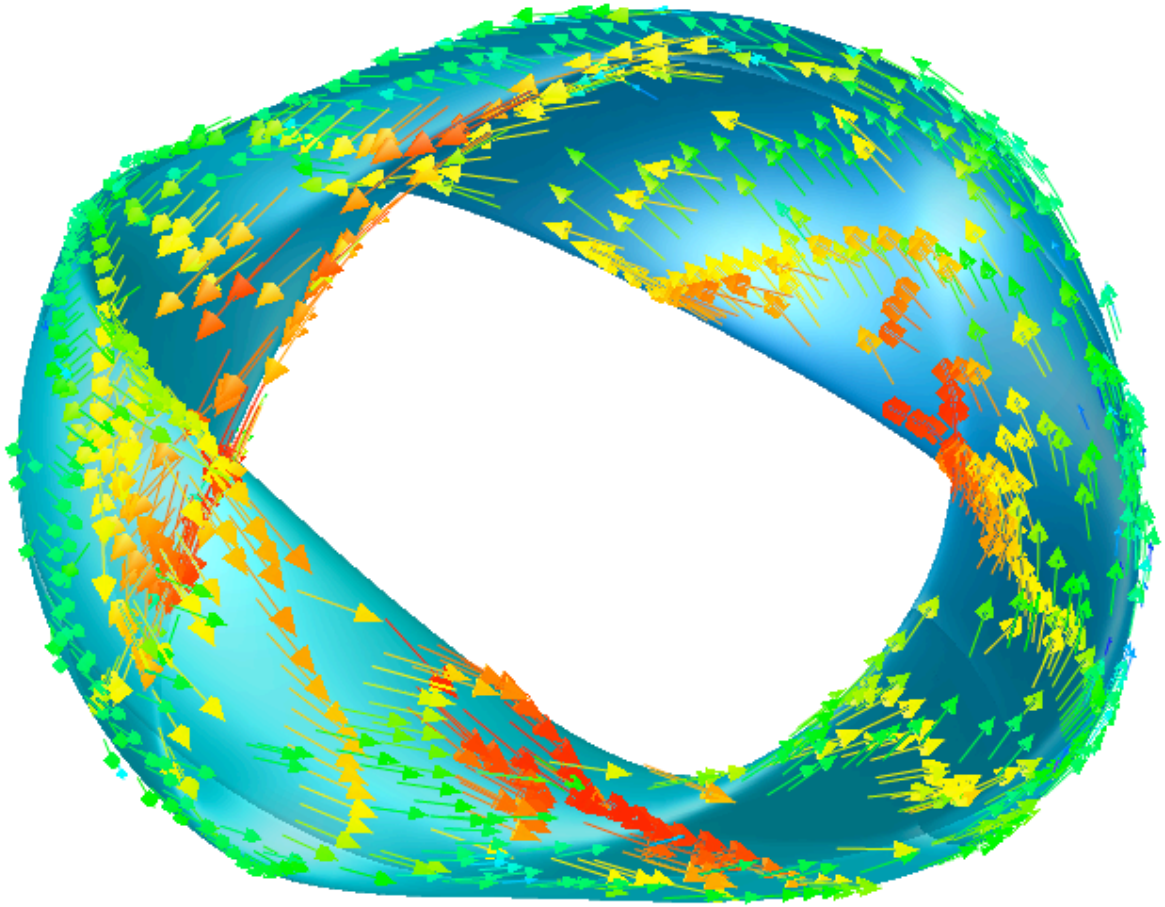


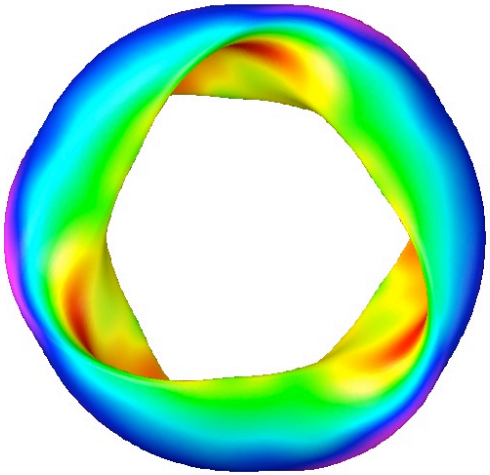
QPS 2D-flow variation

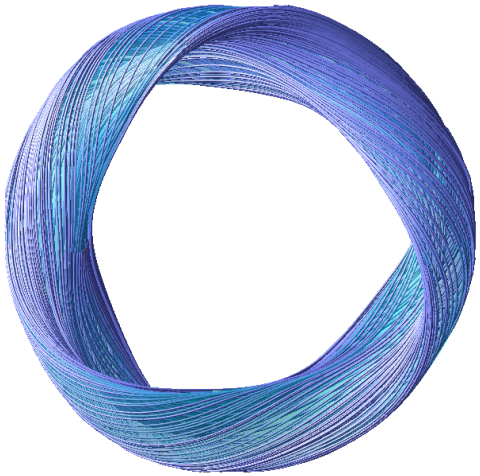




NCSX 2D-flow variation







Plasma flow velocity

$$\vec{V} = \frac{X_1}{eB^2}\vec{\nabla}s \times \vec{B} + \frac{\langle u_{\parallel} B \rangle}{\langle B^2 \rangle}\vec{B} + \frac{X_1}{e}\tilde{U}\vec{B} \quad \text{where } X_1 \equiv -\frac{1}{n}\frac{\partial p}{\partial s} - e\frac{\partial \Phi}{\partial s}$$

diamagnetic and E x B

neoclassical parallel flow

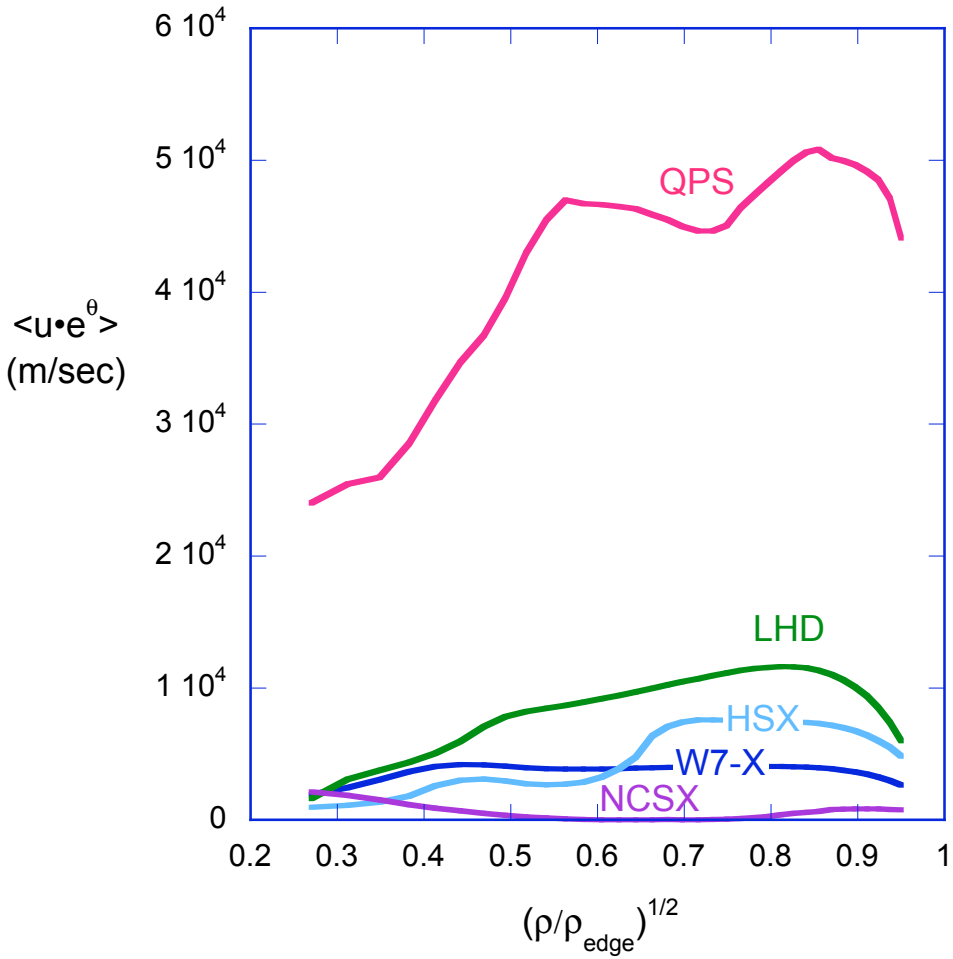
Pfirsch-Schlüter - for $\vec{\nabla} \bullet \left(\rho \vec{V}\right) = 0$

- Components taken: $e^{\theta} = \vec{\nabla}\theta / \left|\vec{\nabla}\theta\right|, \; and \; e^{\zeta} = \vec{\nabla}\zeta / \left|\vec{\nabla}\zeta\right|$

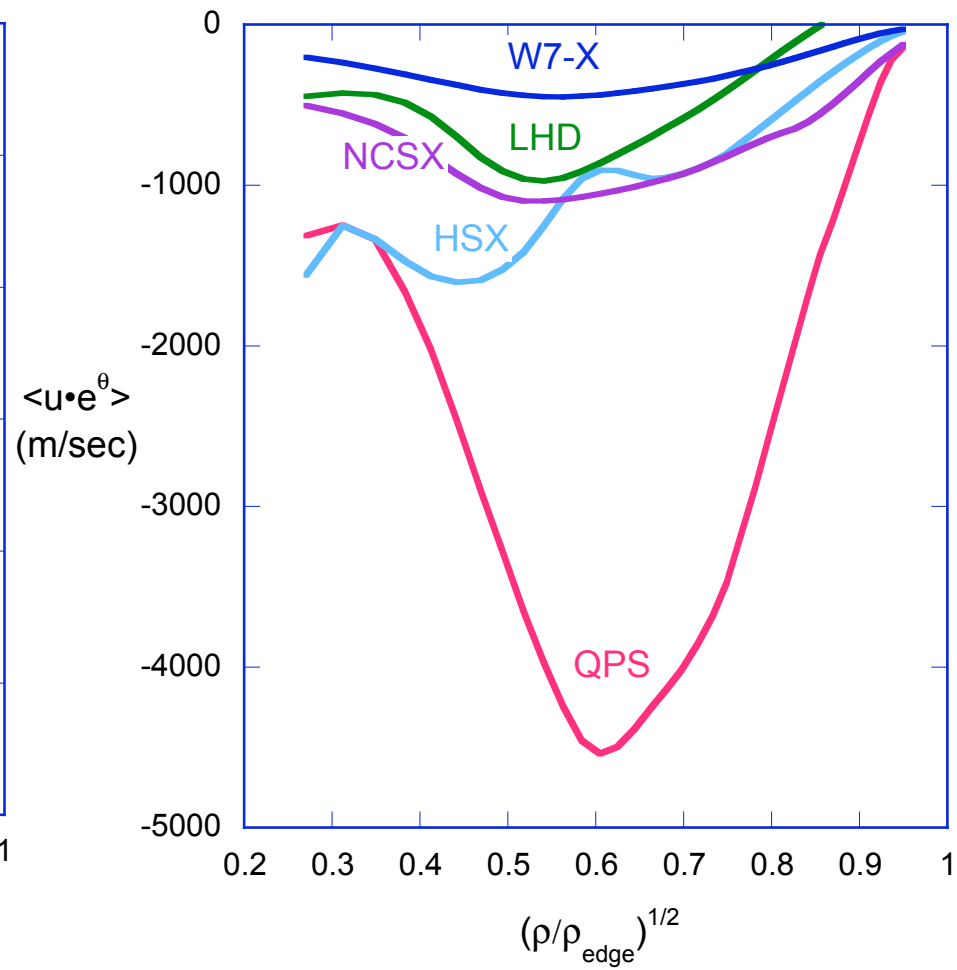
- Reduction to 1D - flux surface average: $\langle \ldots \rangle = \frac{1}{V'} \iint d\theta d\zeta \sqrt{g} (\ldots)$

Comparison of flux-averaged poloidal flow components (contra-variant) among devices indicates QPS has largest poloidal flows

ECH parameters



ICH parameters



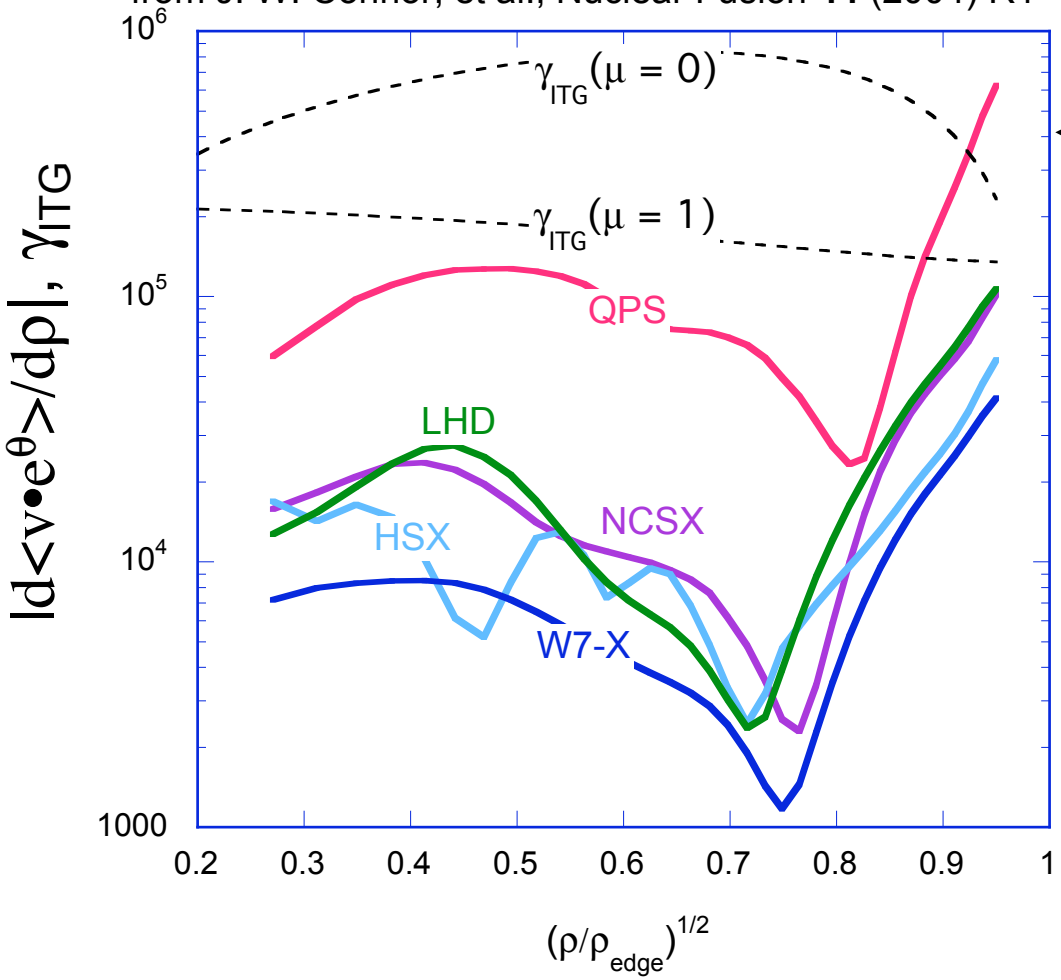
Comparison of shearing rates from ambient flows with ITG growth rates:

Transport barrier condition: shearing rate > γ_{ITG}

$\gamma_{ITG} = (C_S / L_T)(L_T/R)^{\mu}$

where $0 < \mu < 1$, C_S = sound speed

from J. W. Connor, et al., Nuclear Fusion **44** (2004) R1



Recent stellarator DTEM-ITG growth rates

from G. Rewoldt, L.-P. Ku, W. M. Tang, PPPL-4082, June, 2005

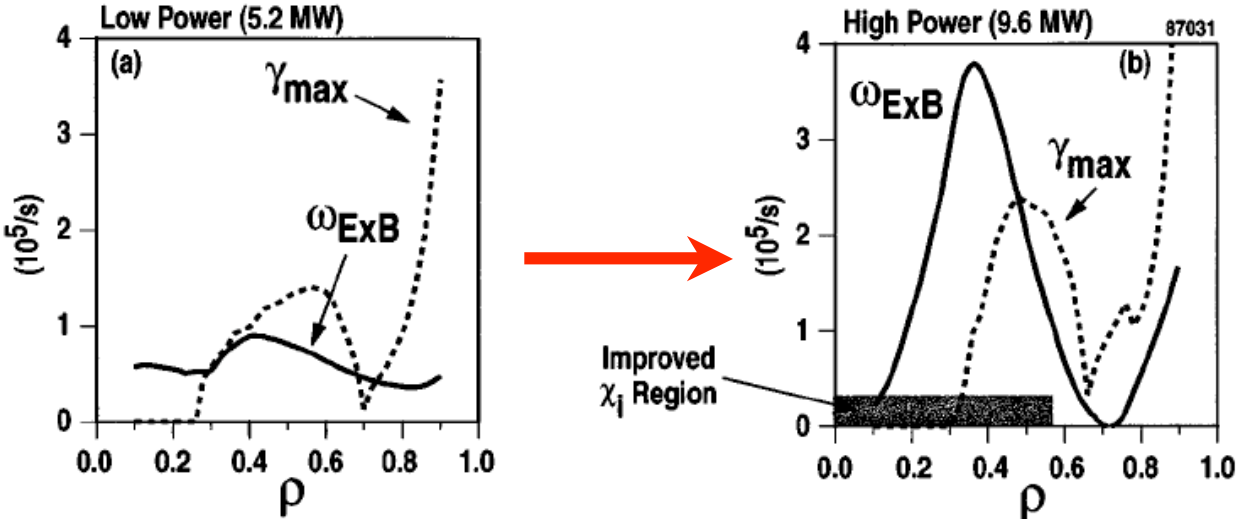
HSX
$\gamma = 4 \times 10^5$ sec^{-1}

QPS, NCSX, W7-X, LHD
$\gamma = 0.2$ to 1.6×10^5 sec^{-1}

Even without external torque, QPS, NCSX, LHD, W7-X shearing rates can exceed $\gamma_{DTEM\text{-}ITG}$ at edge region

DIIID reversed shear (w/ NBI momentum):

$\omega_{ExB} \sim \gamma \Rightarrow$ core confinement transition

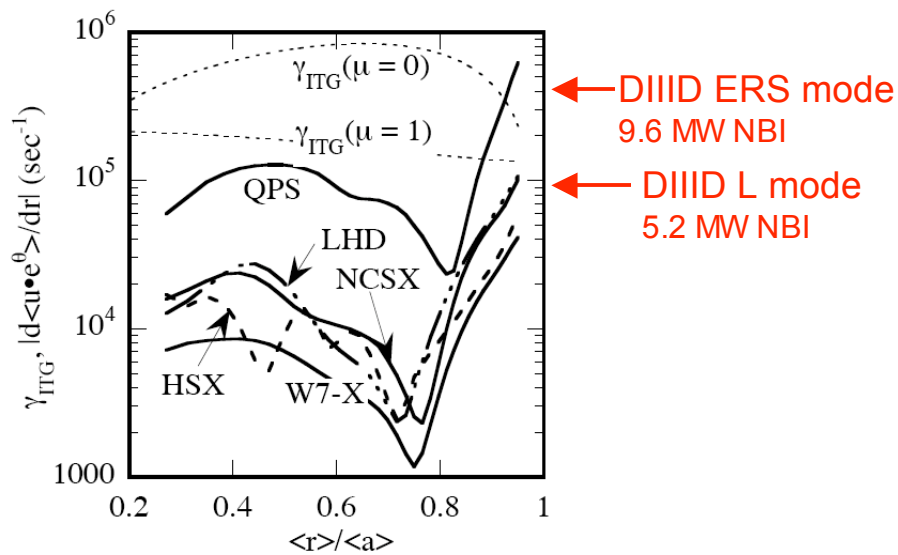


K. Burrell
PoP 1997

QPS: quasi-poloidal symmetry

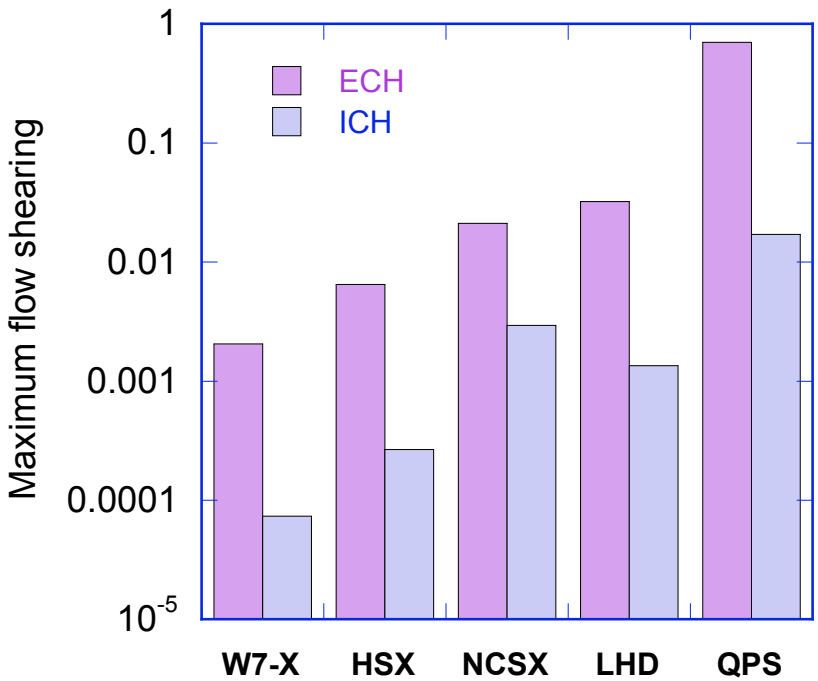
$\omega_{ExB} \sim \gamma_{ITG}$ with ICH

no momentum input



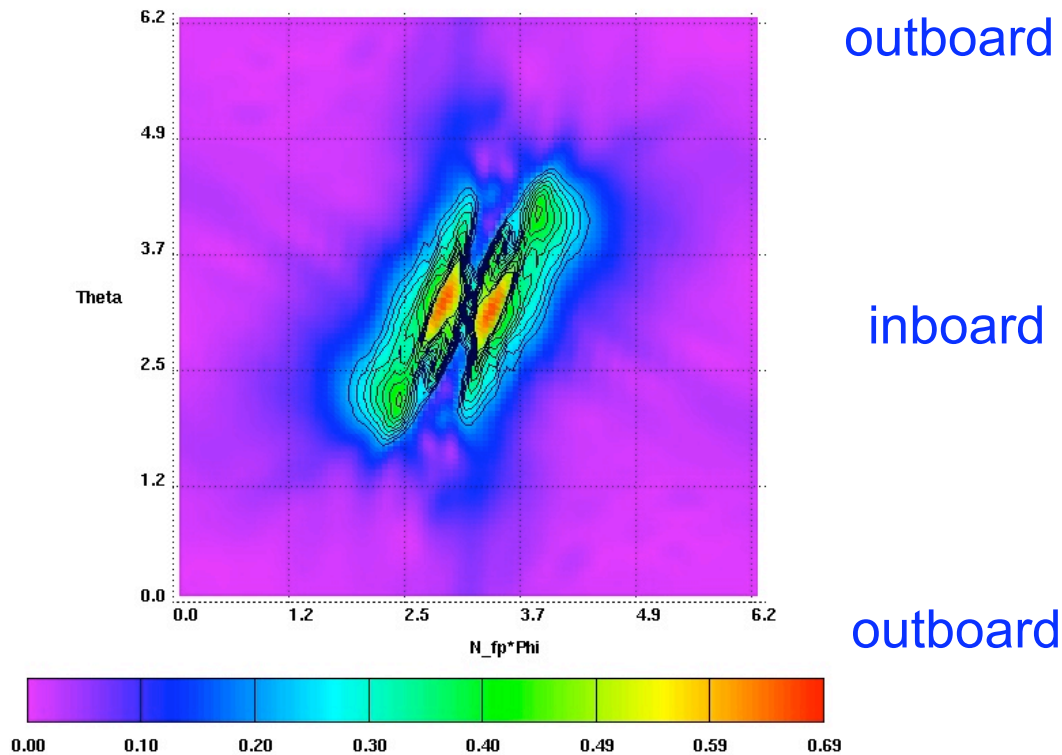
Flow variations within flux surfaces can also impact MHD ballooning/interchange thresholds:

- Maximum parallel flow shearing rates are ~0.5 of Alfvén time
- Could influence MHD stability thresholds



QPS flow shearing along **B**

$$\tau_{A0}\left|\left(\hat{b}\bullet\vec{\nabla}\right)\vec{v}\right| \quad \text{where } \tau_{A0} = \langle R\rangle / v_{A0}$$

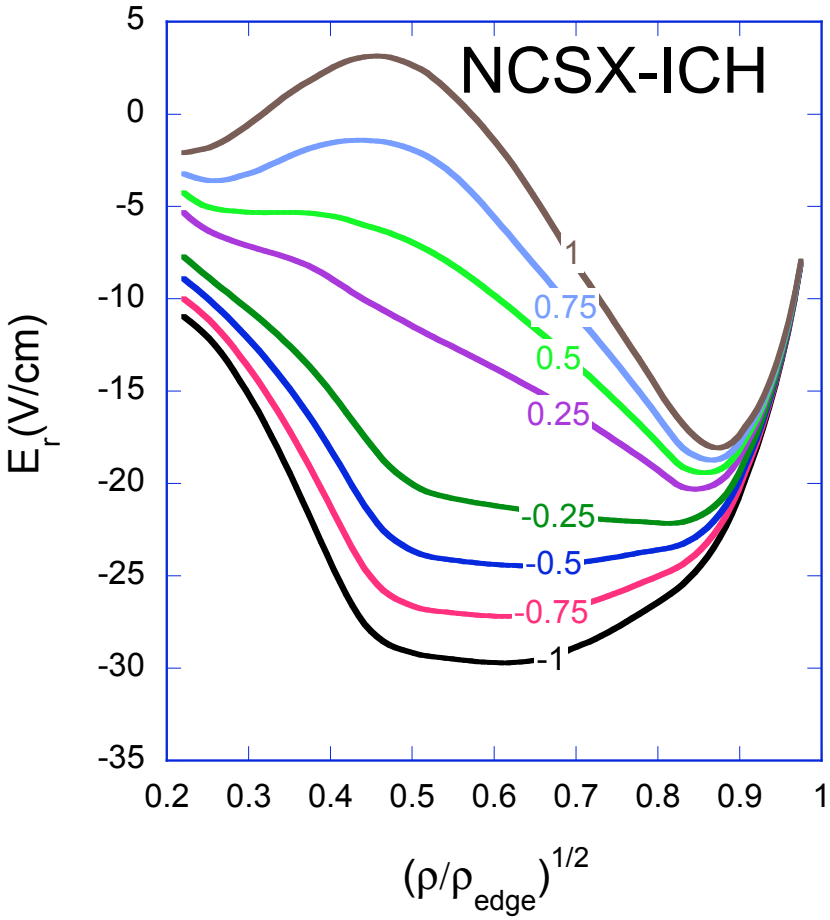


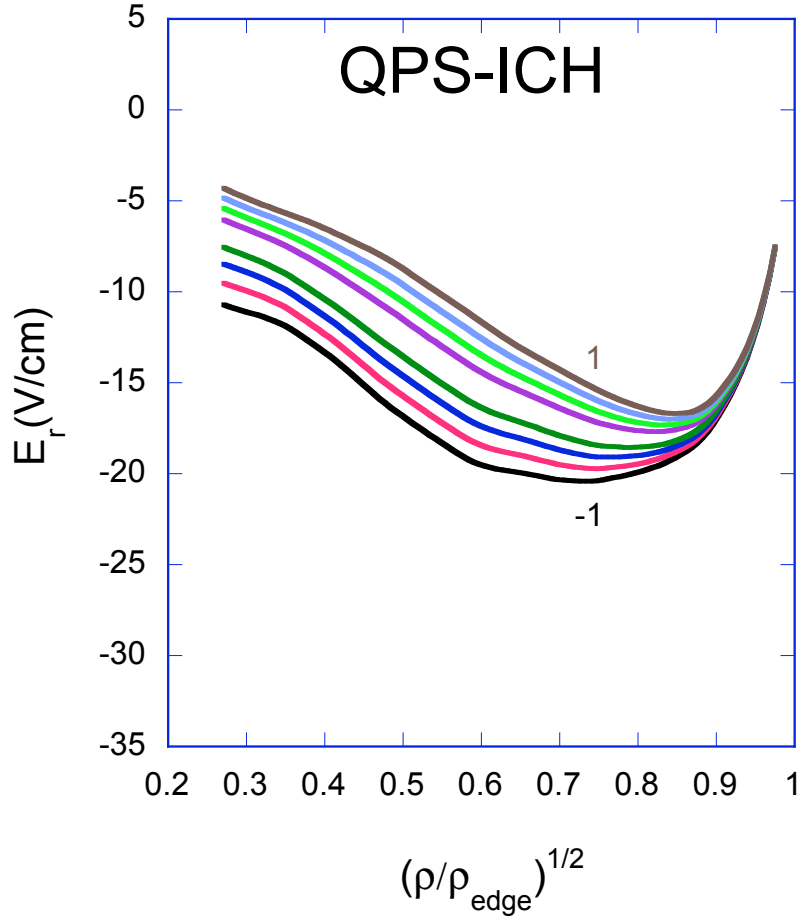
E_r is modified by parallel momentum source

(40 keV H^0 beam, $F_{||}^{(i,e)}$ = -1.0, -0.75, -0.5, -0.25, 0.25, 0.5, 0.75, 1.0 Nt/m^3)

Same viscosity coefficients that relate gradients in n, T, ϕ to $u_{||}$ also imply that changes in $u_{||}$ modify Γ.

$$\begin{bmatrix} \left\langle \vec{B}\cdot\left(\vec{\nabla}\cdot\overleftrightarrow{\Pi}_a\right)\right\rangle \\ \left\langle \vec{B}\cdot\left(\vec{\nabla}\cdot\overleftrightarrow{\Theta}_a\right)\right\rangle \\ \Gamma_a \\ Q_a/T_a \end{bmatrix} = \begin{bmatrix} M_{a1} & M_{a2} & N_{a1} & N_{a2} \\ M_{a2} & M_{a3} & N_{a2} & N_{a3} \\ N_{a1} & N_{a2} & L_{a1} & L_{a2} \\ N_{a2} & N_{a3} & L_{a2} & L_{a3} \end{bmatrix} \begin{bmatrix} \langle u_{||a}B\rangle / \langle B^2\rangle \\ \frac{2}{5p_a}\langle q_{||a}B\rangle / \langle B^2\rangle \\ -\frac{1}{n_a}\frac{\partial p_a}{\partial s} - e_a\frac{\partial\Phi}{\partial s} \\ -\frac{\partial T_a}{\partial s} \end{bmatrix}$$





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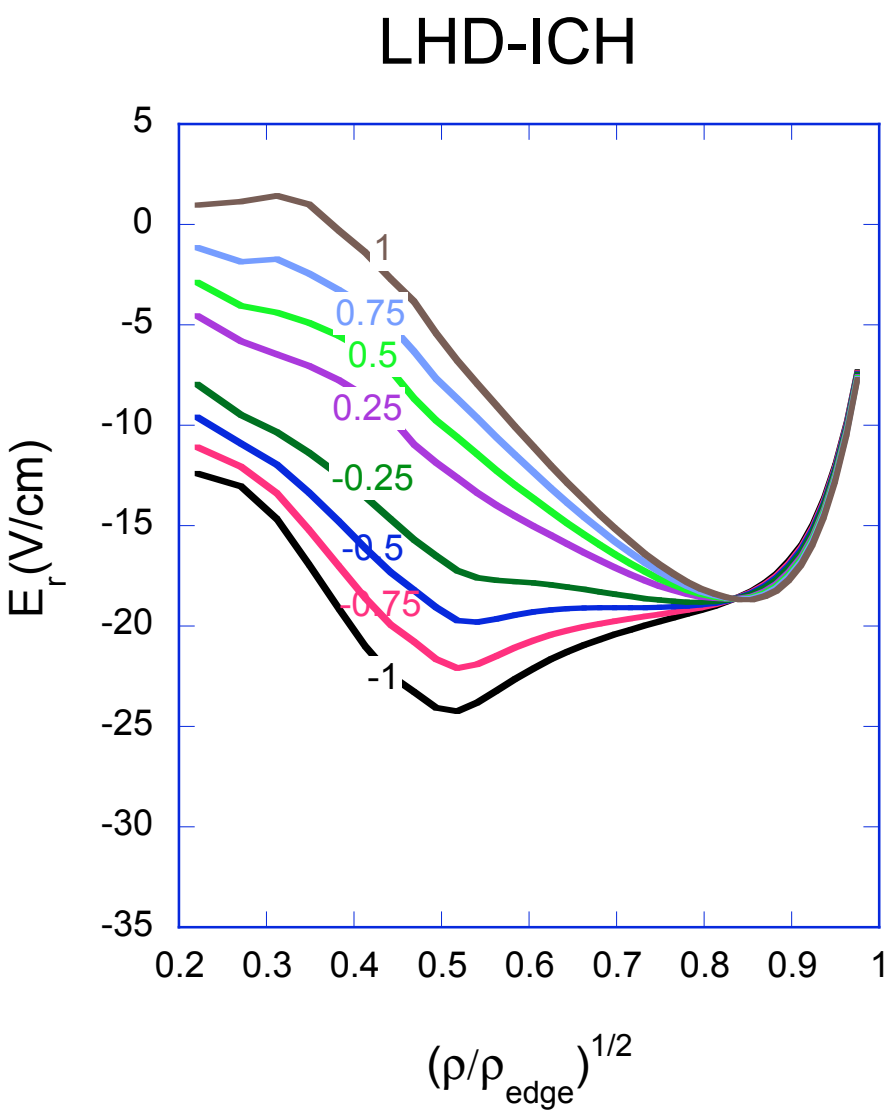
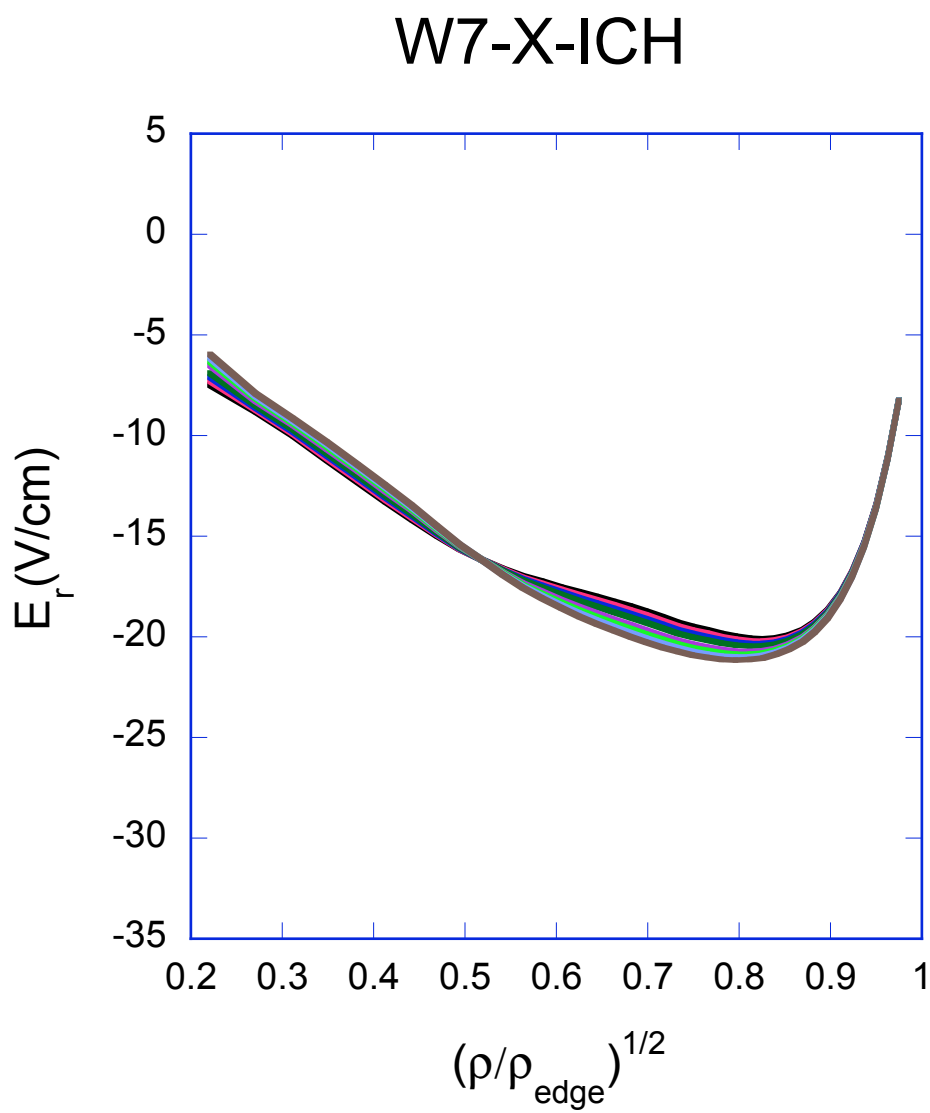
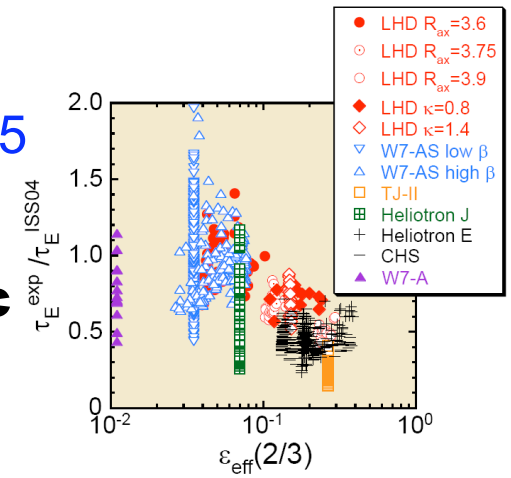




Conclusions

- **Stellarator ambient (no momentum input) shearing rates approach turbulence suppressing levels**
  - ITG, ballooning: $\omega_{ExB}/\gamma_{ITG} \sim (\rho_{ion}/\langle a\rangle)(T_{ion}/T_{elec})^{1/2} f(\nu_*, B)$
  - Stellarator-specific gyro-turbulence work needed, with ω_{ExB}
  - Flow-related optimization targets - use $E \times B$ shear (turbulence suppression to complement neoclassical transport reduction
  - Apply to LHD discharges where ripple changes -> 2.5 increase in confinement time, also HSX
- **The sensitivity of flow properties to magnetic structure is an opportunity for stellarators**
  - Experimental tests of flow damping in different directions
  - Possible new hidden variable in confinement scaling data
  - Other stellarator specific "tricks"
    - Electric field bifurcations via profile control
    - Steady state island -> localized high viscosity -> strong flow shear



From ISCDB
-A. Dinklage,
-et al.