Gyrokinetic study of quasi-linear fluxes in LHD-like and axisymmetric configurations

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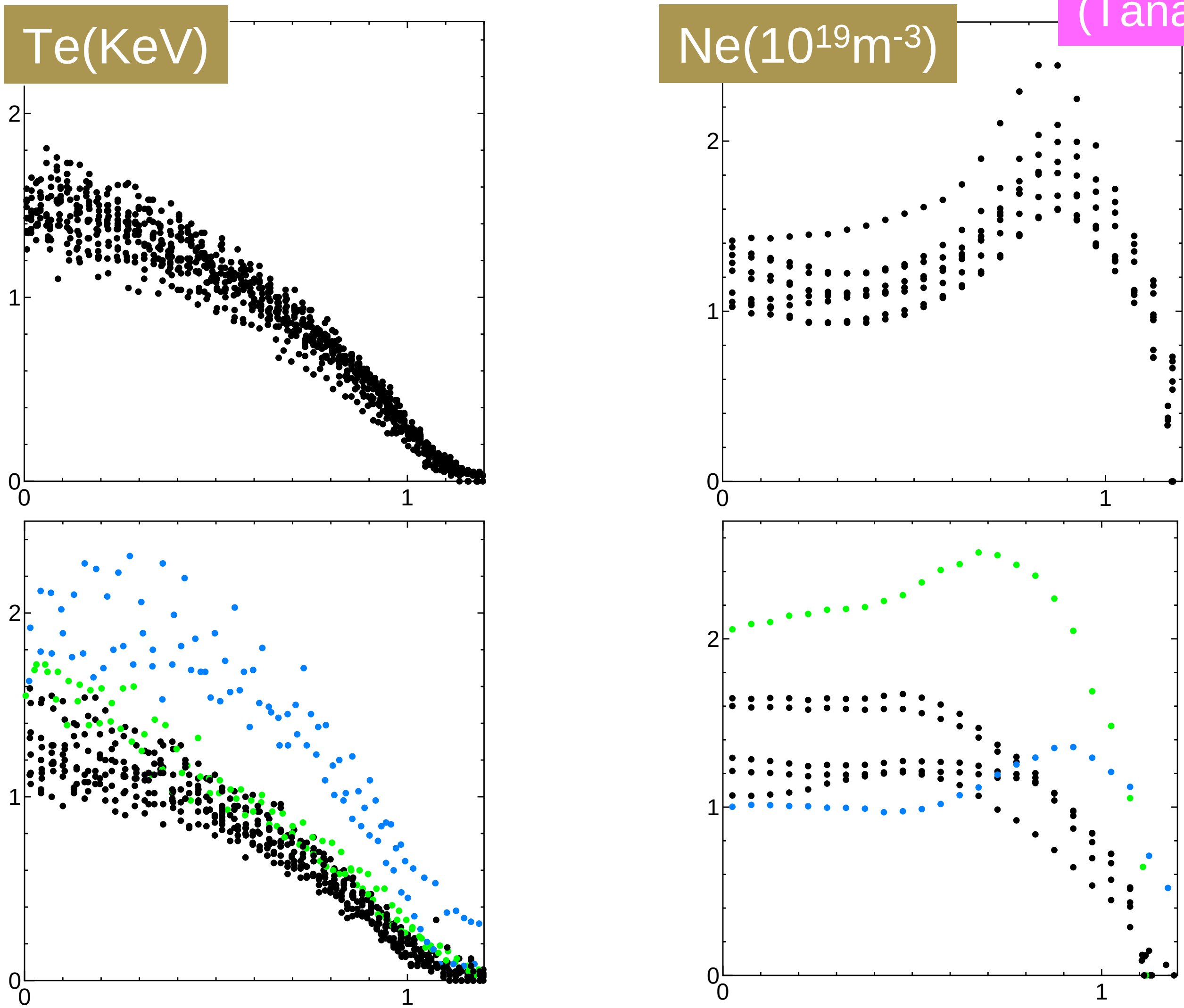
OUTLINE

LHD experiments
(Tanaka, Michael et al.)



R=3.75m

R=3.53m

Various configurations can be realized in the LHD configurations by changing coil currents.
In the typical LHD experiments, the density profile shows hollow profiles.
However recent experiments gradually show the configuration dependence on the density profiles
| typical LHD, for example R=3.75m ⇒HOLLOW | R=3.53m⇒FLAT | with some exceptions

OUTLINE

Early studies

- R=3.53m is found to be neoclassically optimized configuration due to reducing the effective helical ripples [Murakami et.al., NF, 2002]
- Gyrokinetic studies have been done [Rewoldt et al., NF, 2002], and configuration(B) dependence was relatively weak (profile effects are stronger than the configuration effects)
- Experimental results have showed that the typical density profiles in LHD are hollow, while flat profile is observed in R=3.53m configuration

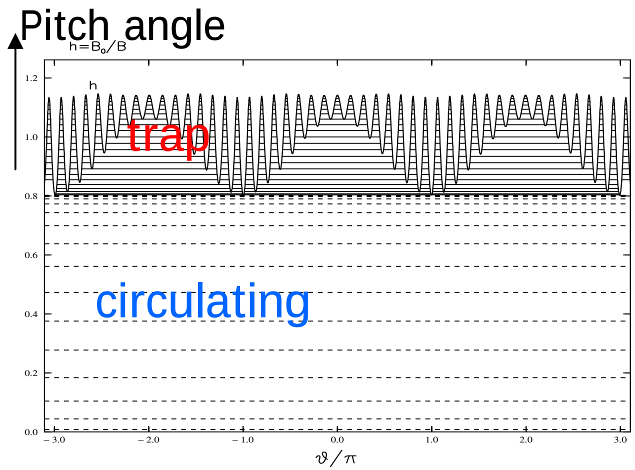
From these we have some questions.
Density shows configuration dependence which should be relevant to anomalous transport.
Nevertheless linear GK did not show strong configuration dependence.
⇒ Why is density profile typically hollow in the LHD?
⇒ What does determine experimental density profiles? neoclassical or anomalous?
⇒ Is the situation different from usual tokamak?

Method

○ Linear electrostatic gyrokinetic equation is solved by GOBLIN code.
Linear Frequencies and quasi-linear (QL) fluxes are estimated.
Concentration is on the **particle** flux by the **ITG** modes in this study

○ The neoclassical fluxes are also estimated by GSRAKE code [Beidler et al., PPCF 1994], which is valid for $1/\nu$ regime (bounce-average type)

GyrOkinetic Ballooning LINear equation solver (GOBLIN)



In the electromagnetic case, Gyrokinetic equation is [1]

$$\left[\omega - \omega_{dj} + i v_{//} \frac{B^{\theta}}{B} \frac{d}{d\theta}\right] h_{\sigma j}(\theta, E, \Lambda) = (\omega - \omega_{*j}^{T}) \frac{e_j}{T_j} F_{Mj} (J_0 \phi - J_0 v_{//} A_{//} - i J_1 v_{\perp} A_{\perp}).$$

The formal solution can be written as [2],

$$h_{\sigma j} = h_{\sigma j}(\theta, E, \Lambda; \omega, \phi, A_{//}, A_{\perp}).$$

Then, Poisson equation, parallel and perpendicular Ampare's law are governing equations,

$$\int d^3v \sum_j e_j \left(\frac{-e_j \phi}{T_j} F_{Mj} + (h_+ + h_-)_j J_0 \right) = \varepsilon_0 k_{\perp}{}^2 \phi,$$

$$\int d^3v \sum e_j |v_{//}| (h_+ - h_-)_j J_0 = \frac{1}{\mu_0} k_{\perp}{}^2 A_{//},$$

$$\int d^3v \sum e_j v_{\perp} (h_+ + h_-)_j J_1 = i \frac{1}{\mu_0} k_{\perp}{}^2 A_{\perp}.$$

To solve, Ritz method is applied, by expanding $\phi = \sum_{l=1}^{L} (h_{l-1}/h) \phi_l$ (and $A_{//}, A_{\perp}$ also),

and integrating with $\frac{T_e}{e_e^2 n_e} \int d\theta h h_{l'}$, leading to a matrix eigenvalue problem [2],

$$\sum_{l=1}^{L} M_{l'l}^{Poisson}(\omega)(\phi_l, A_{//L+l}, A_{\perp 2L+l}) = 0,$$

$$\sum_{l=1}^{L} M_{l'l}^{Amp:para}(\omega)(\phi_l, A_{//L+l}, A_{\perp 2L+l}) = 0,$$

$$\sum_{l=1}^{L} M_{l'l}^{Amp:perp}(\omega)(\phi_l, A_{//L+l}, A_{\perp 2L+l}) = 0, \qquad l' = 0, \ldots, L-1.$$

- Kinetic integrals are very exact [2] (for both circulating/trapped particles)
- Some approximations:
  - Ballooning representation
  - Collisionless
  - $F_0 = F_M$, $E_0 = 0$
- Equilibrium quantities are estimated by VMEC [3], which are entered through ω_d, $k_{\perp}$, B, and so on
- Linear frequencies, eigenfunction, quasi-linear fluxes are obtained

In this study electrostatic assumption is used

[1] J.B.Taylor, et al., Plasma Physics **10**, 479 (1968)
[2] G.Rewoldt, et al., Phys. Fluids **25**, 480 (1982)
[3] S.P.Hirshman, Phys. Fluids **26**, 3553 (1983)

Quasi-linear flux (electro-static)

$$\frac{\delta E\times B}{B^2}\cdot\nabla r$$

$M^{j}_{l'l}$: matrix in calc.

$$\Gamma_j=\left\langle \delta n_j \delta V_r\right\rangle_s=\left(\frac{\chi'}{V'}\frac{\sqrt{g}B^2}{\chi' B_0^2}\right)_{Boz}\left(\frac{k_\alpha}{d\chi/dr}\right)\left(\frac{|e|n_e}{T_e}\frac{1}{Z_j}\right)\mathrm{Re}\left[i\int d\theta\sum_{l'}hh_{l'}\phi_{l'}\left(\int d^3v\,\delta f_j\right)\right]\frac{|\delta\phi|^2}{\sum_l|\phi_l|^2}$$

$$Q_j=\frac{3}{2}\left\langle \delta P_j \delta V_r\right\rangle_s=\left(\frac{\chi'}{V'}\frac{\sqrt{g}B^2}{\chi' B_0^2}\right)_{Boz}\left(\frac{k_\alpha}{d\chi/dr}\right)\left(\frac{|e|n_e}{T_e}\frac{T_j}{Z_j}\right)\mathrm{Re}\left[i\int d\theta\sum_{l'}hh_{l'}\phi_{l'}\left(\int d^3v(v/v_{thj})^2\,\delta f_j\right)\right]\frac{|\delta\phi|^2}{\sum_l|\phi_l|^2}$$

$$\frac{1}{\left\langle 1/B^2\right\rangle_{\theta\varsigma}B_0^2}$$

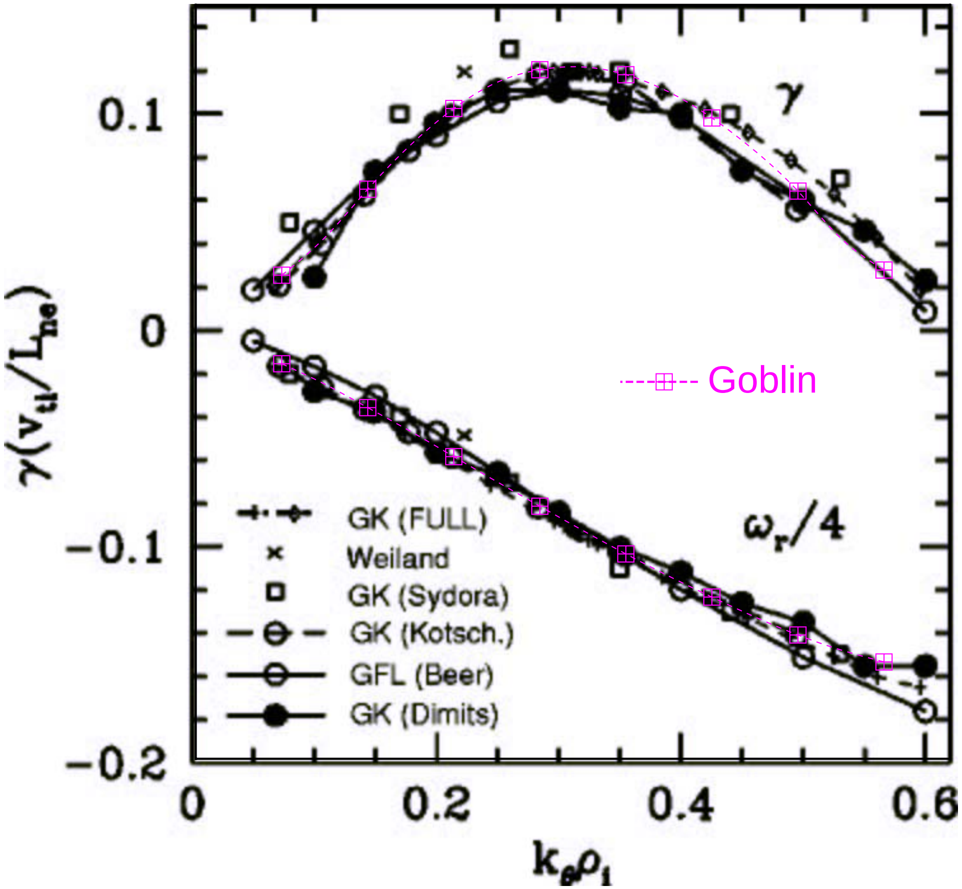
[Rewoldt, PoF 1987]

The absolute value is undetermined.
$|\delta\Phi|$ is sometimes estimated by mixing length assumption, but not used here.
If $|\delta\Phi|$ is given from experiments, the absolute value can be obtained

Recent GK simulations [Jenko, PPCF 2005: Dannert, PoP 2005]
showed that the QL fluxes can give good agreement with the nonlinear fluxes
because the phase between the $\delta\Phi$ and δn or δp is not so different
in the linear and nonlinear phase.

⇒ If so, QL flux is very useful to obtain physics insight

BENCH MARK



(Electrostatic)

$\varepsilon_t = r / R = 0.18$

$q = 1.4$

$\bar{s} = 0.78, \quad \bar{\alpha} = 0$

$R / L_n = 2.2, \quad R / L_T = 6.9$

$\eta = L_n / L_T = 3.114$

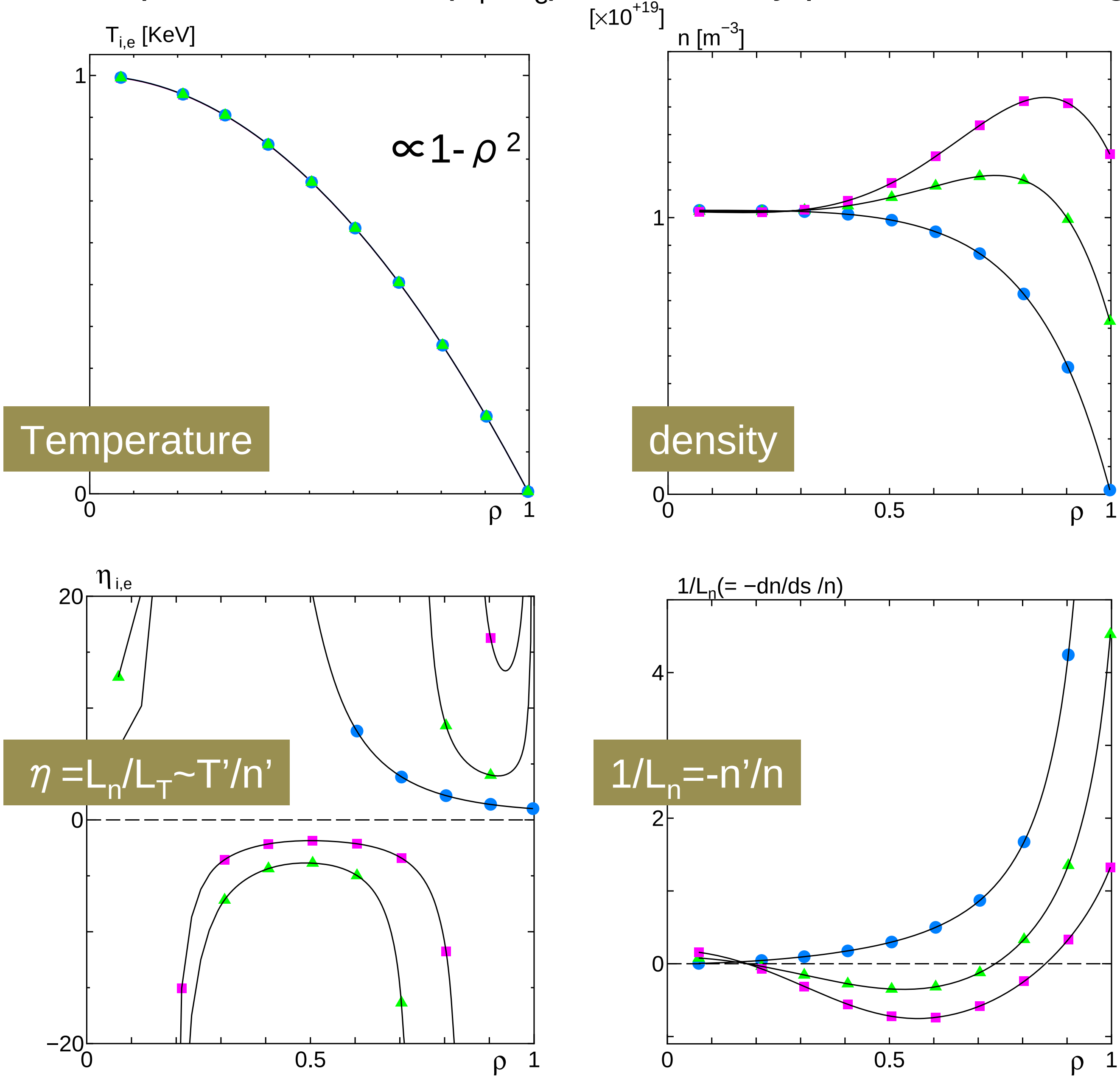
$m_i / m_e = 3670, \quad T_i / T_e = 1$

s-α equilibrium model is used.
Electron is assumed to be **adiabatic**.

(Dimits et al., Phys. Plasmas 7,969(2000), Fig.1)

Profiles

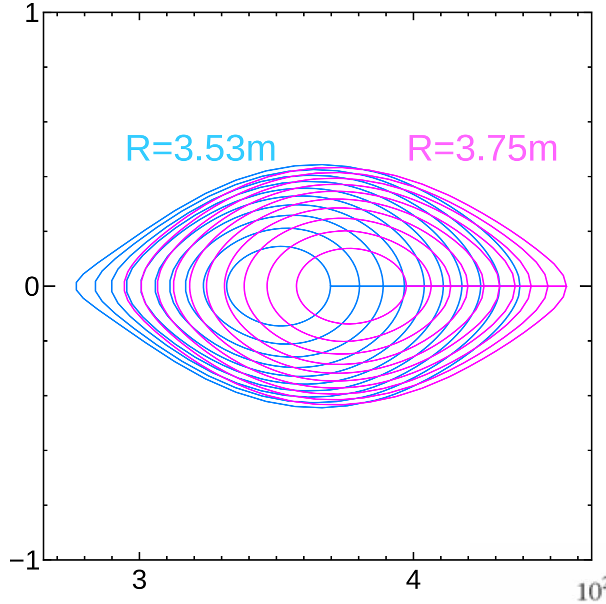
Temperature is fixed (T_i=T_e), and density profiles are changed from peaky to hollow.



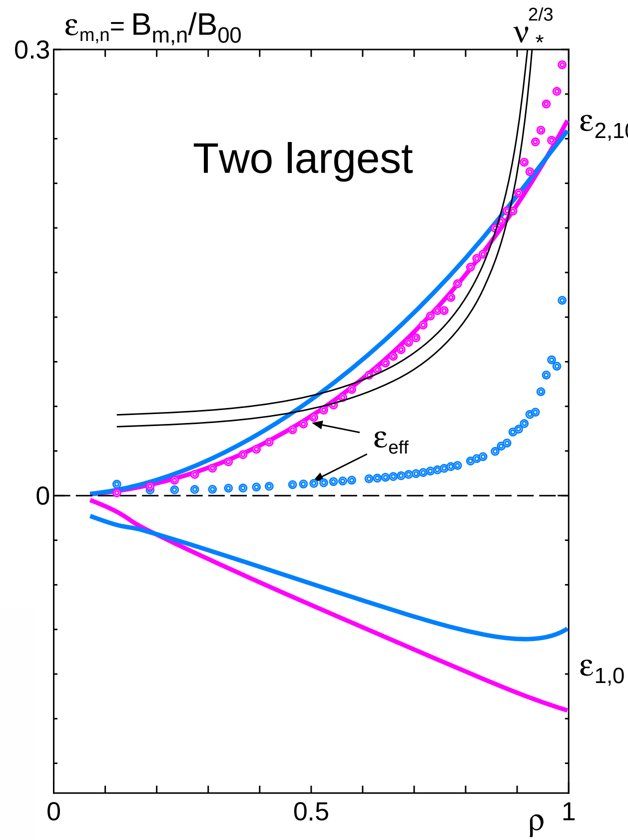
Values
T(0)=1[KeV]
n(0)=1x10^{19}[m^{-3}]
B_0=1[T]

magnetic configurations

MHD equilibrium is calculated by **VMEC** code:

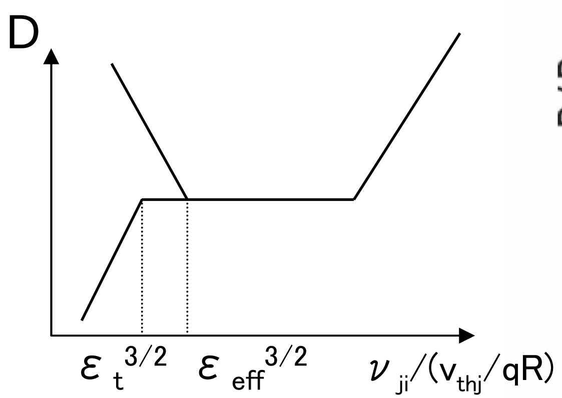


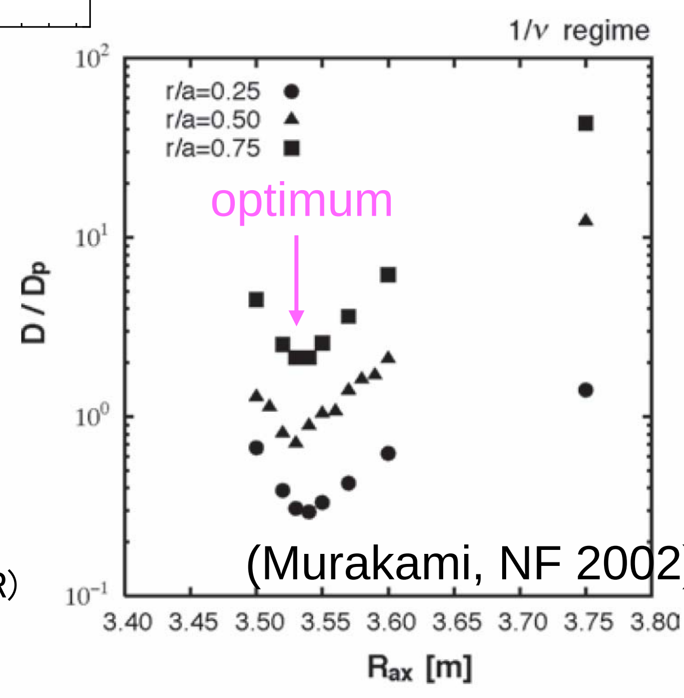
Translate to Boozer Coordinates (**NEWBOZ** code)



B_{LM} is not so different but effective helical ripple is sufficiently small at R=3.53m.

ε_{eff} estimated is defined in (Nemov, PoP 1999)



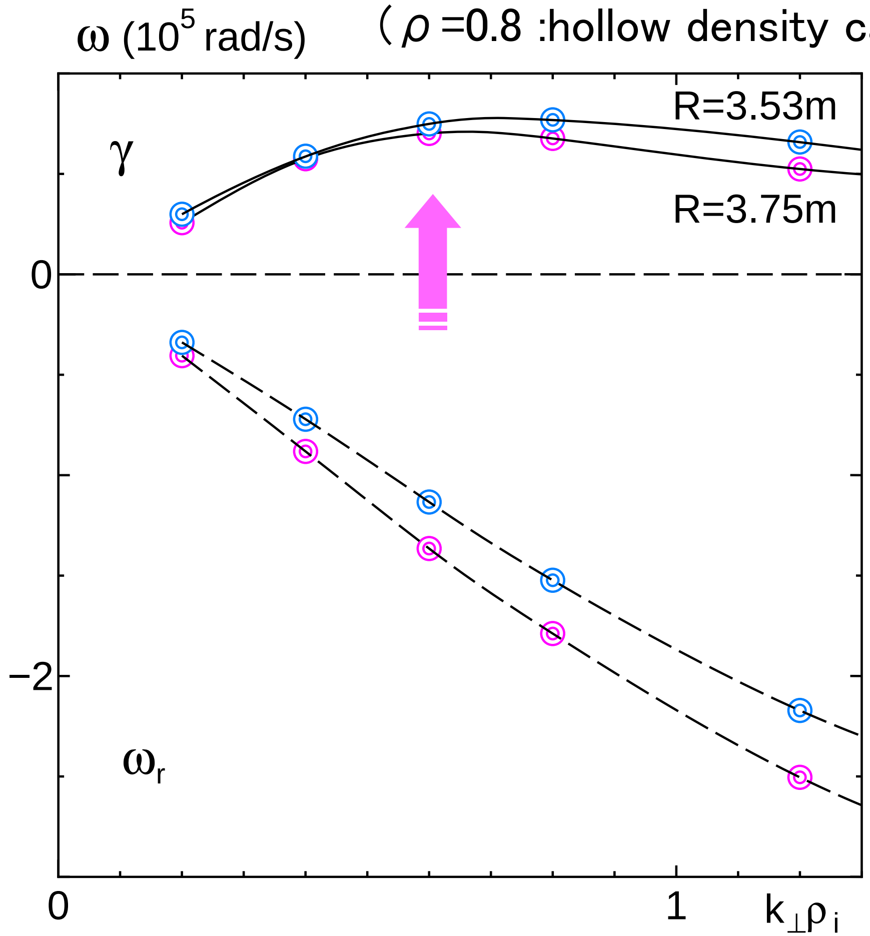


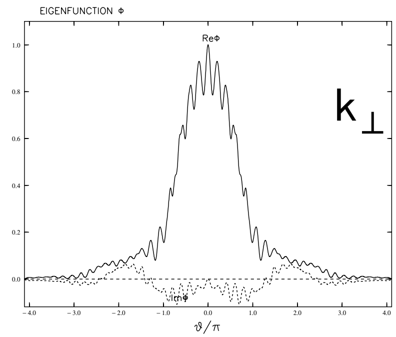
(Murakami, NF 2002)

Quasi-linear anomalous

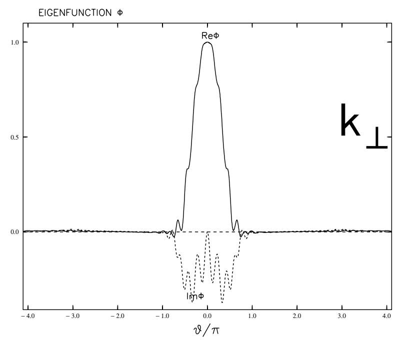
ITG frequencies as a function of krho

(ρ=0.8 :hollow density case)





$k_\perp \rho_i$=0.2

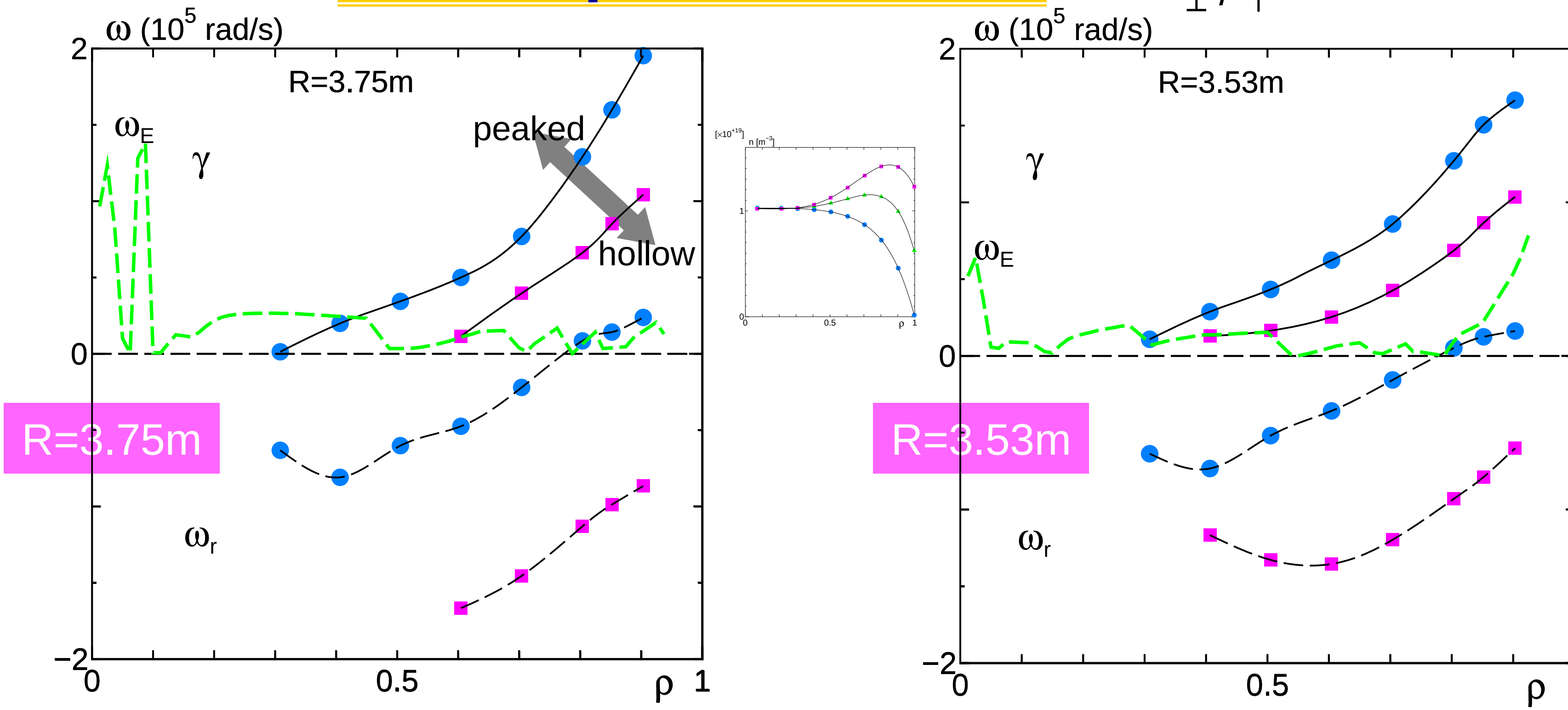


$k_\perp \rho_i$=0.5

The linear peak is at $k_\perp \rho_i \sim 0.6$.
In the following, we take $k_\perp \rho_i$=0.5 fixed
Also $\theta_k = \alpha = 0$ fixed

ITG frequencies in LHD

$k_\perp \rho_i = 0.5$: fixed



Linear growth rate and real frequencies in both configurations are similar.
Profile effect is stronger than the magnetic configuration effect.

$$\omega_E \equiv (2\pi)^2 \left| \frac{(RB_\theta)^2}{B} \frac{d}{d\psi_{pol}} \left(\frac{E_r}{RB_\theta} \right) \right| \approx 2\pi \left| \frac{\rho}{aqB_0} \frac{d}{d\rho} \left(\frac{qE_r}{\rho} \right) \right| \text{ [rad/s]} \quad \left(q = \frac{rB_{tol}}{RB_{pol}} = \frac{\psi'_{tol}}{\psi'_{pol}} \Rightarrow \underline{\frac{\psi'_{pol}}{2\pi} = RB_{pol} = \frac{rB_0}{q}} \right)$$

Heuristic stabilizing condition $\gamma_{lin}(E_r=0) \leqq \omega_E$ [Hahm, Burrell, PoP (1995)]
To estimate ω_E, neoclassical ambipolar E_r from GSRAKE code is used.

Neoclassical E_r shear is insufficient to stabilize γ_{ITG}

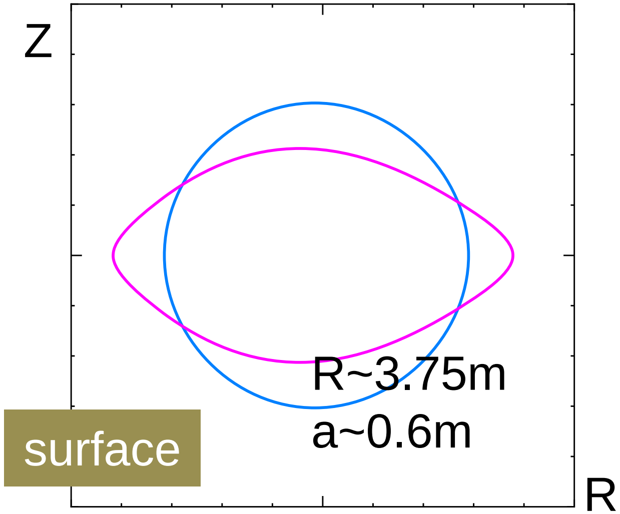
magnetic configurations

Since the configuration dependence of linear GK is too weak in LHD,
We also consider a tokamak with comparable aspect ratio

MHD equilibrium is calculated by **VMEC** code:

1) LHD-like: last closed surface (R_{mn} ,Z_{mn})

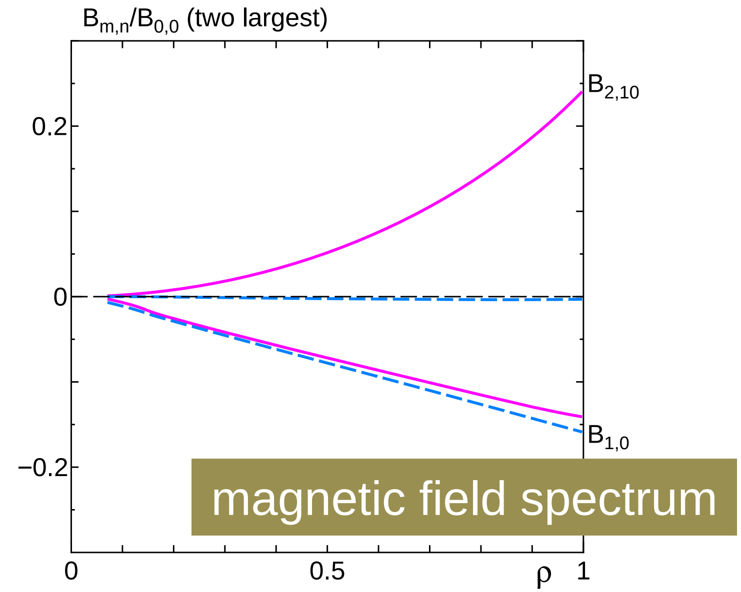
2) Axisymmetric: drop n/=0 components of LHD



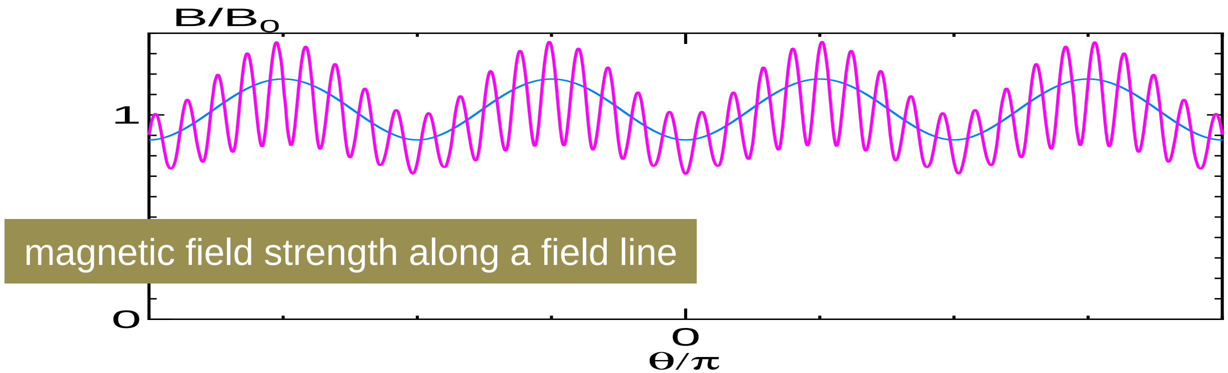
Translate to Boozer Coordinates (**NEWBOZ** code)

Axisymmetric: almost B_1 only like $B=B_0(1-r/R\cos\theta)=B_0+B_1\cos\theta$

LHD: large helical component(2,10)

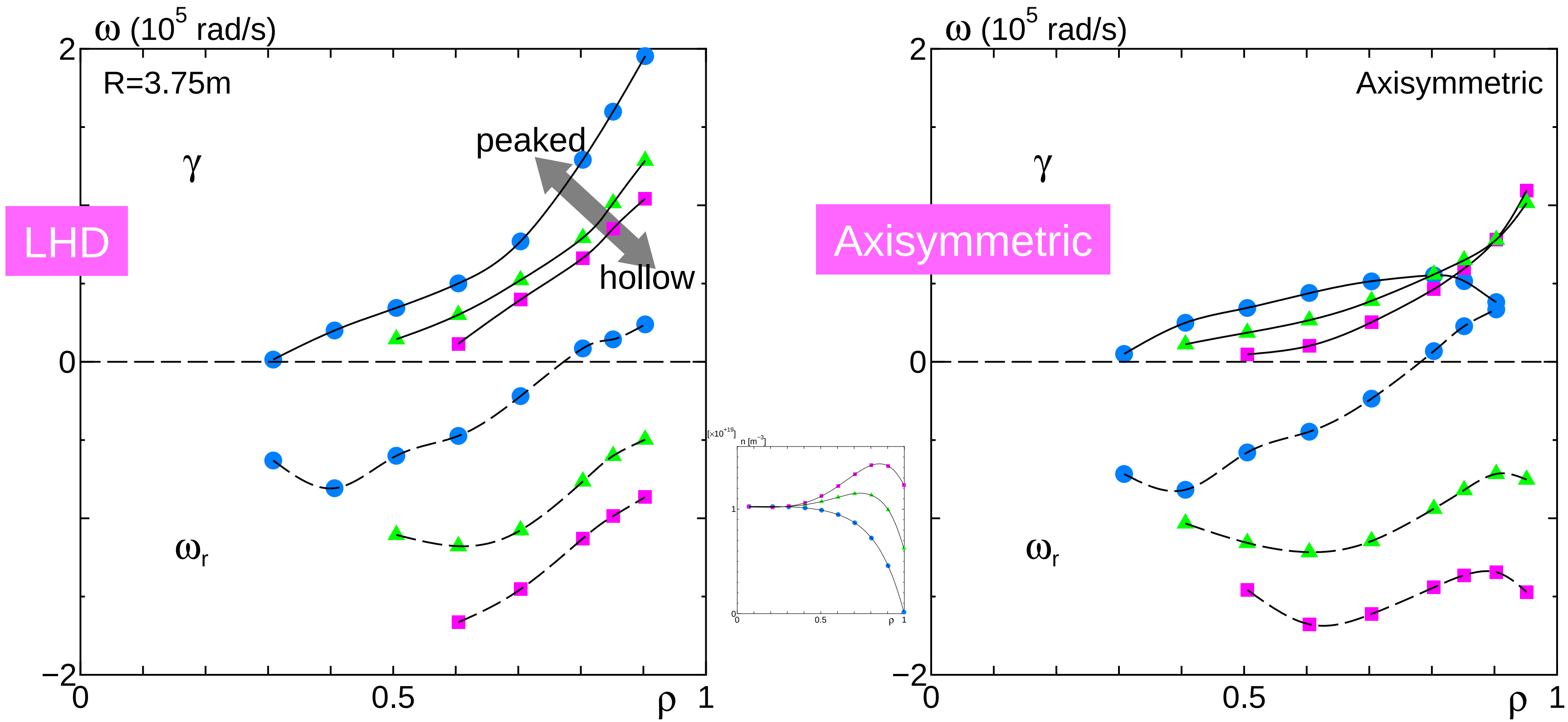


- Net current free is assumed for LHD (iota is determined)
- The iota profile obtained for the LHD is given for Axisymmetric case



Comparison with tokamak

$k_\perp \rho_i = 0.5$: fixed



-LHD

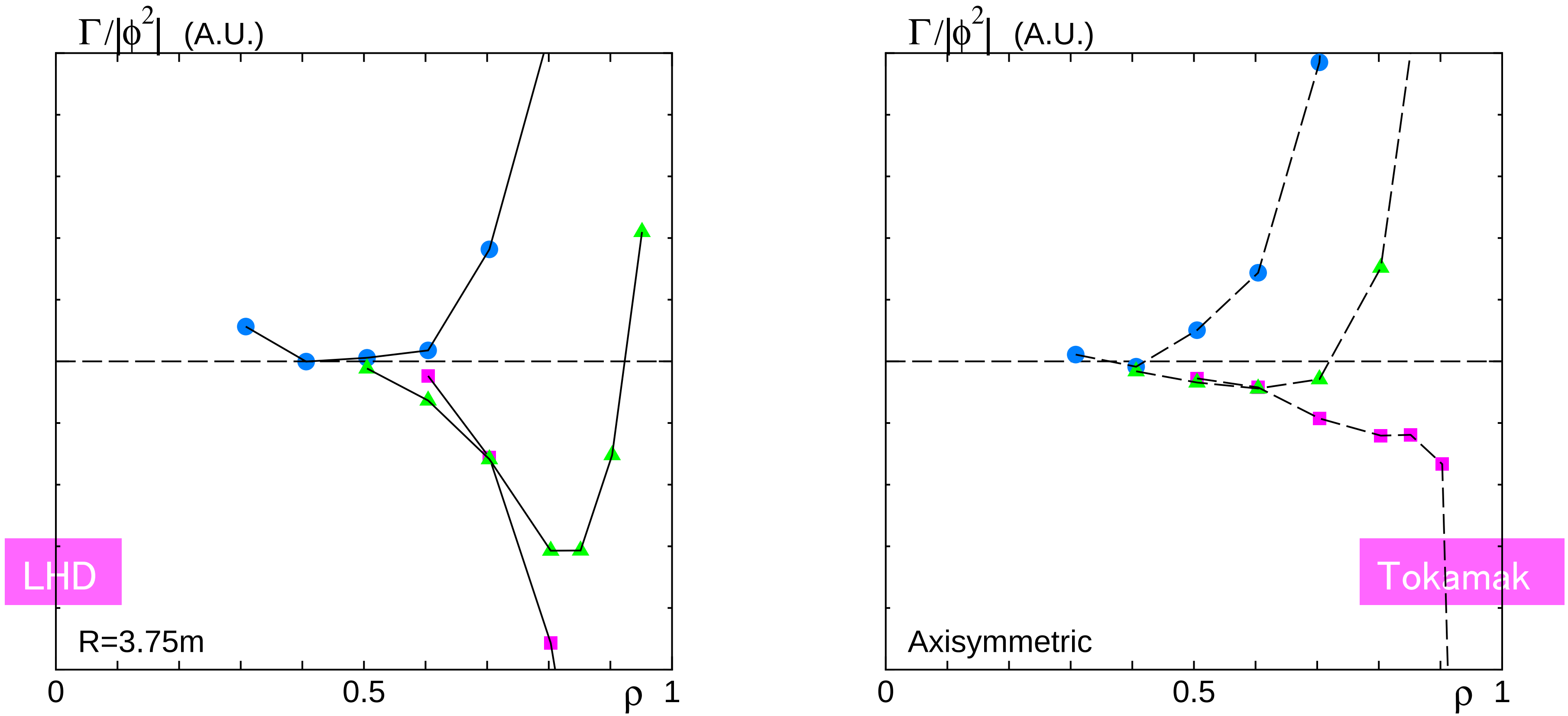
- In all cases, growth rate becomes large as ρ increases.
- Growth rate is larger in peaked profile case.
- Real frequency becomes positive (TEM-drive becomes strong) for peaked case due to $1/L_n$.

-Axisymmetric

- Growth rate becomes large as ρ increases for hollow profile case, while it becomes small in the peaked case (TEM-drive does not connect with ITG-drive well).

➡ Helical ripple amplify the growth rate through the TEM-ITG hybrid mechanism

Quasi-linear particle flux



○Tendency is the same in the LHD and axysmmetric case;
the flux tends to be negative as density profile tends to be hollow

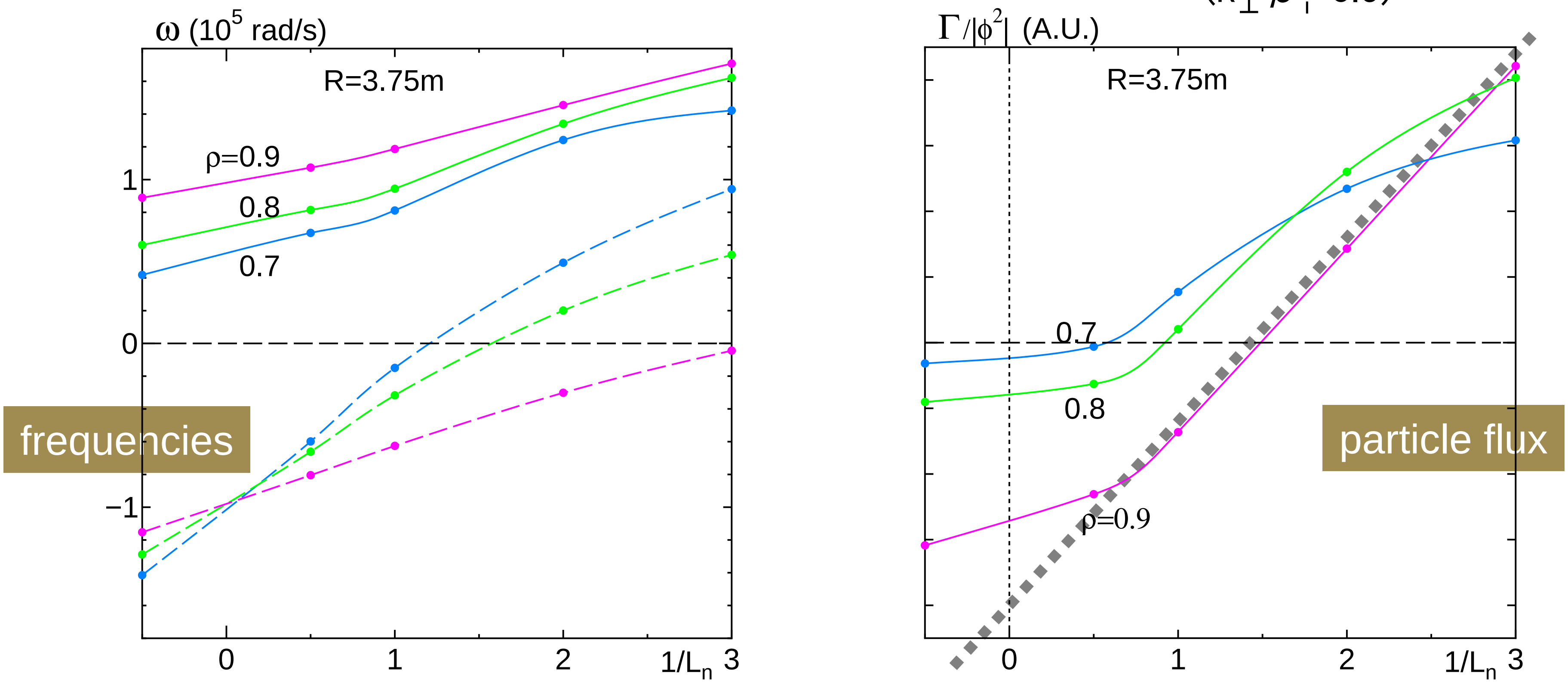
○This result cannot be explained by only the sign of $1/L_n$,
because sign of Γ does not completely correspond to sign of $1/L_n$

○The flux in LHD is more negative than the tokamak (for example in green)

What makes the flux negative? ⇒ hollow (positive ∇n) profile + helical ripple?

$1/L_n$ change in LHD (artificial)

($k_\perp \rho_i$=0.5)



- Particle flux changes from negative to positive with increasing $1/L_n$. The change of sign occurs at some positive $1/L_n$ value.
- In the core, flux tends to be small at $1/L_n$ ~0.
- Near the edge where trapped particles fraction becomes large, sufficiently negative flux remains even at small $1/L_n$.

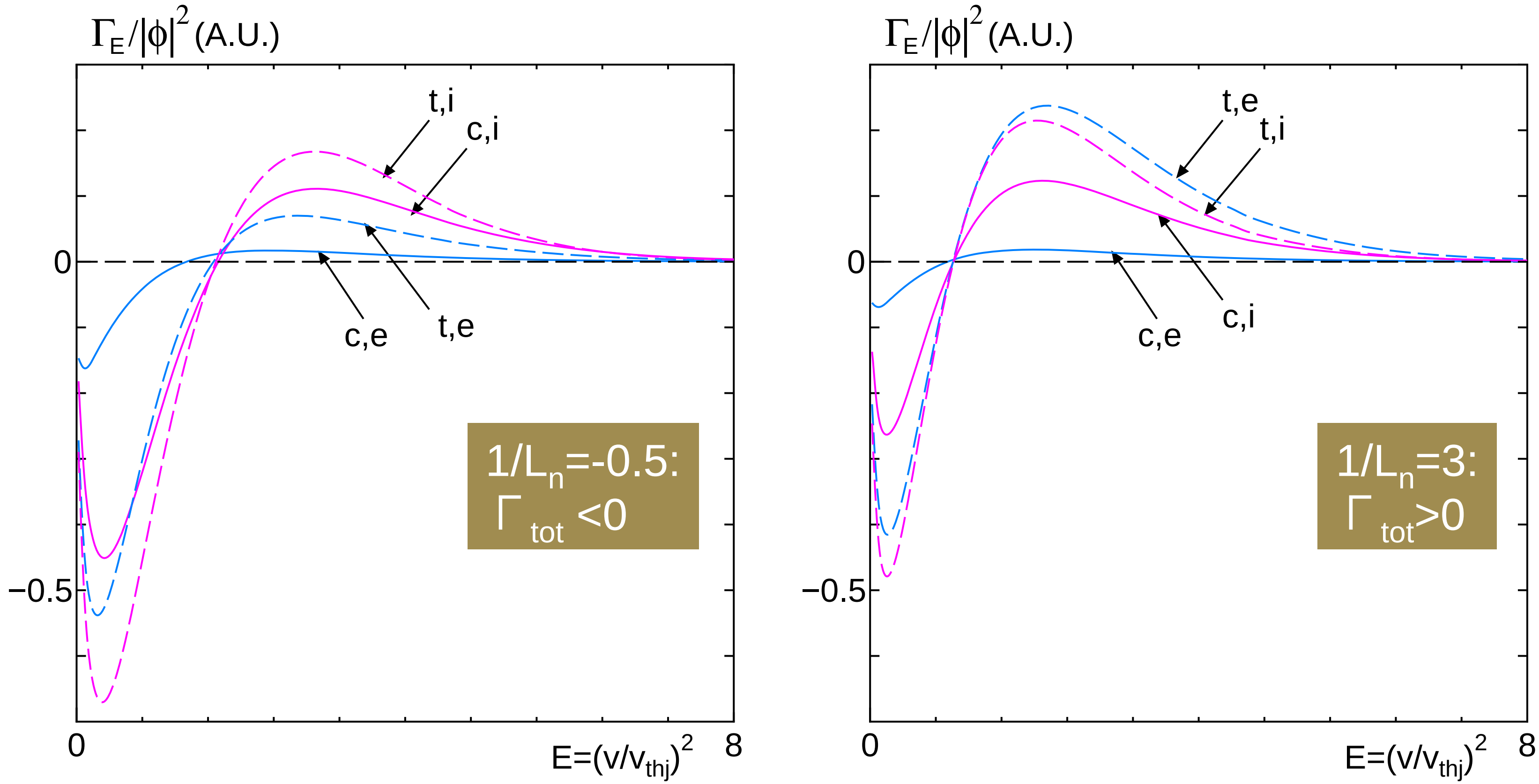
If $\Gamma = -D(dn/dr) + nV$ ⇒ $\Gamma/n = D(1/L_n) + V = 0$ at $1/L_n$ ~1.5 ⇒ $V/D \sim -1.5$

Not only diffusive flux but also convective flux exists near the edge.

In the core Γ seems to be diffusive.

$1/L_n$ change (artificial)

$(k_\perp \rho_i = 0.5)$



In order to see what makes the flux negative, flux is plotted as a function of E

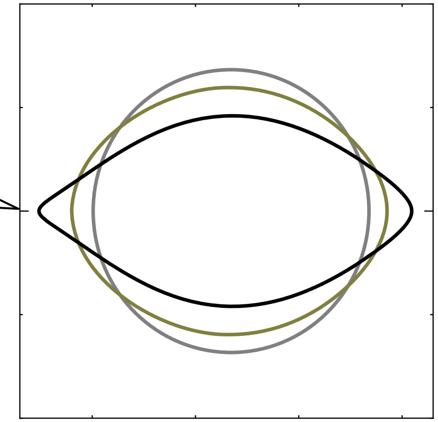
$$\Gamma_{\text{total}} = \Gamma_{\text{circulating}} + \Gamma_{\text{trap}} = \int dE \ [\Gamma_{E \ \text{circulating}} + \Gamma_{E \ \text{trap}}]$$

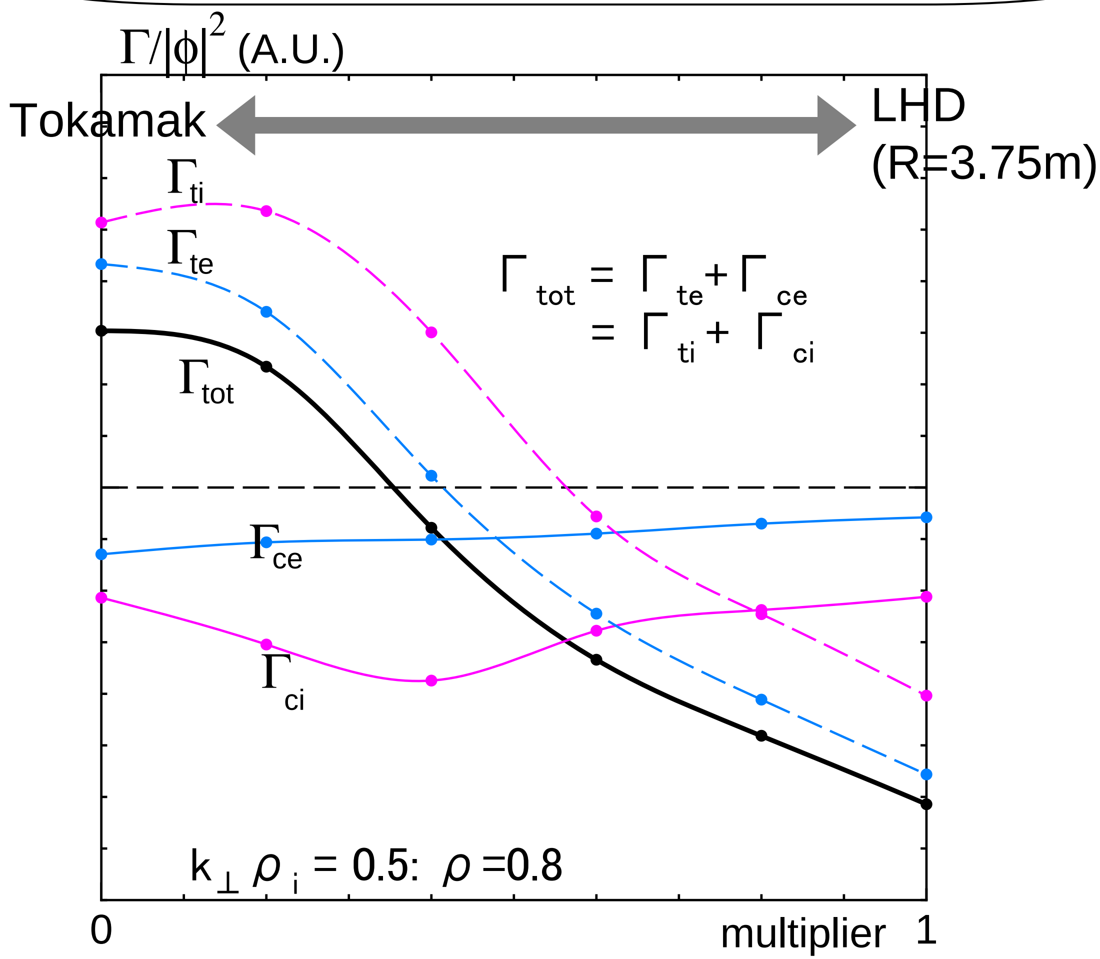
- ○ Slow (fast) particles compared to v_{th} tend to contribute the negative (positive) Flux.
- ○ Increase of $1/L_n$ reduce the negative Γ_E region in E, making the total flux more positive.
- ○ Γ_{trap} are more affected by $1/L_n$, while circulating Γ_{circ} is not sensitive.

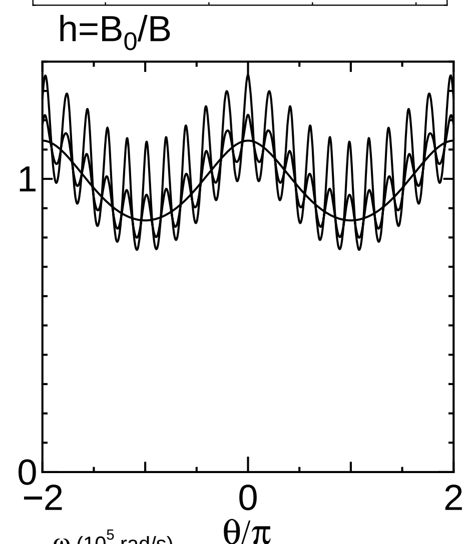
⇒ Trapped particles contribution change the sign of Γ through $1/L_n$ value

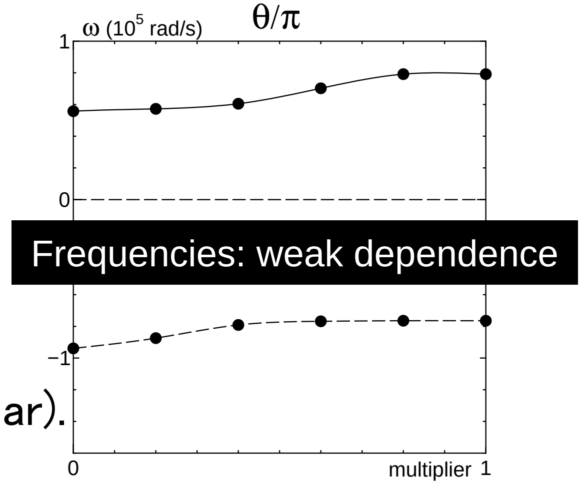
Helical ripple effect

- Multiply a factor to n≠0 components of LHD surface in VMEC
- q,T,n are assumed as the same as original case (multiplier=1)





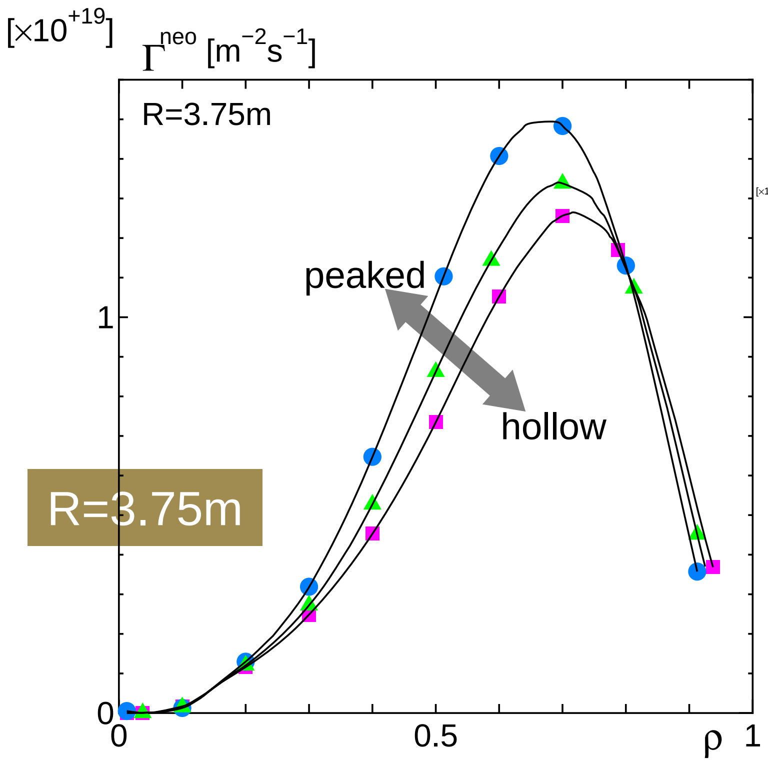


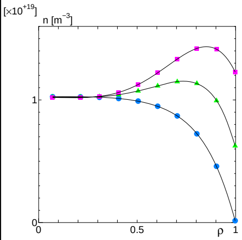


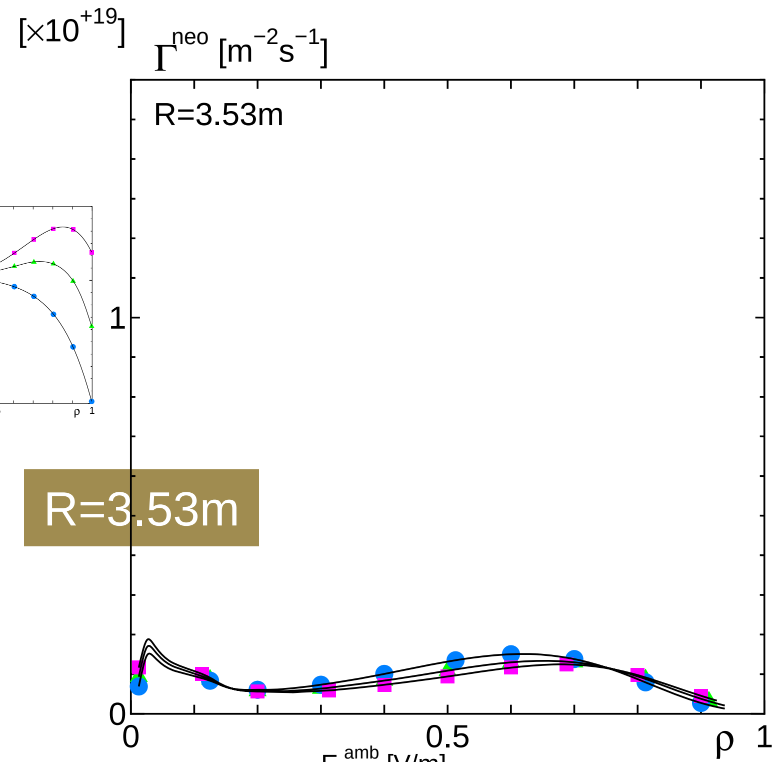
- Circulating flux is independent of the helical ripples.
- Trapped flux is strongly changed with helical ripples, which tends to make Γ more negative (ion/electron are similar).

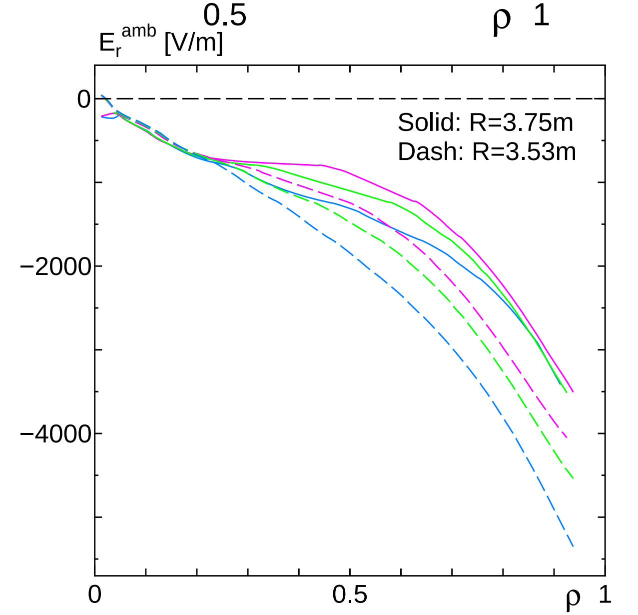
Neoclassical

Neoclassical particle flux (GSRAKE)









- Neoclassical ambipolar particle flux is determined together with ambipolar E_r, in the absolute unit.
- The configuration effect is very strong, in contrast to QL flux. The reason is responsible for the reduction of effective ripple.
- The profile effect is weak, in contrast to the QL flux.
- These are not explained by E_r, as they does not show strong configuration dependence.

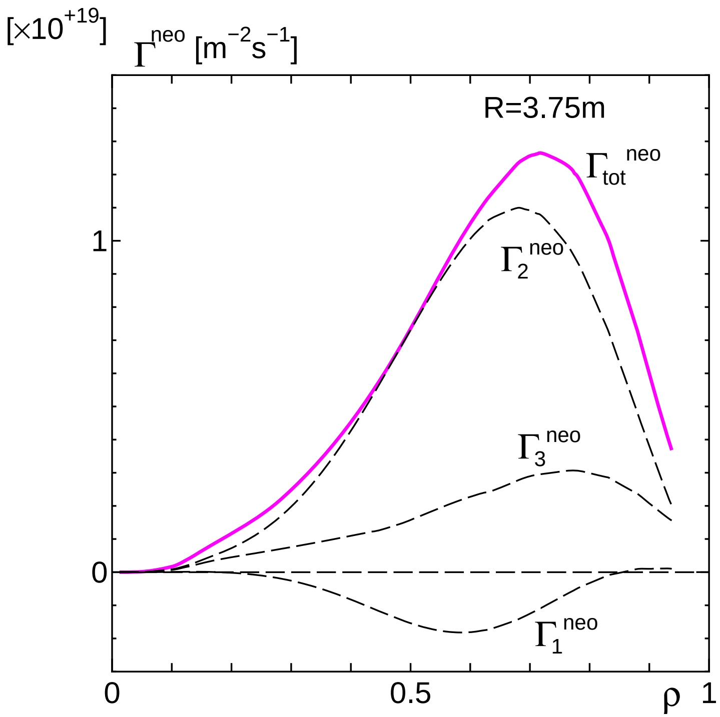
Neoclassical particle flux

Strong configuration dependence can be explained by the difference of effective ripples in R=3.75m and 3.53m (thus R=3.53m is found optimum)

Why profile (density profile change) effect is weak for fixed T profile?
Even in the hollow profile, Γ^{nc} is positive,
which indicates the Fick's law is not satisfied at all.

What is main contribution can be seen by separate Γ^{nc} as,

$$\Gamma^{nc} = -D_{11}\frac{dn}{d\rho} - D_{12}\frac{n}{T}\frac{dT}{d\rho} - D_{11}\frac{ne}{T}\frac{d\phi}{d\rho} = \Gamma_1^{nc} + \Gamma_2^{nc} + \Gamma_3^{nc}$$



Γ_2^{nc} proportional to dT/dr
is large positive,
which determine the flux
dominantly.

In this study dT/dr is fixed,
which is also the experimental case.
Thus, positive Γ^{nc} is robust for
changing density profile.
Its absolute value is affected by ε_{eff}

Why Γ_2^{nc} is dominant in $1/\nu$ regime?

Revisit the bounce - averaged drift kinetic equation in $1/\nu$ regime (not strict, rough flow chart)

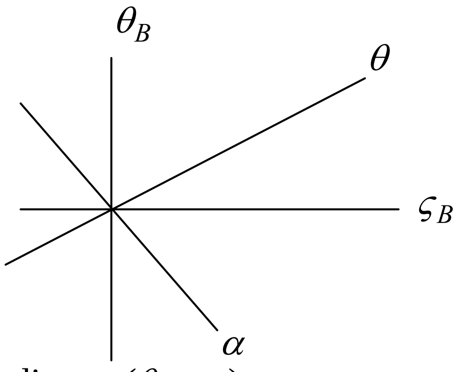
$$\boxed{v_{//} b \cdot \nabla f_1 + v_D \cdot \nabla \rho \frac{\partial f_M}{\partial \rho} = C(f_1) :} \quad v_E \cdot \nabla f_1 \text{ is not considered for simplicity}$$

$$\hat{E} = (v / v_{thj})^2, \quad \frac{v_{//}}{v} = \sigma \sqrt{1 - \Lambda / h}, \quad dv^3 = \pi d\hat{E} d\Lambda \sqrt{2\hat{E}} \frac{1}{h\sqrt{1-\Lambda/h}}, \quad f_{Mj} = \frac{1}{(\sqrt{\pi} v_{thj})^3} \exp(-\hat{E})$$

$$C_{jk}(f_1) = 2\nu_{jk}(\hat{E}) h \frac{v_{//}}{v} \frac{\partial}{\partial \Lambda}\left[\Lambda \frac{v_{//}}{v} \frac{\partial f_1}{\partial \Lambda}\right] = C_{\hat{E}} C_{\Lambda}(f_1) : \text{ Lorents operater is assumed}$$

$$\mathrm{C}_{\hat{\mathrm{E}}} = \nu_{jk} \nu(\hat{E}) = \frac{4\pi n_k Z_j^2 Z_k^2 e^4 \ln \Lambda_{jk}}{m_j^2 v_j^3} \hat{E}^{-3/2}\left[\left(1 - \frac{1}{2\hat{E}}\right)\Phi\sqrt{\hat{E}} + \frac{1}{\sqrt{\pi \hat{E}}} e^{-\hat{E}}\right],$$

$$\Phi(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-y^2} dy,$$



In the following the coordinates (θ, α) is considerd, which is related to the Boozer coordinates (θ_B, ς_B) as

$\theta = \theta_B :$ $\quad : (\partial / \partial \theta)_\alpha = \partial / \partial \theta_B + q \partial / \partial \varsigma_B$

$\alpha = \varsigma_B - q\theta_B : \ (\partial / \partial \alpha)_\theta = \partial / \partial \varsigma_B :$ (negligible in the axisymmetric case)

Then, $v_D \cdot \nabla \rho = v_D^\rho$ include the derivative along and perpendicular to the magnetic field, $\partial / \partial \theta$, $\partial / \partial \alpha$.

We can divide it as $v_D \cdot \nabla \rho = v_D^\rho = v_{//} \frac{B^\theta}{B}\left(\frac{\partial v_D^{*\rho}}{\partial \theta} + \frac{\partial v_D^{*\rho}}{\partial \alpha}\right)$, then the drift kinetic equation becomes a form like

$$\boxed{v_{//} \frac{B^\theta}{B} \frac{\partial}{\partial \theta}(f_1 + v_D^{*\rho}) + v_{//} \frac{B^\theta}{B} \frac{\partial v_D^{*\rho}}{\partial \alpha} \frac{\partial f_M}{\partial \rho} = \mathrm{C}_{\hat{\mathrm{E}}} \mathrm{C}_{\Lambda}(\mathrm{f}_1) :}$$

Taking the bounce - average, and assuming $\langle f_1 \rangle_b = f_1$ (independent of θ) yields,

$$f_1 = \frac{H_1}{C_E H_2}\frac{\partial f_M}{\partial \rho}, \quad \text{where } H_1(\Lambda) = \left\langle v_{//} \frac{B^\theta}{B}\frac{\partial v_D^{\rho *}}{\partial \alpha}(\Lambda) \right\rangle_b, \ \mathrm{H}_2(\Lambda) = \langle C_\Lambda \rangle_b,$$

Here the bounce - average is $\langle A \rangle_b = \oint \frac{dl}{|v_{//}|} A \Big/ \oint \frac{dl}{|v_{//}|} = \oint \frac{d\theta}{|v_{//}|B} A \Big/ \oint \frac{d\theta}{|v_{//}|B},$

and the first term of DKE is removed by considering appropriate boundary condtion at banana tip.

We need $\Gamma_j = \langle n_j V_\rho \rangle_s = \left\langle \int dv^3 v_D \cdot \nabla\rho\, f_1 \right\rangle_s$

where the flux - surface average is resemble to the bounce - average, $\langle A \rangle_s = \int_{-\infty}^{\infty} \frac{d\theta}{B^2} A \Big/ \int_{-\infty}^{\infty} \frac{d\theta}{B^2}.$

Then we obtain the particle flux in the following form,

$$\Gamma_j = -D_{11}\frac{dn_j}{d\rho} - D_{12}\frac{n_j}{T_j}\frac{dT_j}{d\rho}$$

$$= (\text{factor})\left(\int d\Lambda \frac{(H_1)^2}{H_2} \right) \int d\hat{E} \frac{\left(\sqrt{\hat{E}}\right)^2}{C_E}\left[\frac{1}{n_j}\frac{dn_j}{d\rho} - \frac{3}{2}\frac{1}{T_j}\frac{dT_j}{d\rho} + \hat{E}\frac{1}{T_j}\frac{dT_j}{d\rho} \right] f_{Mj}$$

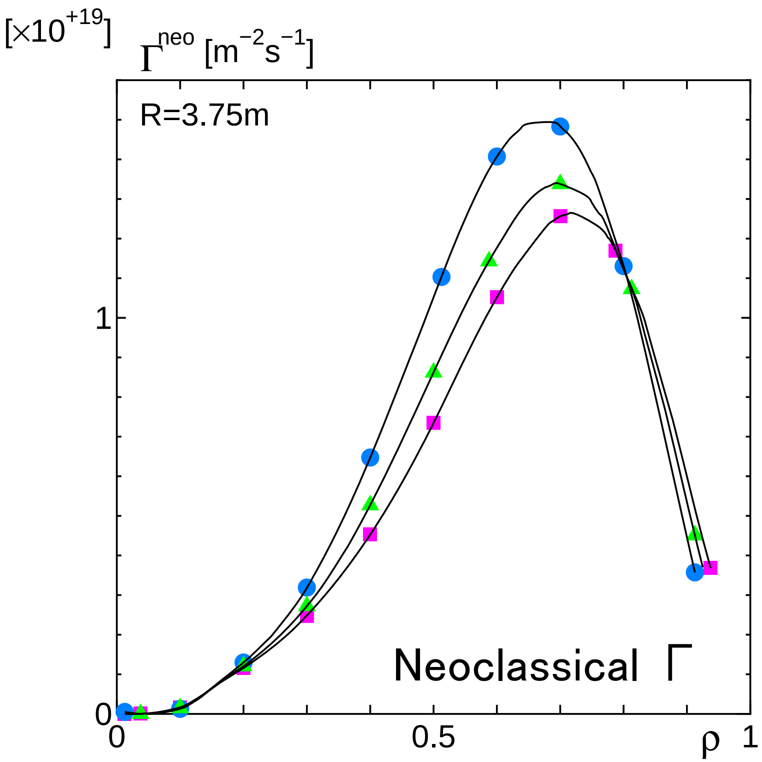
$$= \underbrace{\Gamma_{\text{geometrical}}}_{\propto\, \varepsilon_{eff}^{3/2}} / \nu_{jk} \times \left[\left(\int d\hat{E}\hat{E}\frac{\nu_{jk}}{\nu(\hat{E})} e^{-\hat{E}} \right)\left(\frac{1}{n_j}\frac{dn_j}{d\rho} - \frac{3}{2}\frac{1}{T_j}\frac{dT_j}{d\rho} \right) + \left(\int d\hat{E}\hat{E}^2 \frac{\nu_{jk}}{\nu(\hat{E})} e^{-\hat{E}} \right)\frac{1}{T_j}\frac{dT_j}{d\rho} \right]$$

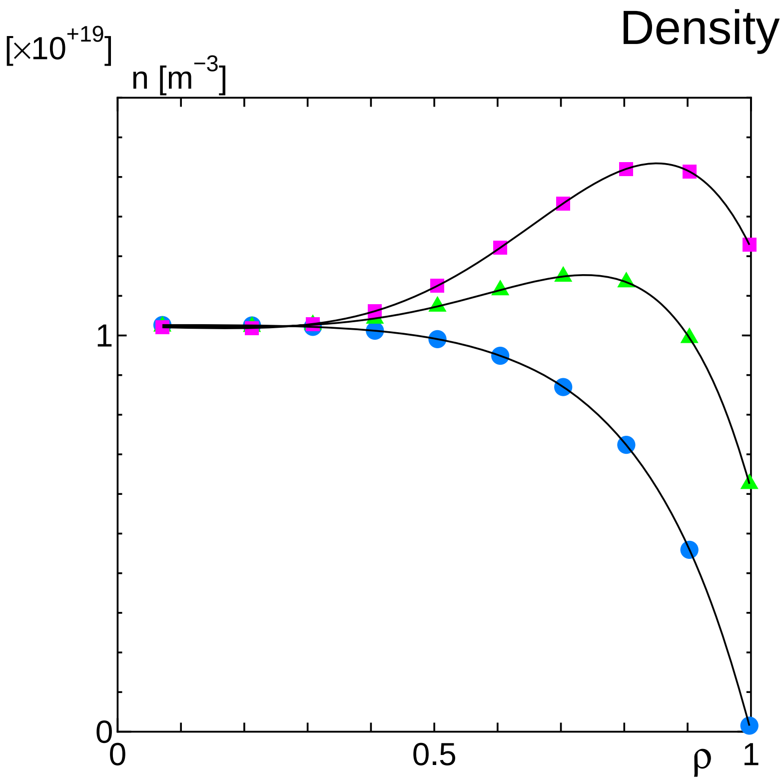
$$\frac{2}{\sqrt{\pi}}\int_0^\pi d\hat{E}\frac{\hat{E}^{5/2}e^{-\hat{E}}}{2\pi\nu^*(E)} = 4.9236, \qquad \frac{2}{\sqrt{\pi}}\int_0^\pi d\hat{E}\frac{\hat{E}^{7/2}e^{-\hat{E}}}{2\pi\nu^*(E)} = 23.9622, \ \text{where } \nu^*(\hat{E}) = \left(1 - \frac{1}{2\hat{E}}\right)\Phi\sqrt{\hat{E}} + \frac{1}{\sqrt{\pi\hat{E}}}\mathrm{e}^{-\hat{E}}$$

(values are from Nakajima's memo, 15 years ago)

Thus D_{12} is always much larger than D_{11} in $1/\nu$ regime

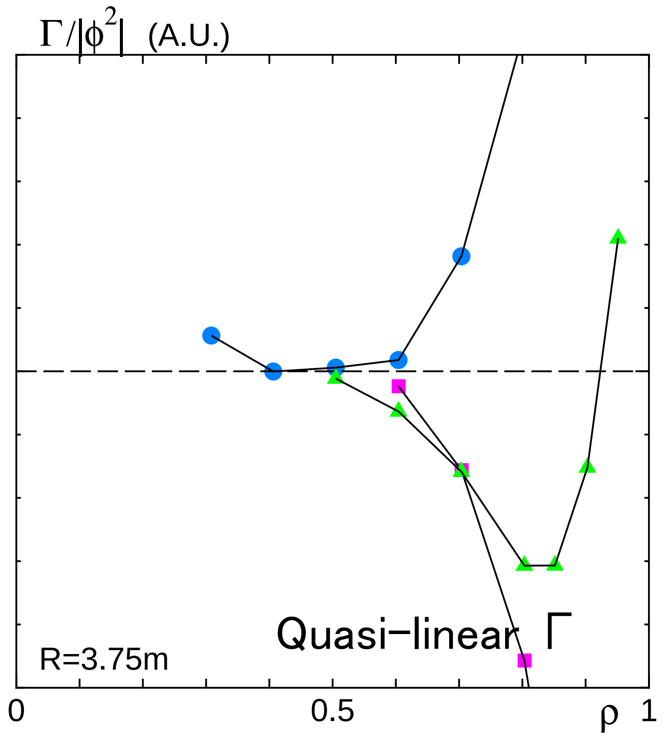
SUMMARY: R=3.75m

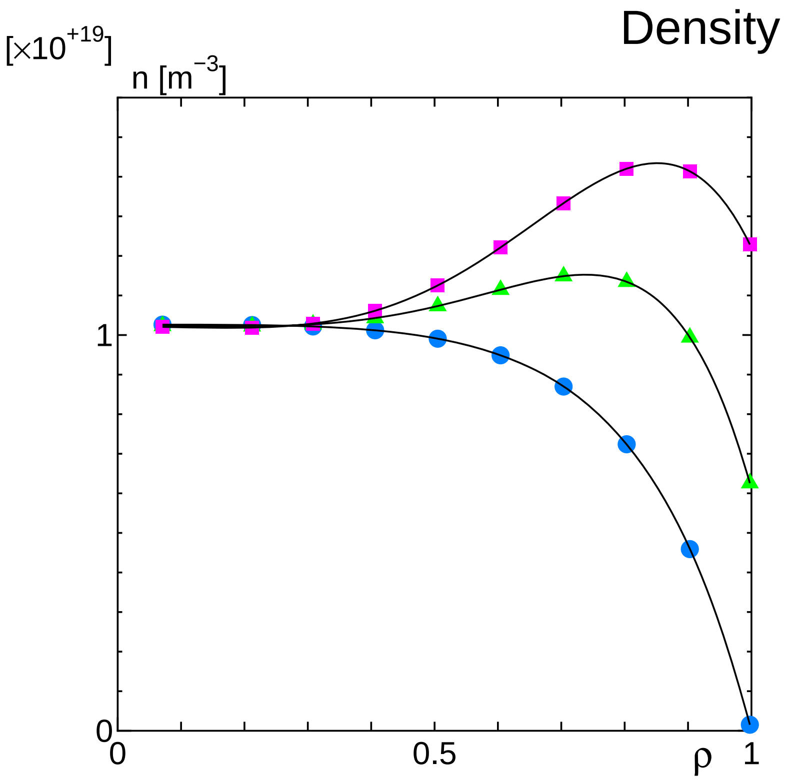




○For given density profiles, neoclassical flux is robast, positive

SUMMARY: R=3.75m

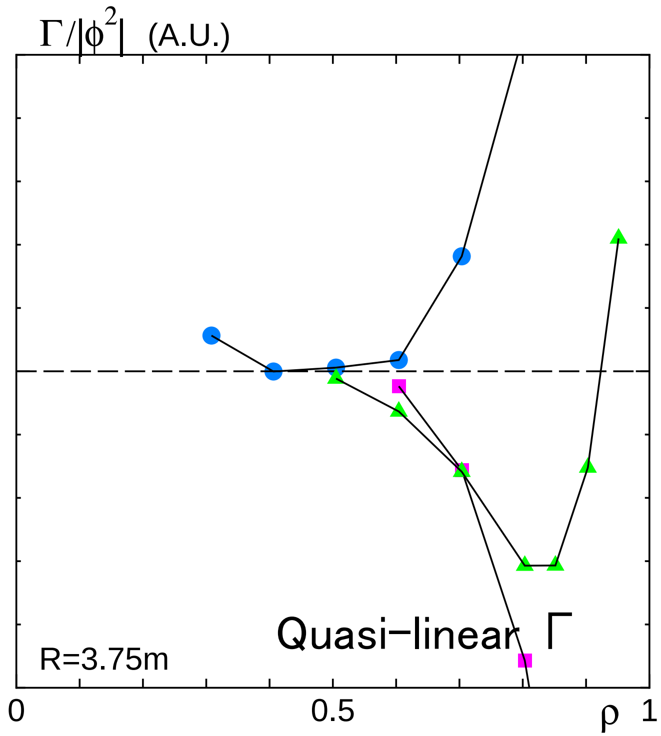


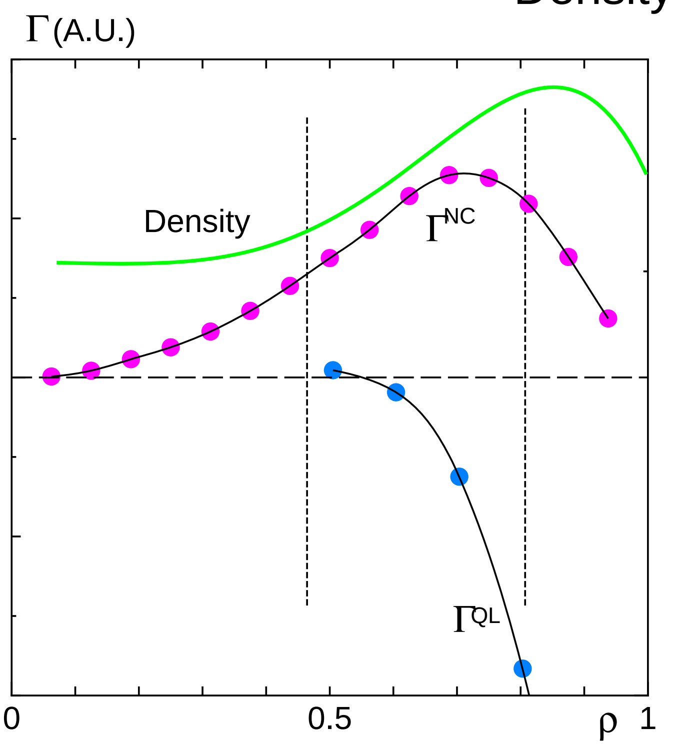


○For given density profiles, neoclassical flux is robast, positive

○Quasi-linear flux is strongly dependent on the profiles

SUMMARY: R=3.75m

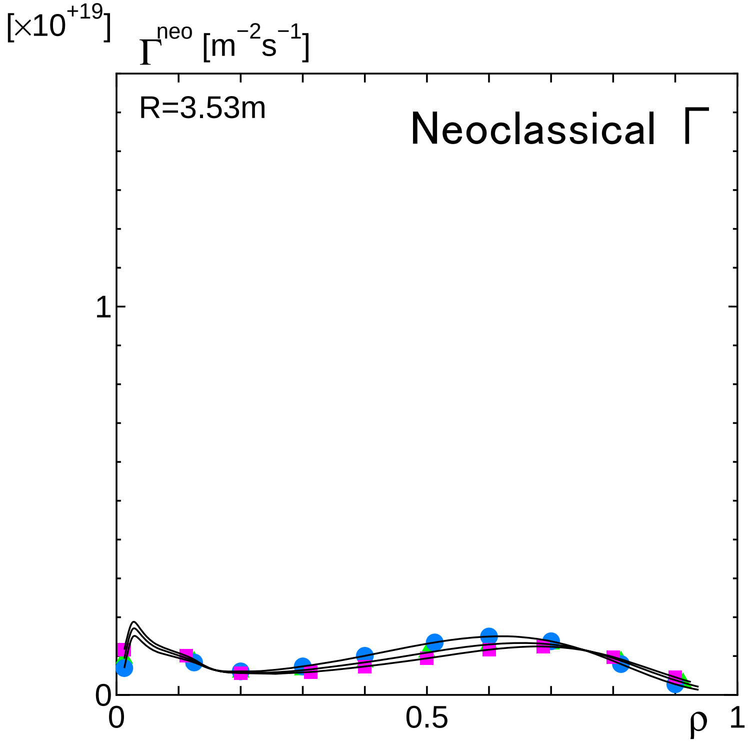


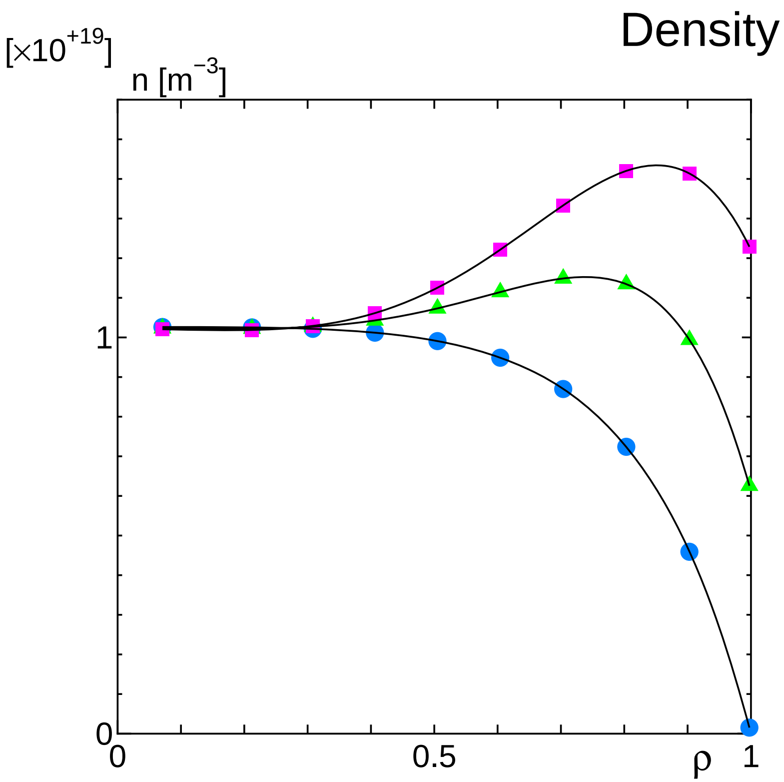


- ○For given density profiles, neoclassical flux is robast, positive
- ○Quasi-linear flux is strongly dependent on the profiles

In the steady state, in the core where particle source is negligible, the particle balance $\frac{\partial n}{\partial t} + \nabla \cdot \Gamma = S$ imposes $\Gamma^{NC}+\Gamma^{QL}=0$, indicating that the density profile **should be hollow** to make Γ^{QL} negative!

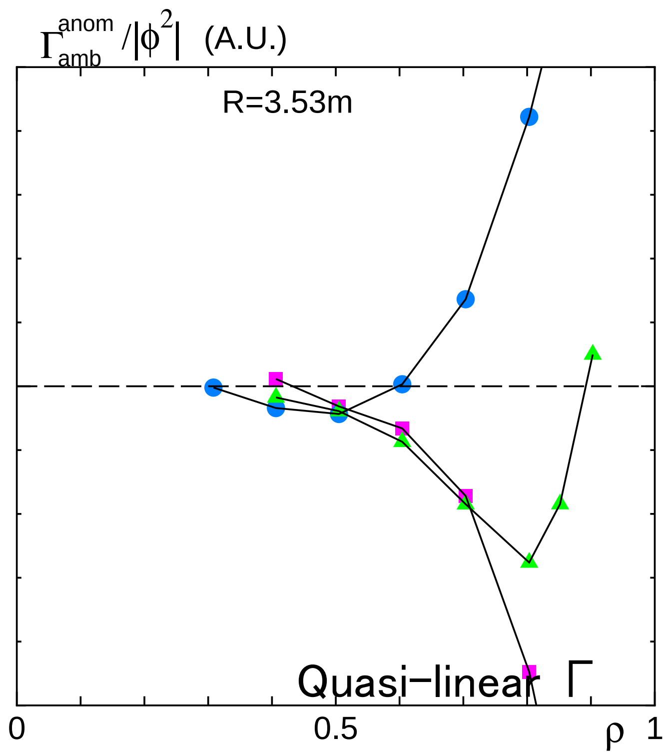
SUMMARY: R=3.53m

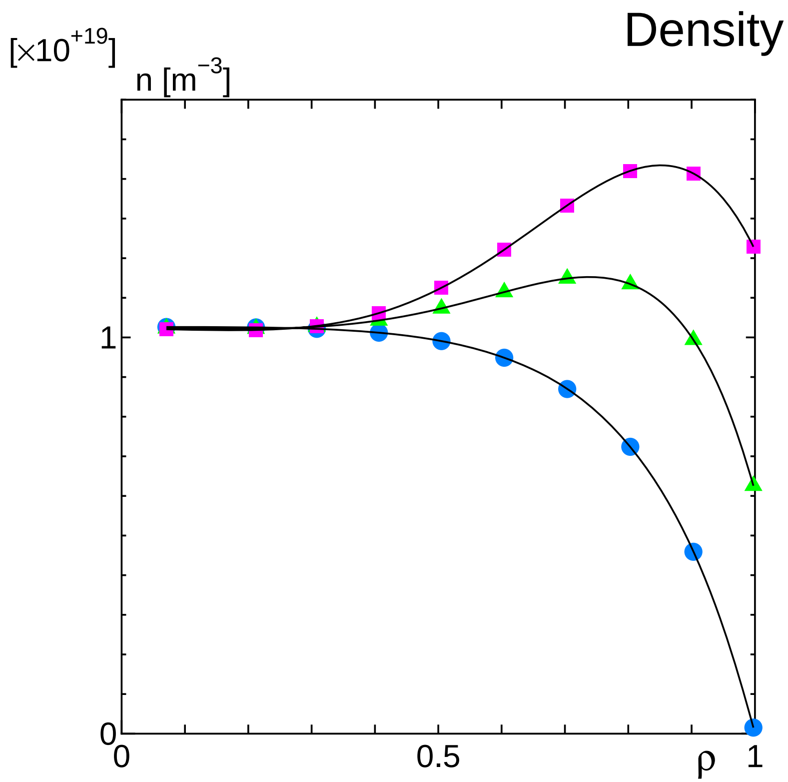




○ Neoclassical flux is strongly reduced due to effective ripple optimization

SUMMARY: R=3.53m

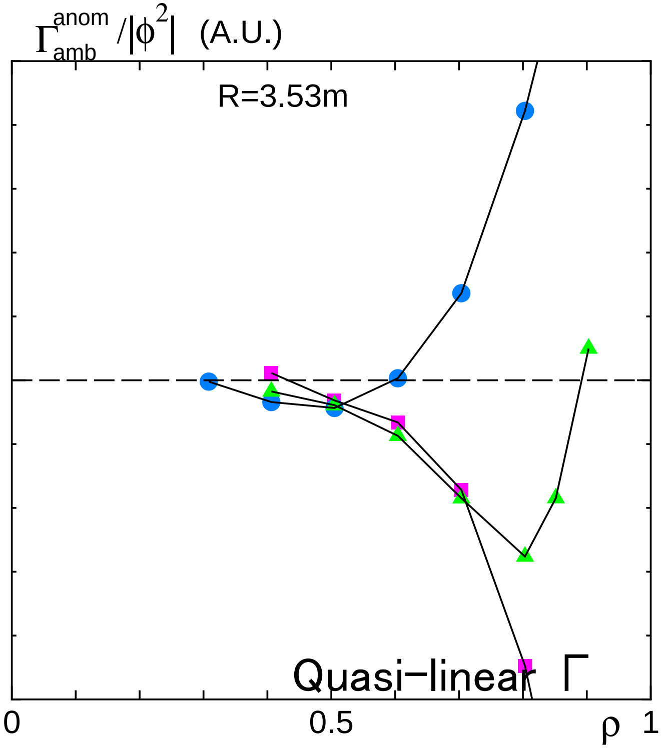


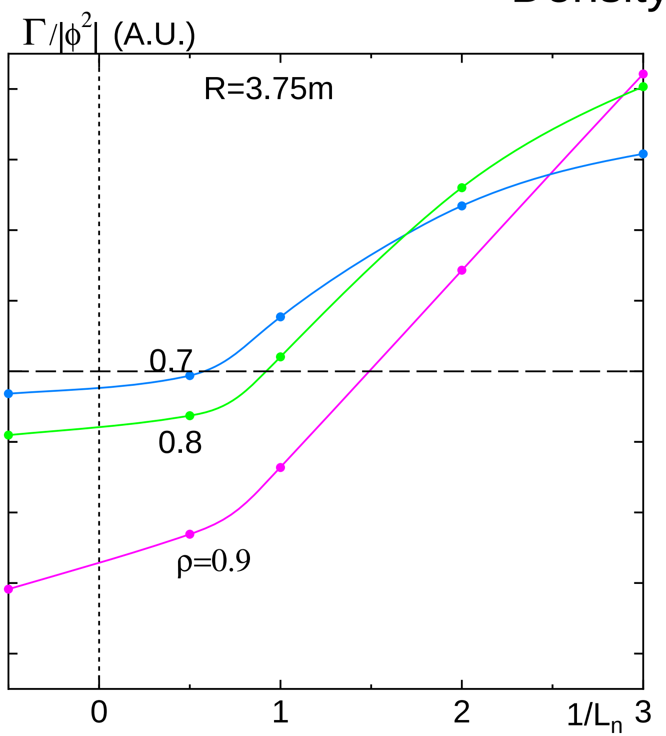


○ Neoclassical flux is strongly reduced due to effective ripple optimization

○ Quasi-linear flux does not show the configuration dependence.
Although absolute value of flux cannot estimated,
the growth rate is not different, indicating
the anomalous flux does not change in any LHD configulations

SUMMARY: R=3.53m





○ Neoclassical flux is strongly reduced due to effective ripple optimization

○ Quasi-linear flux does not show the configuration dependence.
Although absolute value of flux cannot estimated,
the growth rate is not different, indicating
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Then, the particle balance $\Gamma^{NC}+\Gamma^{QL}=0$, indicating $\Gamma^{QL}=0$.
This situation maybe resemble to the usual tokamak.
We saw that the QL flux disappears at positive $1/L_n$
(and also it is larger toward edge due to the increasing trapped particle's effect)
indicating the density profile should be FLAT or PEAKY

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