evolve

Minimize "energy" via a differential flow.

[called by: notopt.] [calls: knotxx.]

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1.1 overview

1. The geometry of the coils is "evolved" under the "energy gradient flow", where the energy function, $E(\mathbf{x})$, defined below, is considered to be a function of the coil geometry, $\mathbf{x} = \{x^i_{c,m}, x^i_{s,m}, y^i_{c,m}, y^i_{s,m}, z^i_{c,m}, z^i_{s,m}\}$, where the $x^i_{c,m}$ etc. are the Fourier harmonics of the i-th coil, see e.g. iccoil.

2. The evolution is described mathematically as a system of coupled, first-order equations:

$$\frac{\partial \mathbf{x}}{\partial \tau} = -\frac{\partial E}{\partial \mathbf{x}}, \tag{1}$$

where τ is an artifical time.

3. The integration is performed using NAG:D02BJF.

1.2 energy functional and derivatives

1. The "energy" is defined as the "quadratic-flux" on a given "plasma boundary",

$$\begin{aligned} E &= \int_{\mathcal{S}} \frac{1}{2}(B_n)^2 \, ds \\ &= \Delta\theta \, \Delta\zeta \sum_{j,k} \sqrt{g}_{j,k} \frac{1}{2} \left(n^x_{j,k} \sum\nolimits_i B^{x,i}_{j,k} + n^y_{j,k} \sum\nolimits_i B^{y,i}_{j,k} + n^z_{j,k} \sum\nolimits_i B^{z,i}_{j,k} \right)^2 \end{aligned} \tag{2}$$

where $B^{x,i}_{j,k}$, $B^{y,i}_{j,k}$ and $B^{z,i}_{j,k}$ are the Cartesian components of the magnetic field, which depend explicity on the geometry of and the current in the i-th coil, The normal vector to the plasma boundary at the angular location $(\theta_{j,k}, \zeta_{j,k})$ is $\mathbf{n}_{j,k} \equiv n^x_{j,k}\,\mathbf{i} + n^y_{j,k}\,\mathbf{j} + n^z_{j,k}\,\mathbf{k}$. (This is pre-computed in rbsurf.)

2. The resolution of the discretized surface integral is given by `Nteta` and `Nzeta` (see global and rbsurf).

3. The magnetic field at $\mathbf{x} \equiv x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ is given by the Biot-Savart integral,

$$\mathbf{B} \equiv I \int_{\mathcal{C}} \frac{d\mathbf{l} \times \mathbf{r}}{r^3}, \tag{3}$$

where $\mathbf{r} \equiv \mathbf{x} - \bar{\mathbf{x}}$.

4. The derivatives of E with respect to the coil parameters, e.g. α^i, take the form:

$$\frac{\partial E}{\partial \alpha^i} = \Delta\theta \, \Delta\zeta \sum_{j,k} \sqrt{g}_{j,k} \left(n^x_{j,k} \sum\nolimits_i B^{x,i}_{j,k} + n^y_{j,k} \sum\nolimits_i B^{y,i}_{j,k} + n^z_{j,k} \sum\nolimits_i B^{z,i}_{j,k} \right) \left(n^x_{j,k} \frac{\partial B^{x,i}_{j,k}}{\partial \alpha^i} + n^y_{j,k} \frac{\partial B^{y,i}_{j,k}}{\partial \alpha^i} + n^z_{j,k} \frac{\partial B^{z,i}_{j,k}}{\partial \alpha^i} \right) \tag{4}$$

5. The required integrals are computed using NAG:D01EAF. The integrands are defined below.

1.3 integrands

1. In component form, Eq.(3) is

$$B^x \equiv I \int_0^{2\pi} \frac{\dot{\bar{y}}(z-\bar{z}) - \dot{\bar{z}}(y-\bar{y})}{r^3} dt, \tag{5}$$

$$B^y \equiv I \int_0^{2\pi} \frac{\dot{\bar{z}}(x-\bar{x}) - \dot{\bar{x}}(z-\bar{z})}{r^3} dt, \tag{6}$$

$$B^z \equiv I \int_0^{2\pi} \frac{\dot{\bar{x}}(y-\bar{y}) - \dot{\bar{y}}(x-\bar{x})}{r^3} dt, \tag{7}$$

where $\dot{\bar{x}} \equiv d\bar{x}/dt$, etc.