torflux

Based on the theory of MHD, the toroidal flux on each plasma flux surface should be identical. Thus, we can put a constraint on the total toroidal flux, rather than fixing the currents in coils.

[called by: denergy.]

contents

1.1 Toroidal flux

1. Toroidal flux at a poloidal surface which is produced by cutting the plasma surface with a $\zeta =$ constant surface equals the line integral of magnetic vector potential over the boundary, based on the Stokes' theorem. That is,

$$\Phi_i = \int_{S_i} \vec{B} \cdot d\vec{s} = \int_{S_i} \nabla \times \vec{A} \cdot d\vec{s} = \int_{l_i} \vec{A} \cdot d\vec{l} \tag{1}$$

$$\vec{A} = \sum_{j=1}^{Ncoils} I_j \int_{coil\_j} \frac{d\vec{l}}{r} \tag{2}$$

   Here i is denoted to the poloidal surface label.

2. The total toroidal flux constraints then can be represented as,

$$tflux \equiv \frac{1}{nzeta} \sum_{i=1}^{nzeta} \frac{1}{2} (\Phi_i - \Phi_o)^2 \tag{3}$$

1.2 First derivatives

The first derivatives of toroidal flux cost function $tflux$ are derived as,

$$\begin{aligned} \frac{\partial tflux}{\partial I^j} &= \frac{1}{nzeta} \sum_{i=1}^{nzeta} (\Phi_i - \Phi_o) \frac{\partial \Phi_i}{\partial I^j} \\ &= \frac{1}{nzeta} \sum_{i=1}^{nzeta} (\Phi_i - \Phi_o) \int_{l_i} \int_{coil\_j} \frac{d\vec{l}}{r} \cdot d\vec{l} \end{aligned} \tag{4}$$

$$\begin{aligned} \frac{\partial tflux}{\partial x_n^j} &= \frac{1}{nzeta} \sum_{i=1}^{nzeta} (\Phi_i - \Phi_o) \frac{\partial \Phi_i}{\partial x_n^j} \\ &= \frac{1}{nzeta} \sum_{i=1}^{nzeta} (\Phi_i - \Phi_o) \int_{l_i} I^j \int_{coil\_j} \frac{\partial \frac{d\vec{l}}{r}}{\partial x_n^j} \cdot d\vec{l} \end{aligned} \tag{5}$$

Here, j means the argument is about the jth coil.

1.3 Second derivatives

Similarly, the second derivatives of $tflux$ can be written as,

$$\frac{\partial^2 tflux}{\partial X^j \partial X^k} = \frac{1}{nzeta} \sum_{i=1}^{nzeta} \frac{\partial \Phi_i}{\partial X^j} \frac{\partial \Phi_i}{\partial X^k} + \delta_j^k (\Phi_i - \Phi_o) \frac{\partial^2 \Phi_i}{\partial X^j \partial X^k} \tag{6}$$

Here, X represents all the DoFs, both the currents and geometry parameters.

1.4 Normalization

Since Φ_o is a user sepcified constant and identical at each cross-section, the normalization for toroidal flux cost function can be implemented by dividing all the functions and derivatives with ${\Phi_o}^2$. In order to calculate the flux value first (in which case, target_flux would be reset to zero and can not be divied.), this normalization is finished in costfun subroutine in denergy.