Stepped pressure profile equilibria in cylindrical plasmas via partial Taylor relaxation

HOLE Matthew J.[1], HUDSON Stuart R.[2] and DEWAR Robert L.[1]
1) Research School of Physical Sciences and Engineering, Australian National University, ACT 0200, Australia
2) Princeton Plasma Physics Laboratory, P.O. Box 451, Princeton, New Jersey 08543, U.S.A.
E-mail: matthew.hole@anu.edu.au

1. Introduction

One of the most important advances in magnetically confined fusion plasma physics has been the discovery of high confinement regimes, where at sufficiently high heating power, the plasma self-organises to produce internal transport barriers. Whilst descriptive theories for these transport barriers exist (*e.g.* shear flow suppression of turbulence [1] and chaotic magnetic field line dynamics [2]), little research has been devoted to addressing *why* the plasma self-organises into this state. One possible explanation is that these are constrained minimum energy states, where the plasma within the barrier satisfies ideal MHD, and the plasma between barriers is in a Taylor relaxed state [3].

Taylor relaxation describes plasma with small but finite resistivity and viscosity, in which the magnetic field has evolved to a minimum energy state, subject to the conservation of magnetic helicity and toroidal flux, and the presence of a perfectly conducting wall. In such plasma states the pressure gradient is zero, and the magnetic field $\mathbf{B}$ satisfies the Beltrami equation

$$\nabla \times \mathbf{B} = \mu \mathbf{B} \tag{1}$$

with the Lagrange multiplier μ below some critical value μ_T, which depends only on the vessel. By introducing ideal MHD barriers between different Taylor relaxed states, equilibria with stepped pressure profiles can be constructed. Subject to the imposed constraints, these new equilibria are in a relaxed or minimum energy state, and so may explain the existence of transport barriers in toroidal magnetic confinement experiments. The stepped pressure profile model also offers a possible solution to the long-standing 3D equilibrium existence problem [4].

2. Stepped Pressure Plasmas in Cylindrical Geometry : Equilibria

Our working builds principally upon a variational model developed by Spies *et al.*[5], which comprised a plasma/vacuum/conducting wall system. In Spies [5] the theory is applied to a plasma slab equilibrium, with boundary conditions designed to simulate a torus. Later analysis by Spies [6] extended the plasma model to include finite pressure. More recently, Kaiser and Uecker [7] analysed the finite pressure model in cylindrical geometry. In this work, we generalise the analysis of Kaiser and Uecker [7] to an arbitrary number of Taylor relaxed states, each separated by an ideal MHD barrier. For this system, the energy functional can be written

$$W = U - \Sigma_{i=1}^{n} \mu_i H_i/2 - \Sigma_{i=1}^{n} \nu_i M_i \tag{2}$$

Setting the first variation to zero yields the following set of equations :

$$\mathscr{P}_i \;:\; \nabla \times \mathbf{B} = \mu_i \mathbf{B}, \qquad p_i = \text{const.} \tag{3}$$

$$\mathscr{I}_i \;:\; \mathbf{n} \cdot \mathbf{B} = 0, \qquad < p_i + 1/2B^2 >= 0 \tag{4}$$

$$\mathscr{V} \;:\; \nabla \times \mathbf{B} = 0, \qquad \nabla \cdot \mathbf{B} = 0 \tag{5}$$

$$\mathscr{W} \;:\; \mathbf{n} \cdot \mathbf{B} = 0 \tag{6}$$

where $\mathscr{P}_i, \mathscr{I}_i$ are the i'th plasma region and interface, and $\mathscr{V}, \mathscr{W}$ are the vacuum region and wall, respectively. Also, μ_i is the Lagrange multiplier in each region, p_i the pressure in each region, $\mathbf{n}$ a unit vector in the radial direction, and $< x >= x_{i+1} - x_i$ denotes the change in quantity x across the interface $\mathscr{I}_i$. Figure 1 shows an example with 5 ideal barriers (shown as solid circles in panel (a), interior to the boundary). In addition to the poloidal flux, the safety factor is plotted, which decreases in Taylor relaxed regions, and undergoes step discontinuities at the ideal MHD barriers.

3. Stability Analysis

To investigate stability, we minimize $\delta^2 W$ with respect to the perturbed vector potential $\mathbf{a}$, while keeping the displacement ξ_i of the interfaces constant. This reduces the energy functional $\delta^2 W$ to a summation of interface integrals. Solutions for the cylinder are obtained by using a Fourier decomposition in the poloidal and axial directions for the perturbed field $\mathbf{b} = \nabla \times \mathbf{a}$ and the displacements ξ_i. In general, the solutions for both the equilibrium and perturbed field are Bessel functions, and modified Bessel functions, respectively. The use of interface conditions relates the perturbed magnetic field amplitude to the equilibrium quantities. Finally, stability is assessed by computing the free energy summed over the different interfaces. The space of unstable solutions is mapped numerically, and compared to systems with fewer constraints.

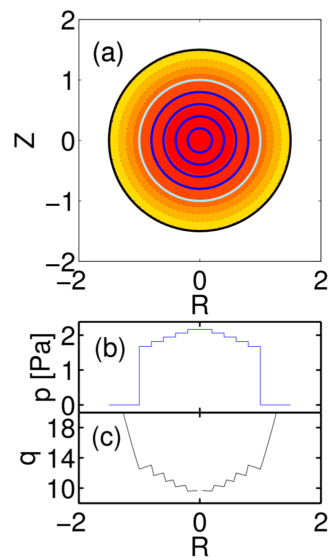

Figure 1: Example of a stepped-pressure plasma profile, with five ideal MHD barriers, showing : (a) a contour plot of the polodial flux, (b) the pressure profile, and (c) the safety factor.

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References

[1] K. H. Burrel, Phys. Plas. 4(5), pp. 1499-1518, 1997.
[2] J. H. Misguich, Phys. Plas. 8(5), pp. 2132-2138, 2001.
[3] J. B. Taylor, Rev. Mod. Phys. 58(3), pp. 741-763, 1986.
[4] H. Grad, Phys. Fluids, 10(1), pp. 137-154, 1967.
[5] G. O. Spies, D. Lortz and R. Kaiser, Phys. Plas. 8(8), pp. 3652-3663, 2001.
[6] G. O. Spies, Phys. Plas., 10(7), pp. 3030-3031, 2003.
[7] R. Kaiser and H. Uecker, 57(1), pp. 1-17, Q. Jl Mech. Appl. Math., 2004.