Field aligned coordinates (for integrable and chaotic fields)

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Field aligned coordinates can increase the numerical accuracy when treating strongly anisotropic quantities; however, toroidicity and chaos can create problems . . .

A brief introduction to these matters will be given . . .

Consider first an integrable magnetic field . . .

1) When field lines lie on nested toroidal flux surfaces, magnetic coordinates are possible.

2) Straight field line coordinates (s, θ, ζ) are particular choices of angles such that $\mathbf{B} = \nabla s \times \nabla \left(\theta - \iota \zeta \right)$, where $\iota = \iota$(s) is the rotational transform.

3) Field aligned coordinates : $\alpha = \theta - \iota \zeta$, then $\mathbf{B} = \nabla s \times \nabla \alpha$.
In (s,α,ζ) coordinates, $\sqrt{g}\mathbf{B} \cdot \nabla = \partial_\zeta$: "grid-points" lie along field lines, which is optimal for separating parallel and perpendicular effects

4) However, the (s,α,ζ) coordinates are not toroidally periodic.
One must "cut" the toroidal domain, say $\zeta \in [-\pi, \pi]$.
This complicates the periodic boundary condition.

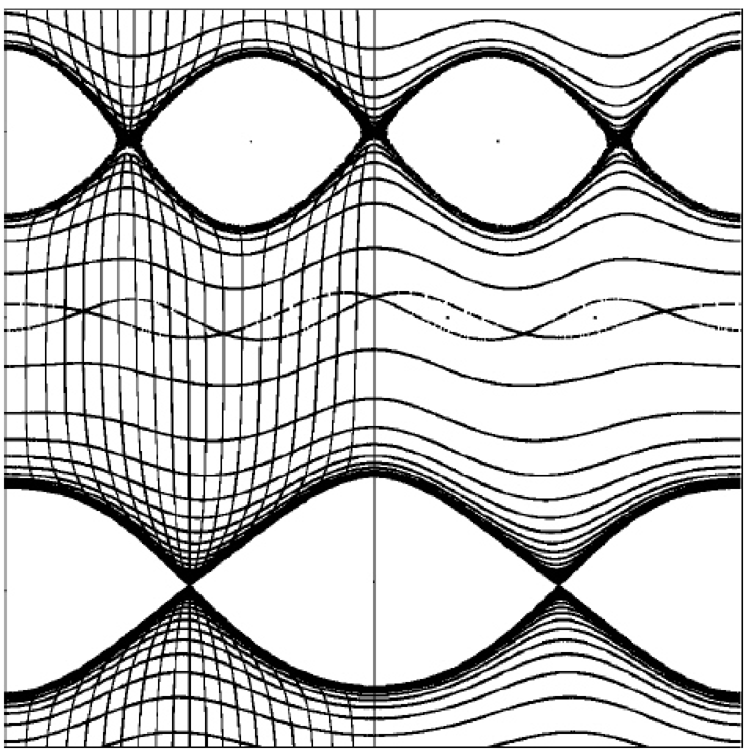
When the field goes chaotic the continuous magnetic coordinates are broken, but isolated KAM surfaces still exist

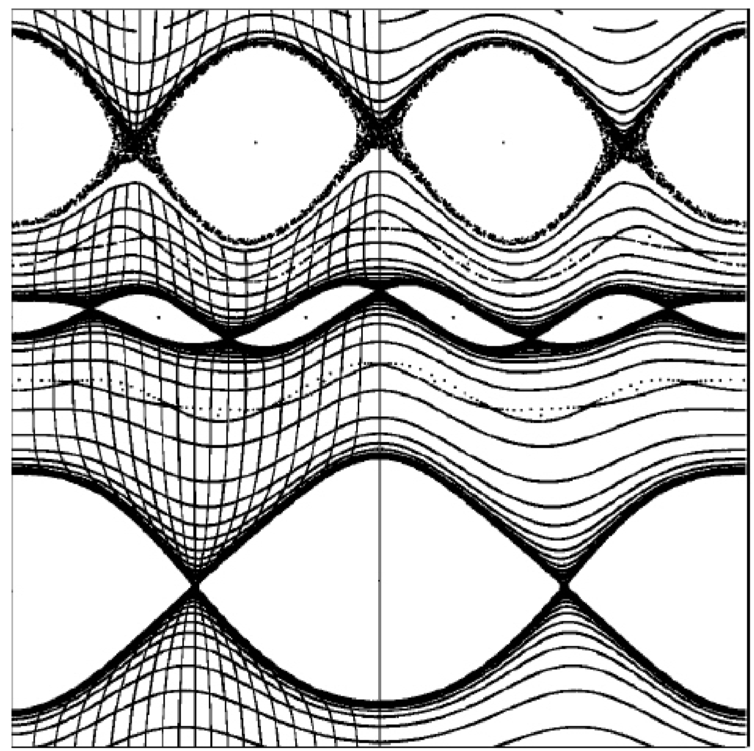
1) If one wishes to adapt the coordinates to the evolving field, then

  a) identify and locate existing KAM surfaces

  b) construct "discrete" magnetic coordinates on a discrete set of KAM surfaces

  [eg. Destruction of invariant surfaces and magnetic coordinates for perturbed magnetic fields. S.R.Hudson. Physics of Plasmas 11(2):677,2004.]





One can also construct “pseudo-magnetic-coordinates” .

1) The presence of a chaotic field does not wash out all structure (in the solution to the heat-conduction for example).

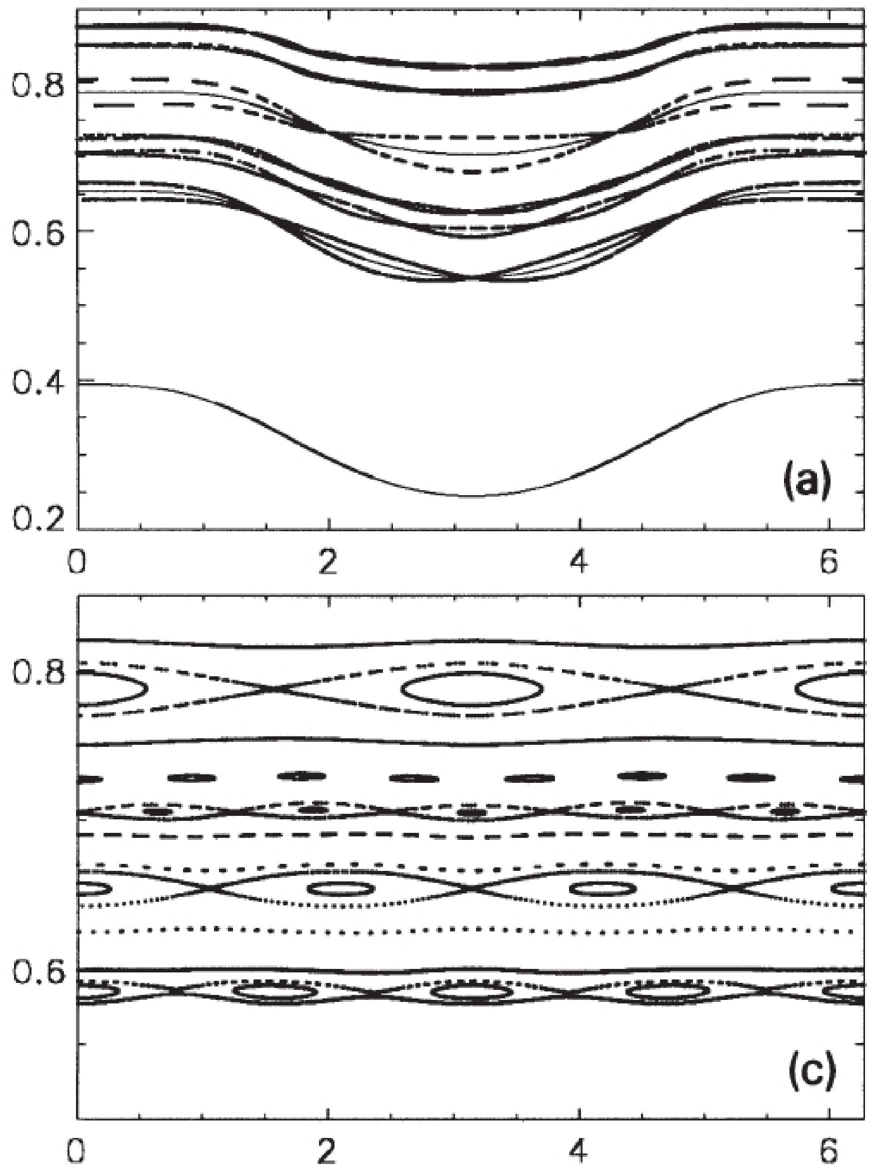
2) "Chaotic coordinates", or pseudo magnetic coordinates, are based on a set of pseudo flux surfaces, such as the (rational) quadratic-flux minimizing surfaces and ghost-surfaces.

minimize B_n^2

slide periodic curve from O curve to X curve

These surfaces pass directly through island chains, and correspond to rational flux surfaces of nearby integrable field. They provide a natural construction of nearby integrable field, and can be chosen to correspond to structures that inhibit transport in chaotic fields.

Example : construction of nearby integrable coordinates



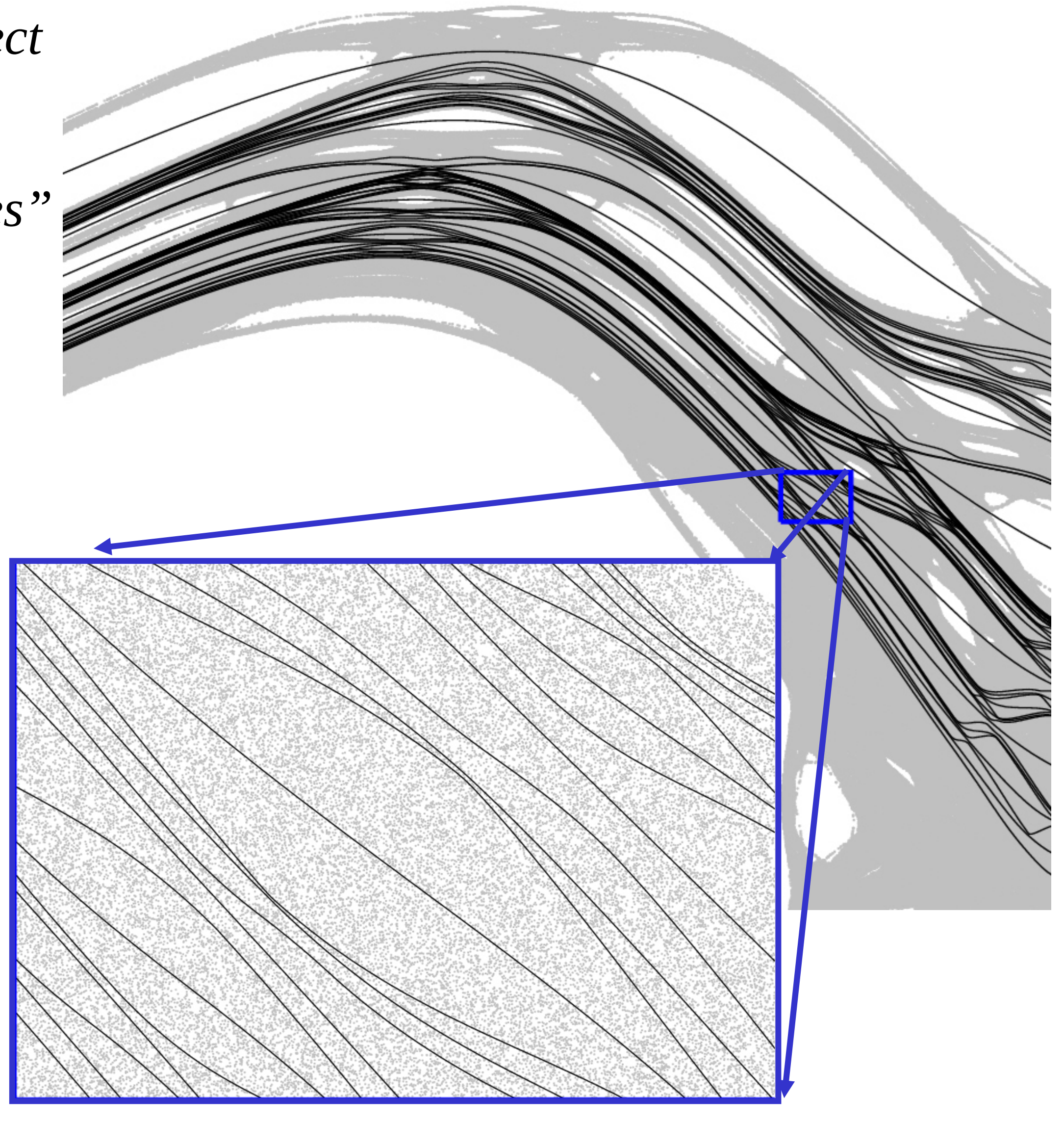
Poincare plot shown in background coordinates

Magnetic field for JT60U calculated by PIES

Poincare plot shown in nearby-integrable coordinates

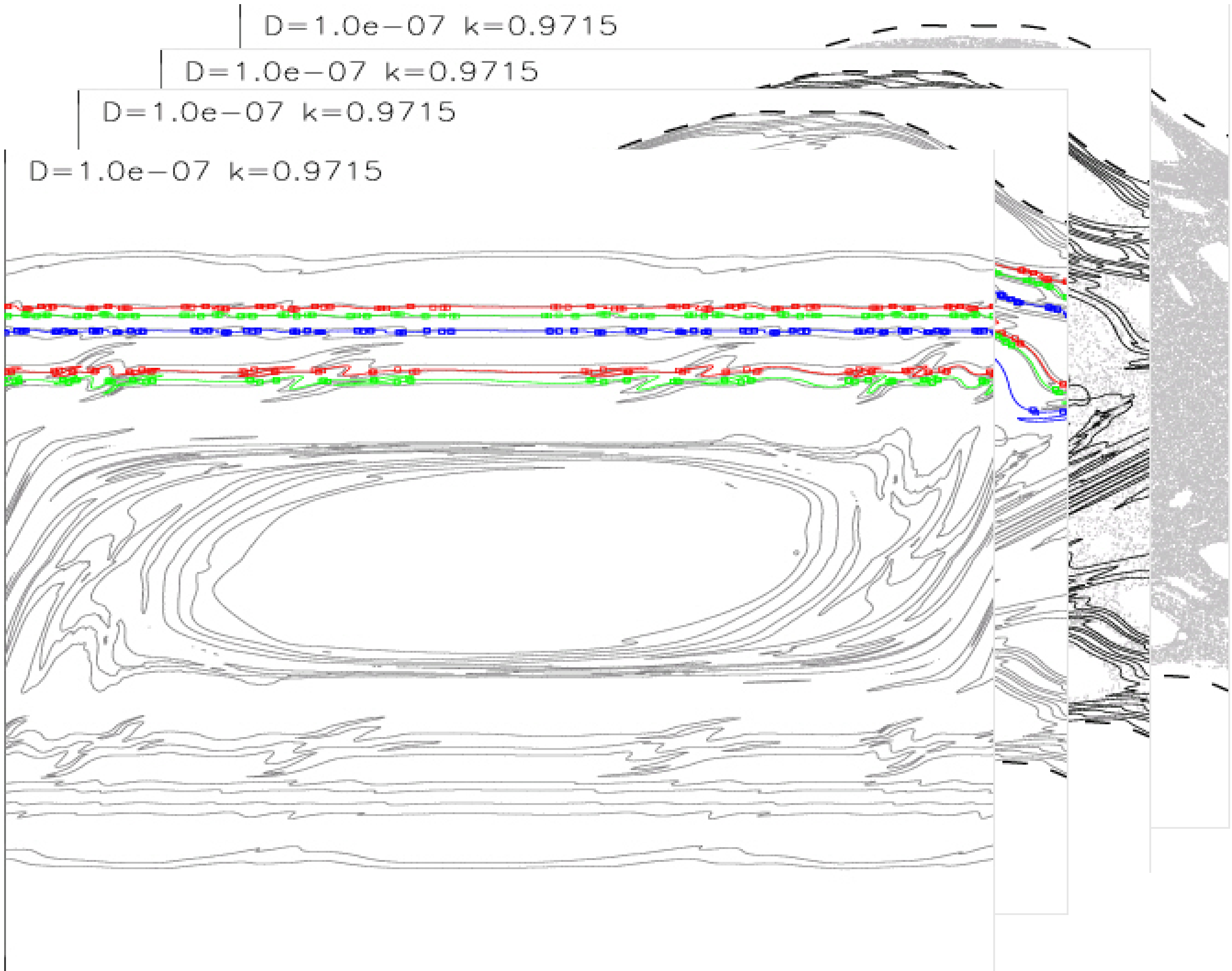
Ghostcurves do not intersect, and may be used to form a chaotic coordinate grid

- *different ghostcurves don't intersect*
  - *careful selection of (p,q) required*
- *can construct "chaotic-coordinates"*
  - *coordinates cannot straighten chaos,*
  -- but coordinates that capture the invariant periodic sets come close



Ongoing work : can chaotic-ghost-coordinates simplify description of chaotic field ?

the advection diffusion equation $\frac{\partial \mathrm{T}}{\partial \mathrm{t}} + v \cdot \nabla T = D\nabla^2 T$ is solved in a chaotic flow



Comments

1) A variety of magnetic / pseudo-magnetic coordinates for integrable, slightly perturbed and strongly chaotic fields are possible.

2) The invariant structures at the heart of these coordinates (ie. invariant flux/KAM surfaces, periodic orbits, cantori) are all very quick to calculate.

3) It is likely that field adapted coordinates may be very useful for numerical simulation; which type of coordinates depends on the application.