Almost-Invariant Tori In The Hamiltonian Dynamics Of 3-D Magnetic Fields

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Recent calculations of heat diffusion along chaotic magnetic field lines [1] show that the isotherms correspond very closely with approximate magnetic surfaces, constructed from a sequence of *ghost surfaces*, cutting through high-order magnetic island chains. These toroidal surfaces capture the islands' "X-point" and "O-point" closed field lines, which are *invariant* under flow along field lines. Thus ghost surfaces form a class of *almost-invariant tori*.

Closed field lines extremize the magnetic action, $\oint \mathbf{A}\cdot d\mathbf{l}$, where $\mathbf{A}$ is the vector potential, while almost-invariant surfaces are constructed from a continuous family of "pseudo field lines," i.e. paths that come as close to extremizing action as possible in some suitable sense.

In the case of ghost surfaces the pseudo field lines are evolved between true closed field lines via a flow down the action-gradient, whereas *quadratic-flux-minimizing* (QFMin) surfaces minimize the angle-averaged square of the action gradient, thus forming an alternative class of almost-invariant tori.

Hudson and Dewar [2] recently showed that QFMin surfaces approximate ghost tori up to and including terms of order ε when the magnetic field $\mathbf{B}$ is of the form $\nabla\psi\times\nabla\theta+\nabla\zeta\times\nabla(\chi_0+\varepsilon\chi_1)$, where θ and ζ are poloidal and toroidal angles, respectively, ψ is the toroidal flux function and the unperturbed poloidal flux function $\chi_0(\psi)$ plays the role of an integrable field-line Hamiltonian, while the perturbation $\chi_1(\psi,\theta,\zeta)$ in general gives rise to magnetic islands and chaos. They also showed perturbatively that, by redefining both QFMin and ghost surfaces in terms of a new poloidal angle Θ, the two approaches to defining almost-invariant tori can be unified, that is, made to agree to higher order in ε.

In order to investigate whether unification can be made exact we study the case of Beltrami fields, which are force-free fields satisfying $\nabla\times\mathbf{B}=\mu\mathbf{B}$, with μ constant over a region, arising naturally from Taylor relaxation and its multiregion generalization [3]. Specifically, in a Cartesian coordinate system x,y,z and taking $\theta=y$ and $\zeta=z$, we use as a test case the *ABC* model $\mathbf{B}=(A\sin\zeta+C\cos\theta)\mathbf{e}_x+(B\sin x+A\cos\zeta)\mathbf{e}_y+(C\sin\theta+B\cos x)\mathbf{e}_z$. This is a three-dimensional equilibrium field with magnetic shear, analytically integrable when $C=0$ [4].

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