Partially-relaxed, topologically-constrained MHD equilibria with chaotic fields

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The commonly used equation of ideal force balance, $\nabla p = \mathbf{j} \times \mathbf{B}$, is pathological when the magnetic field, $\mathbf{B}$, is chaotic. This is because $\mathbf{B} \cdot \nabla p = 0$ implies that any continuous pressure, p, will have a gradient everywhere discontinuous or zero. Recently [1], formulation of the three-dimensional equilibrium problem has been proposed that combines elements of both ideal MHD, and thus allows non-trivial pressure profiles, and Taylor relaxation, so that the magnetic field may reconnect. A key element of this model that allows some a-priori control of the pressure (an input quantity) while making minimal assumptions regarding the topology of the chaotic field is to specify the pressure and rotational-transform profiles discretely.
Consider a plasma region comprised of a set of N nested annular regions which are separated by a discrete set of toroidal interfaces, $\mathcal{I}_l$. In each volume, $\mathcal{V}_l$, bounded by the $\mathcal{I}_{l-1}$ and $\mathcal{I}_l$ interfaces, the plasma energy, U_l, the global-helicity, H_l, and the "mass", M_l, are given by the integrals:

$$U_l = \int_{\mathcal{V}_l} \left(\frac{p}{\gamma - 1} + \frac{B^2}{2\mu_0} \right) dv, \qquad H_l = \int_{\mathcal{V}_l} \mathbf{A} \cdot \mathbf{B} \, dv, \qquad M_l = \int_{\mathcal{V}_l} p^{1/\gamma} \, dv, \tag{1}$$

where $\mathbf{A}$ is the vector potential, $\mathbf{B} = \nabla \times \mathbf{A}$.
The equilibrium states that we seek minimize the total plasma energy, subject to the constraints of conserved helicity and conserved mass in each annular region. We allow arbitrary variations in both the magnetic field in each annulus and the geometry of the interfaces, except that we assume the magnetic field remains tangential to the interfaces which we consider to act as ideal barriers. The Euler-Lagrange equations show [1] that that in each annulus the magnetic field satisfies $\nabla \times \mathbf{B} = \mu \mathbf{B}$ and the pressure is constant, and across each interface the total pressure is continuous, $[[p + B^2/2]] = 0$.
We have implemented this model in a code, the Stepped Pressure Equilibrium Code (SPEC), and numerical results will be presented. The SPEC code uses a mixed Fourier-Finite element representation for the vector potential. Quintic polynomial basis functions give rapid convergence in the radial discretization, and the poloidal angle is adjusted to minimize a "spectral-width". For given interface geometries the Beltrami fields in each annulus are constructed in parallel, and a Newton method (with quadratic-convergence) is implemented to adjust the interface geometry to satisfy force-balance.

[1] Relaxed plasma equilibria and entropy-related plasma self-organization principles, Dewar *et al.*, Entropy 10:621, 2008.