# Advanced MHD models of anisotropy, flow and chaotic fields

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**22nd International Toki Conference**

19-22 November 2012

**Acknowledgement:** Australian Research Council, DIISRTE

# Outline

*"Cross-validation of Experiment and Modelling for fusion and astrophysical plasmas"*

- Bayesian inference framework (see P3-4, von Nessi)
  - Used to infer flux surface geometry with uncertainties
  - Provides model validation (equilibrium and mode structure)
  - Can be used to identify faulty diagnostics & optimise systems
  - Harnessed to infer properties of plasma (e.g. fast particle pressure)
- Anisotropy: equilibrium and stability
  - Development of anisotropy into EFIT++
  - Determine impact of anisotropy on plasma stability
- Multiple Relaxed Region MHD model (see P1-1, S. Hudson)
  - resolves chaotic field regions, islands, flux surfaces in fully 3D plasmas
  - Stepped Pressure Equilibrium Code.
  - Applied to DIIID RMP coils as illustration.

# Bayesian equilibrium modelling

Jakob Svensson, Gregory von Nessi, Matthew Hole, Lynton Appel,

$$P(\mathbf{H}|\mathbf{D}) = P(\mathbf{D}|\mathbf{H})P(\mathbf{H})/P(\mathbf{D})$$

$$\mathbf{H} = \{J_\phi(R,Z), p'(\psi), f(\psi), \rho(\psi,R), \Omega(\psi)\}$$

$$\mathbf{D} = \{P_i(R,Z), F_i(R,Z), \tan\gamma_i(R,Z), I_p, \underbrace{P_{s,e}, S_e(k,\omega)}_{\text{TS spectra}}, S_C(\nu)\}$$

| | | | | | |
|---|---|---|---|---|---|
| Pick-up coils | Flux loops | MSE signals | Plasma current | TS spectra | CXRS spectra |

Aims

(1) Improve equilibrium reconstruction

(2) Validate different physics models

Two fluid with rotation

[McClements & Thyagaraja Mon. Not. R. Astron. Soc. 323 733–42 2001]

Ideal MHD fluid with rotation

[Guazzotto L et al, Phys. Plasmas 11 604–14, 2004]

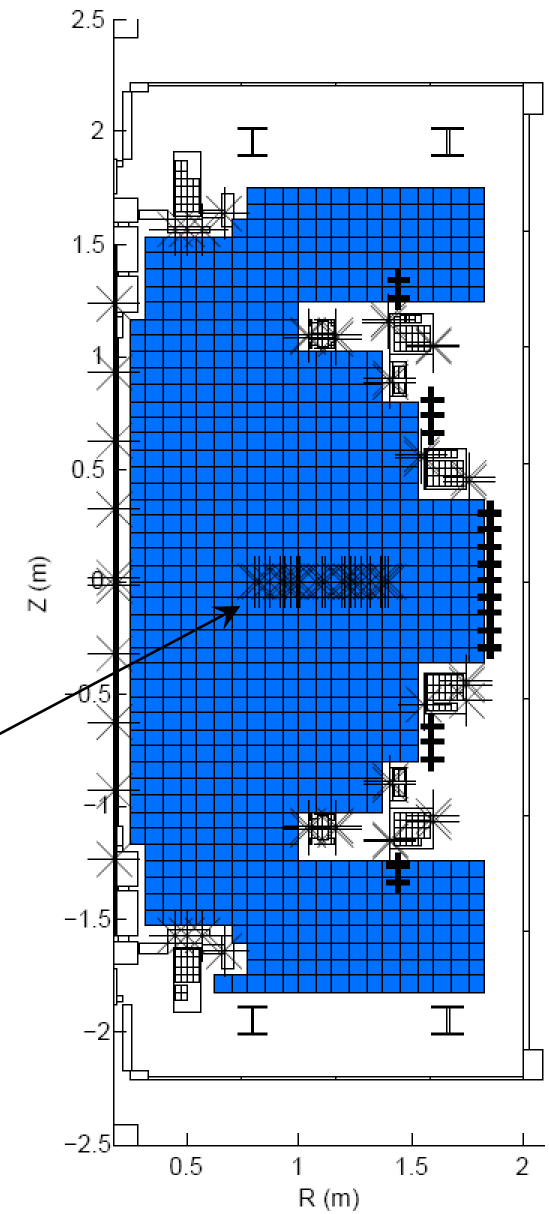
Energetic particle resolved multiple-fluid

[Hole & Dennis, PPCF 51 035014, 2009]

(3) Infer poorly diagnosed physics parameters

# “Analytic” current tomograhy

- Model the MAST plasma current as a cluster of rectangular, toroidal current beams that fill out the limiter region.
- Aim is to infer the distribution for each of these plasma beam currents (ie. **H** = vector of currents, I).
- Constraints:
  - Pick up coils data, $P_i$ (**+**)
  - Flux loops data, $F_i$ (*)
  - MSE data, tan $\gamma_i$




[ Svensson J and Werner A *Plasma Phys. Control. Fusion* 50 085002 , 2008]

# Forward models for magnetics and MSE

- Forward model describes predicted signal given plasma parameters (ie. **D**|**H** in P(**D**|**H**)). For pickup coils $P_i$, flux loops $F_i$ and polarisation angle $\gamma_i$

Angle between coil normal and midplane

$$
\begin{aligned}
\bar{F}_P\left(\bar{I}_L; R, Z\right) &= B_R\left(R, Z; I\right)\cos\left(\theta_i\right) + B_Z\left(R, Z; I\right)\sin\left(\theta_i\right) \\
\bar{F}_F\left(\bar{I}_L; R, Z\right) &= \psi\left(R, Z; I\right) \\
\bar{F}_M\left(\bar{I}_L; R, Z\right) &= \tan\gamma_i\left(R, Z; I\right) = \frac{A_0 B_Z\left(R, Z; I\right) + A_1 B_R\left(R, Z; I\right) + A_2 B_\phi\left(R, Z; I\right)}{A_3 B_Z\left(R, Z; I\right) + A_4 B_R\left(R, Z; I\right) + A_5 B_\phi\left(R, Z; I\right)} \\
\bar{F}_{TP}\left(\bar{I}_L; R, Z\right) &= \sum_i I_{L,i}
\end{aligned}
$$

$$
B_R(\bar{I}_L; R, Z) = -\frac{1}{R}\frac{\partial\psi}{\partial Z}, \quad B_Z(\bar{I}_L; R, Z) = \frac{1}{R}\frac{\partial\psi}{\partial R}, \quad B_\phi(\bar{I}_L, \bar{f}_c; R, Z) = \frac{\mu_0 f}{2\pi R}
$$

- MSE viewing optics on midplane $\Rightarrow$ $A_2$=$A_3$=$A_4$≈0.

# Mean in posterior gives flux surfaces

- If current beams *I* have a Gaussian pdf $\Rightarrow$ inference analytic

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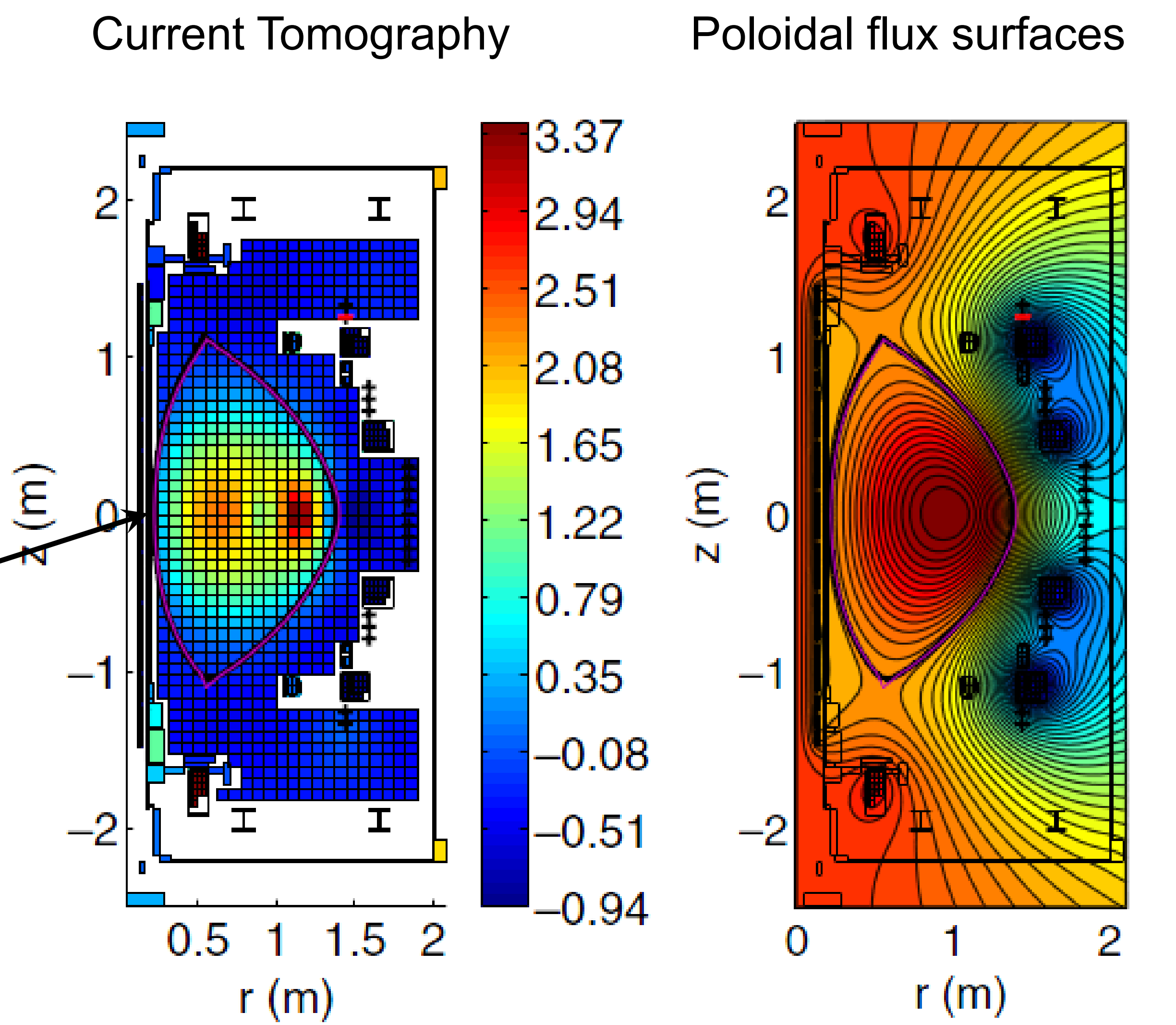
- MAST #24600 @280ms
- D plasma, 3MW NB heating
- $I_p$ = 0.8MA, $\beta_n$=3



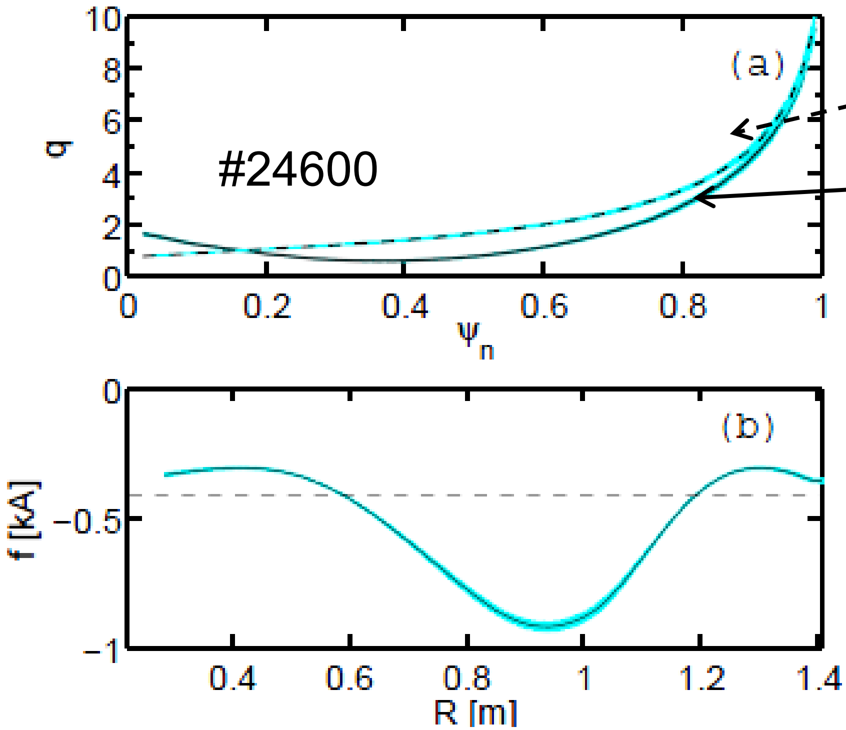

Last closed flux surface of MSE& EFIT

$J_\phi$ and $\psi$ surfaces plotted for currents corresponding to the maximum of the posterior

[M.J. Hole, G. von Nessi, J. Svensson, L.C. Appel, Nucl. Fusion 51 (2011) 103005]

# Sampling of posterior gives distribution

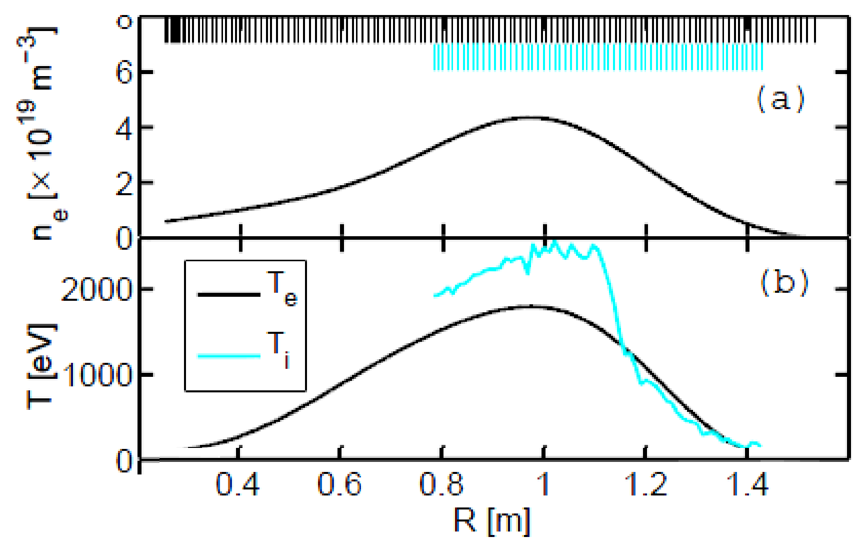
- Distributions generated by sampling, e.g. q profile




No poloidal currents

Inference of poloidal currents: allow $f(\psi)$ to be a $4^{th}$-order polynomial in $\psi$

**Errors < 5%, but are model dependant**




- Bayesian models for TS and CXRS

# Bayesian Equilibrium Analysis & Simulation Tool

*Submitted 14/09/2012 Journal of Physics A: Mathematical and Theoretical*

Gregory von Nessi

- Fold in Force balance model as a weak constraint by technique of split observations.
- Allows quantification of agreement of force-balance through evidence

Biot-Savart link to diagnostics

$$\underbrace{-\frac{R}{2\pi\mu_0}\nabla\cdot\left[\left(\frac{\nabla\psi}{R^2}\right)\right]}_{J_L} = J_\phi = \underbrace{-2\pi R p'(\psi) + \frac{\mu_0}{2\pi R} f(\psi) f'(\psi)}_{J_R}$$

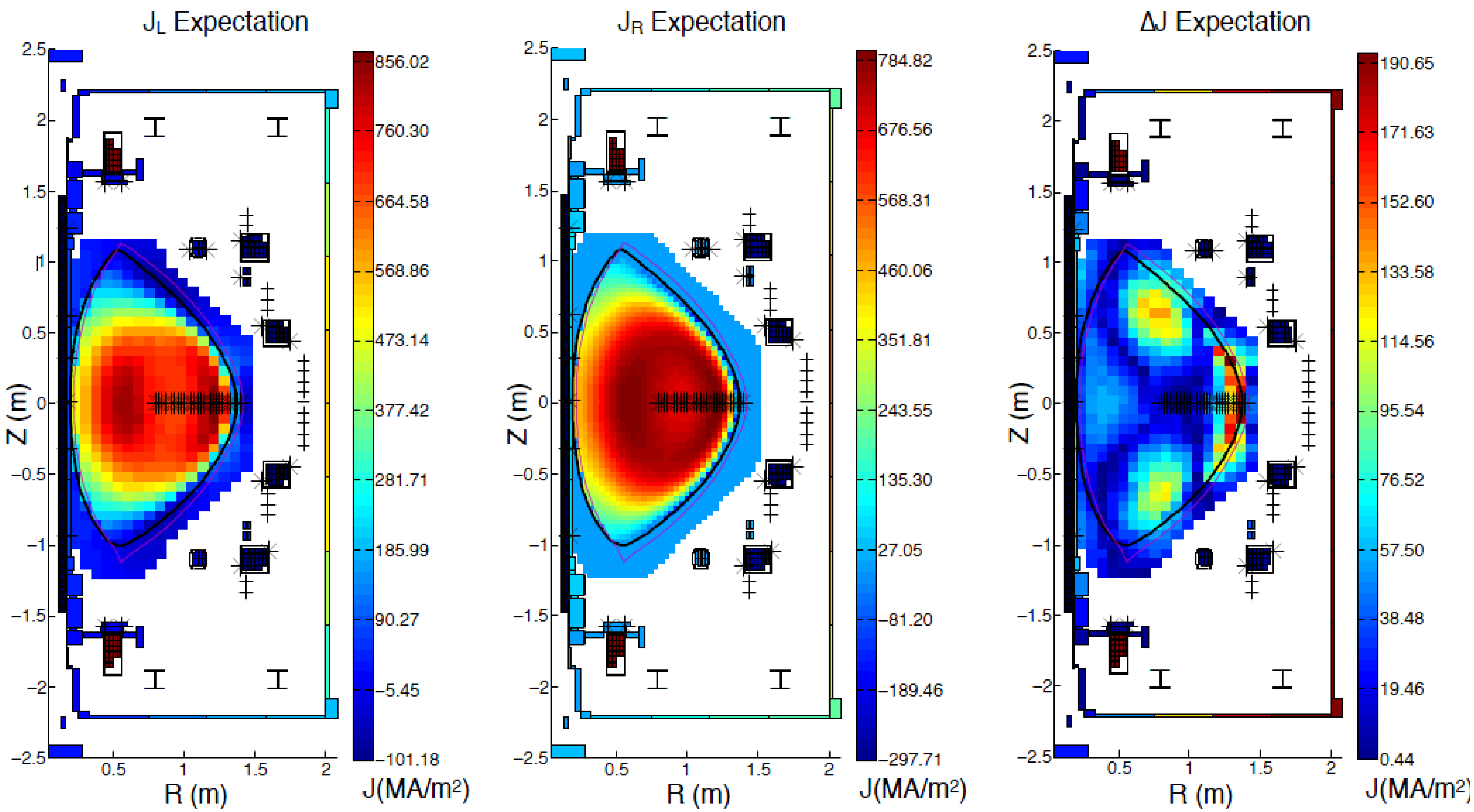
$$P(\mathbf{H}|\mathbf{D}) = \frac{P(\mathbf{D}|\mathbf{H})P(\mathbf{H})}{P(\mathbf{D})} = \frac{P_{1/2}(\mathbf{D}|\mathbf{H}_L)P_{1/2}(\mathbf{D}|\mathbf{H}_R)P(\mathbf{H})}{C_{1/2}P(\mathbf{D})}$$

- Grad-Shafranov equation is non-linear:
  Computational challenges overcome by nested sampling.

# Validation of force balance

MAST #24600 at 265ms

Gregory von Nessi




(a) (b) (c)

- Discrepancy between LHS & RHS ⇒ model not consistent with observations
- Agreement quantified by evidence ln(P(D))=-1290.037 ± 1.129
- BEAST: $\beta_p + l_i/2 = 0.6873 \pm 0002$; EFIT: $\beta_p + l_i/2 = 1.0782$

# Energetic pressure inference

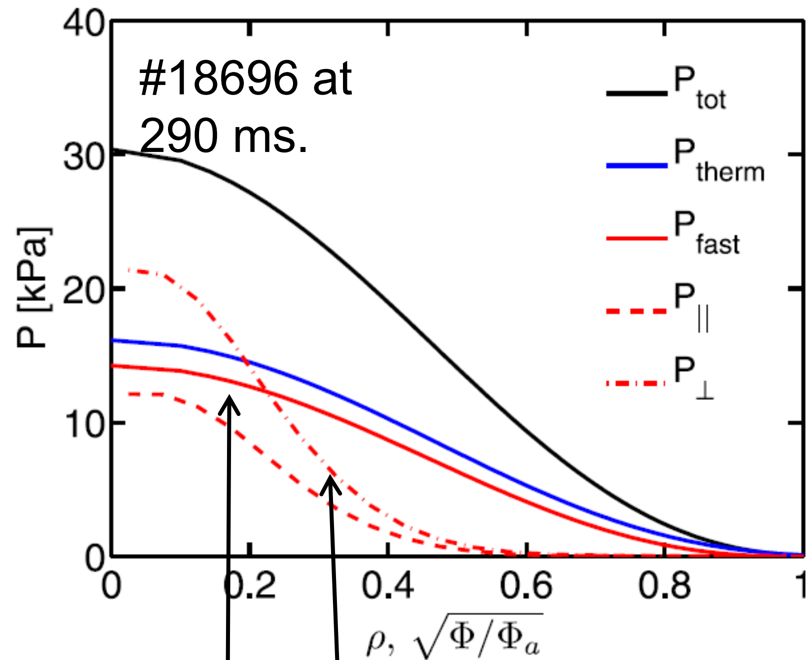
- Add polynomial parameterisations of $P_{total}$, $P_{therm}$ to *H*, and add analysed Thomson scattering data to *D*

- Assume
  $P_{therm} = (n_i T_i + n_e T_e) \sim n_e T_e$
  $f(\psi) \alpha\ \psi$

- Apply force-balance constraint $\Rightarrow$

  $P_{fast} = P_{tot} - P_{therm}$

- Work with CCFE: implementing FIDA into Bayesian framework




inferred $P_{fast}$ ~ $(P_{\perp} + P_{||})/2$ computed in NUBEAM.

[M. J. Hole, G von Nessi, M Fitzgerald and the MAST team, Plasma Phys. Control. Fusion 54 (2012), accepted]

# Bayesian tools on MAST

- Analytic current tomography; CAR prior
  [Hole et al J. Plasma Fusion Res. SERIES, Vol. 9 (2010)]
  [Hole et Rev. Sci. Instrum. 81, 10E127 2010]
- Analytic current tomography; Gaussian process prior
  [ Svensson, submitted to *IEEE Imaging)*,
- Evidence based cross-validation
  [G. T. von Nessi *et al* Phys. Plasmas 19, 012506 (2012)]
- BEAST: Model validation and equilibrium inference
  [G. T. von Nessi *et al*, lodged *Journal of Physics A: Mathematical and Theoretical*]
- Thomson scattering …paper in progress
- Energetic particle pressure inference
  [M.J. Hole, G. von Nessi, J. Svensson, L.C. Appel, Nucl. Fusion 51 (2011) 103005]
  [M. J. Hole, G von Nessi, M Fitzgerald , the MAST team, PPCF 54 (2012), accept.]

  ... FIDA in progress
- *Connect toroidal rotation*
- “Scheduler” service for probablistic equilibrium inference; q profile and uncertainty.

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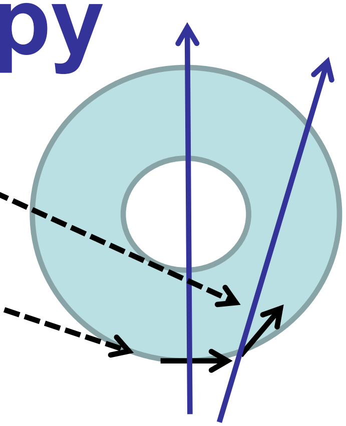
# Expected impact of anisotropy

- Small angle $\theta_b$ between beam, field $\Rightarrow p_{||} > p_{\perp}$
- Beam orthogonal to field, $\theta_b = \pi/2 \Rightarrow p_{\perp} > p_{||}$

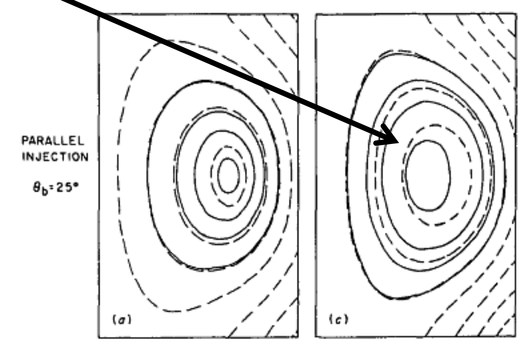


- If $p_{||}$ sig. enhanced by beam, $p_{||}$ surfaces distorted and displaced inward relative to flux surfaces

  [Cooper et al, Nuc. Fus. 20(8), 1980]

Peaked pressure profile

Broad pressure profile




Parallel pressure contours(solid)

Flux surfaces (dashed)

- If $p\perp > p_{||}$, an increase will occur in centrifugal shift :

  [R. Iacono, A. Bondeson, F. Troyon, and R. Gruber, Phys. Fluids B 2 (8). August 1990]

- Compute $p_{\perp}$ and $p_{||}$ from moments of distribution function, computed by TRANSP

[M J Hole, G von Nessi, M Fitzgerald, K G McClements, J Svensson, PPCF 53 (2011) 074021]

- Infer $p_{\perp}$ from diamagnetic current $\mathbf{J}_{\perp}$

[see V. Pustovitov, PPCF 52 065001, 2010 and references therein]

# MHD with rotation & anisotropy

- Inclusion of anisotropy and flow in equilibrium MHD equations

[R. Iacono, et al Phys. Fluids B 2 (8). 1990]

$$\nabla \cdot (\rho \mathbf{v}) = 0, \qquad \rho \mathbf{v} \cdot \nabla \mathbf{v} = \mathbf{J} \times \mathbf{B} - \nabla \cdot \overline{\mathbf{P}}, \qquad \nabla \cdot \mathbf{B} = 0$$

$$\mu_0 \mathbf{J} = \nabla \times \mathbf{B}, \qquad \nabla \times (\mathbf{v} \times \mathbf{B}) = 0,$$

$$\overline{\mathbf{P}} = p_\perp \overline{\mathbf{I}} + \Delta \mathbf{B}\mathbf{B}/\mu_0 , \qquad \Delta = \frac{\mu_0 (p_{||} - p_\perp)}{B^2}$$

# MHD with rotation & anisotropy

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$$\nabla\cdot(\rho\mathbf{v})=0,\qquad \rho\mathbf{v}\cdot\nabla\mathbf{v}=\mathbf{J}\times\mathbf{B}-\nabla\cdot\overline{\mathbf{P}},\qquad \nabla\cdot\mathbf{B}=0$$

$$\mu_0\mathbf{J}=\nabla\times\mathbf{B},\qquad \nabla\times(\mathbf{v}\times\mathbf{B})=0,$$

$$\overline{\mathbf{P}}=p_\perp\overline{\mathbf{I}}+\Delta\mathbf{B}\mathbf{B}/\mu_0\,,\qquad \Delta=\frac{\mu_0\left(p_{||}-p_\perp\right)}{B^2}$$

- Frozen flux gives velocity plus axis-symmetry

$$\mathbf{v}=\frac{\psi_M'(\psi)}{\rho}\mathbf{B}-R\phi_E'(\psi)\mathbf{e}_\varphi.$$

Equilibrium eqn becomes:

$$\boxed{\nabla\cdot\left[\tau\left(\frac{\nabla\psi}{R^2}\right)\right]=-\frac{\partial p_\parallel}{\partial\psi}-\rho H_M'(\psi)+\rho\frac{\partial W}{\partial\psi}-I_M'(\psi)\frac{I}{R^2}-\psi_M''(\psi)\mathbf{v}\cdot\mathbf{B}+R\rho v_\phi\phi_E''(\psi)}$$

$$I=RB_\varphi$$

$$I_M(\psi)=\tau I-\mu_0R^2\psi_M'(\psi)\phi_E'(\psi)$$

$$H_M(\psi)=W_M(\rho,B,\psi)-\frac{1}{2}\left[R\phi_E'(\psi)\right]^2+\frac{1}{2}\left[\frac{\psi_M'(\psi)B}{\rho}\right]^2,$$

$$\left\{I_M(\psi),\psi_M(\psi),\phi_E(\psi),H_M(\psi),\frac{\partial p_\parallel}{\partial\psi},\frac{\partial W}{\partial\psi}\right\}$$

**Set of 6 profile constraints**

$$\tau=1-\Delta-\mu_0\left(\psi_M'\right)^2/\rho,$$

# Neglect poloidal flow

- Suppose $\mathbf{v} = -R\phi_E'(\psi)\mathbf{e}_\varphi = R\Omega(\psi)\mathbf{e}_\varphi \quad \Rightarrow \quad F(\psi) = I_M(\psi)/\tau$

and equilibrium eqn becomes:

$$\boxed{\nabla\cdot\left[(1-\Delta)\left(\frac{\nabla\psi}{R^2}\right)\right] = -\frac{\partial p_\parallel}{\partial\psi} - \rho H'(\psi) + \rho\frac{\partial W}{\partial\psi} - \frac{F'(\psi)F'(\psi)}{R^2(1-\Delta)} + R^2\rho\Omega(\psi)\Omega'(\psi)}$$

**Set of 5 profile constraints**

$$\left\{F(\psi),\Omega(\psi),H(\psi),\frac{\partial p_\parallel}{\partial\psi},\frac{\partial W}{\partial\psi}\right\}$$

- $\partial W/\partial\psi$: different for MHD/ double-adiabatic/ guiding centre

- If two temperature Bi-Maxwellian model chosen

$$p_\parallel(\rho, B\psi) = \frac{k_B}{m}\rho T_\parallel(\psi) \qquad p_\perp(\rho, B\psi) = \frac{k_B}{m}\rho T_\perp(\psi) = \frac{k_B}{m}\rho T_\parallel(\psi)\frac{B}{B-\theta(\psi)T_\parallel}$$

$$\left\{F(\psi),\Omega(\psi),H(\psi),T_\parallel(\psi),\theta(\psi)\right\}$$

# Constraining the flux functions to transport codes or experiment

$$\{F(\psi), \Omega(\psi), H(\psi), T_{\|}(\psi), \theta(\psi)\}$$

[ M. Fitzgerald, L.C. Appel, M.J. Hole *to be submitted* J. Comp Phys]

- TRANSP computes $f(E,\lambda)$: Moments give $p_\perp$, $p_{||}$, $u_{||}$,
- Dependency of flux functions on (R,Z) mesh

$$T_{\|}(R_i, Z_i) = \frac{p_{\|}(R_i,Z_i)}{\left(\frac{k}{m}\right)\rho(R_i,Z_i)}$$

$$F(R_i, Z_i) = R_i B_\phi(R_i, Z_i)[1 - \Delta(R_i, Z_i)]$$

$$\Omega(R_i, Z_i) = \frac{v_\phi(R_i,Z_i)}{R_i}$$

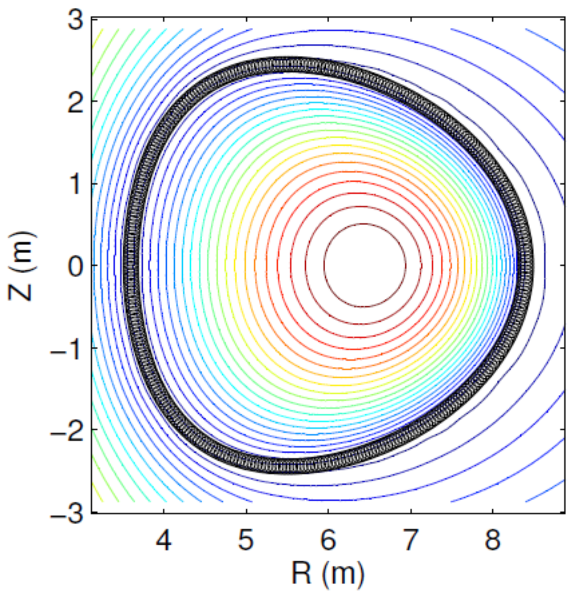
$$H(R_i, Z_i) = \frac{p_{\|}(R_i,Z_i)}{\rho(R_i,Z_i)} \ln\left(\frac{\rho(R_i,Z_i) p_{\|}(R_i,Z_i)}{\rho_0 p_\perp(R_i,Z_i)}\right) - \frac{v_\phi^2(R_i,Z_i)}{2}$$

$$\theta(R_i, Z_i) = \frac{\left(\frac{k}{m}\right)\rho(R_i,Z_i) B(R_i,Z_i)}{p_{\|}(R_i,Z_i)} - \frac{\left(\frac{k}{m}\right)\rho(R_i,Z_i) B(R_i,Z_i)}{p_\perp(R_i,Z_i)}$$

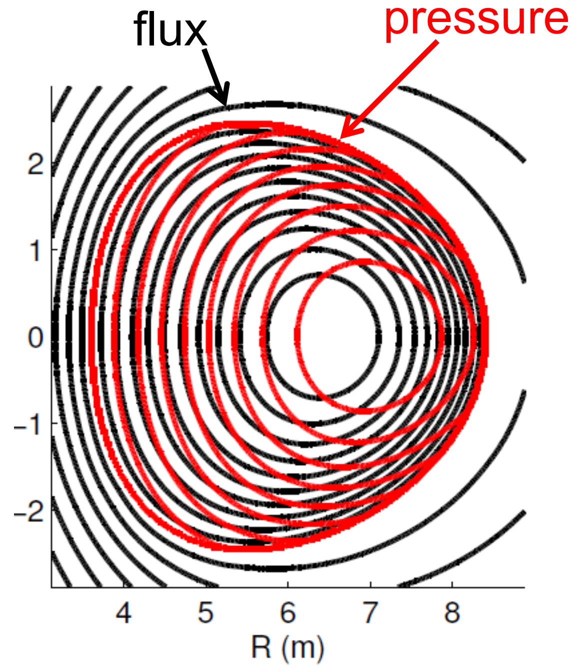
# Code benchmarked

- So far tested against MAST #13050, #18696
- Able to use the same constraints as existing EFIT++
- Converges at same speed as existing EFIT++
- Soloviev benchmarks have been computed for isotropic, anisotropic and flow cases.

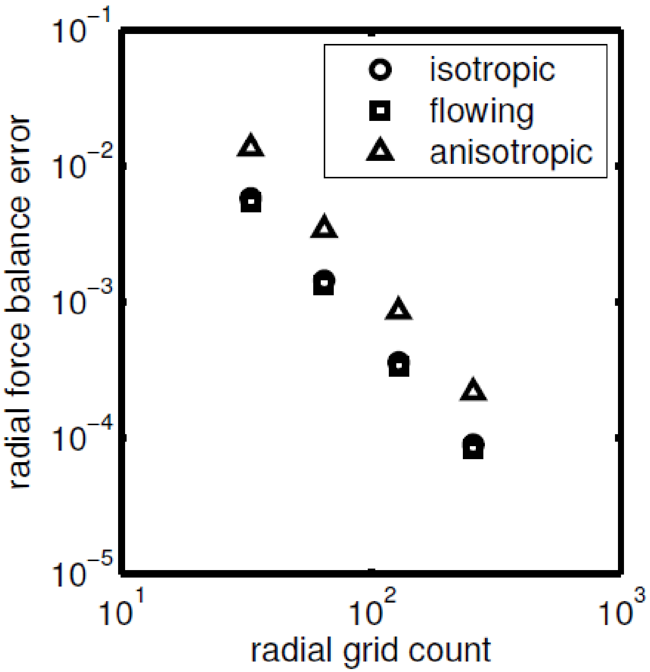
[ M. Fitzgerald, L.C. Appel, M.J. Hole, *to be submitted* J. Comp Phys]




Soloviev:
$\beta_t$=0.07



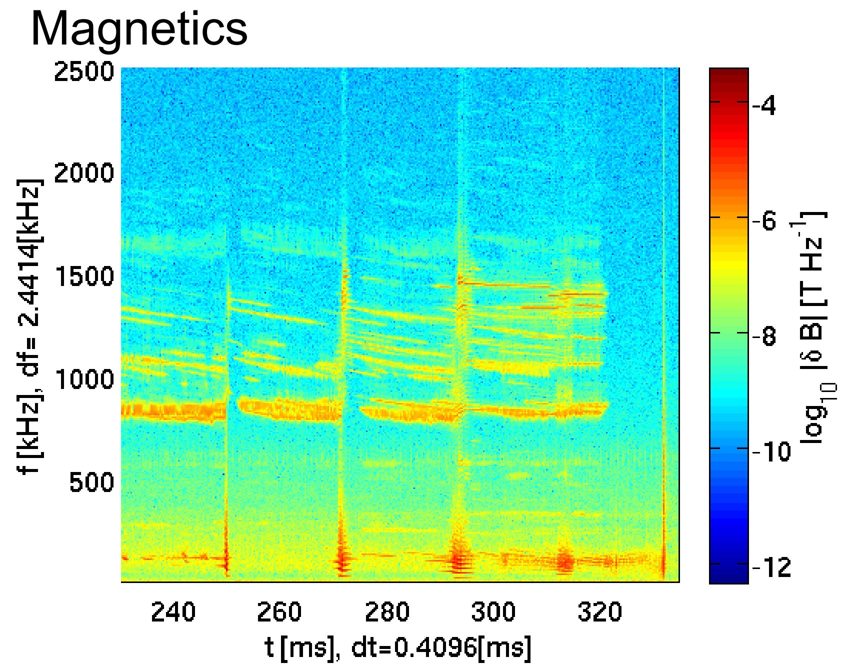

Extended Soloviev:
$\beta_t$=0.07, $M_\phi$=0.8, $\Delta$=0.004,



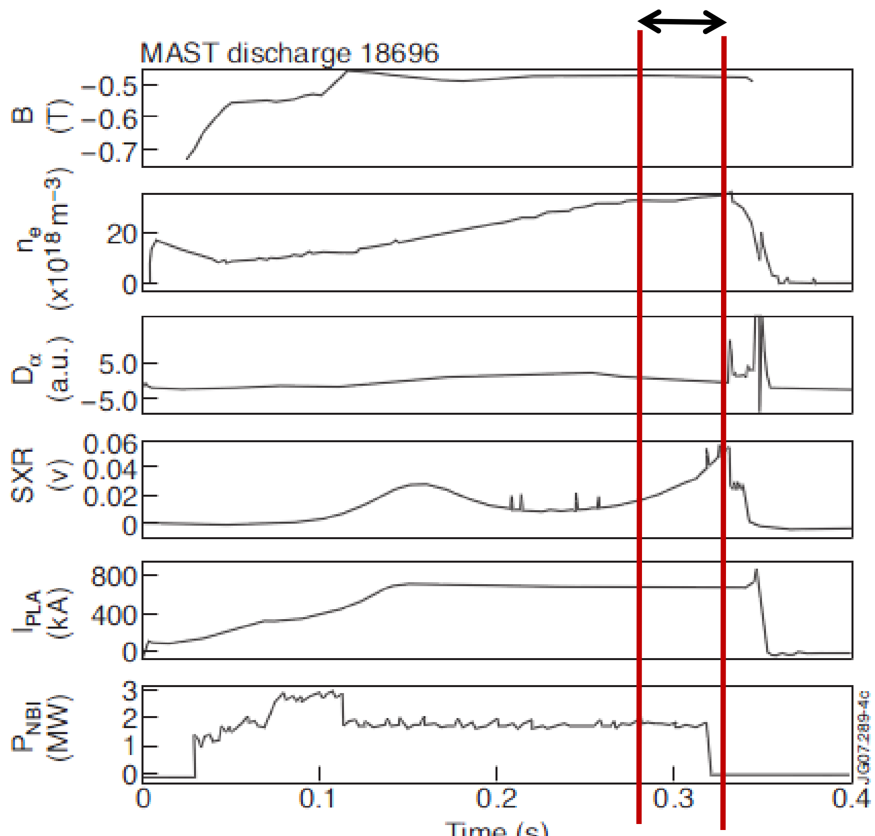

Solution Convergence

# Anisotropy on MAST

- MAST #18696
- 1.9MW NB heating
- $I_p$ = 0.7MA, $\beta_n$=2.5
- TRANSP simulation available
- Magnetics shows CAEs



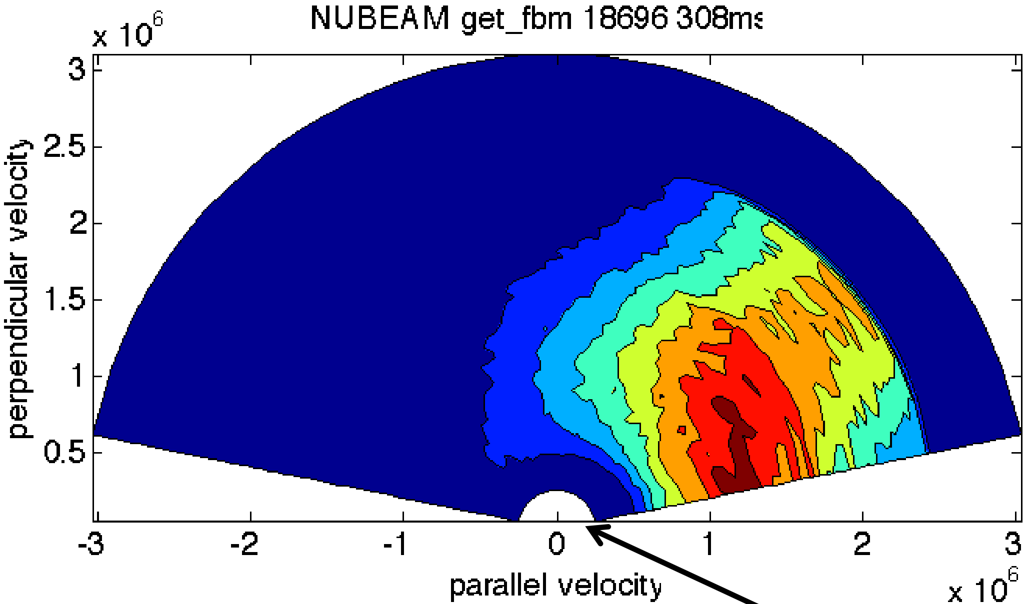

[M.P. Gryaznevich et al, Nuc. Fus. 48, 084003, 2008.; Lilley *et al* 35th EPS Conf. Plas.Phys. 9 - 13 June 2008 ECA Vol.32D, P-1.057]




- What is the impact on q profile due to presence of anisotropy and flow?

# $p_{||}$, $p_{\perp}$, flow from $f(E,\lambda)$ moments

r/a=0.25




$$E = 0.5mv^2, \quad v_{\|} = v\cos\lambda$$

$$n = \int_0^\infty \int_{-1}^1 \hat{f}(E,\lambda)\, d\lambda\, dE$$

$$nu_{\|} = \int_0^\infty \int_{-1}^1 v_{\|} \hat{f}(E,\lambda)\, d\lambda\, dE$$

$$p_{\|} = m \int_0^\infty \int_{-1}^1 (v_{\|} - u_{\|})^2 \hat{f}(E,\lambda)\, d\lambda\, dE$$

$$p_{\perp} = \frac{m}{2} \int_0^\infty \int_{-1}^1 v_{\perp}^2 \hat{f}(E,\lambda)\, d\lambda\, dE.$$

[35th EPS 2008; M.K.Lilley et al]

Thermal population

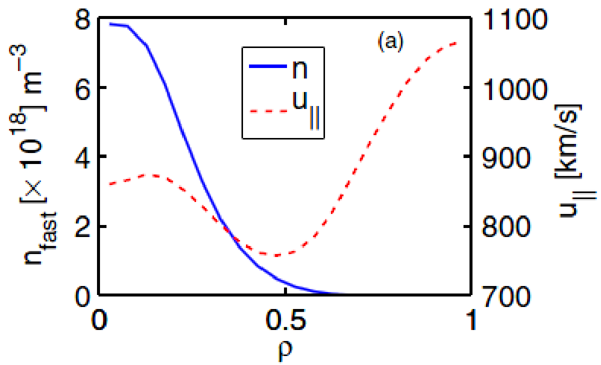
$v_{||} > v_{\perp}$ in disitribution function, *however...*

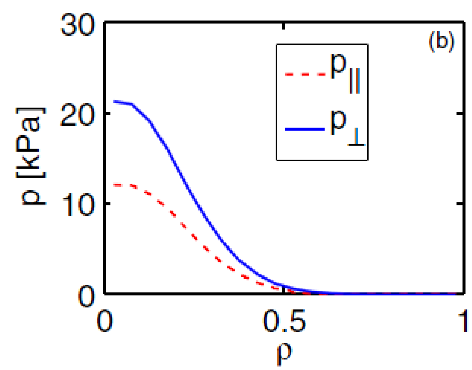
$p_{||}$ computed with subtracted $u_{||} \Rightarrow p_{||} < p_{\perp}$

In single fluid limit, need to add thermal species and recompute moments to get complete anisotropy.

[M J Hole, G von Nessi, M Fitzgerald, K G McClements, J Svensson, PPCF 53 (2011) 074021]

# In absence of thermals... $p_\perp/p_{||} \approx 1.7$




$p_\perp/p_{||} \approx 1.7$

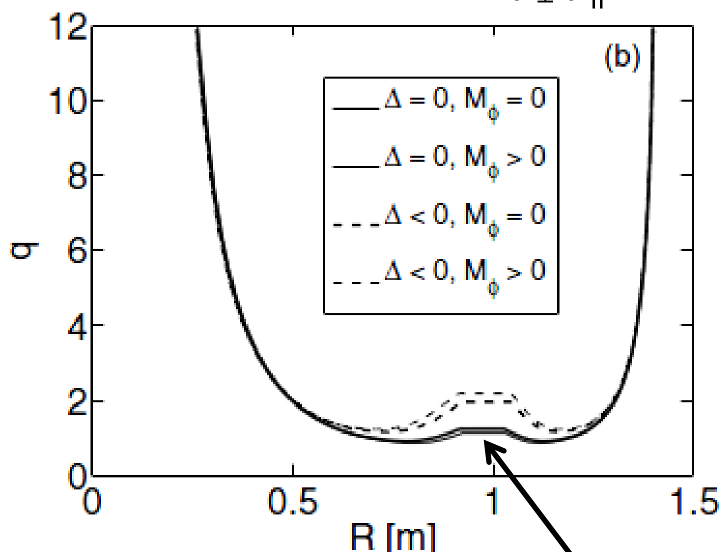
$$\rho = \sqrt{\Phi / \Phi_0}$$

$\Phi$ = toroidal flux

## Impact on plasma computed using FLOW, EFIT TENSOR

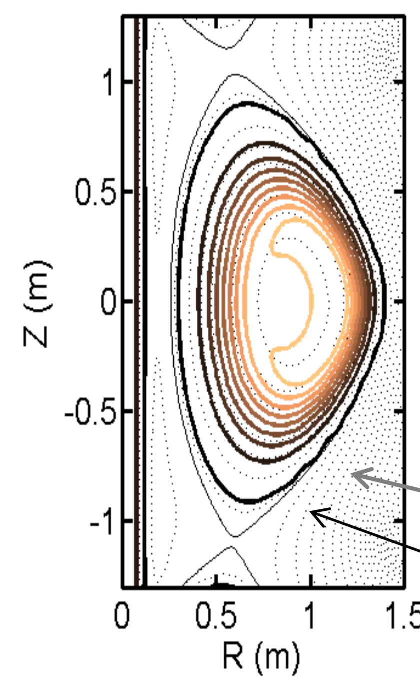
FLOW scans $\Delta < 0$: $p_\perp/p_{||} \approx 1.7$

$\Delta = 0$: $p_\perp/p_{||} = 1$




Low grid resolution of FLOW at core

EFIT++ (TENSOR)




Calculation of MAST #18696 at 290ms.

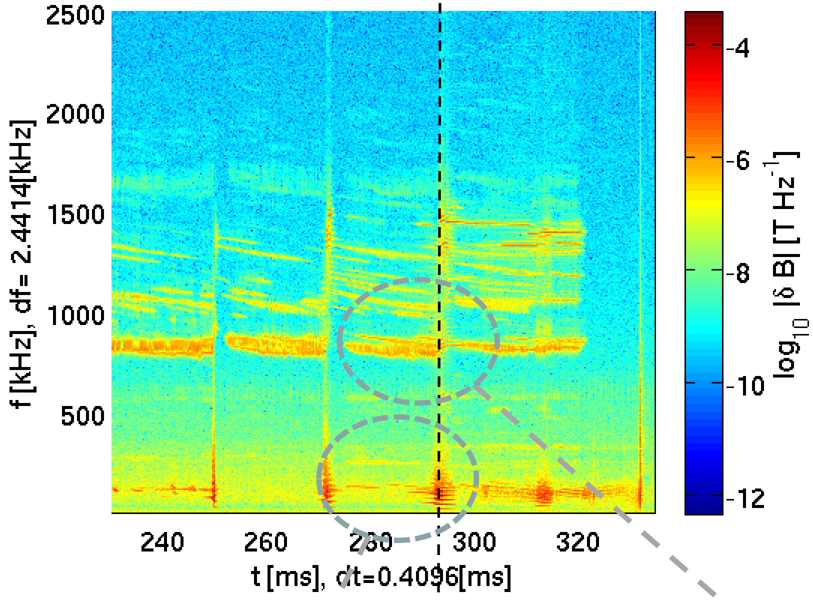
$p_\perp / p_{||} \sim 1.7$

(slowing down beam particles)

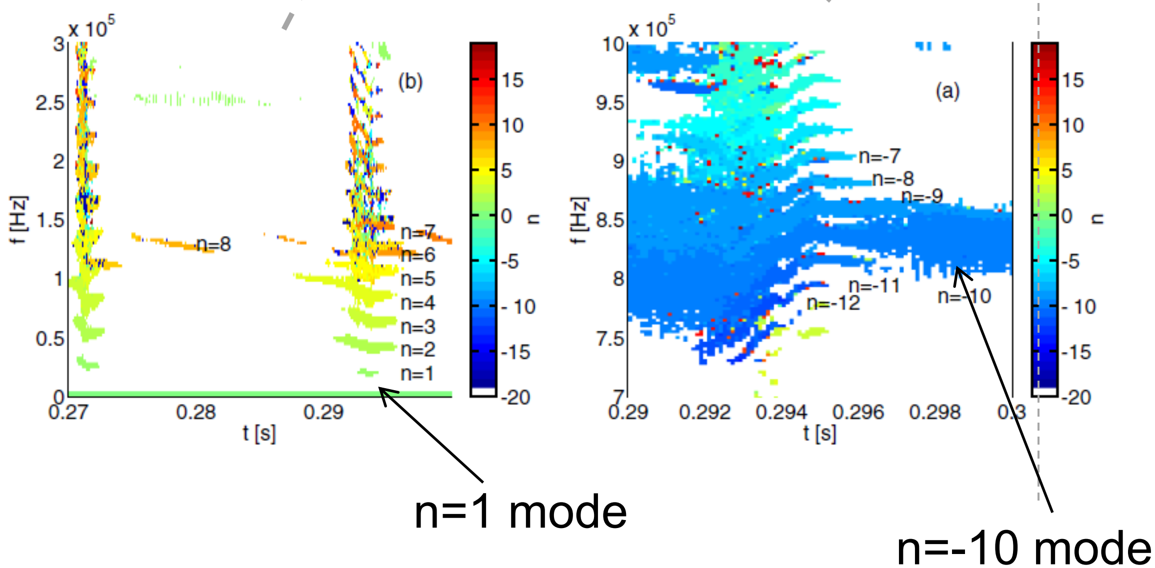
poloidal flux

surfaces of constant $p_{||}$.

# Impact of anisotropy on wave modes

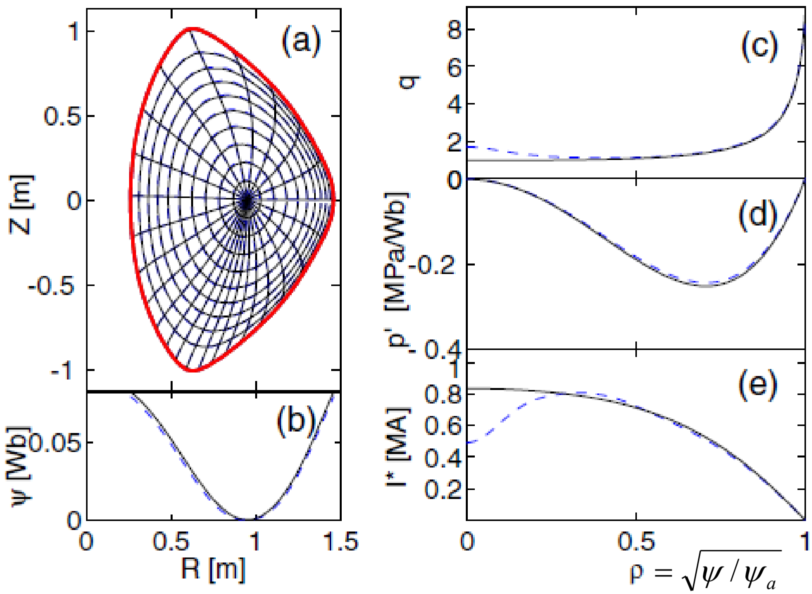



- How do predicted mode frequencies change due to changes in *q* produced by anisotropy and flow?
- Calculation of change in stability due to anisotropy in progress.




- Appetiser: What is the change in ideal MHD stability of n=1 TAE?

# Increased shear gives multiple TAEs

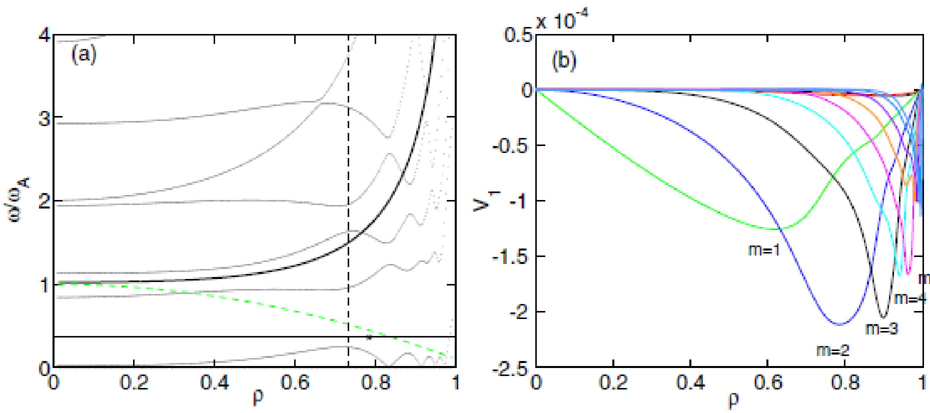



- Reshape plasma to have larger reverse shear

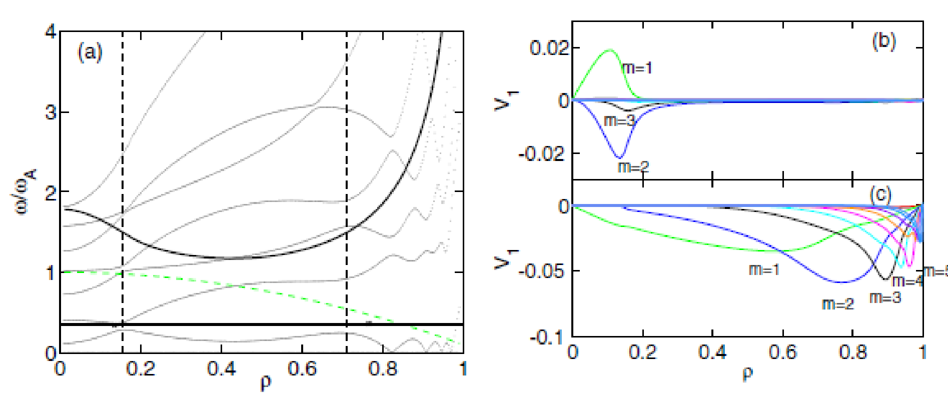
$$I^*(s) \rightarrow I^*(s) + \underbrace{I_0 \exp\left[-\frac{(s-s_0)^2}{2\sigma_0^2}\right]}_{\text{core}} + \underbrace{I_1 \exp\left[-\frac{(s-s_1)^2}{2\sigma_1^2}\right]}_{\text{reverse shear}}$$

$I_0$, $I_1$ varied to match $q_0$=1.7, $q_{min}$=1.24

[ M J Hole, G von Nessi, M Fitzgerald and the MAST team, accepted, PPCF 54 (2012)]




Single global TAE at (m,n) = (1,1)




Reverse shear produces second (m,n)) = (1,1) odd TAE resonance in the core

# Anisotropy ongoing work

- Kinetic constraints from TRANSP.
- Configuration physics:
  - scan of configurations with significant anisotropy.
  - experiments with varying beam parameters (MAST, DIIID?)
- Formulate stability in presence of anisotropy, flow
- Implement anisotropy extensions of MISHKA or PHOENIX
  - generation of MISHKA straight field line metric directly from (R,Z) metric (✓ Kieran Woolfe – Honours student)
- Couple HAGIS to EFIT TENSOR and MISHKA or PHOENIX
- Extend CSCAS to include anisotropy.
- Feed anisotropy inputs into ANIMEC to explore impact of anisotropy in 3D (no flow).

# Outline

*"Cross-validation of Experiment and Modelling for fusion and astrophysical plasmas"*:

- Probabilistic (Bayesian) inference framework
  - Used to infer flux surface geometry with uncertainties
  - Provides model validation (equilibrium and mode structure)
  - Can be used to identify faulty diagnostics & optimise systems
  - Harnessed to infer properties of plasma (e.g. fast particle pressure)
- Anisotropy: equilibrium and stability
  - Development of anisotropy into EFIT++
  - Determine impact of anisotropy on plasma stability
- Multiple Relaxed Region MHD model
  - resolves chaotic field regions, islands, flux surfaces in fully 3D plasmas
  - Stepped Pressure Equilibrium Code.
  - Applied to DIIID RMP coils and ITER ELM coils as illustration.

# Toroidal plasma equilibrium in 3D

- Simplest model to approximate global, macroscopic force-balance is magnetohydrodynamics (MHD).

$$\nabla p = \mathbf{J} \times \mathbf{B}, \qquad \nabla \times \mathbf{B} = \mathbf{J}, \qquad \nabla \cdot \mathbf{B} = 0$$

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- Non-axisymmetric magnetic fields generally **do not** have a nested family of smooth flux surfaces, **unless** ideal surface currents are allowed at the rational surfaces.

- If the field is non-integrable (i.e. chaotic, with a fractal phase space), then any **continuous** pressure that satisfies $\mathbf{B}\cdot\nabla p=0$ must have an **infinitely discontinuous gradient,** $\nabla p$.

- Instead, solutions with stepped-pressure profiles are guaranteed to exist. Variational principle called MRXMHD (R. L. Dewar).

- Numerical implementation, SPEC, by S. Hudson (PPPL).

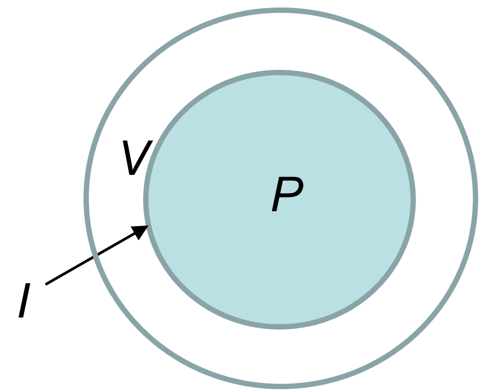
# Taylor Relaxed States

- Zero pressure gradient regions are **force-free** magnetic fields:

- In 1974, Taylor argued that turbulent plasmas with small resistivity, and viscosity relax to a Beltrami field

Internal energy: $$W = \int_{P \cup V} \left( \frac{B^2}{2\mu_0} + \frac{p}{\gamma - 1} \right) d\tau^3$$

Total Helicity : $$H = \int_V (\mathbf{A} \cdot \mathbf{B}) d\tau^3$$




Taylor solved for minimum *W* subject to fixed *H*

i.e. solutions to δF=0 of functional $$F = W - \mu H \ / 2$$

$$P: \quad \nabla \times \mathbf{B} = \mu \mathbf{B}$$

$$I: \quad \left[\left[ \frac{B^2}{2\mu_0} + p \right]\right] = 0$$

$$V: \quad \nabla \times \mathbf{B} = 0$$

Model had a lot of success for toroidal pinches, multipinch, and spheromaks

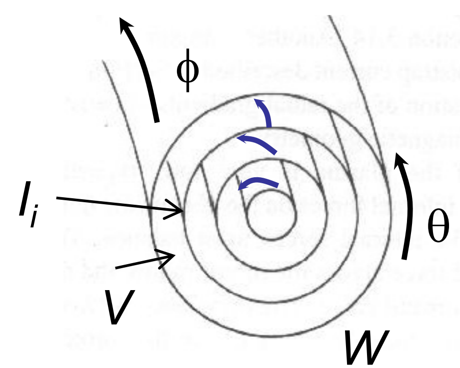
# Generalised Taylor Relaxation:
## Multiple Relaxed Region MHD (MRXMHD)

R. L. Dewar

- Assume each invariant tori $I_i$ act as ideal MHD barriers to relaxation, so that Taylor constraints are localized to subregions.

New system comprises:

- $N$ plasma regions $P_i$ in relaxed states.
- Regions separated by ideal MHD barrier $I_i$.
- Enclosed by a vacuum $V$,
- Encased in a perfectly conducting wall $W$




$$W_l = \int_{R_i} \left( \frac{B_l^2}{2\mu_0} + \frac{P_l}{\gamma - 1} \right) d\tau^3$$

$$H_l = \int_V (\mathbf{A}_l \cdot \mathbf{B}_l) d\tau^3$$

Seek minimum energy state:

$$F = \sum_{l=1}^{N} \left( W_l - \mu_l H_l / 2 \right)$$

$$\begin{array}{ll}
P_l : & \nabla \times \mathbf{B} = \mu_l \mathbf{B} \\
 & P_l = \text{constant} \\
I_l : & \mathbf{B} \cdot \mathbf{n} = 0 \\
 & [[P_l + B^2/(2\mu_0)]] = 0 \\
V : & \nabla \times \mathbf{B} = 0 \\
 & \nabla \cdot \mathbf{B} = 0 \\
W : & \mathbf{B} \cdot \mathbf{n} = 0
\end{array}$$

# Stepped Pressure Equilibrium Code, SPEC

**[POP to appear 2012; PPCF, 54:014005, 2012]**

P1-1 S. Hudson

## Vector potential is discretised using mixed Fourier & finite elements

- Coordinates (s,φ, ζ)
- Interface geometry $R_i = \sum_{l,m,n} R_{lmn} \cos(m\vartheta - n\zeta),\ Z_i = \sum_{l,m,n} Z_{lmn} \sin(m\vartheta - n\zeta)$
- Exploit gauge freedom $\mathbf{A} = A_\vartheta(s,\vartheta,\zeta)\nabla\vartheta + A_\zeta(s,\vartheta,\zeta)\nabla\zeta$
- Fourier $A_\vartheta = \sum_{m,n} \alpha(s)\cos(m\vartheta - n\varsigma)$
- Finite-element $a_\vartheta(s) = \sum_i a_{\vartheta,i}(s)\varphi(s)$

## & inserted into constrained-energy functional $F = \sum_{l=1}^{N}(W_l - \mu_l H_l / 2)$

- Derivatives wrt **A** give Beltrami field $\nabla \times \mathbf{B} = \mu\mathbf{B}$
- Field in each annulus computed independently, distributed across multiple cpu's
- Field in each annulus depends on enclosed toroidal flux, poloidal flux, interfaces ξ

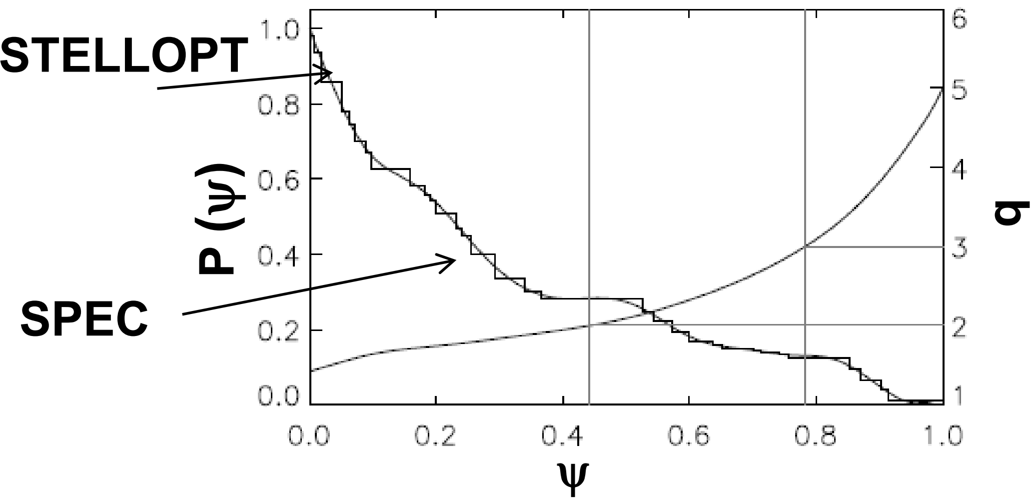
## Force balance solved using multi-dimensional Newton method

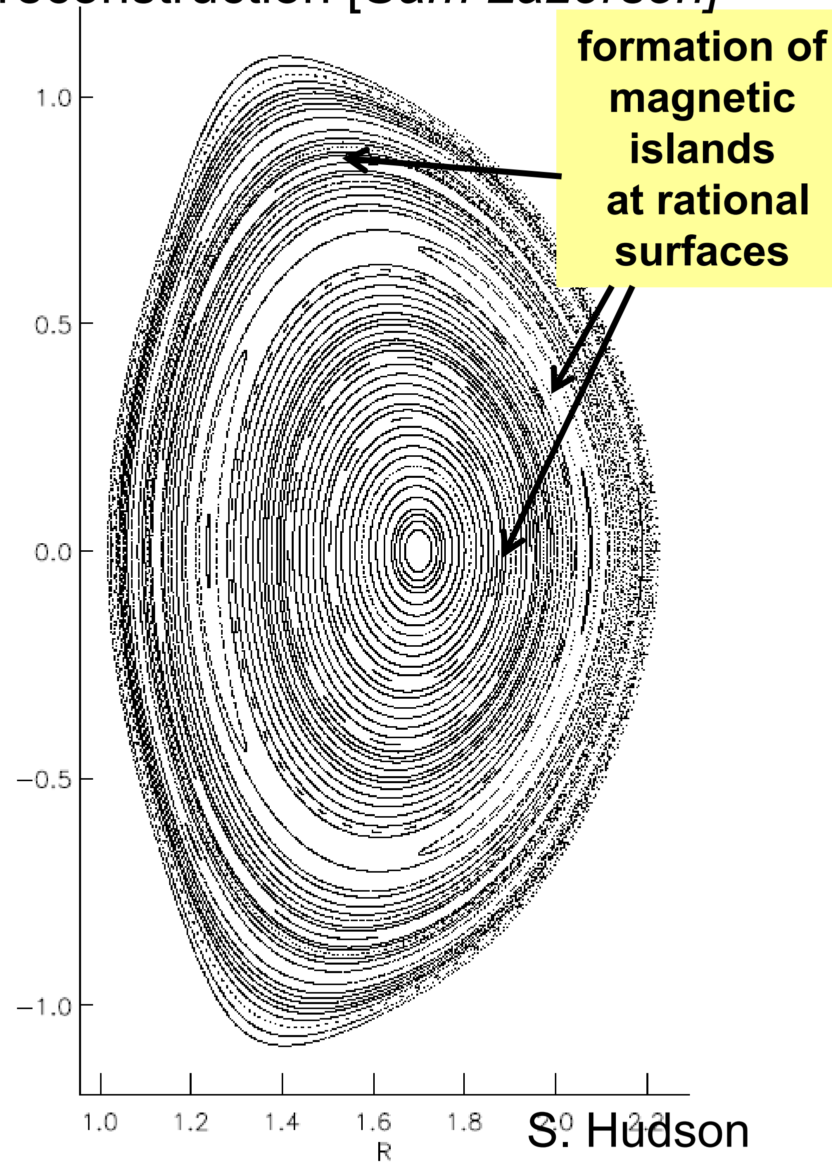
- Interface geometry adjusted to satisfy force balance $\mathbf{F}[\xi] = \{[[p + B^2/2]]_{m,n}\} = 0$
- Angle freedom constrained by spectral condensation,
- Dertivative matrix ∇F[ξ] computed in parallel using finite difference

# Example: DIIID with n=3 applied error field

**[Hudson *et al*, POP to appear 2012]**

- 3D boundary, p, q-profile from STELLOPT reconstruction [*Sam Lazerson]*
- Irrational interfaces chosen to coincide with pressure gradients.




- Island formation is permitted
- No rational “shielding currents” included in calculation.

S. Hudson

# Spontaneously formed helical states

G. Dennis

- The quasi-single helicity state is a a stable helical state in RFP: becomes purer as current is increase

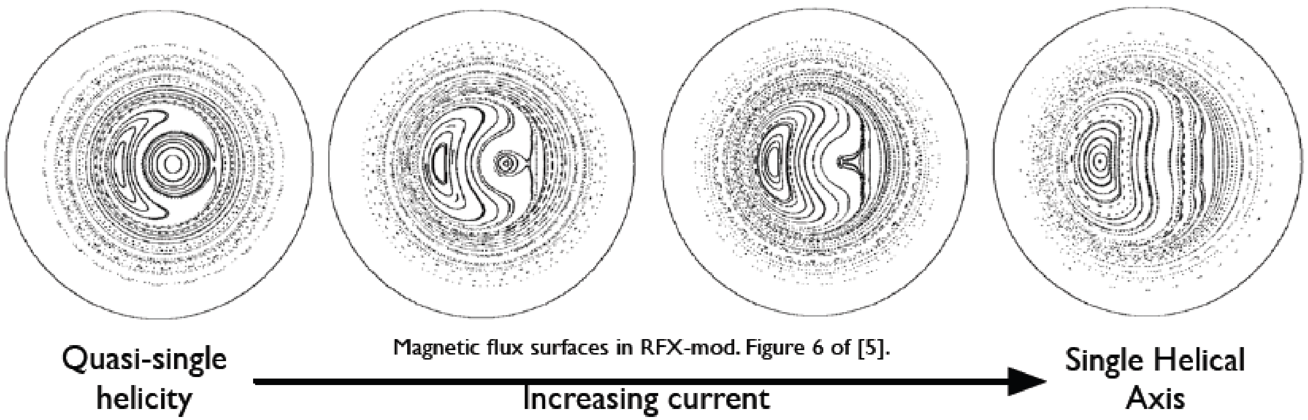
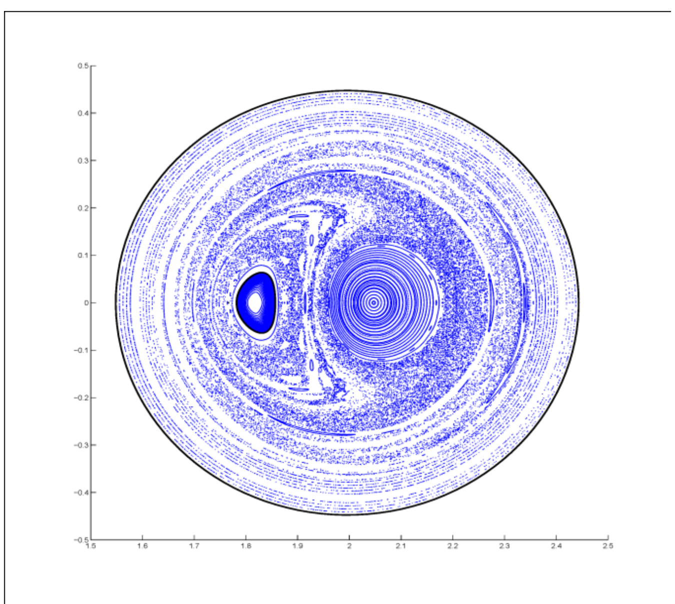



Fig. 6 of P. Martin *et al., Nuclear Fusion* ***49, 104019 (2009)***



- Attempt to describe RFX-mod QSH state by a two-interface minimum energy MRXMHD state
- Calculation of the RFP bifurcated state, with energy lower than the comparable axis-symmetric state
- Both magnetic axes can be reproduced in addition to island structure and significant amounts of chaos

# Summary

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  - Used to infer flux surface geometry with uncertainties
  - Provide model validation (equilibrium and mode structure)
  - Harnessed to infer properties of plasma (e.g. fast particle pressure)
- Anisotropy equilibrium and stability
  - Development of anisotropy into EFIT++
  - Shown impact of anisotropy on equilibrium and plasma stability can be significant
- Multiple Relaxed Region MHD model
  - Introduced a new MHD variational principle to resolve chaotic field regions, islands, flux surfaces in 3D plasmas
  - Demonstrated application of a new code "Stepped Pressure Equilibrium Code." to DIIID RMP coils