Multi-Region, relaXed MHD (MRXMHD)

global, continuous *topological constraint*

Classic MHD: $W = \underbrace{\int_V \left(\frac{p}{\gamma - 1} + \frac{B^2}{2} \right) dv}_{\text{energy}},$ ideal variations $\delta \mathbf{B} = \nabla \times (\boldsymbol{\xi} \times \mathbf{B})$;

1st variation = 0: $\nabla p = \mathbf{j} \times \mathbf{B}$; 2nd variation gives ideal stability;

NO *topological constraint*

Taylor Relaxation: $F = W + \frac{\mu}{2}(H - H_0)$, $H = \underbrace{\int_V (\mathbf{A} \cdot \mathbf{B})\, dv}_{\text{helicity}}$;

1st variation = 0: $\nabla \times \mathbf{B} = \mu \mathbf{B}$; no pressure gradient;

---

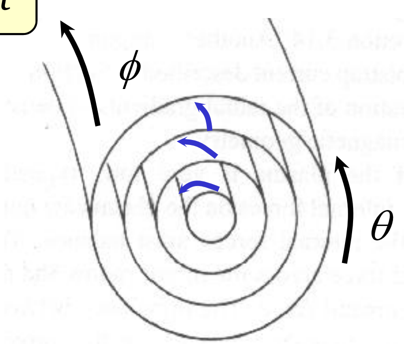
discrete *topological constraint*

MRXMHD: $F = \sum_l [W_l + \frac{\mu_l}{2}(H_l - H_{0,l})]$;

$\rightarrow$ ideal interfaces $\equiv$ flux surfaces forced to coincide with pressure gradients;

1st variation = 0: $\nabla \times \mathbf{B} = \mu \mathbf{B}$, $[[p + B^2/2]] = 0$;

2nd variation gives ideal/resistive stability of partially chaotic equilibria



Existence of Three-Dimensional Toroidal MHD Equilibria with Nonconstant Pressure

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We establish an existence result for the three-dimensional MHD equations

$$\begin{aligned}(\nabla \times \mathbf{B}) \times \mathbf{B} &= \nabla p \\ \nabla \cdot \mathbf{B} &= 0 \\ \mathbf{B} \cdot n|_{\partial T} &= 0\end{aligned}$$

with $p \neq$ const in tori T *without symmetry*. More precisely, our theorems insure the existence of sharp boundary solutions for tori whose departure from axisymmetry is sufficiently small; they allow for solutions to be constructed with an arbitrary number of pressure jumps.

Communications on Pure and Applied Mathematics, Vol. XLIX, 717–764 (1996)

→ this was a strong motivation for pursuing the stepped-pressure equilibrium model

→ how large the "sufficiently small" departure from axisymmetry can be needs to be explored numerically

Stepped Pressure Equilibrium Code, SPEC

[Plasma Physics and Controlled Fusion, 54:014005, 2012]

The vector-potential is discretized using mixed Fourier & finite-elements

geometry $R_l = \sum_{m,n} R_{l,m,n} \cos(m\vartheta - n\zeta), Z_l = \sum_{m,n} Z_{l,m,n} \sin(m\vartheta - n\zeta)$

$$\mathbf{A} = A_\vartheta(s,\vartheta,\zeta)\nabla\vartheta + A_\zeta(s,\vartheta,\zeta)\nabla\zeta$$

$$A_\vartheta = \sum_{m,n} a_\vartheta(s)\cos(m\vartheta - n\zeta)$$

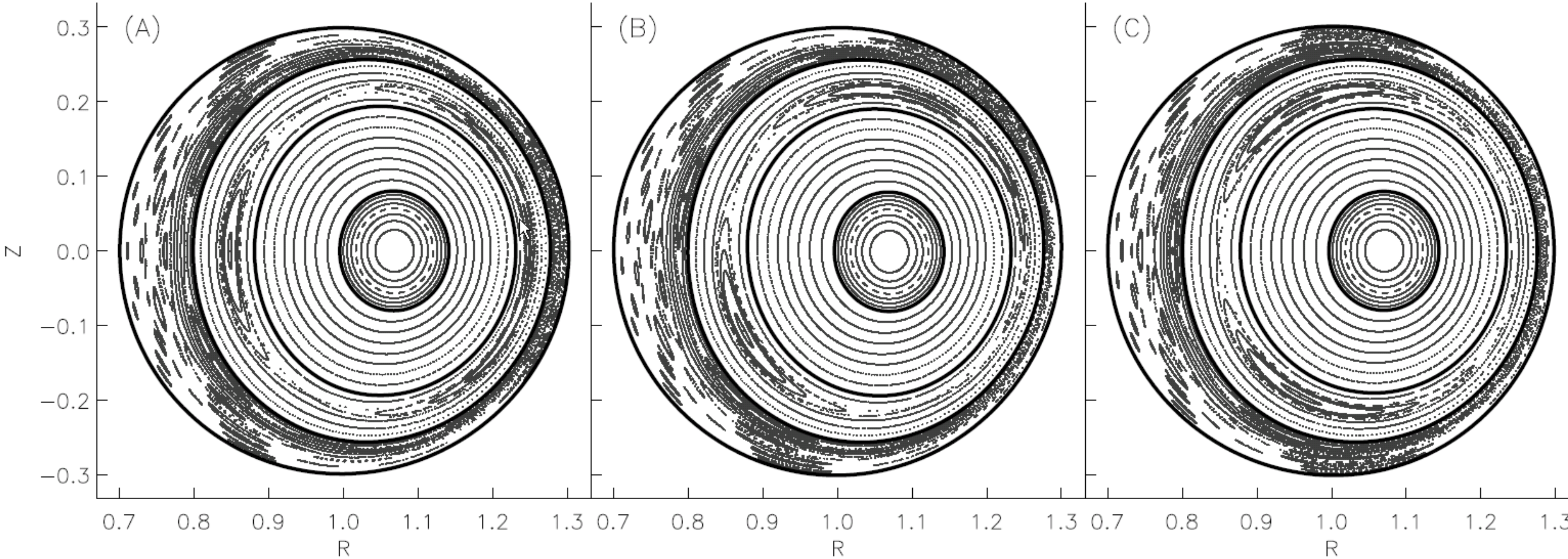
$$a_\vartheta(s) = \sum_i a_{\vartheta,i}(s)\varphi(s)$$

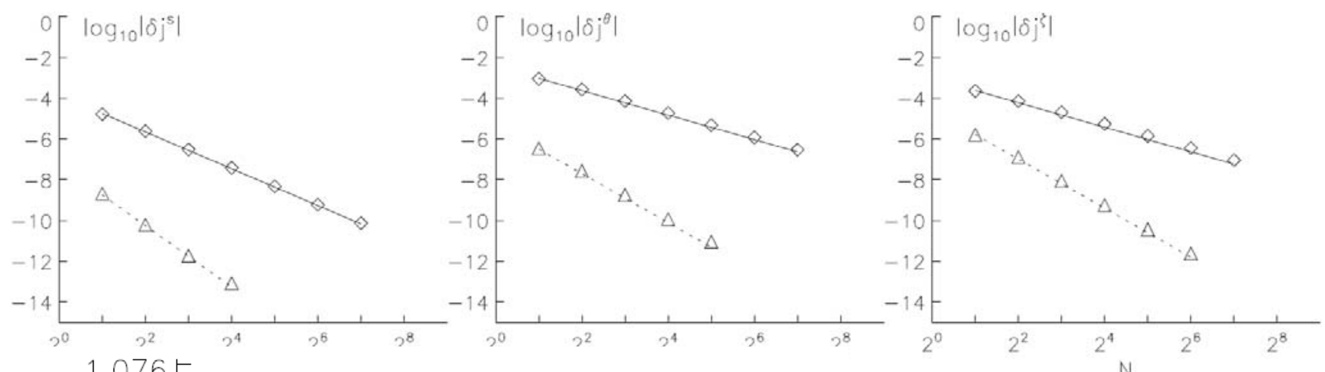
and inserted into constrained-energy functional $F = \sum_{l=1}^{N}\left(W_l - \mu_l H_l / 2\right)$

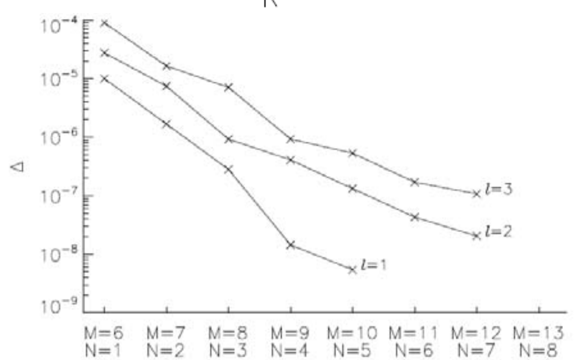
* derivatives w.r.t. vector-potential $\rightarrow$ Beltrami field $\nabla\times\mathbf{B} = \mu\mathbf{B}$

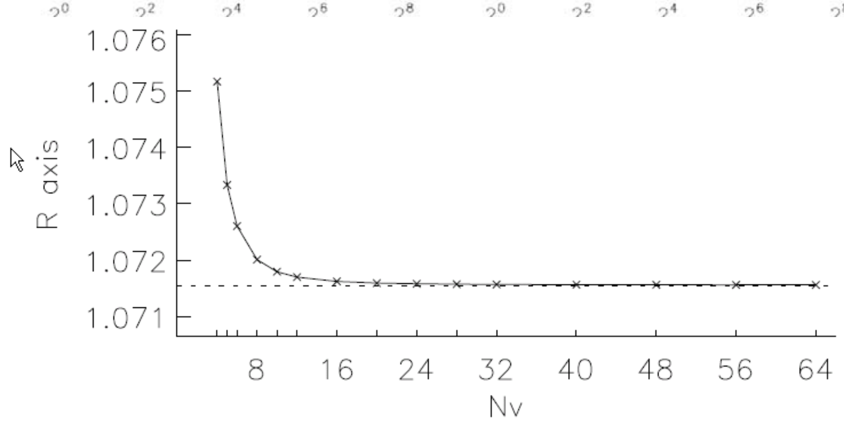
Force balance solved using multi-dimensional Newton method.

* interface geometry is adjusted to satisfy force $\mathbf{F}[\boldsymbol{\xi}] \equiv \left\{[[p + B^2/2]]_{m,n}\right\} = 0$







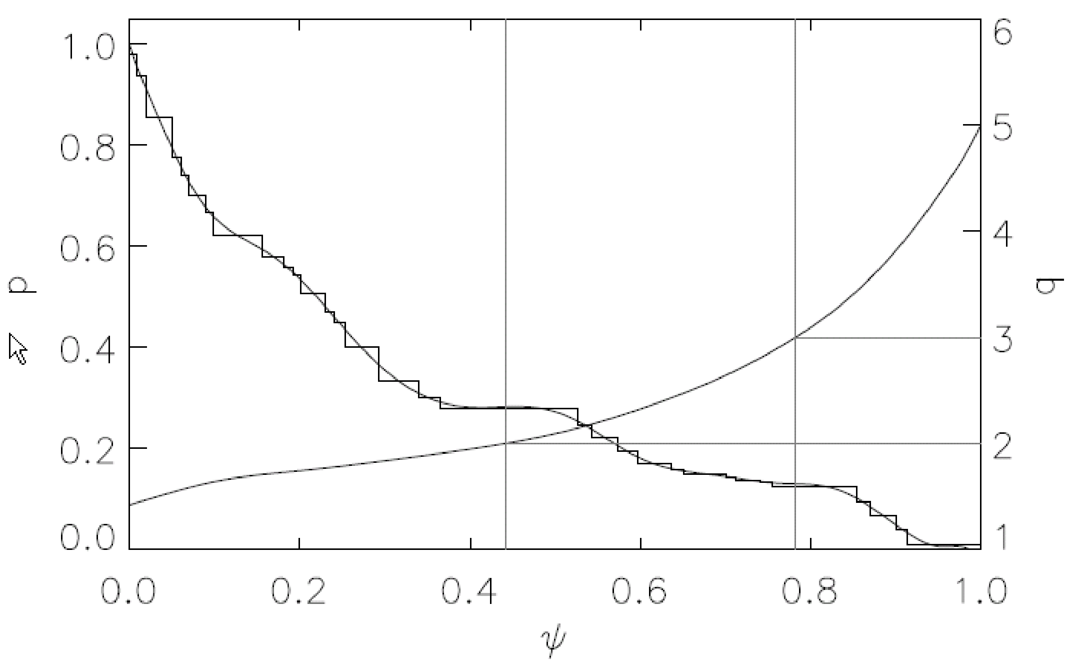


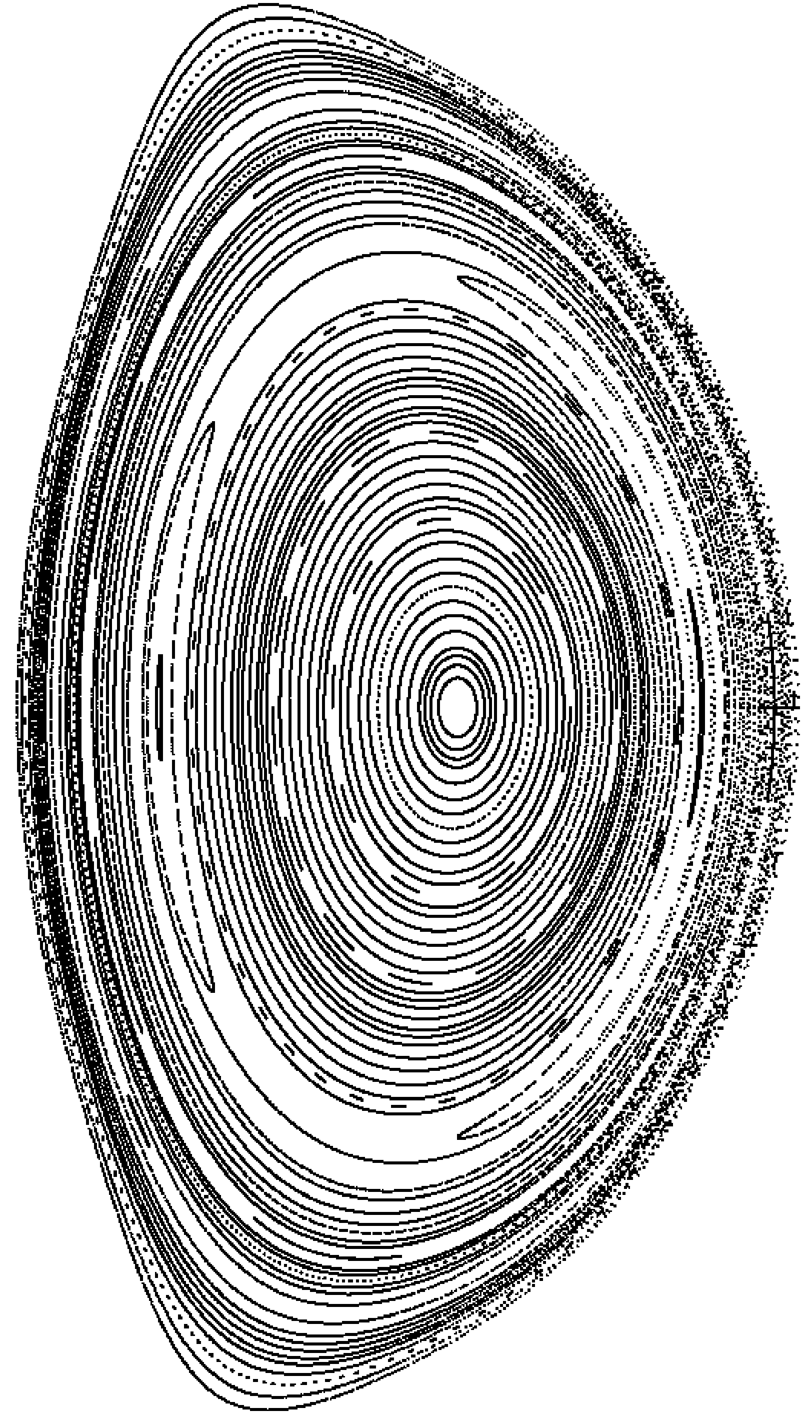
converges as it should . . . .

1) *Radial resolution convergence,*
2) *Fourier resolution convergence,*
3) *Converges to Grad-Shafranov for axisymmetric case*

DIIID Fixed Boundary Equilibrium

Boundary, pressure, q-profile from equilibrium reconstruction
(STELLOPT, Lazerson)





DIIID Free Boundary Equilibrium

(work in progress)

plasma supported by vacuum provided by coils;

generalized boundary condition at computational wall allows for unstable manifold;