A minimally constrained model of self-organized helical states in reversed-field pinches

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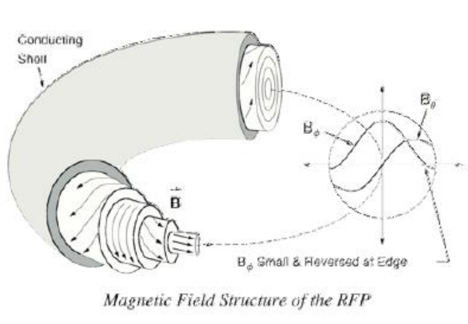
A self-organized helical state has been observed in RFP experiments



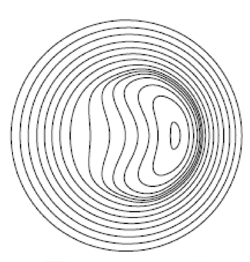
Poor confinement observed in "traditional" *axisymmetric* states

Better confinement now observed when *helical* state forms in RFX-mod

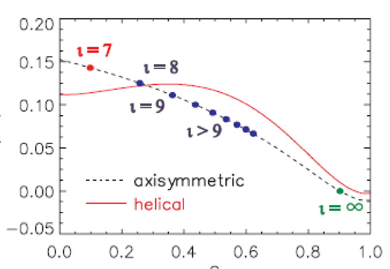
This structure occurs even for an axisymmetric plasma boundary, i.e. it is *self-organized*.

Ideal MHD can model the Single-Helical Axis state

Experimental Poincaré plot[1]



Theoretical reconstruction using VMEC[2]

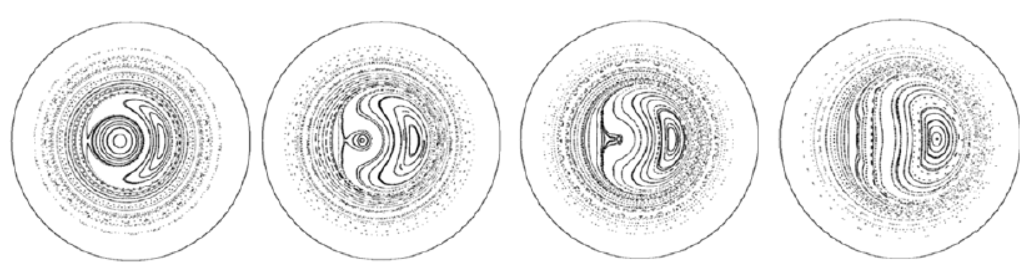


Theoretical safety factor profile[2]

…but the safety factor profile must be carefully chosen

[1] P. Martin et al., *Nuclear Fusion* **49**, 104019 (2009).
[2] D. Terranova et al., *PPCF* **52**, 124023 (2010).

Helical states with non-trivial topology are also observed



Double-Helical Axis state → Single Helical Axis state

Ideal MHD (with assumed nested flux surfaces) *cannot* model the Double-Helical Axis state.

[1] P. Martin et al., *Nuclear Fusion* **49**, 104019 (2009).

We seek a minimally constrained solution for all RFX mod states

Taylor's theory is a good description of axisymmetric Reversed Field Pinches

Taylor's theory: Plasma quantities are only conserved *globally*

Ideal MHD: Plasma quantities conserved on *every flux surface*

Goal: *minimal* description of helical states in RFP

Taylor's theory ✗	Multiple Region Relaxed MHD ?	Ideal MHD ✓

Fewer constraints ← → More constraints

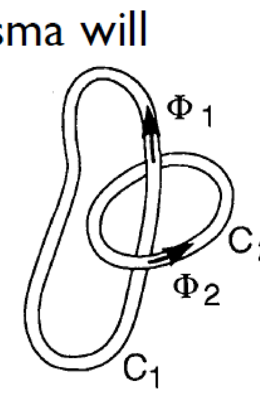
Taylor's theory is that a turbulent plasma will minimize its energy…

$$E = \int \left(\frac{p}{\gamma - 1} + \frac{1}{2} B^2 \right) dV$$

…with conserved magnetic helicity

$$H = \int \mathbf{A} \cdot \mathbf{B} \, dV \quad \text{(+ gauge terms)}$$

…and conserved enclosed fluxes



$$H = \Phi_1 \Phi_2$$

Motivation: with small resistivity, both energy and helicity will decay

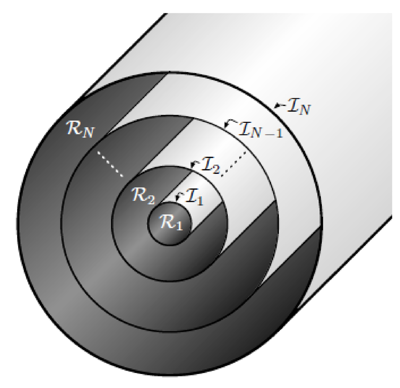
$$\dot{H} = \eta \int \mathbf{J} \cdot \mathbf{B} \, dV \sim \eta \sum_k k^1 \mathbf{B}_k^2$$

$$\dot{E} = \eta \int \mathbf{J} \cdot \mathbf{J} \, dV \sim \eta \sum_k k^2 \mathbf{B}_k^2$$

… but energy more quickly (for short length-scale turbulence)

Multiple-Region Relaxed MHD (MRXMHD) extends Taylor Relaxation

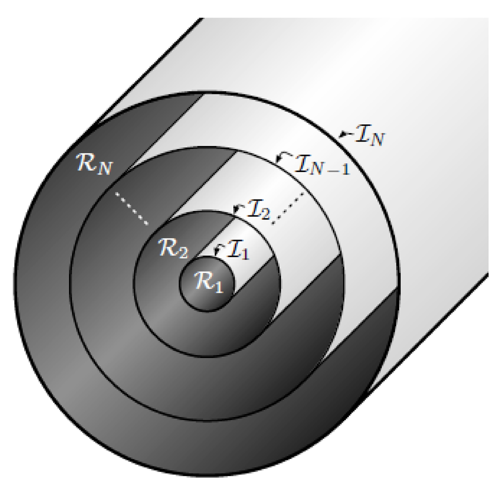
- Relaxed regions $\mathcal{R}_i$ separated by
- nested, ideal, toroidal barriers $\mathcal{I}_i$, which
- independently undergo Taylor relaxation.
- Magnetic islands and chaos are allowed between the toroidal current sheets
- Each plasma region has constant pressure,

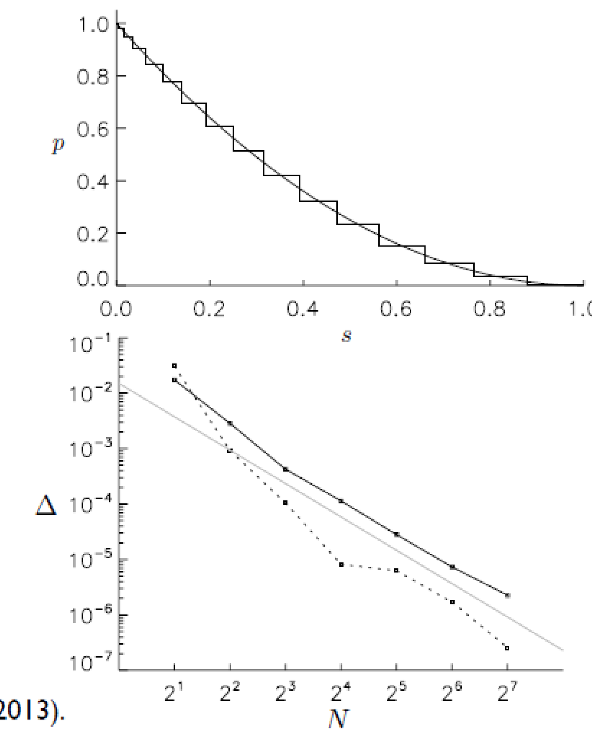


$$W_i = \int_{\mathcal{R}_i} \left(\frac{B_i^2}{2\mu_0} + \frac{P_i}{\gamma - 1} \right) d\tau^3$$

$\mathcal{R}_i$:	$\nabla \times \mathbf{B} = \mu_i \mathbf{B}$	P_i = constant
$\mathcal{I}_i$:	$[[P_i + B^2/(2\mu_0)]] = 0$	$\mathbf{B} \cdot \mathbf{n} = 0$
$\mathcal{V}$:	$\nabla \times \mathbf{B} = 0$	
$\mathcal{W}$:	$\mathbf{B} \cdot \mathbf{n} = 0$	

Multiple-Region Relaxed MHD (MRXMHD) approaches ideal MHD as $N \to \infty$





[1] G. Dennis et al., *Phys. Plasmas* **20**, 032509 (2013).

Goal: *minimal* description of helical states in RFP

Taylor's theory ✗	Multiple Region Relaxed MHD ?	Ideal MHD ✓

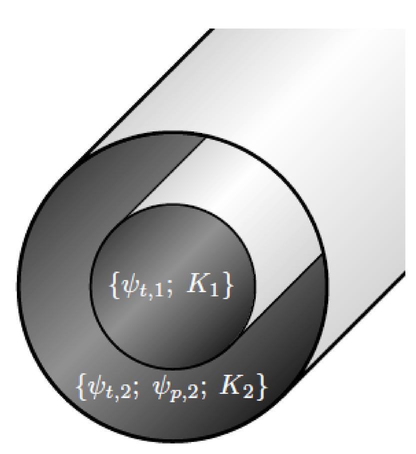
Fewer constraints ← → More constraints

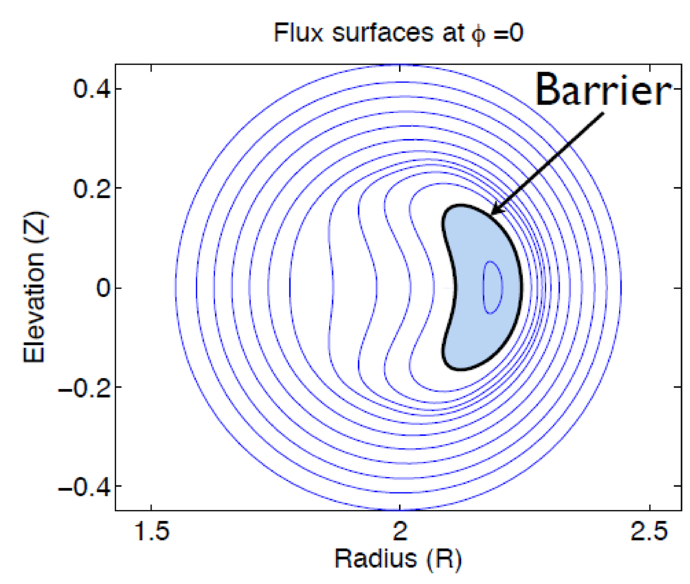
$N = 1$ is Taylor's theory ✗

$N = 2$?

$N = \infty$ is Ideal MHD ✓

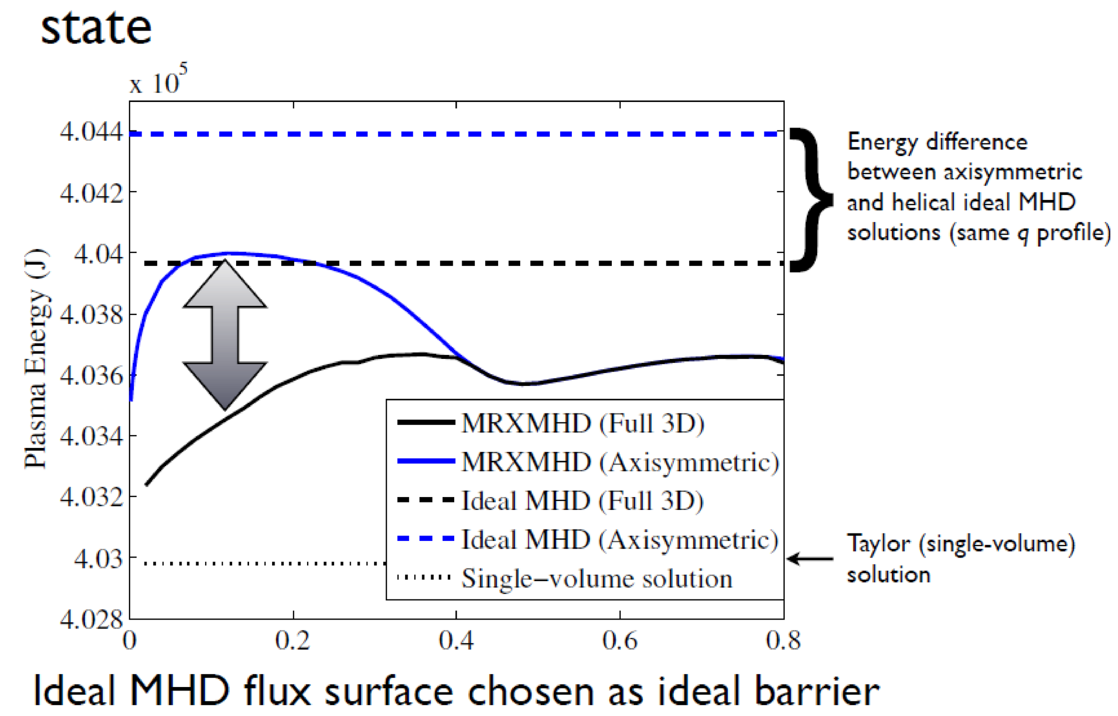
A two-volume MRXMHD model (without pressure) is constrained by 5 parameters





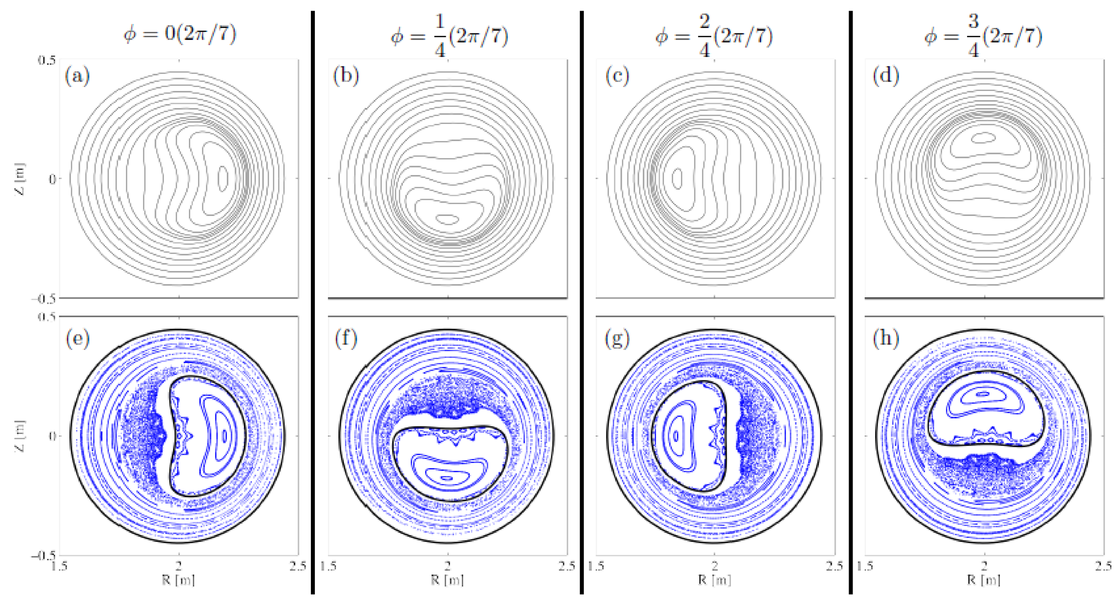
We take an ideal MHD solution and reduce the constraints i.e. examine 1D line in 5D parameter space

The plasma equilibrium is a minimum energy state



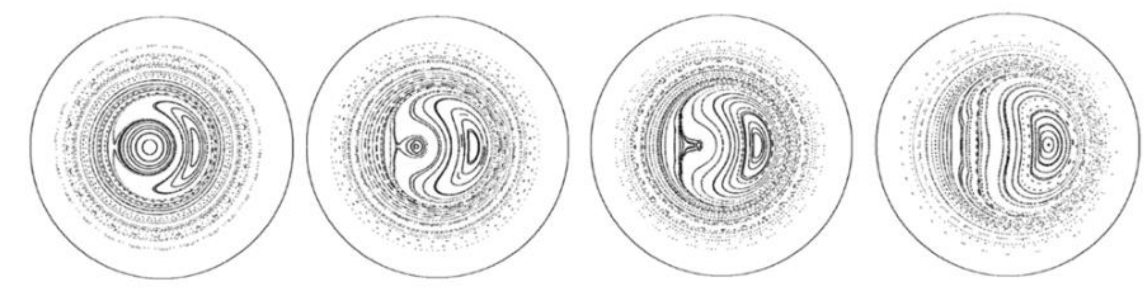
Ideal MHD flux surface chosen as ideal barrier

Comparison of VMEC and SPEC RFX-mod equilibria



- The quasi-single helicity state is a stable helical state in RFP: becomes purer as current is increase

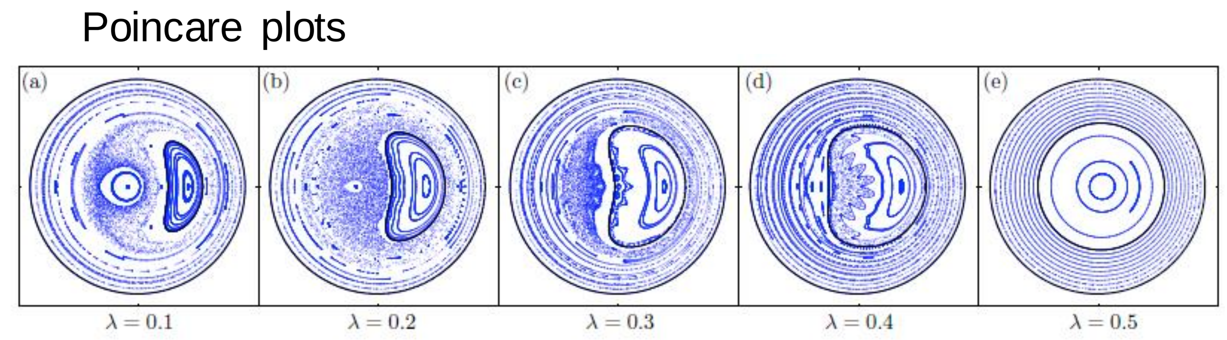
[Fig. 6 of P. Martin et al., Nuclear Fusion 49, 104019 (2009)]

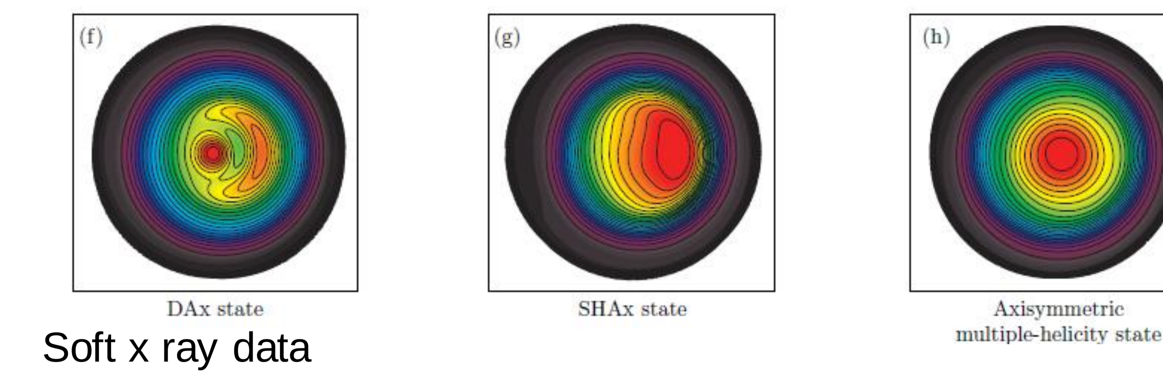


Double-Helical Axis state → Single Helical Axis state

Increasing current

Poincare plots





Soft x ray data

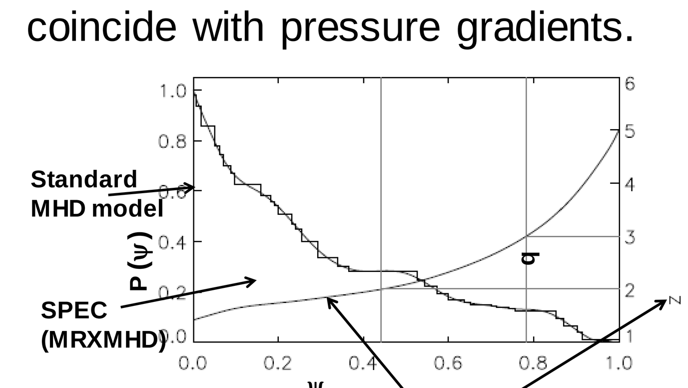
Conclusions

- MRXMHD gives a good qualitative explanation of the
- high-confinement state in Reversed Field Pinches
- With a *minimal* model we reproduced the helical pitch and structure of the Quasi-Single Helicity state in RFP
- MRXMHD is a well-formulated model that lies between Taylor's theory and ideal MHD
- RFP bifurcated state has lower energy (preferred) than comparable axis-symmetric state
- …next steps, error fields, rmp coils, sawteeth

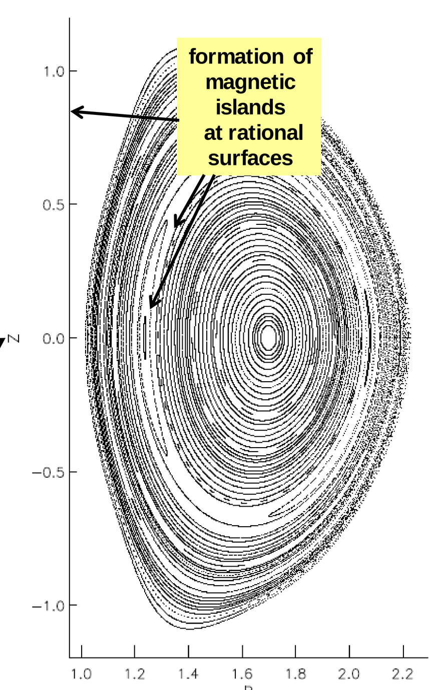
Example: DIIID with n=3 applied error field

[S. R. Hudson et al Phys. Plas. 19, 112502 (2012)]

- 3D boundary, p, q-profile from flux-surfaces-model
- Irrational interfaces chosen to coincide with pressure gradients.



- Islands are now resolved



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