Minimally constrained model of self-organised helical states in RFX

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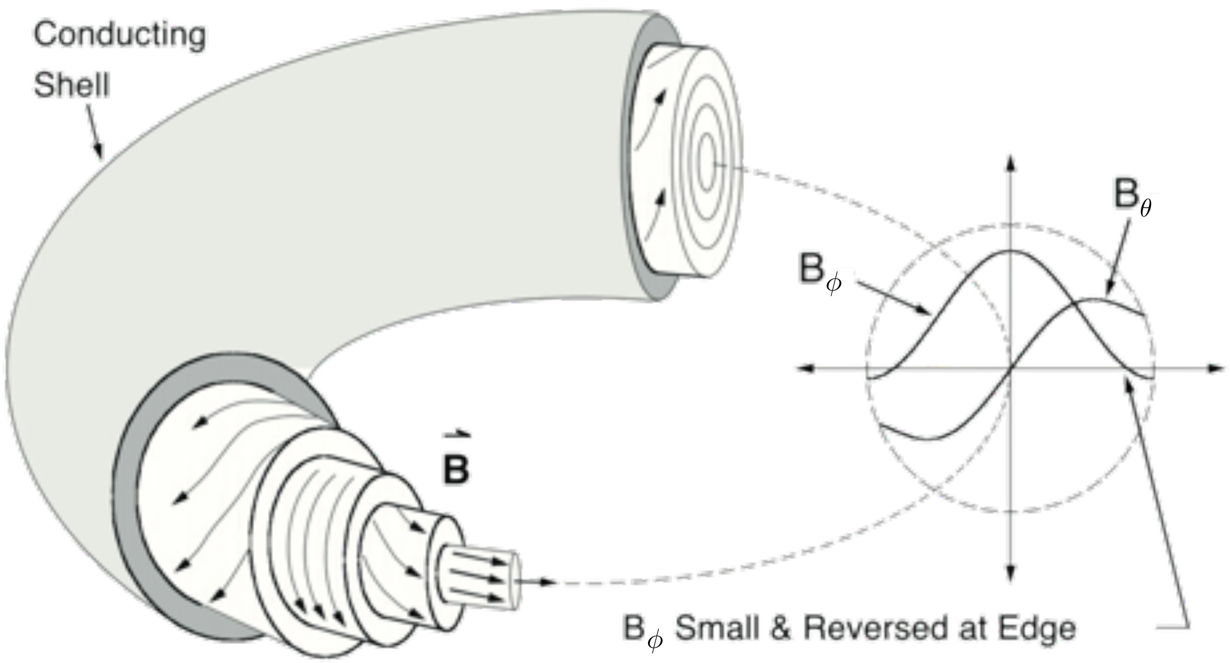
Australian National University

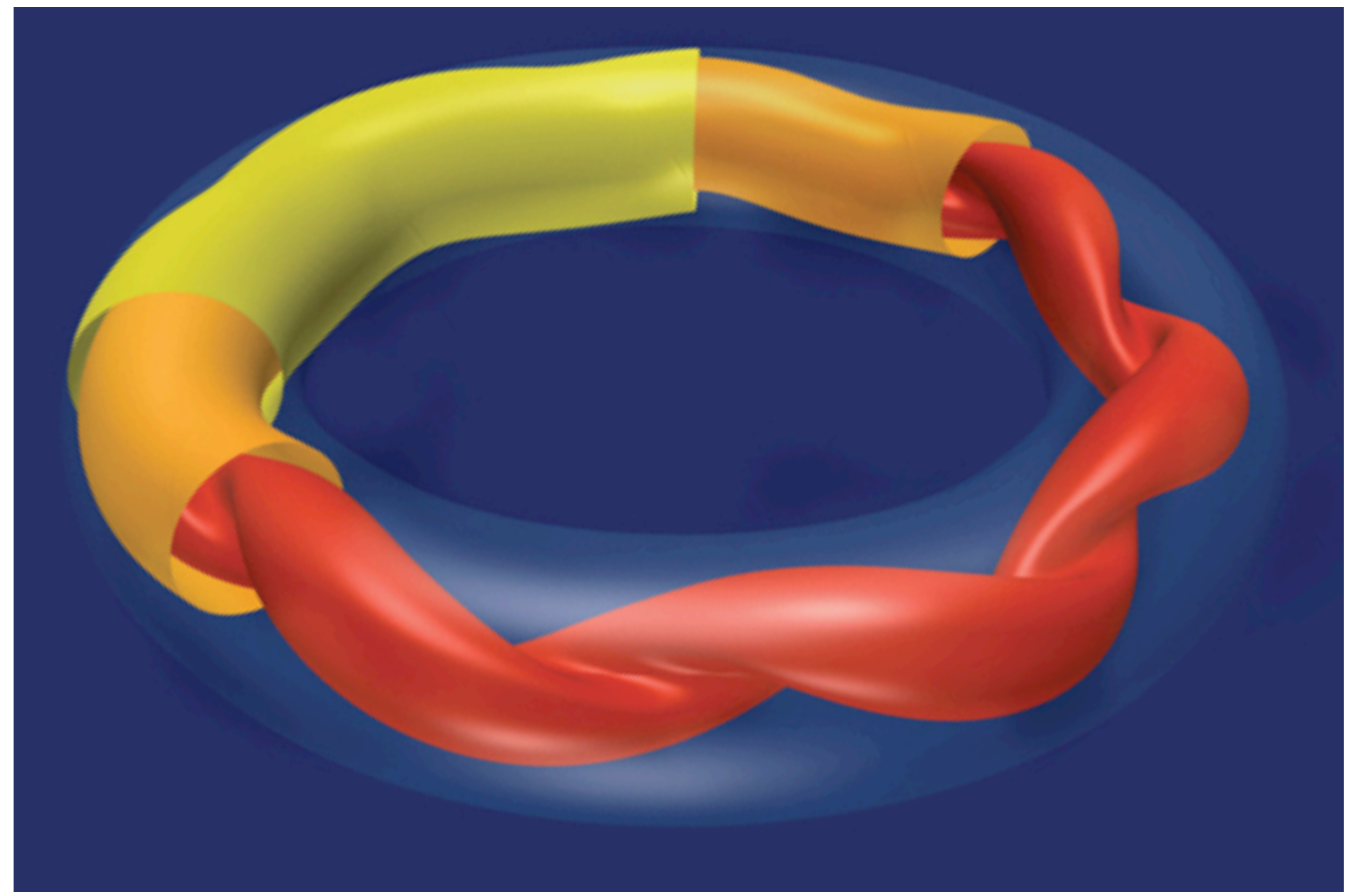
CONSORZIO RFX

PPPL PRINCETON PLASMA PHYSICS LABORATORY

A self-organized helical state has been observed in RFP experiments



Magnetic Field Structure of the RFP



Limited confinement observed in "traditional" *axisymmetric* RFP states

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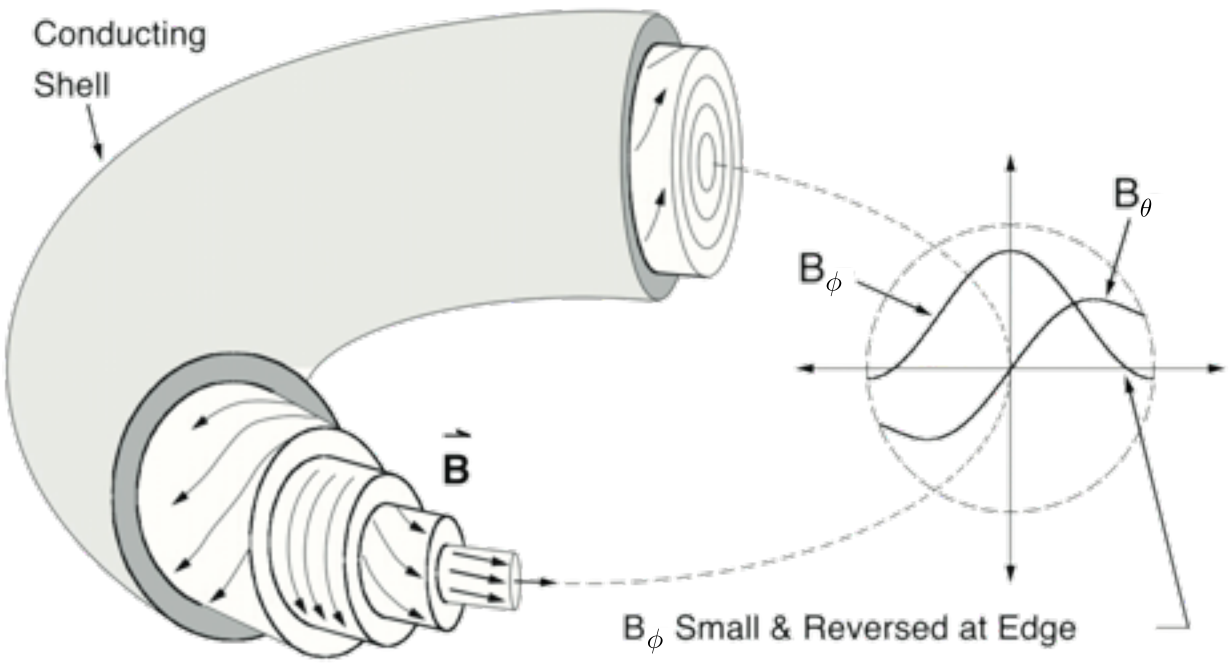

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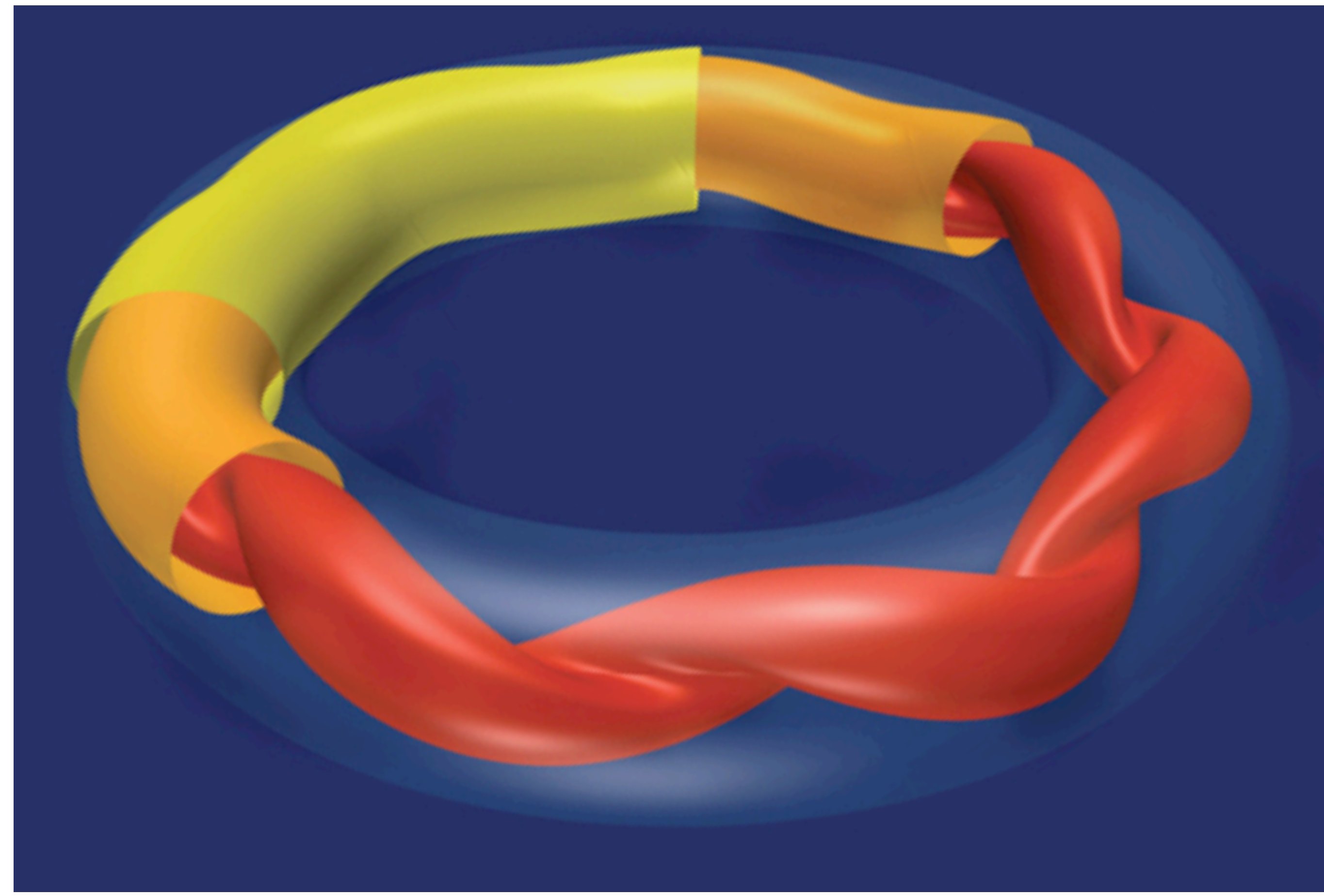
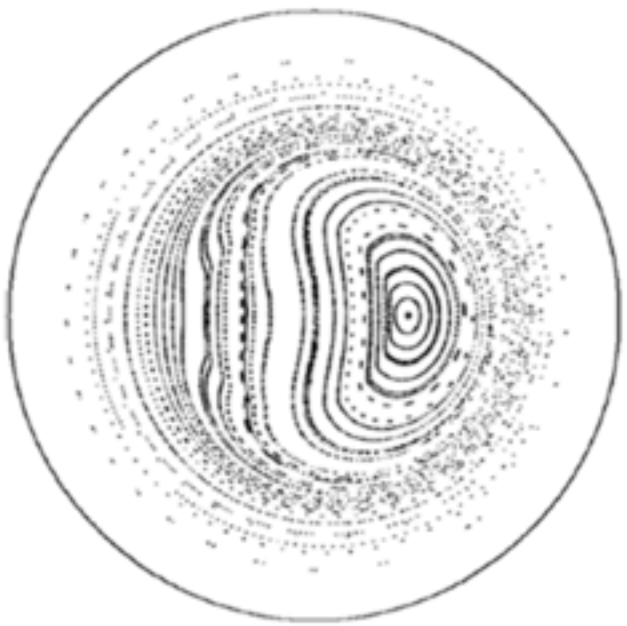


Limited confinement observed in “traditional” *axisymmetric* RFP states

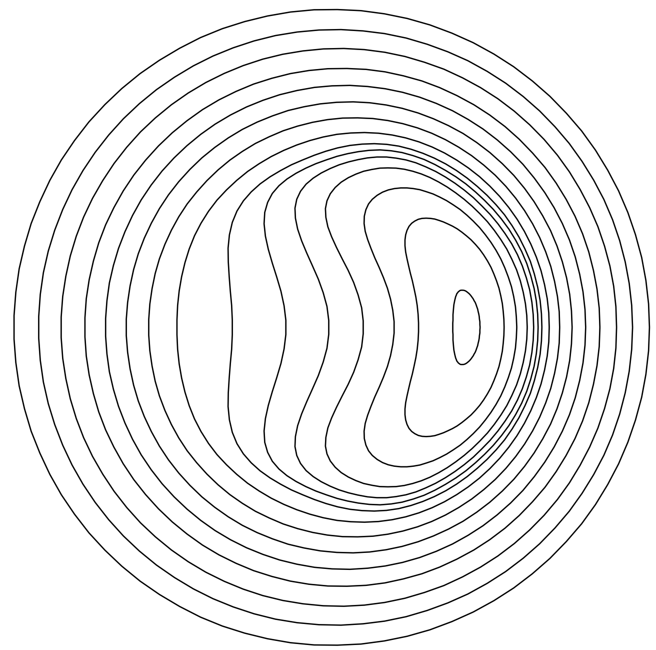
Better confinement now observed when *helical* state forms in RFX-mod

This structure occurs even for an axisymmetric plasma boundary, i.e. it is *self-organized.*

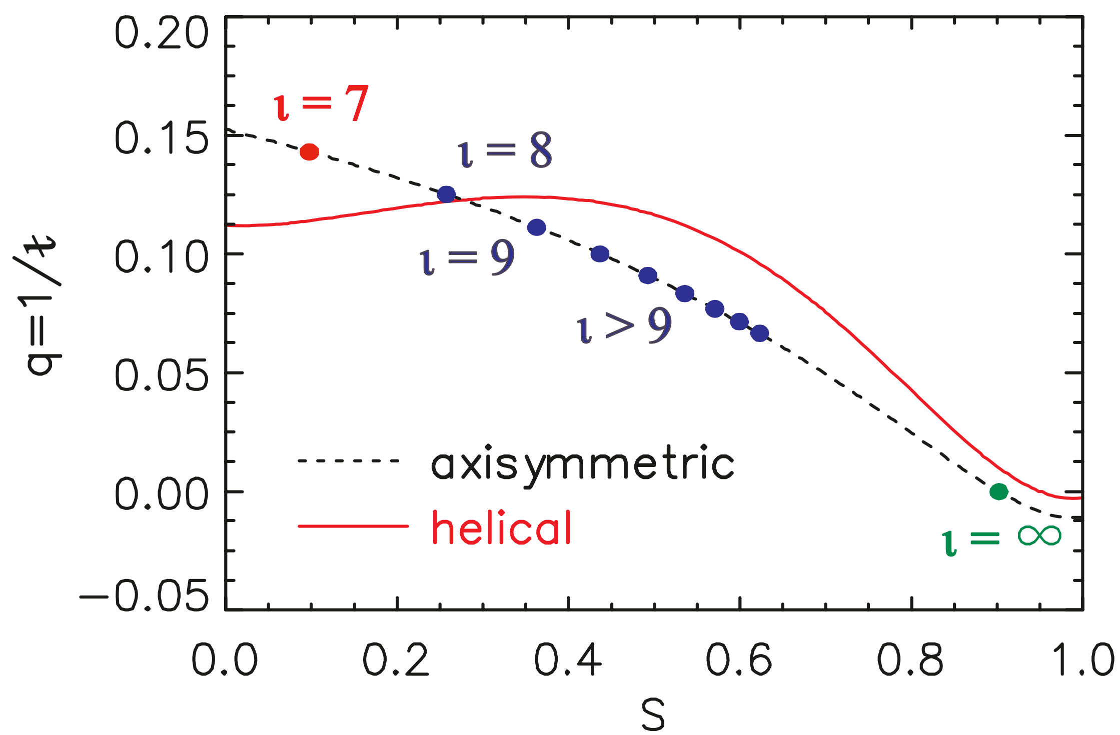
Ideal MHD can model the Single-Helical Axis state



Reconstructed Poincaré plot[1]



Theoretical reconstruction using VMEC[2]



Theoretical safety factor profile[2]

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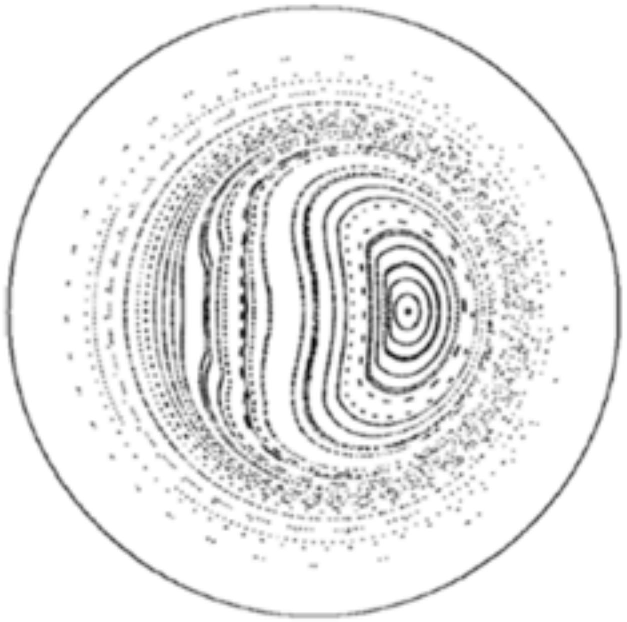

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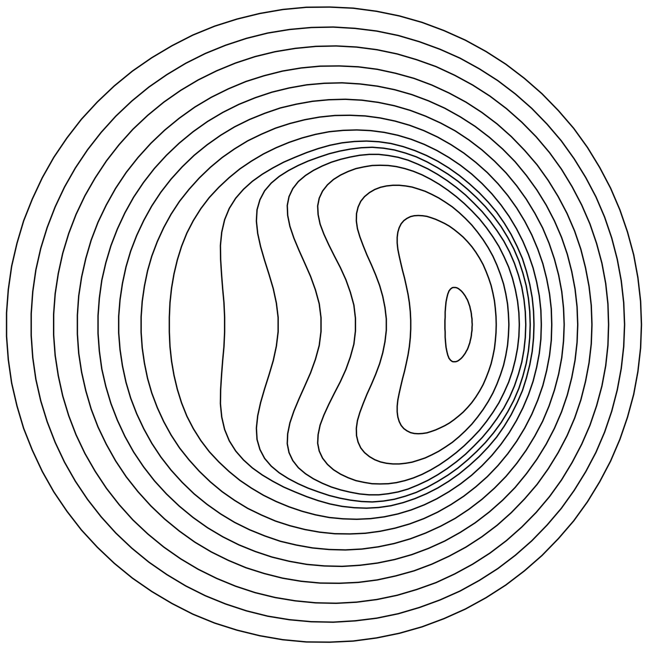

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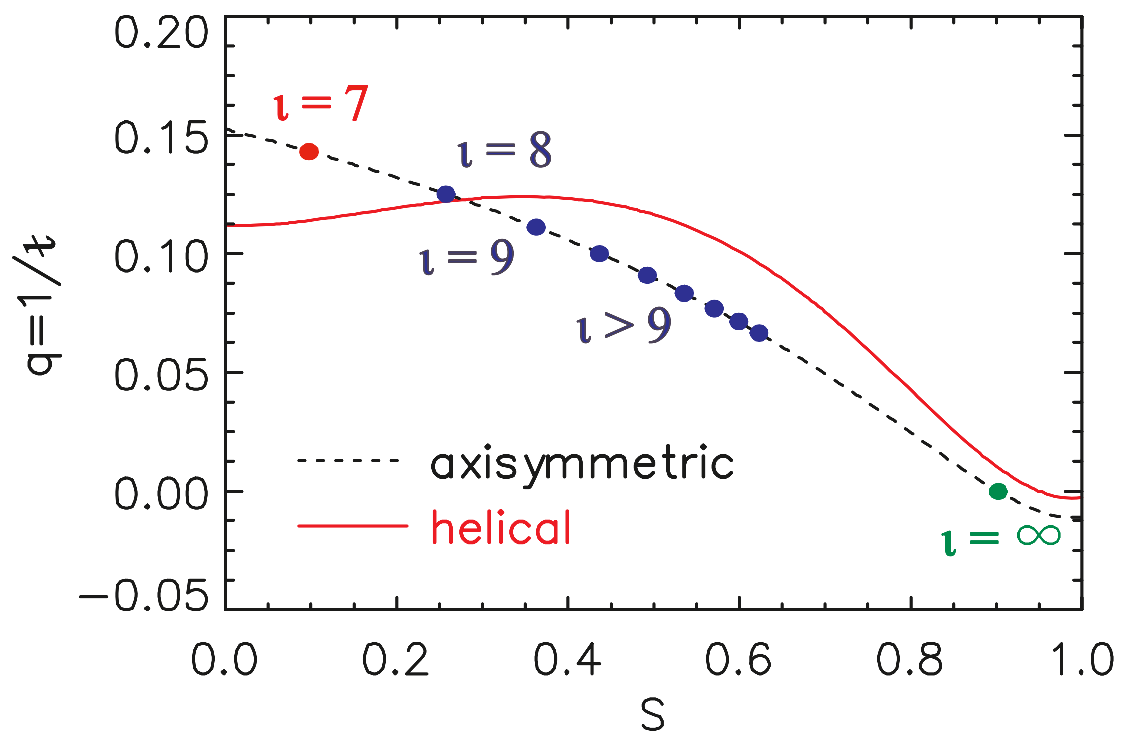

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…but the safety factor profile must be carefully chosen

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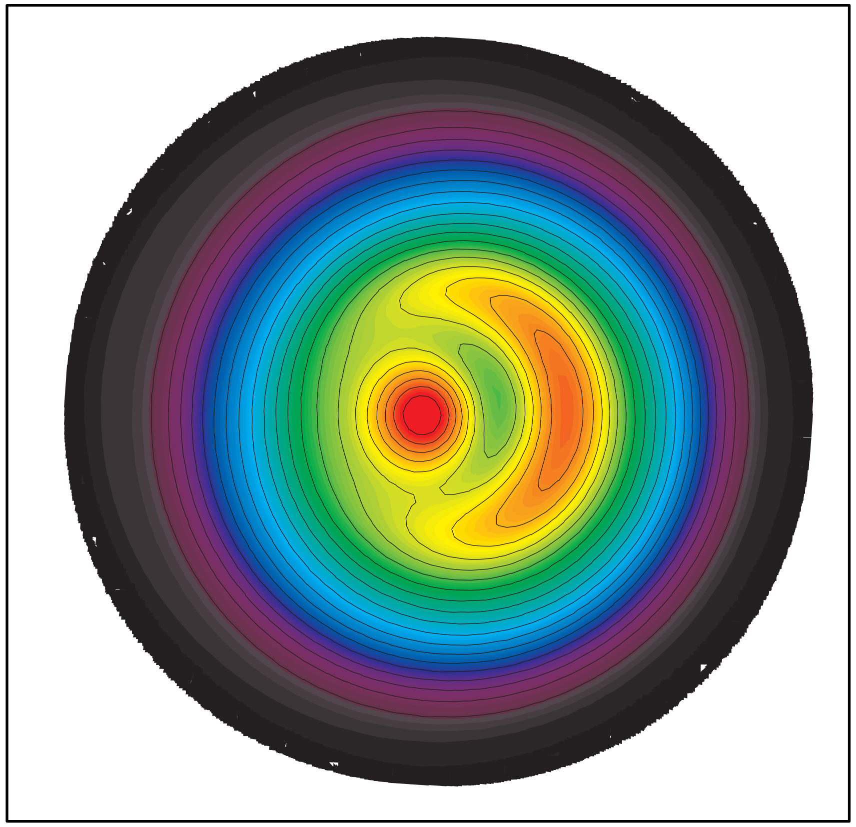
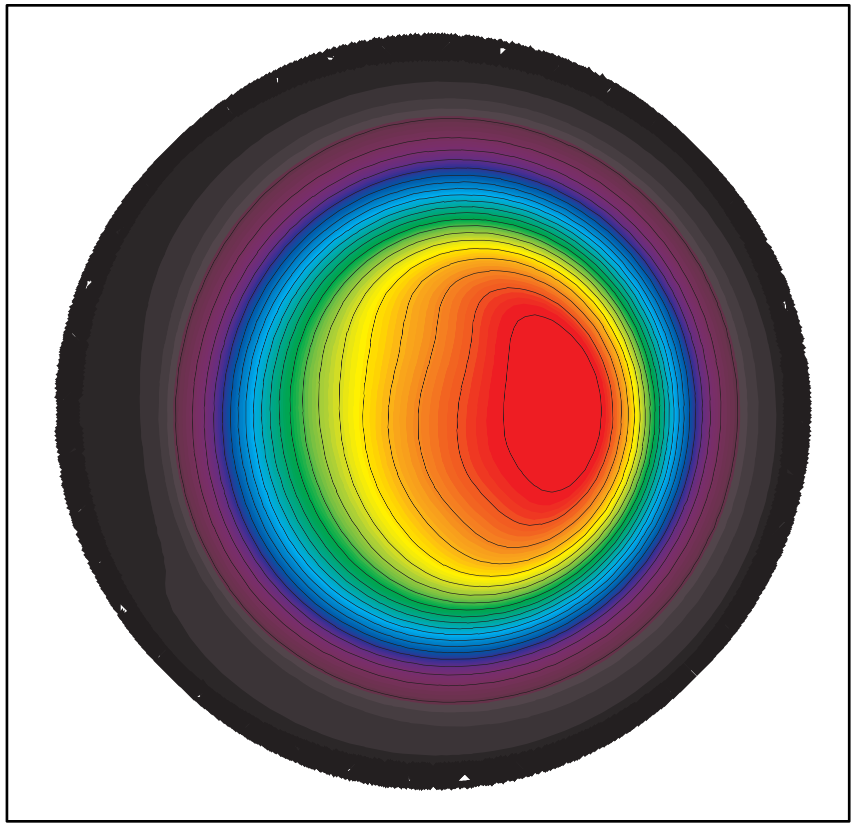
Helical states with non-trivial topology are also observed

Tomographic inversions of soft x-ray imaging



Double-Helical Axis state



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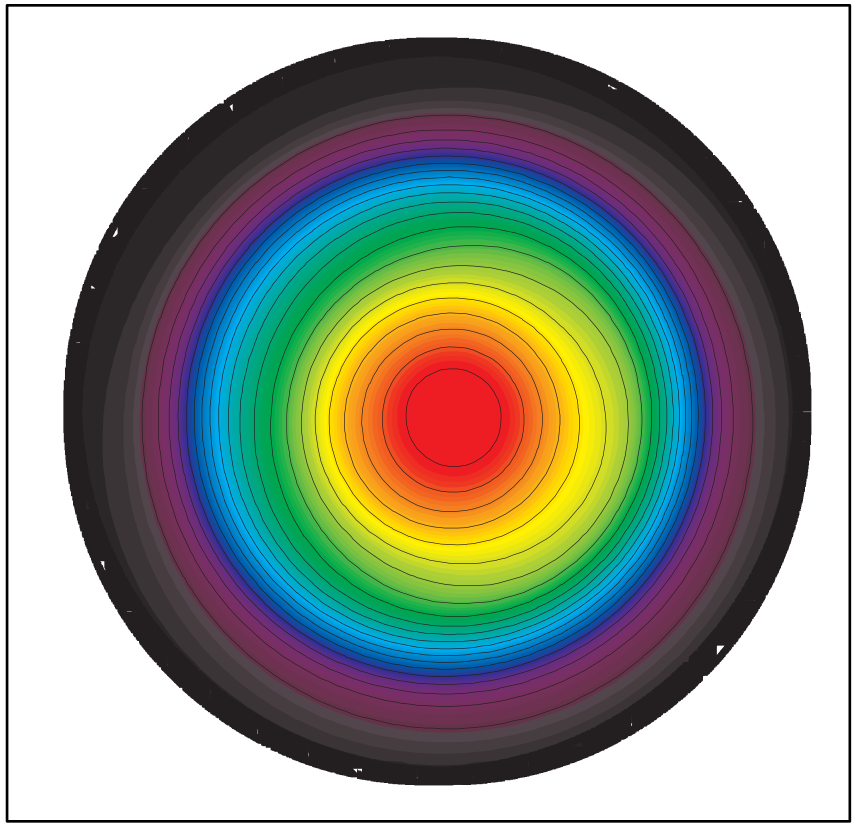

Axisymmetric multiple-helicity state

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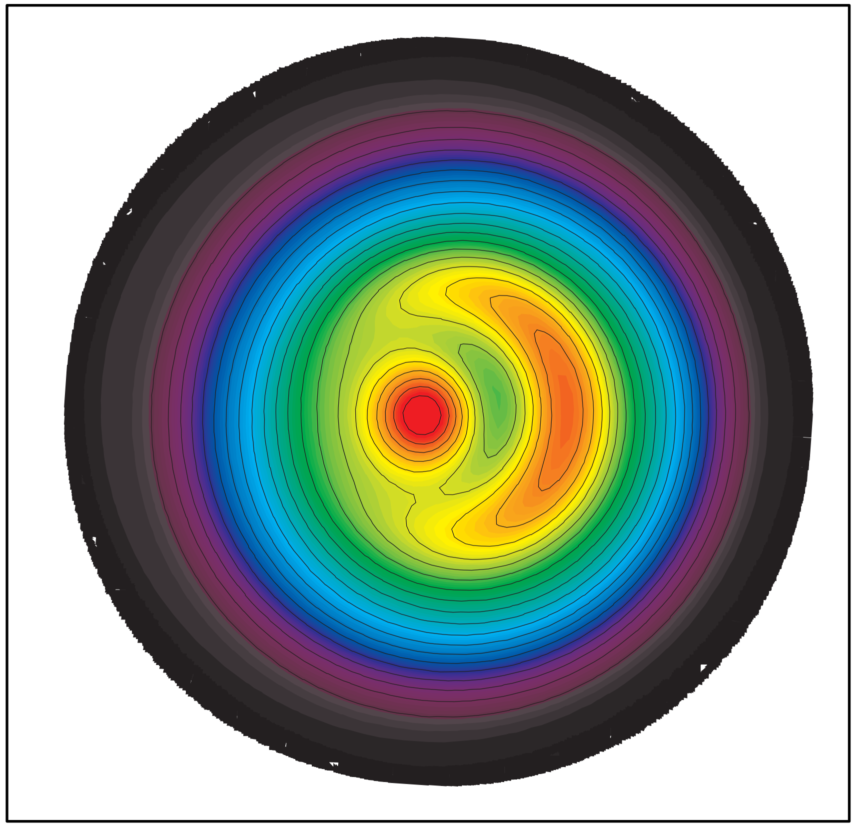

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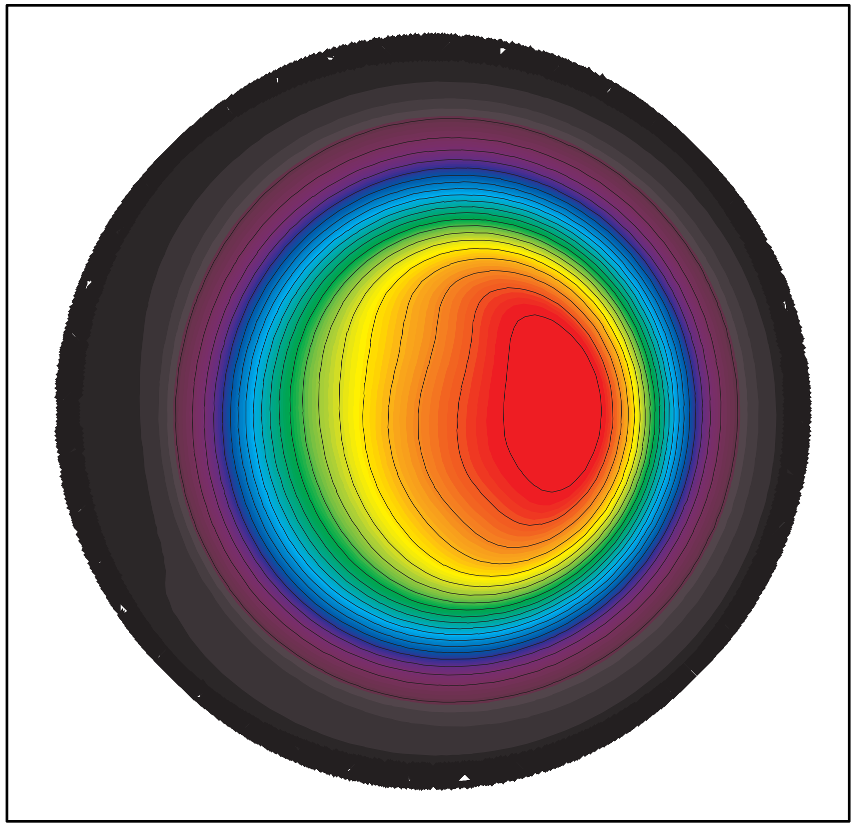

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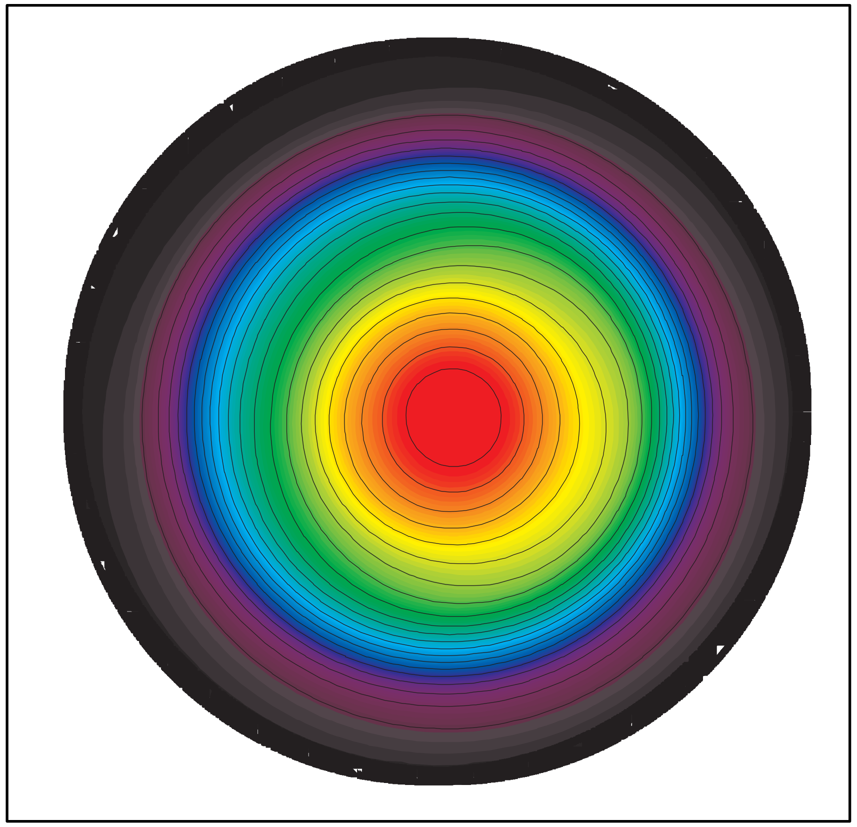

Axisymmetric multiple-helicity state

Ideal MHD (with assumed nested flux surfaces) *cannot* model the Double-Helical Axis state.

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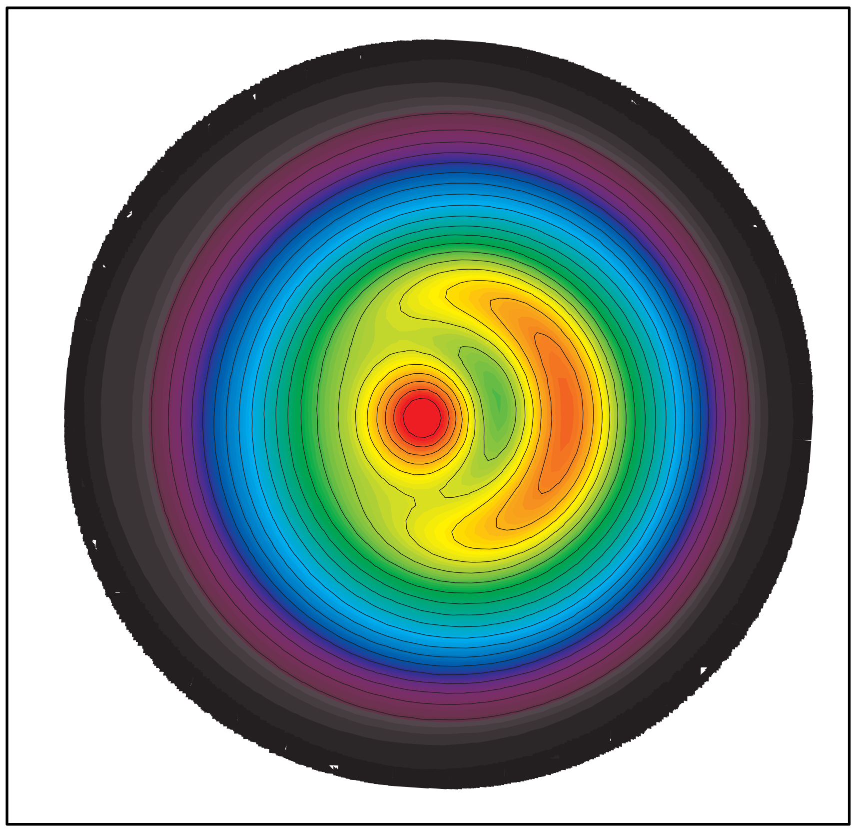

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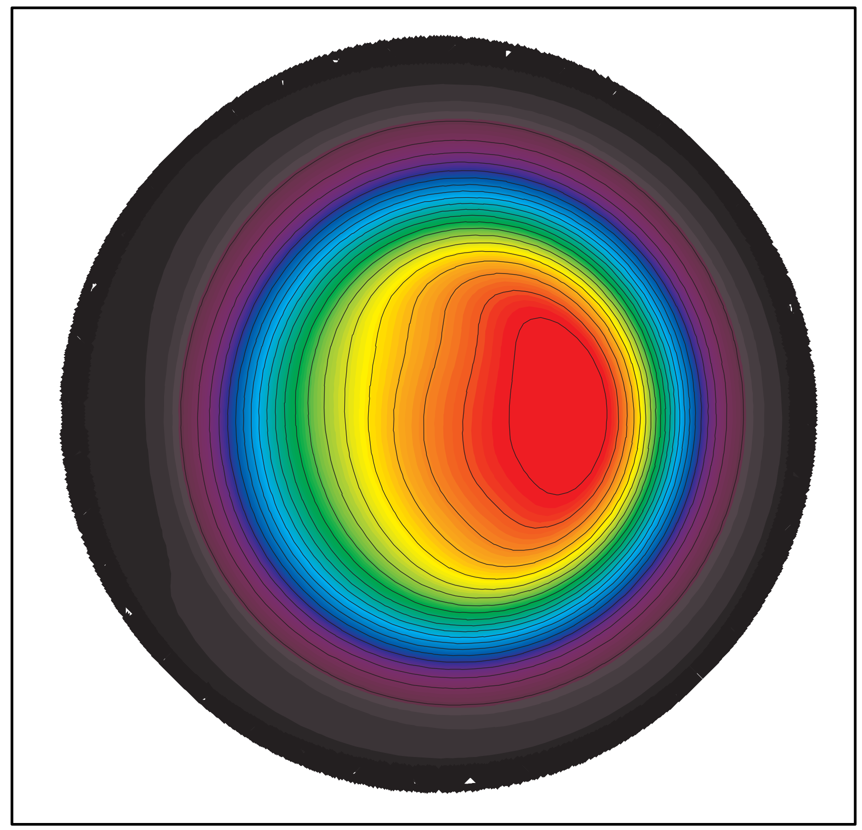

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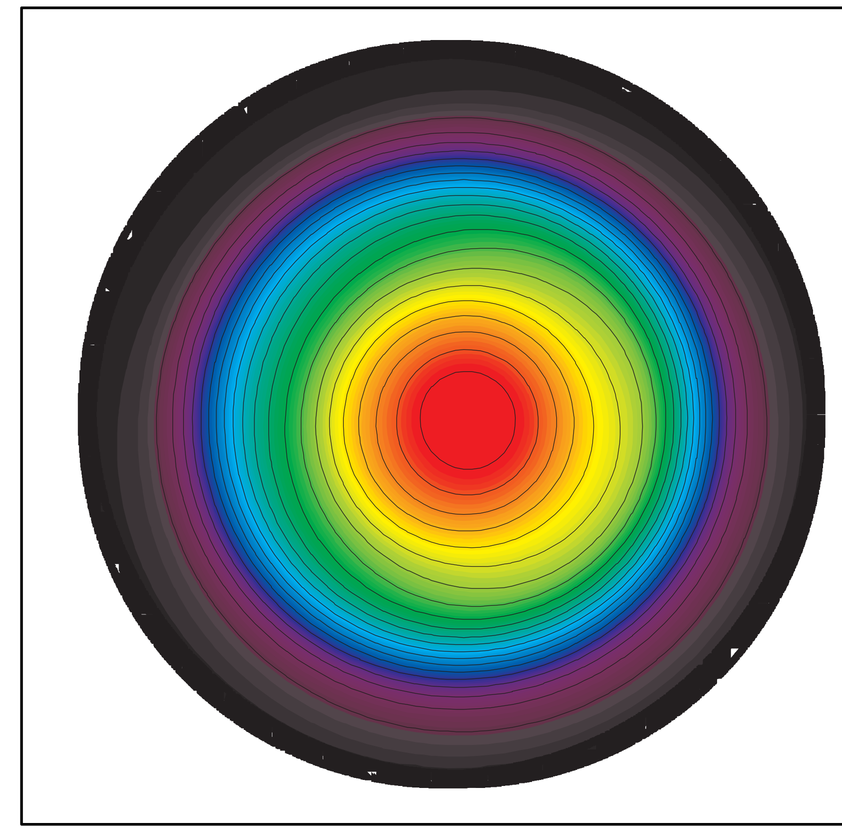

Axisymmetric multiple-helicity state

We seek a *minimally constrained* model for all RFX helical states

Taylor's theory is a good description of axisymmetric Reversed Field Pinches

Taylor's theory: Plasma quantities are only conserved *globally*

Ideal MHD: Plasma quantities conserved on *every flux surface*

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Fewer constraints

More constraints

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Multiregion Relaxed MHD

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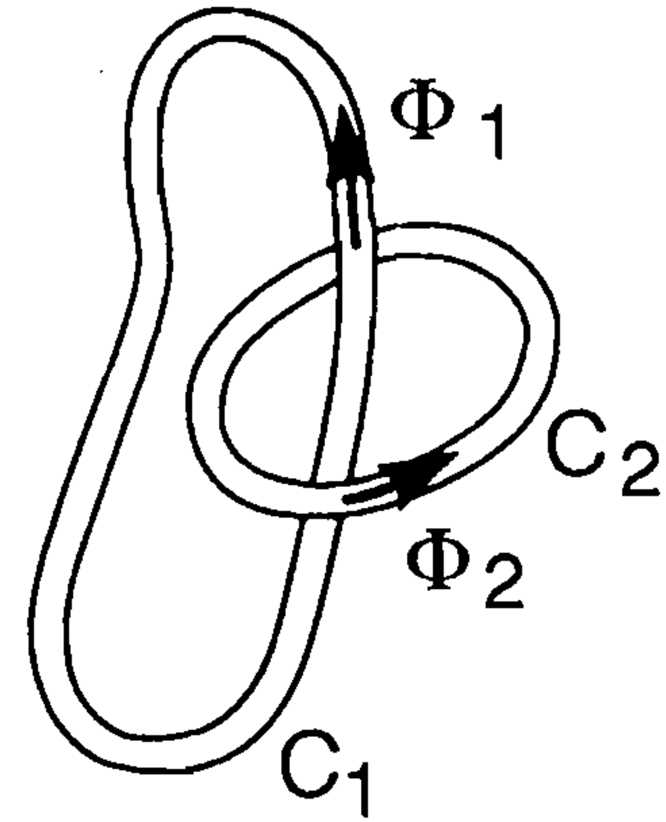

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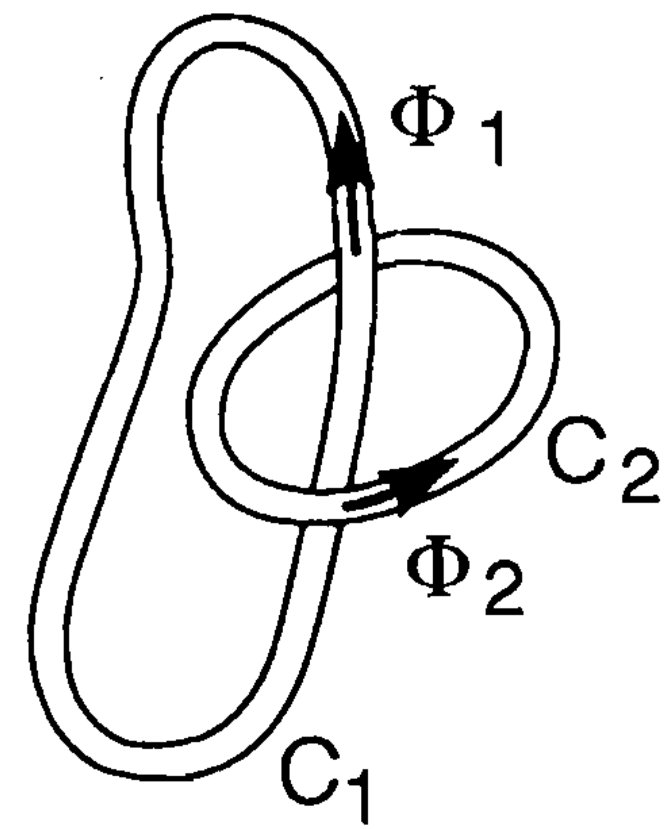

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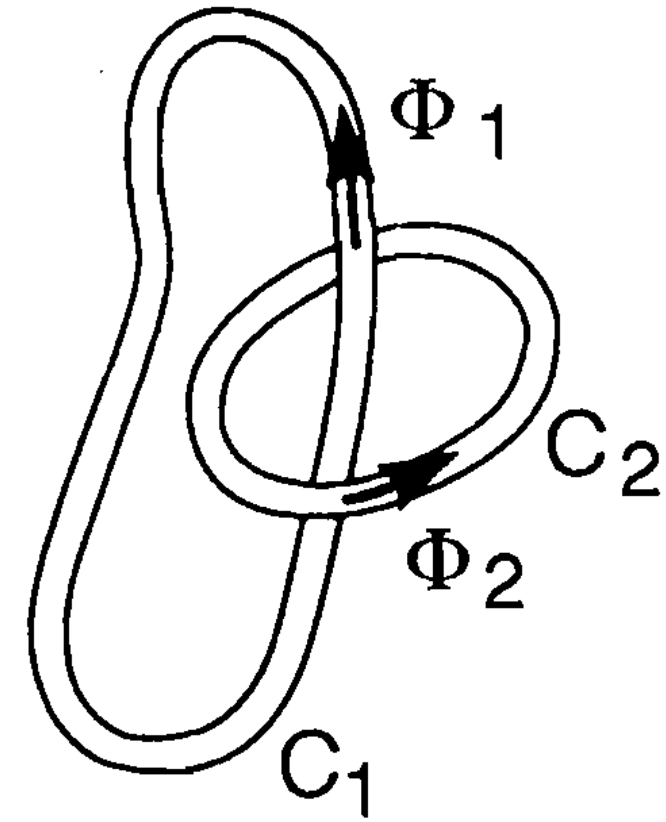

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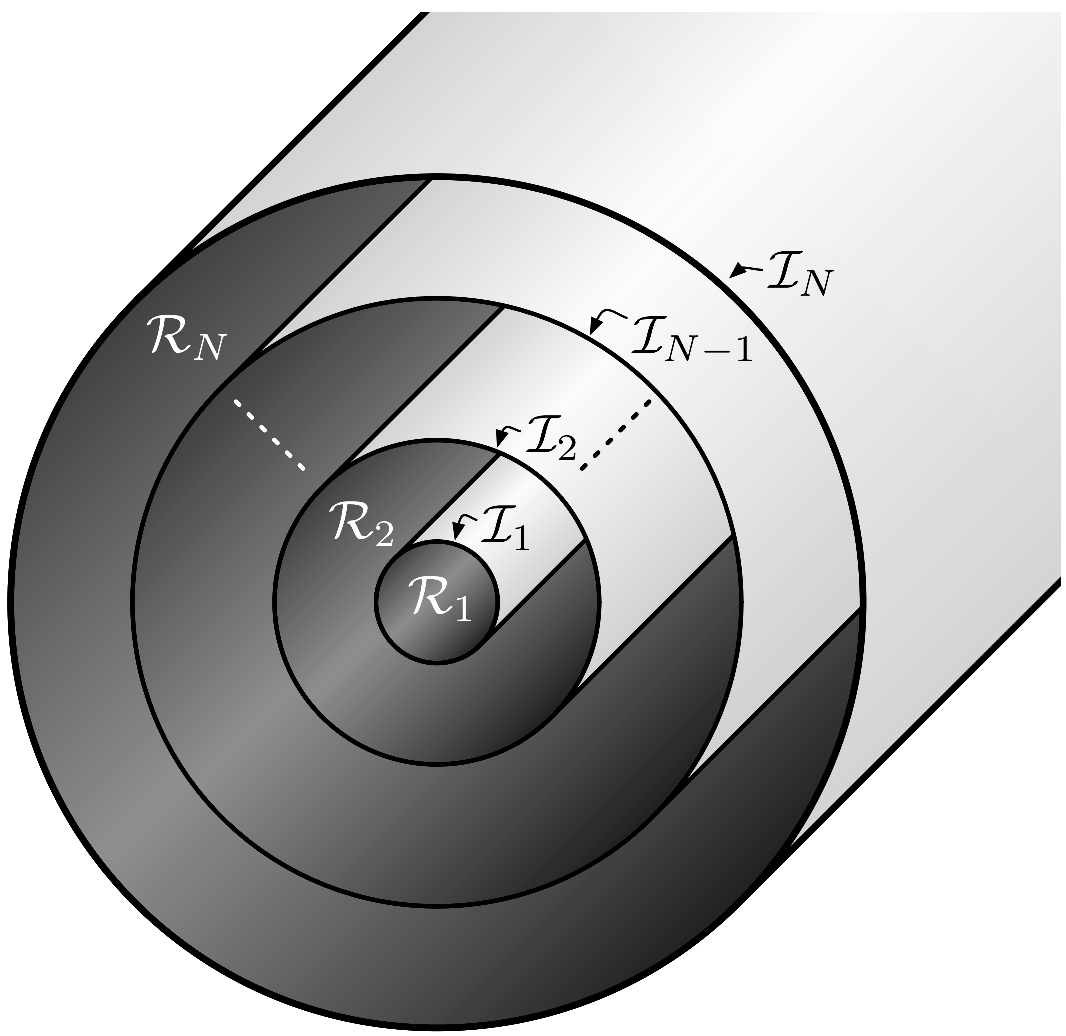
Motivation: with small resistivity, both energy and helicity will decay

$$\dot{H} = \eta \int \mathbf{J} \cdot \mathbf{B} \, dV \sim \eta \sum_k k^1 \mathbf{B}_k^2$$

$$\dot{E} = \eta \int \mathbf{J} \cdot \mathbf{J} \, dV \sim \eta \sum_k k^2 \mathbf{B}_k^2$$

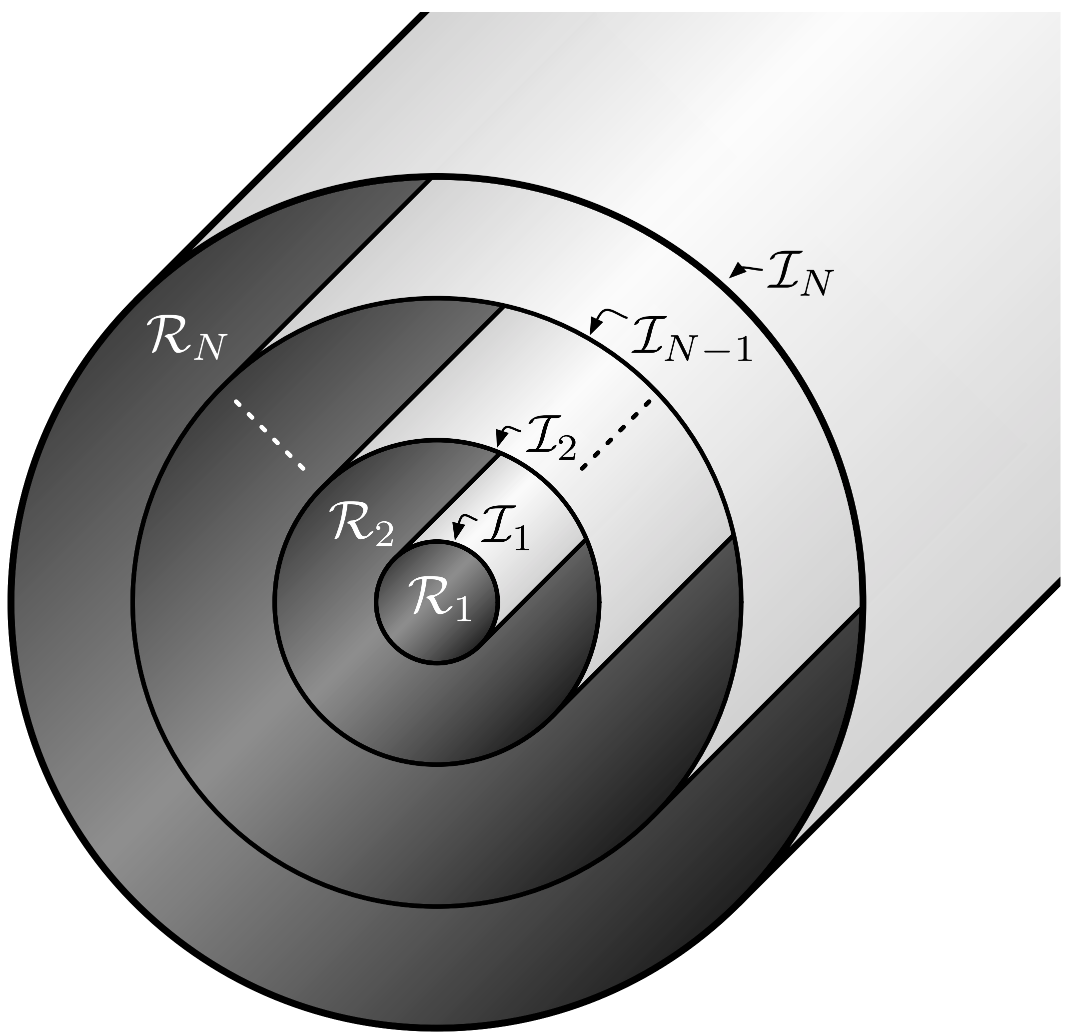
… but energy more quickly (for short length-scale turbulence)

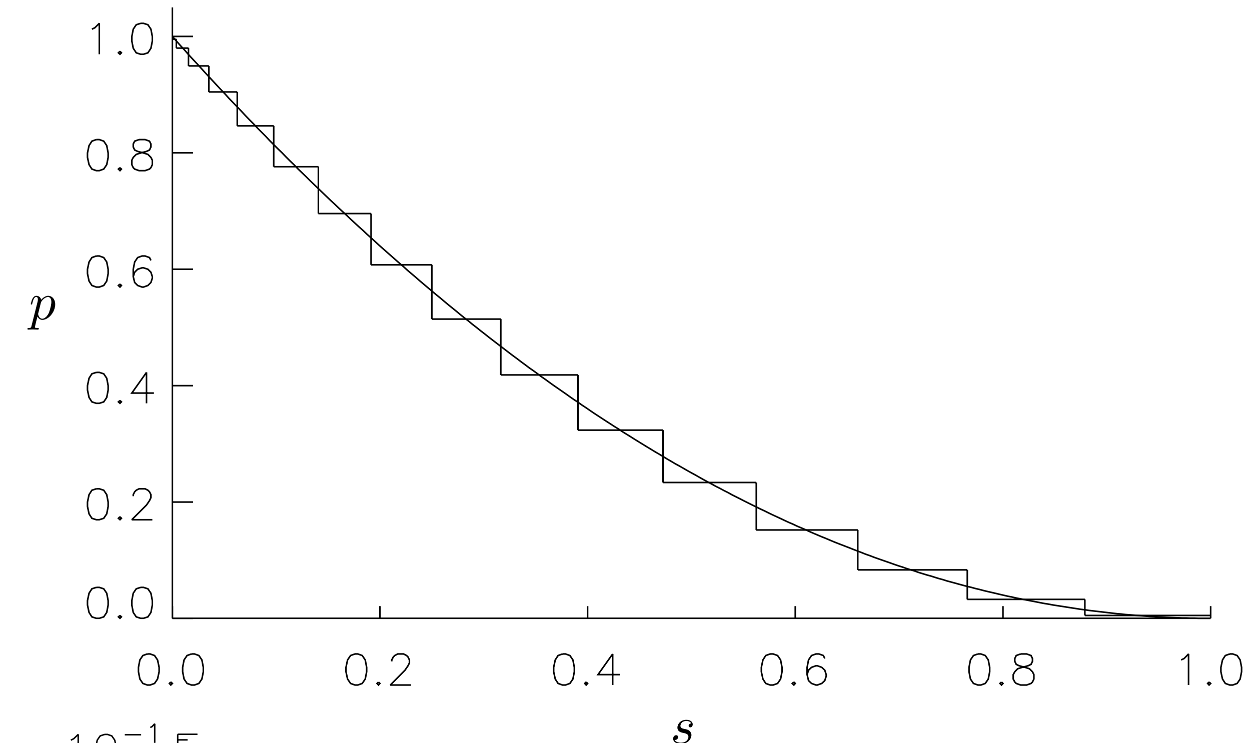
Multiregion Relaxed MHD (MRxMHD) extends Taylor Relaxation

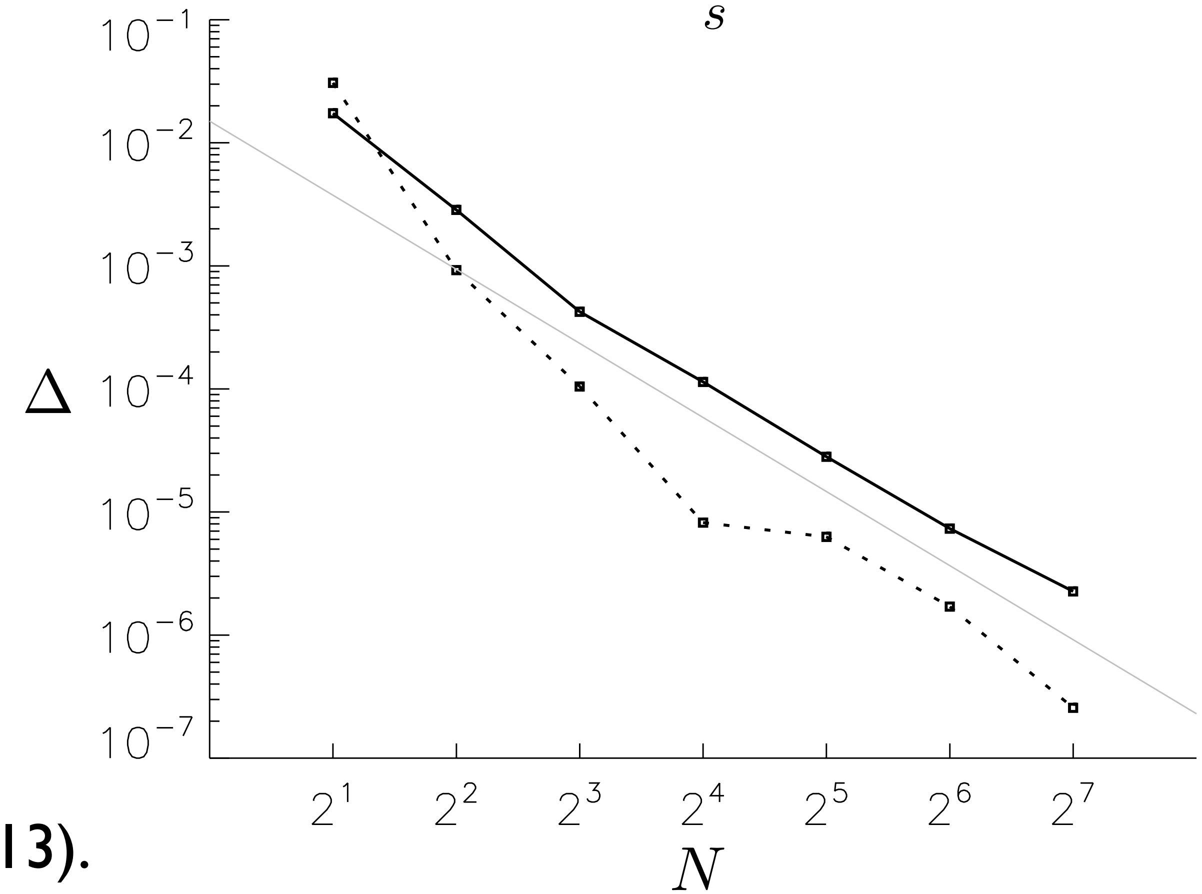


- Relaxed regions $\mathcal{R}_i$, separated by
- nested, ideal, toroidal barrier interfaces $\mathcal{I}_i$, which
- independently undergo Taylor relaxation.
- Magnetic islands and chaos are allowed between the toroidal current sheets
- Each plasma region has constant pressure, creating a piecewise constant pressure profile

Multiregion Relaxed MHD (MRxMHD) approaches ideal MHD as $N \to \infty$







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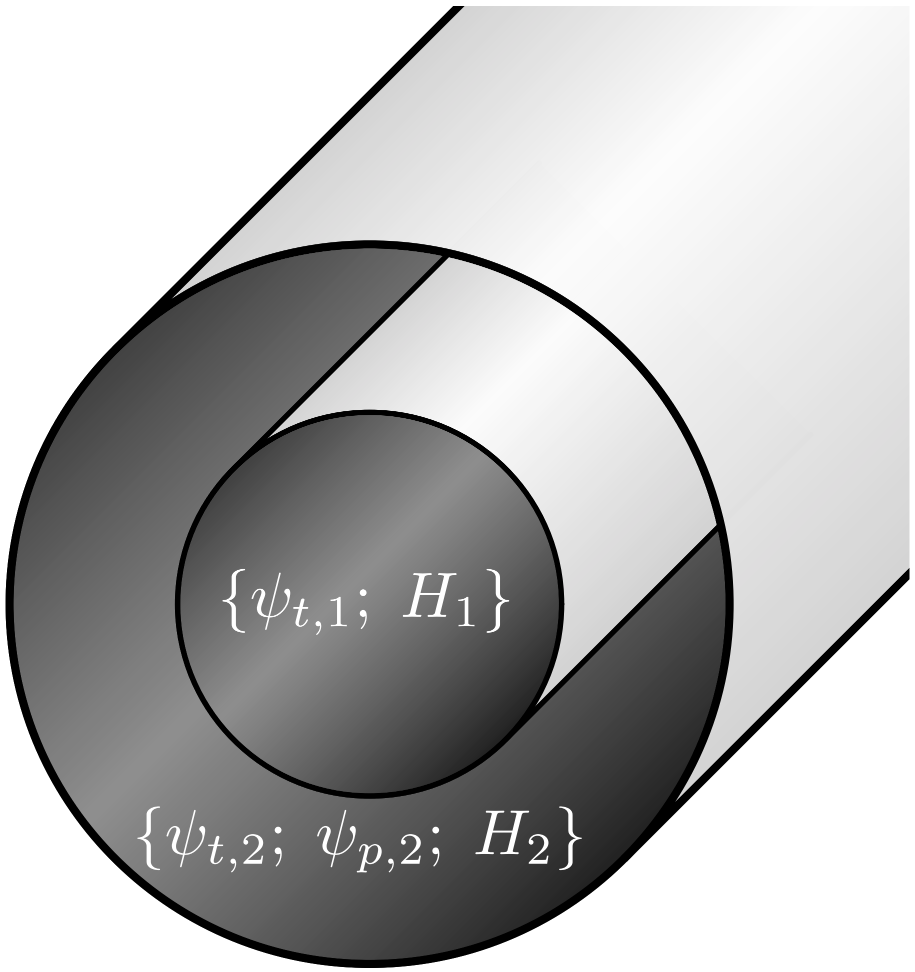
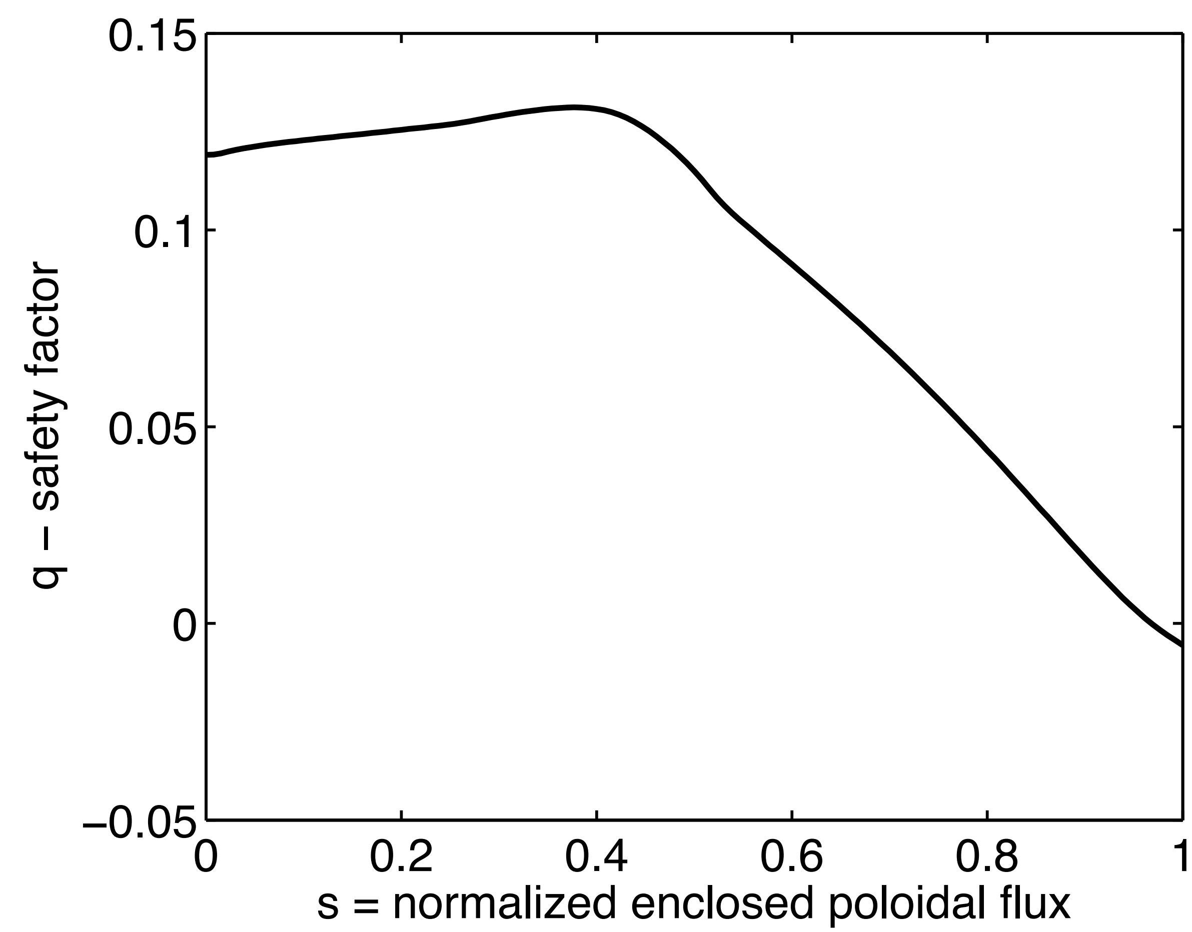
$N = 1$	is Taylor's theory	✗
$N = 2$	?	
$N = \infty$	is Ideal MHD	✓

A two-volume MRxMHD model (without pressure) is defined by 5 parameters





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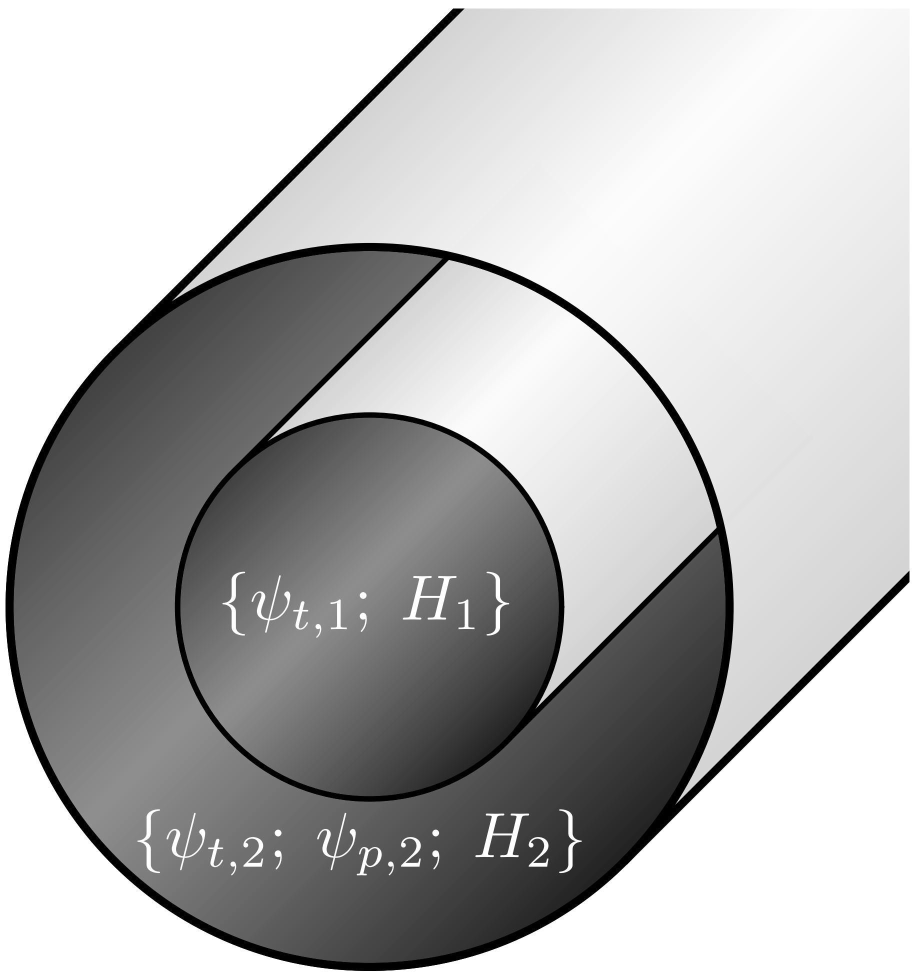

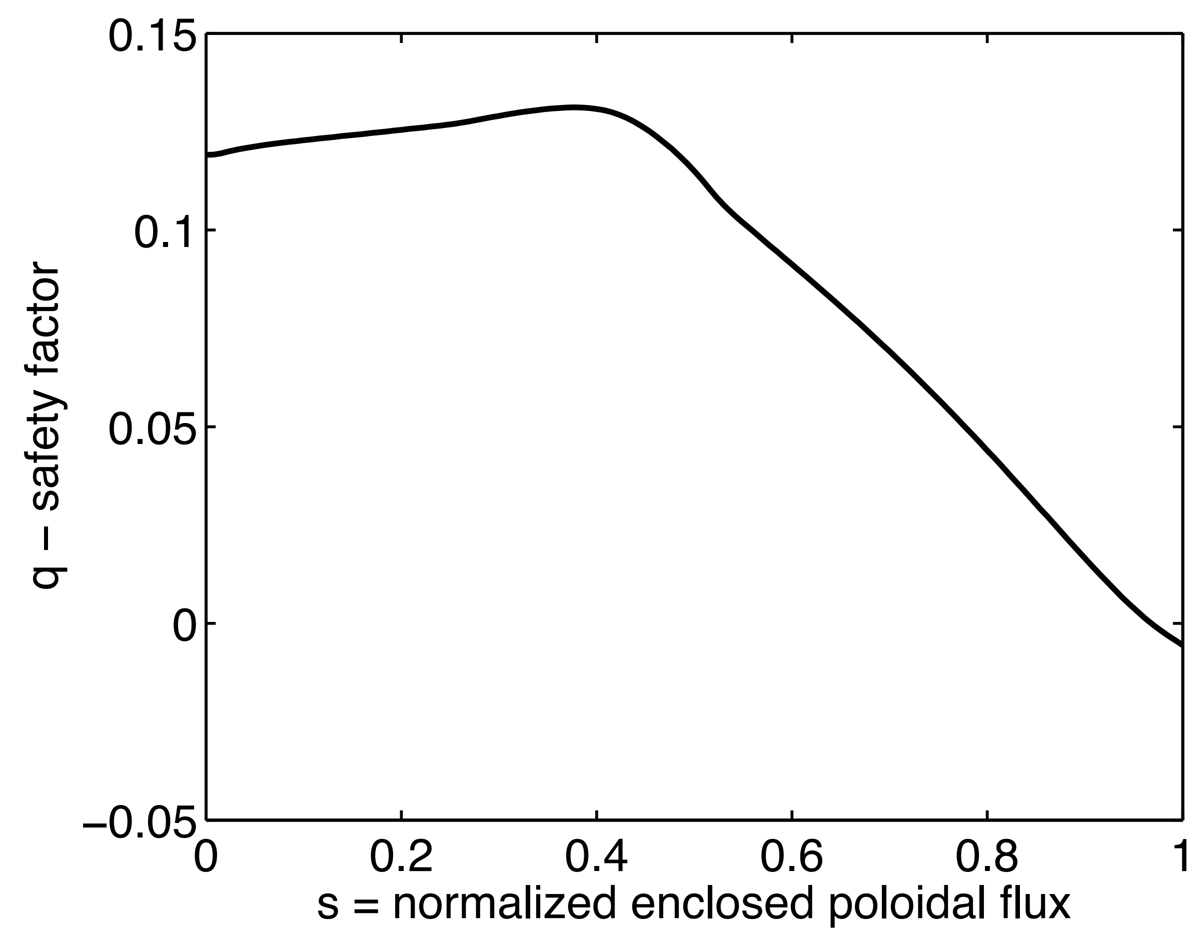

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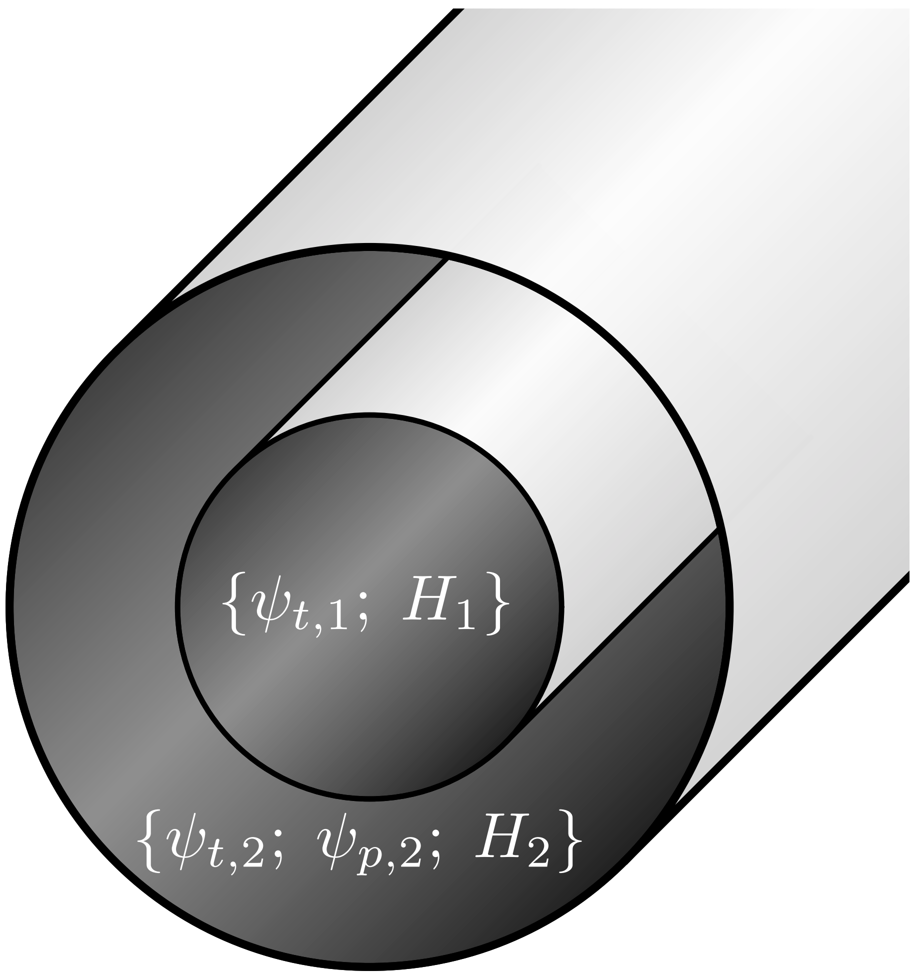

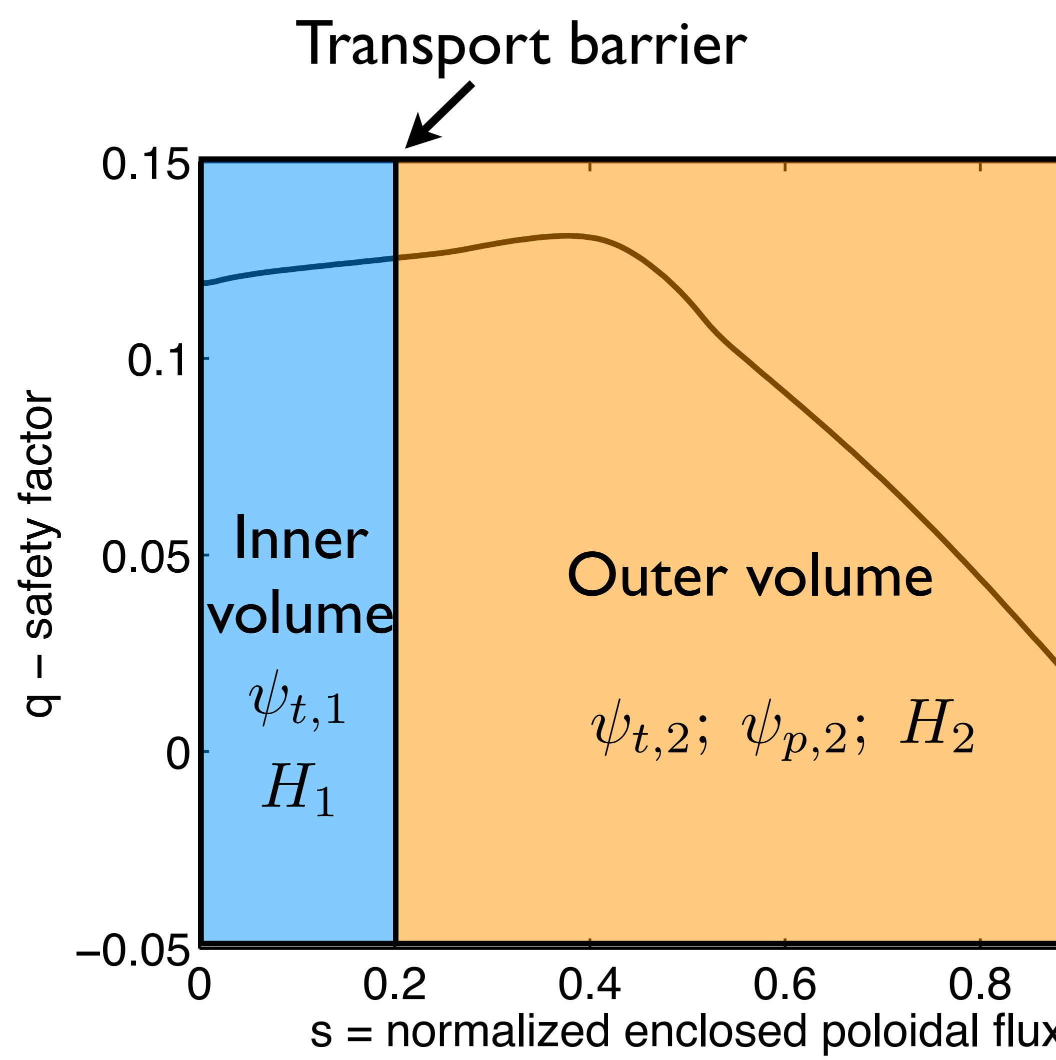

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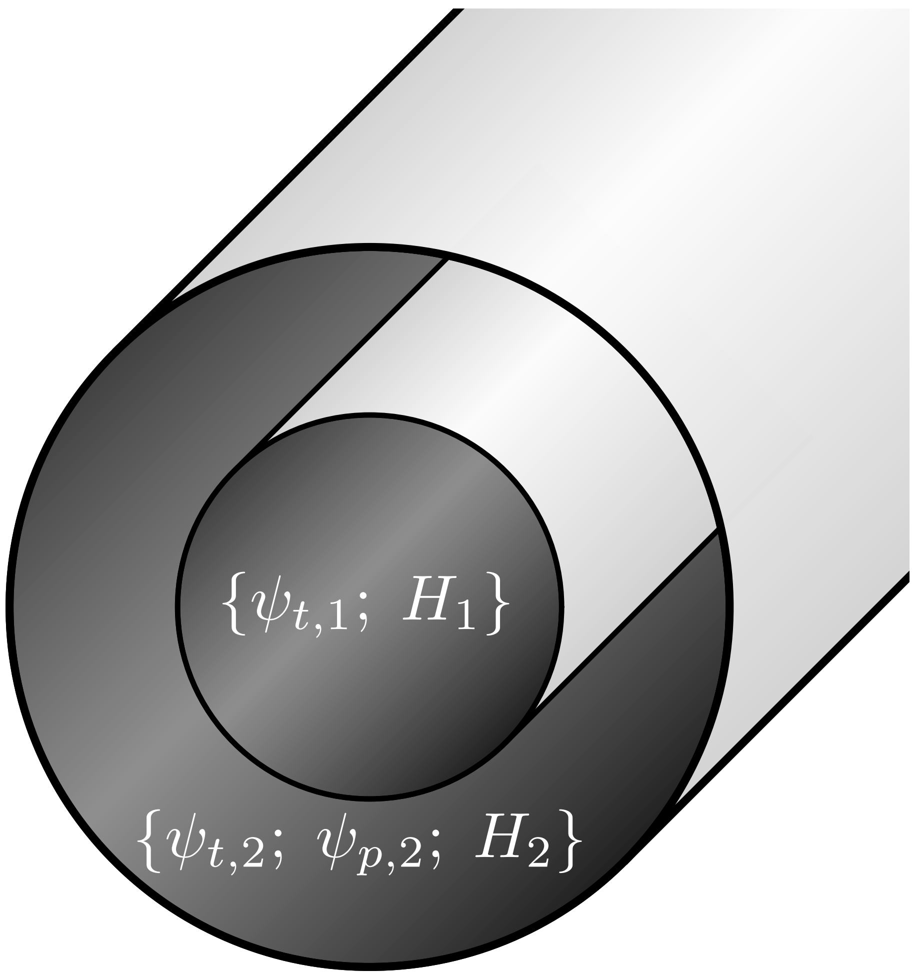

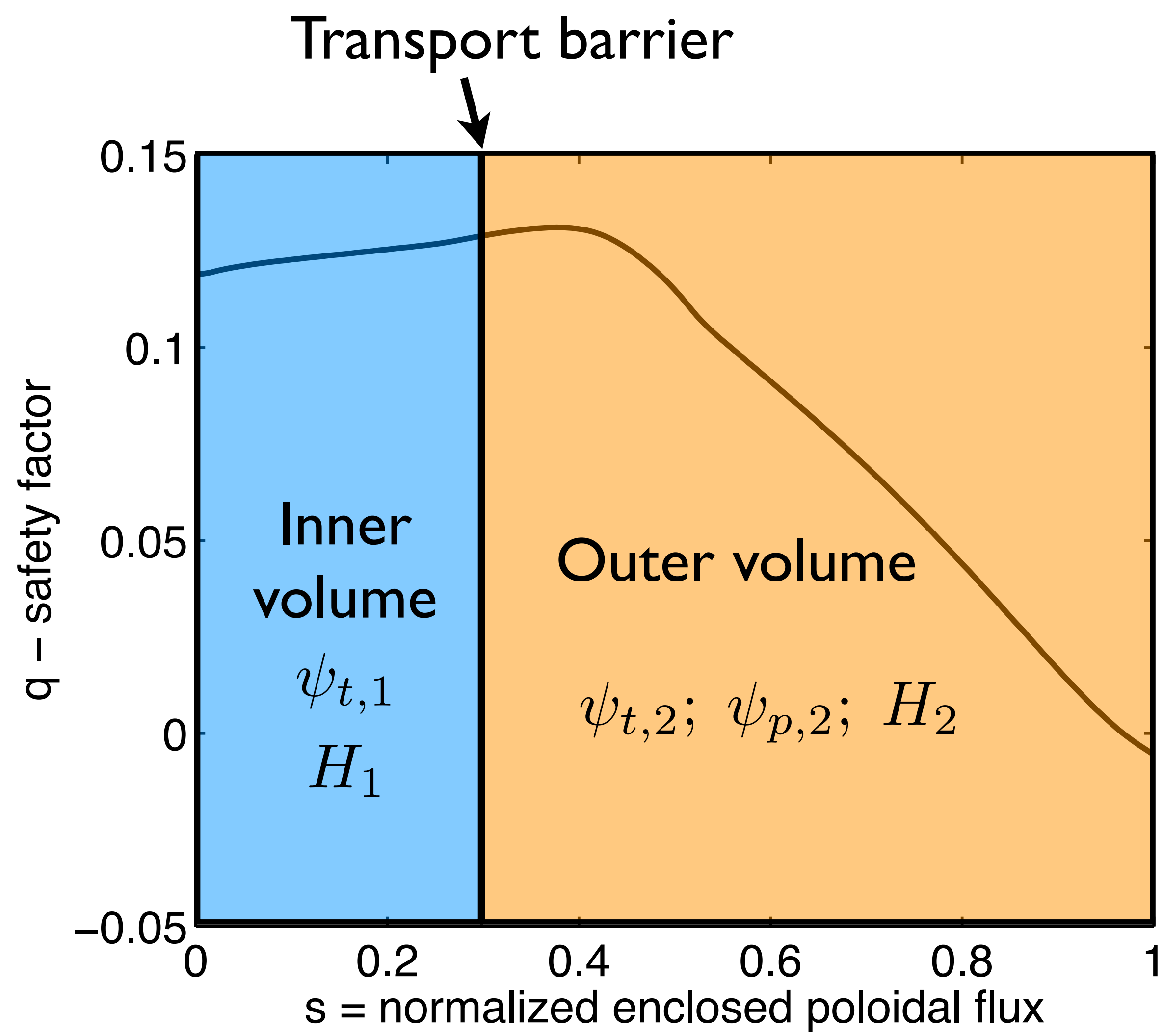

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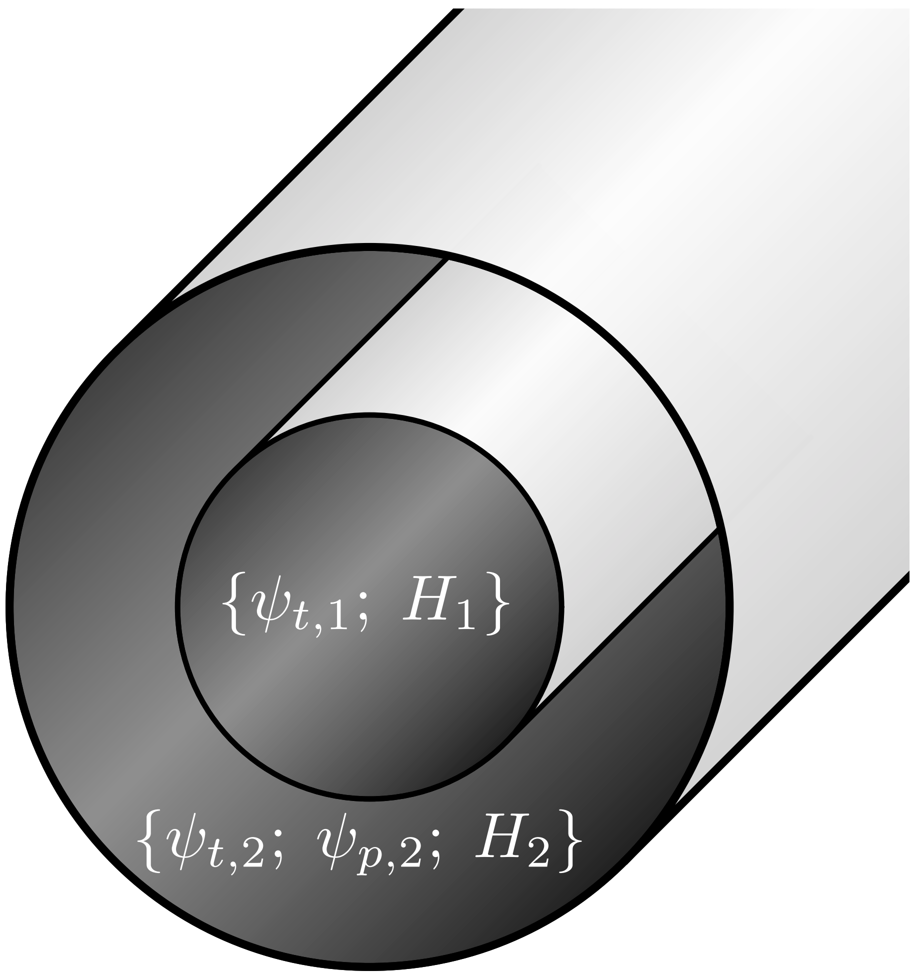

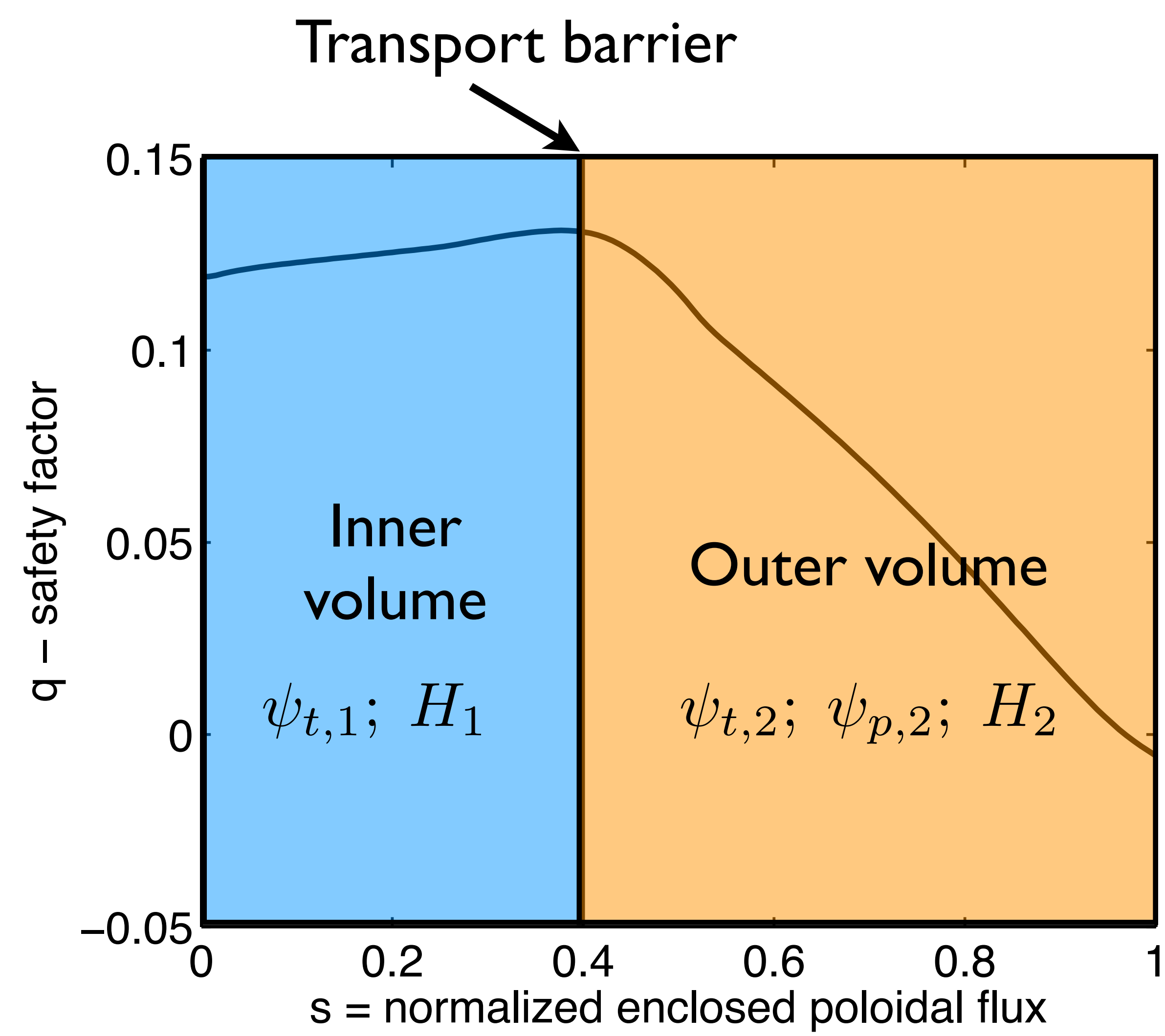

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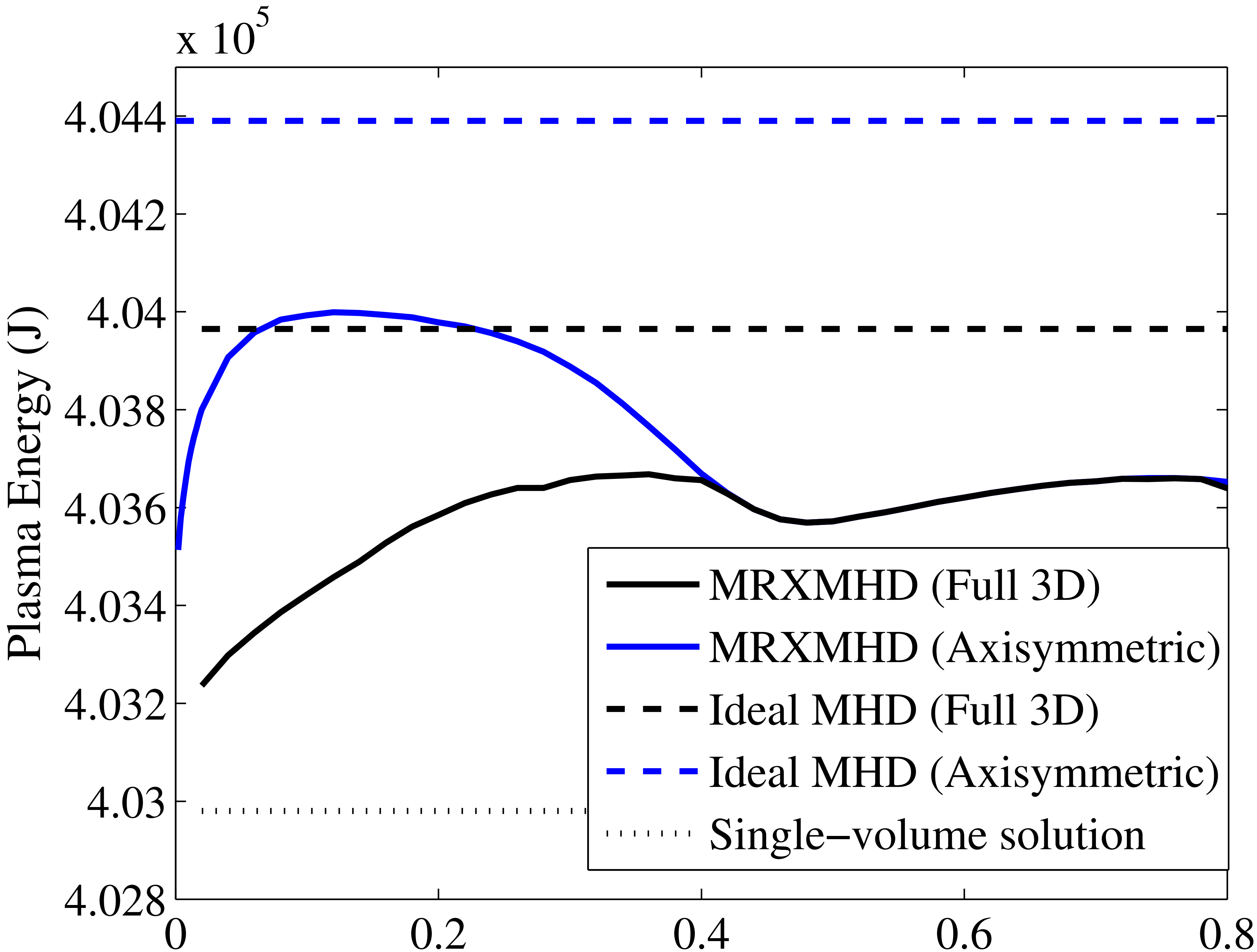
The plasma equilibrium is a minimum energy state



Ideal MHD flux surface chosen as ideal barrier

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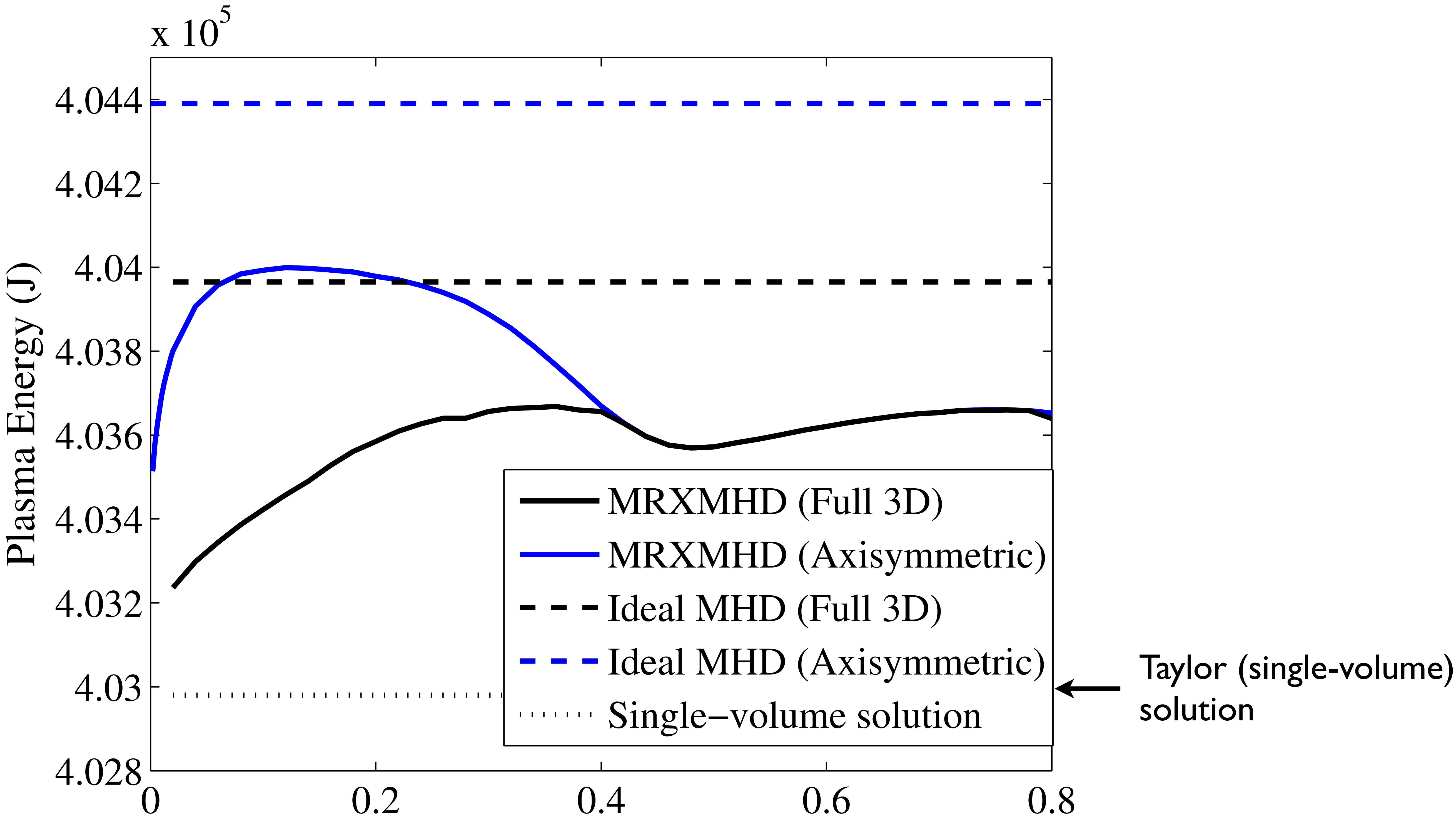

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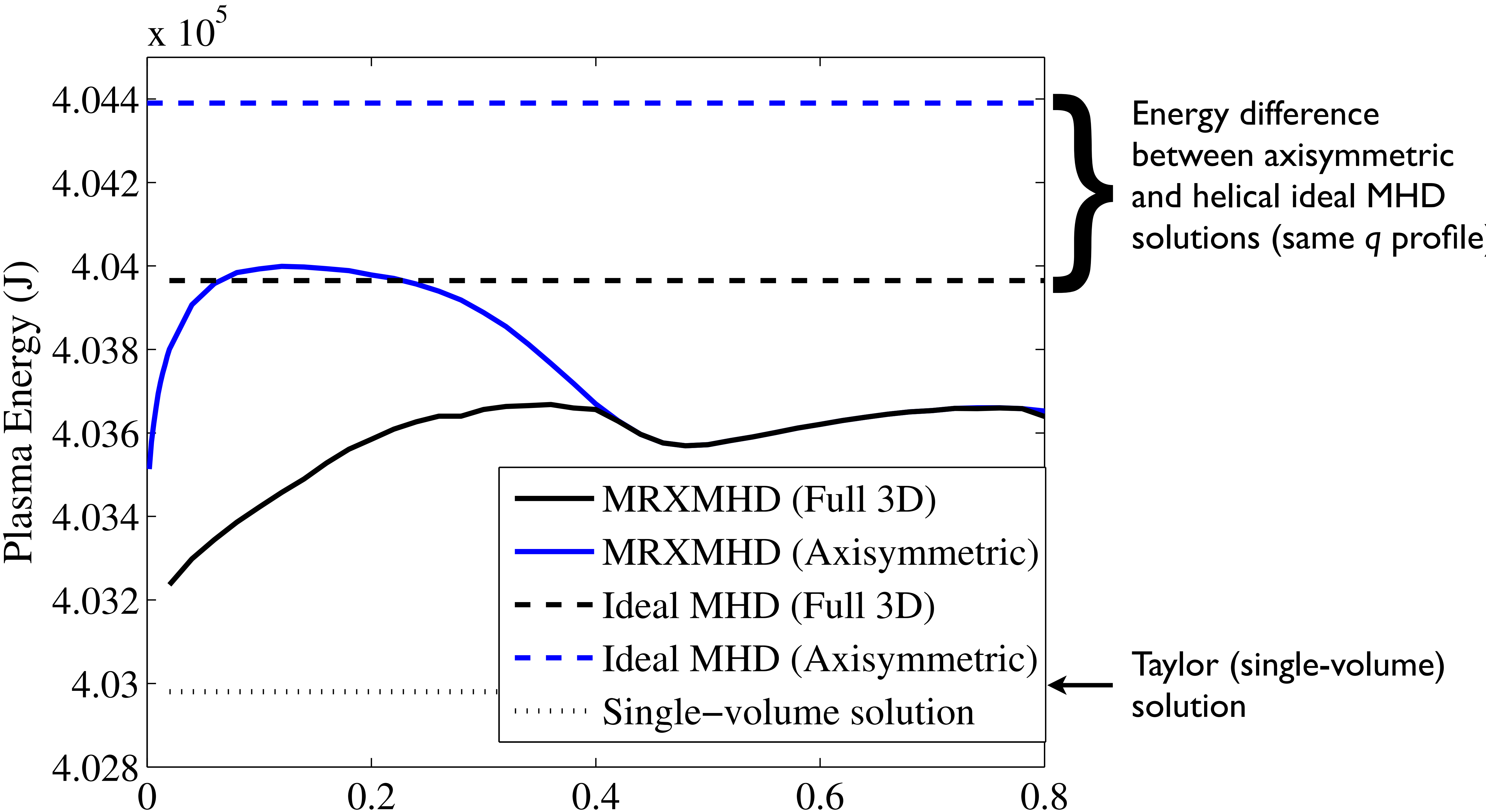

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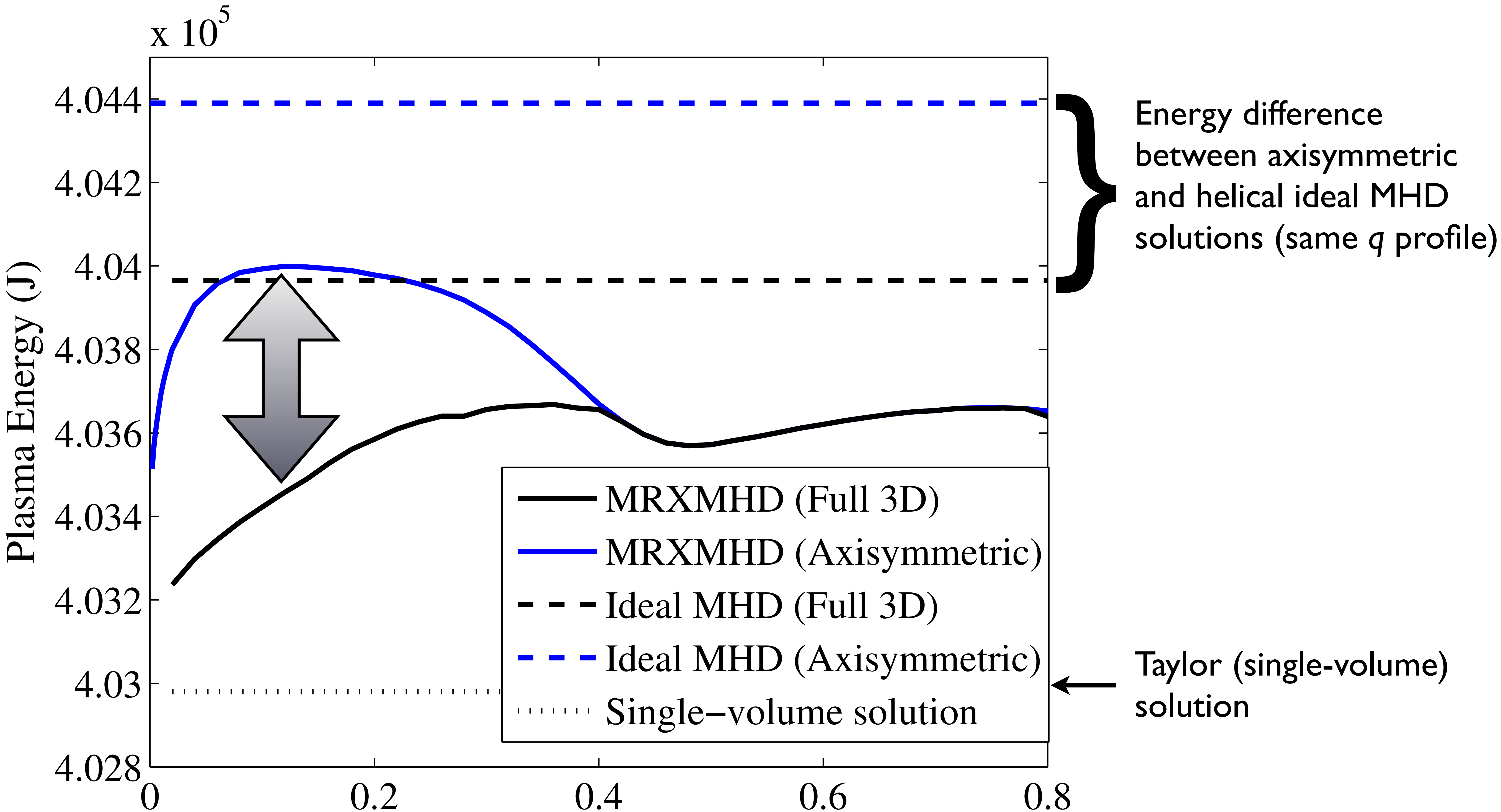

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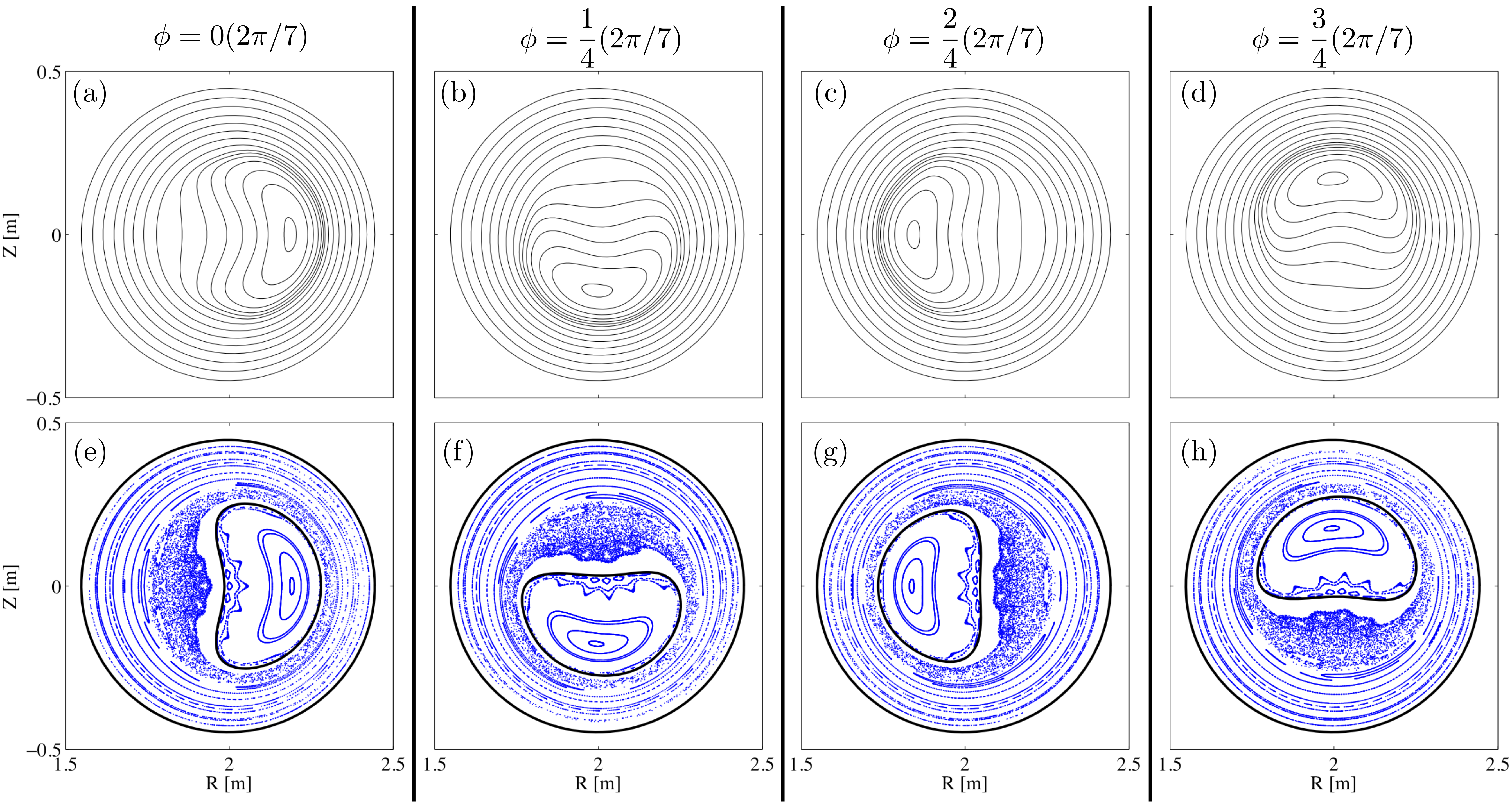
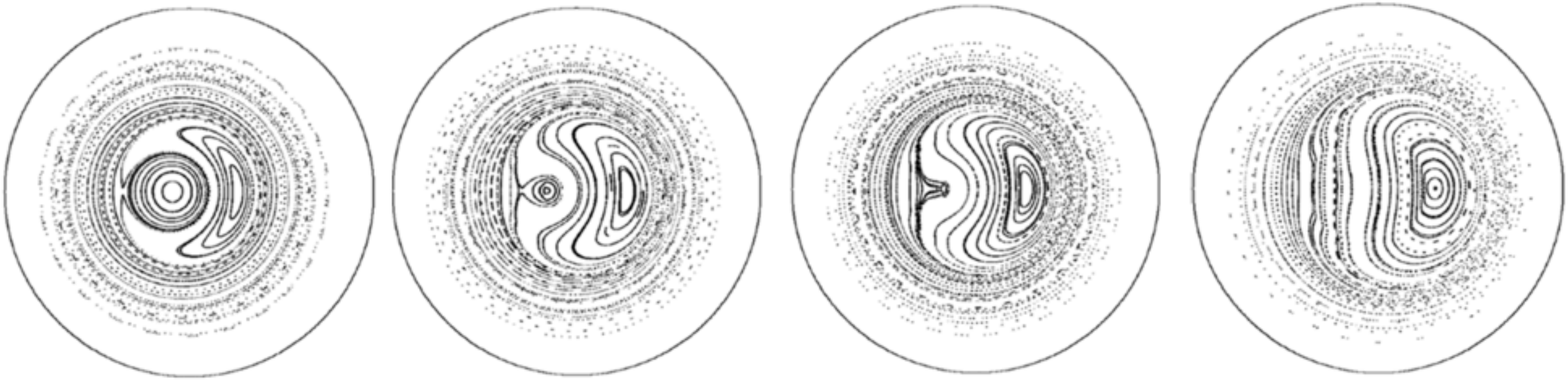
Comparison of VMEC and SPEC RFX-mod equilibria



Reconstructed Poincaré plots



Quasi-single helicity → Single Helical Axis

Top figure source: P. Martin *et al.*, *Nuclear Fusion* **49**, 104019 (2009).

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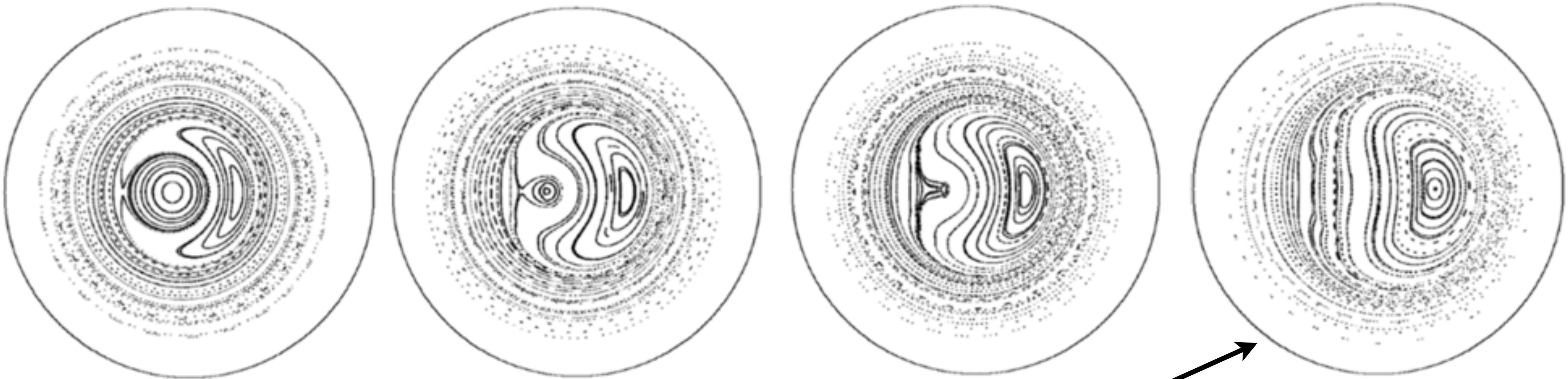
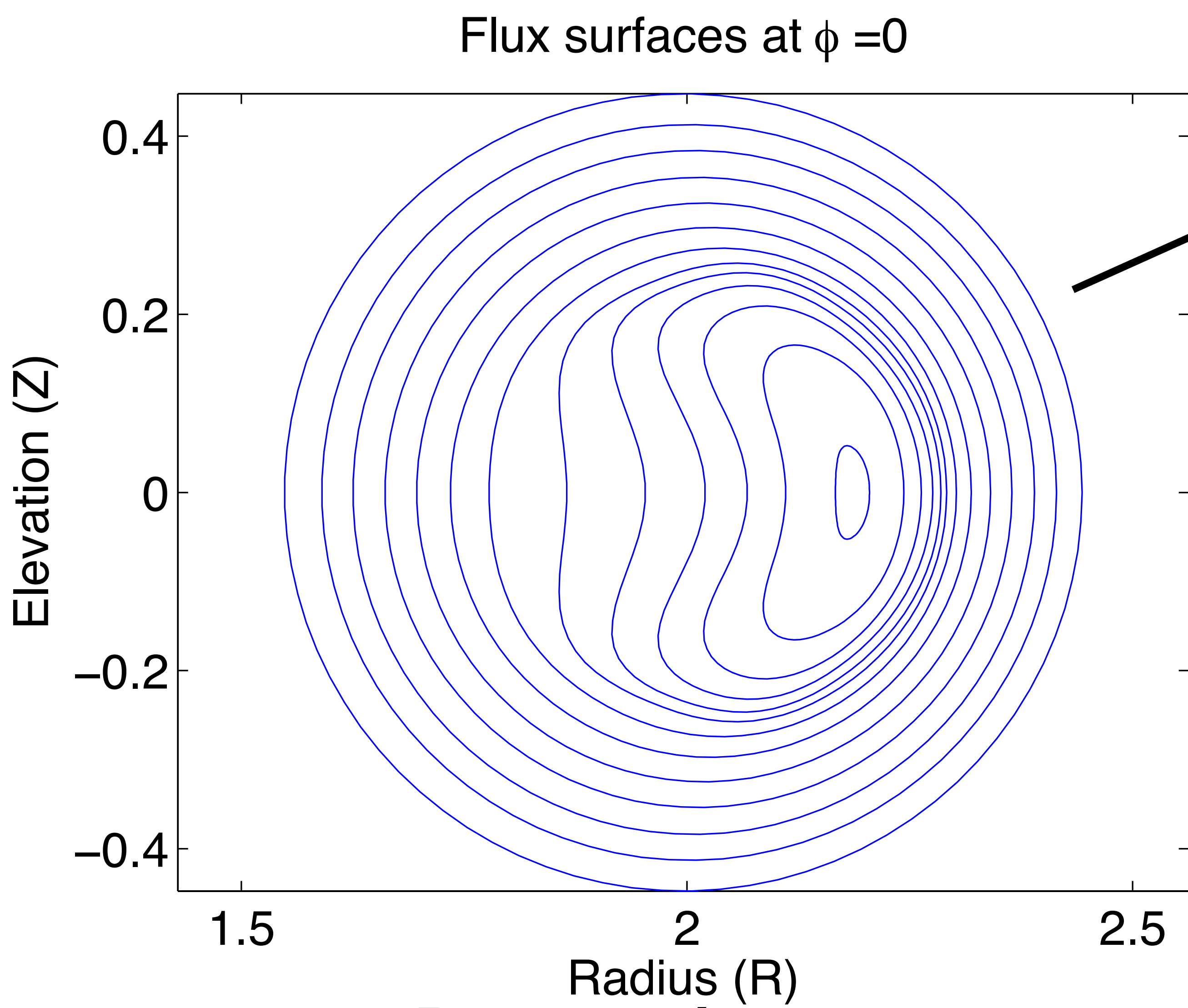
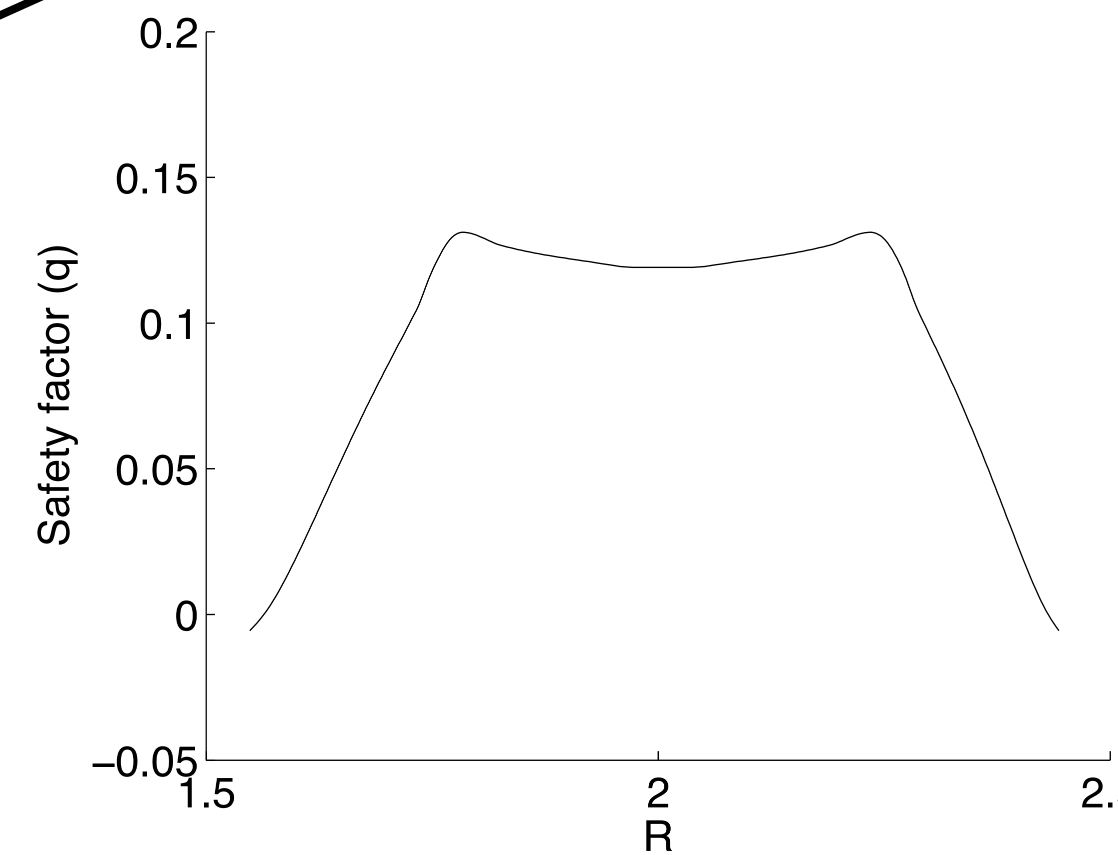




Poincaré section



Safety factor profile (q)

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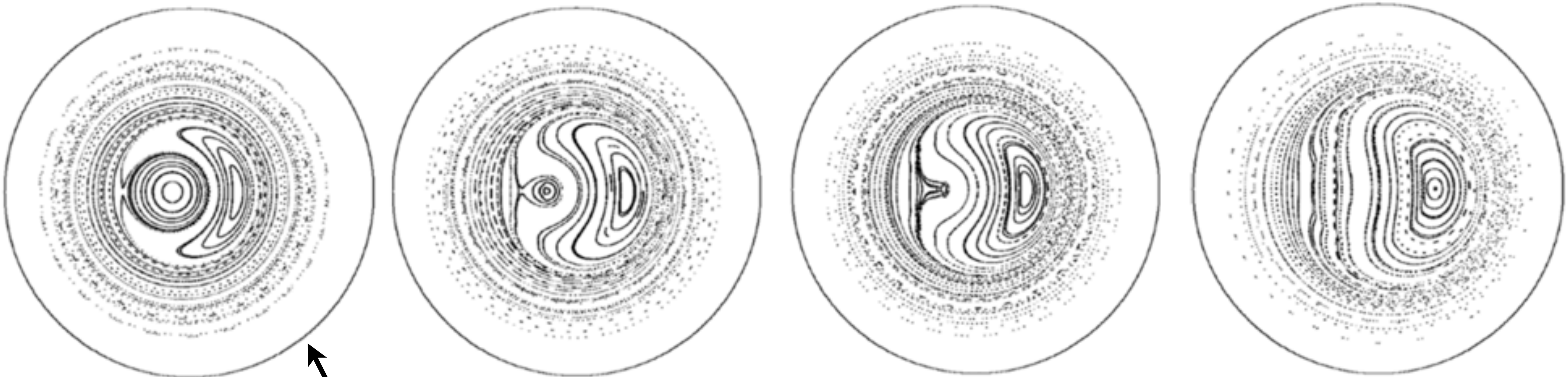

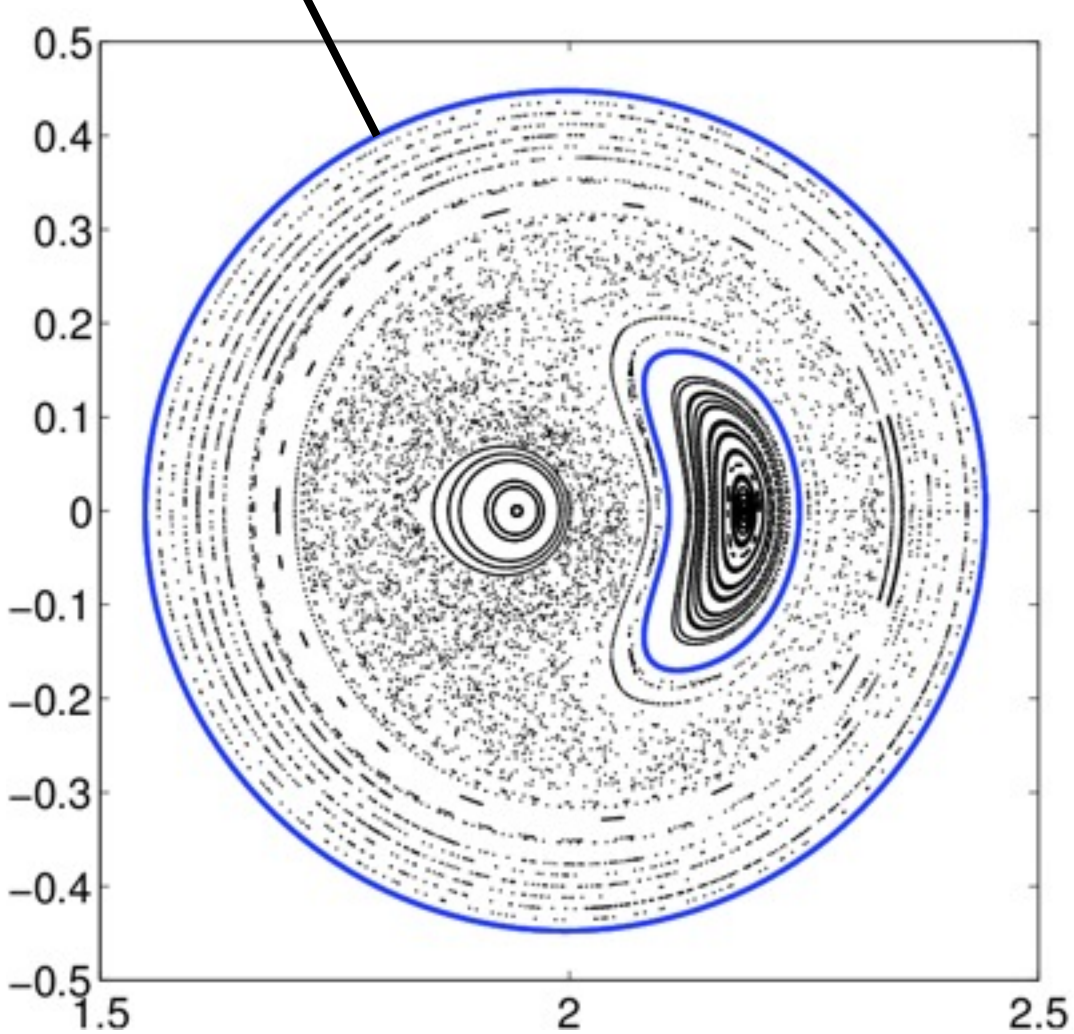

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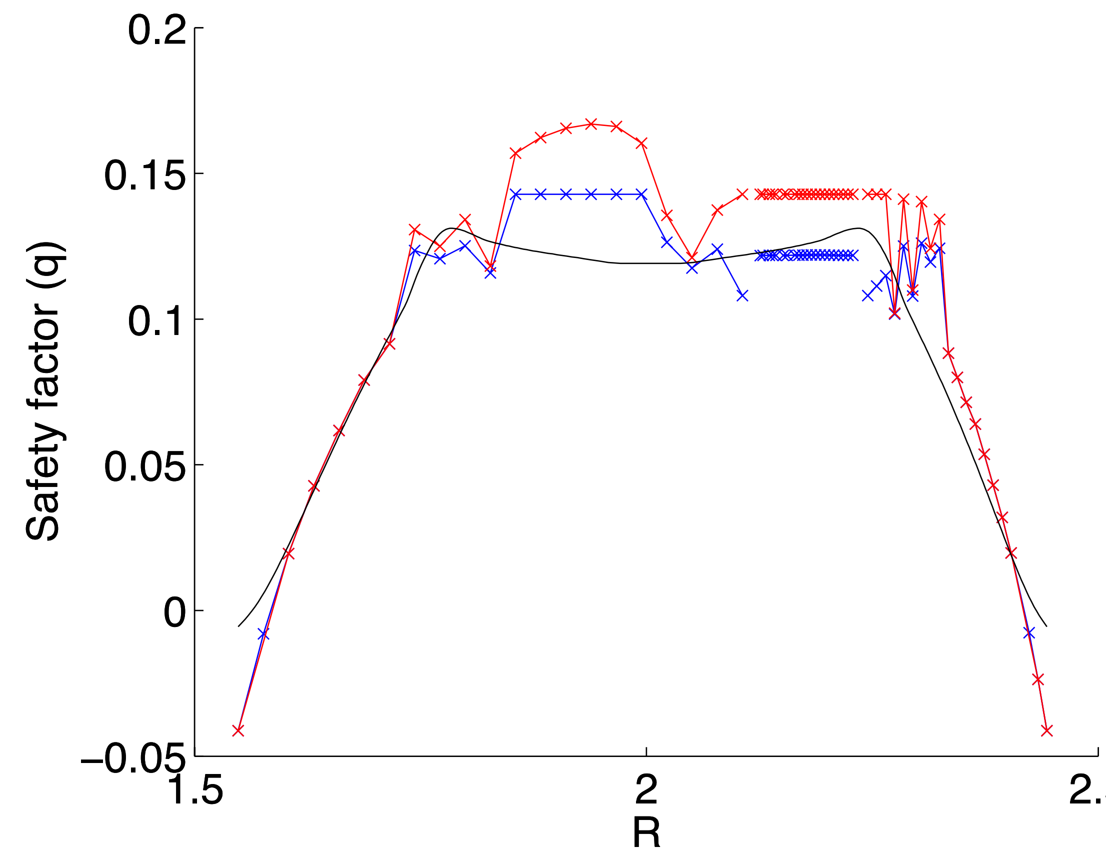

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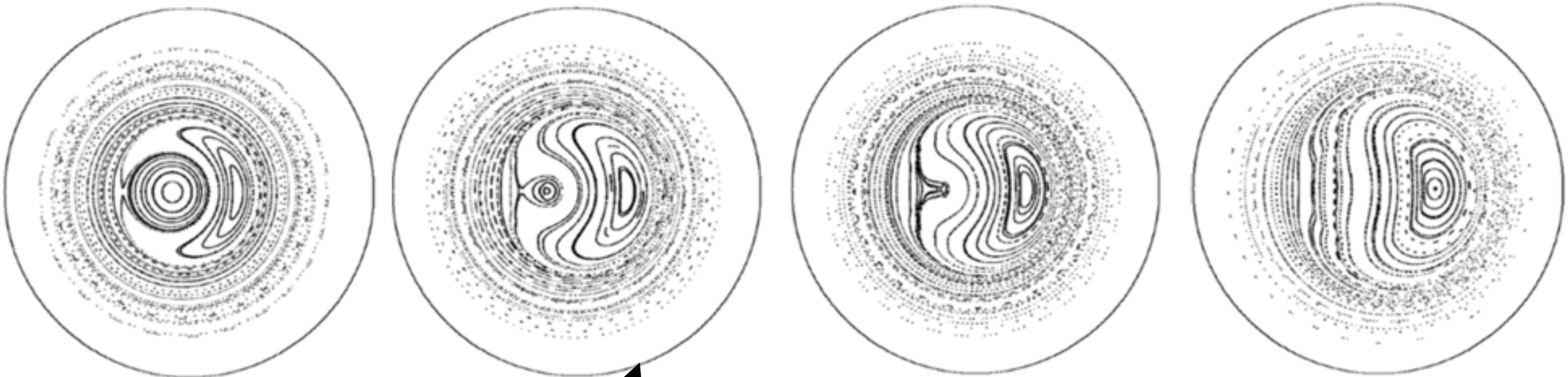

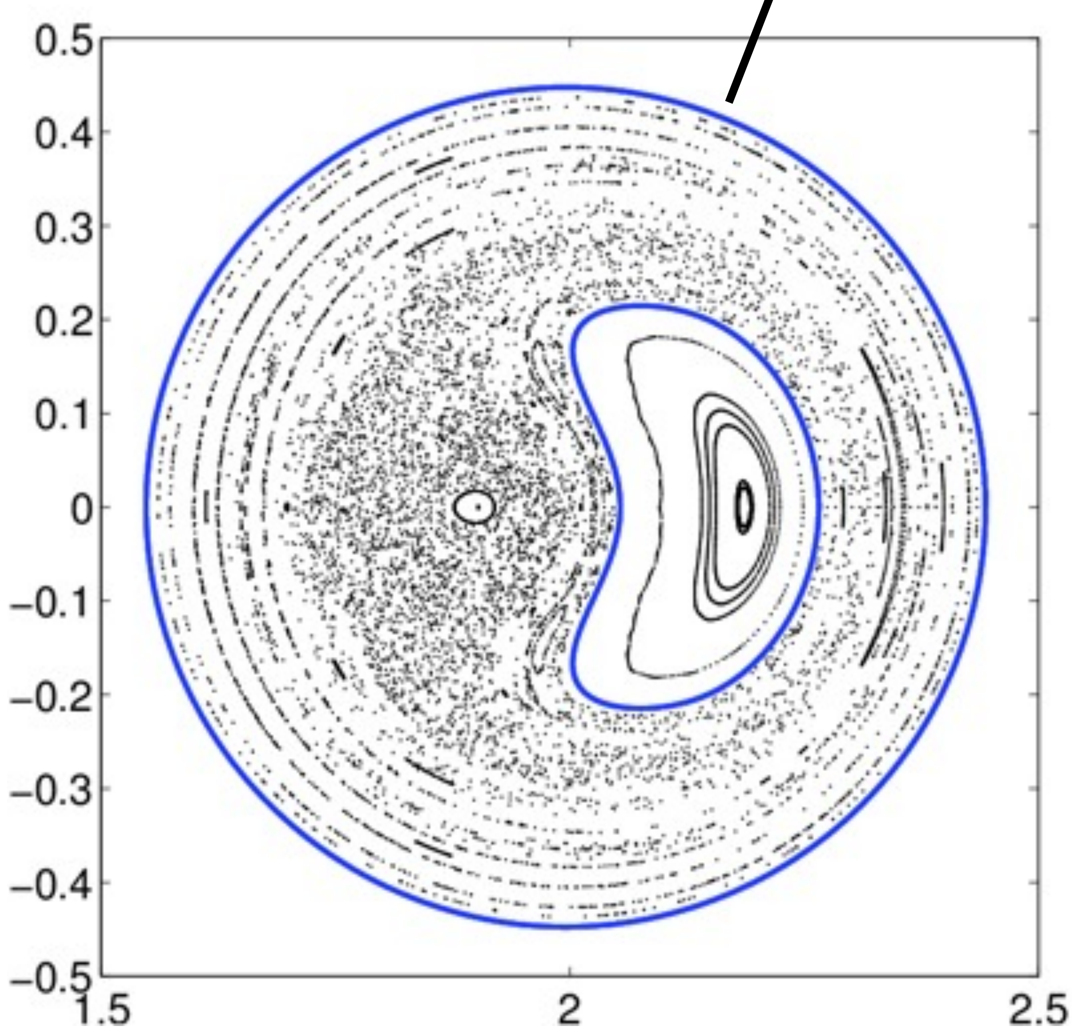

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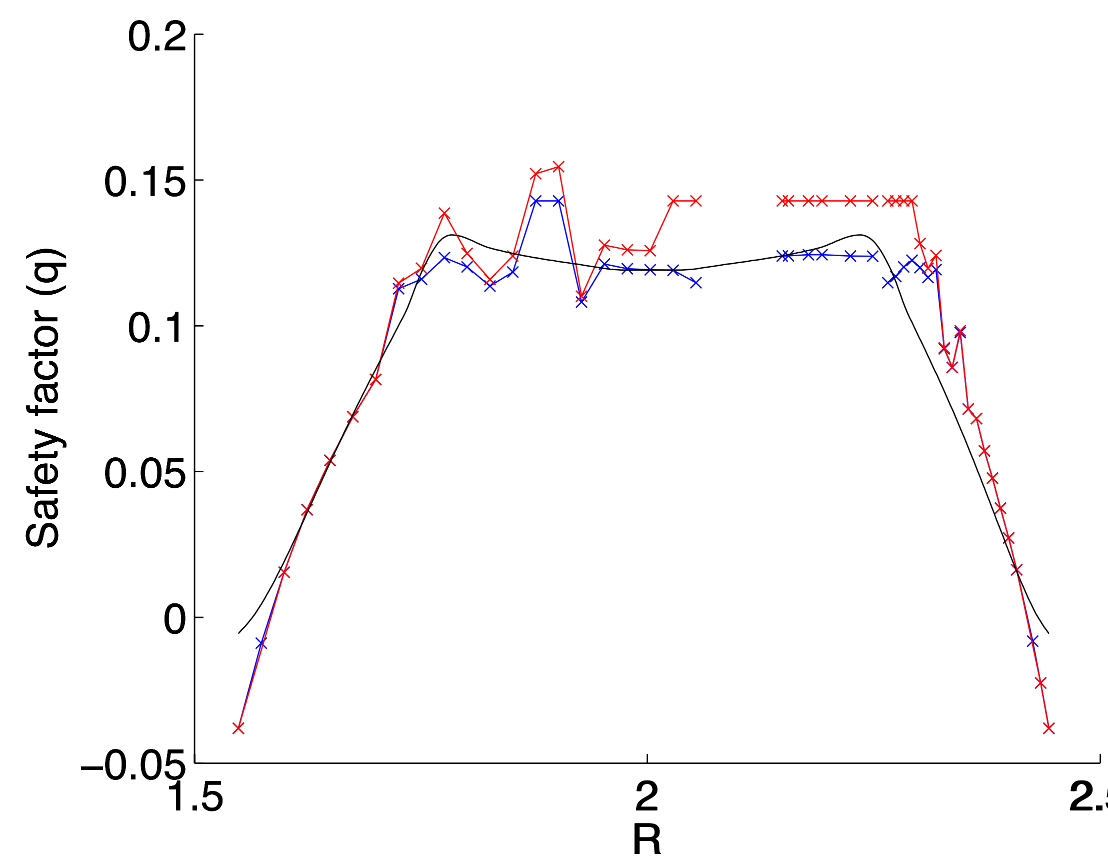

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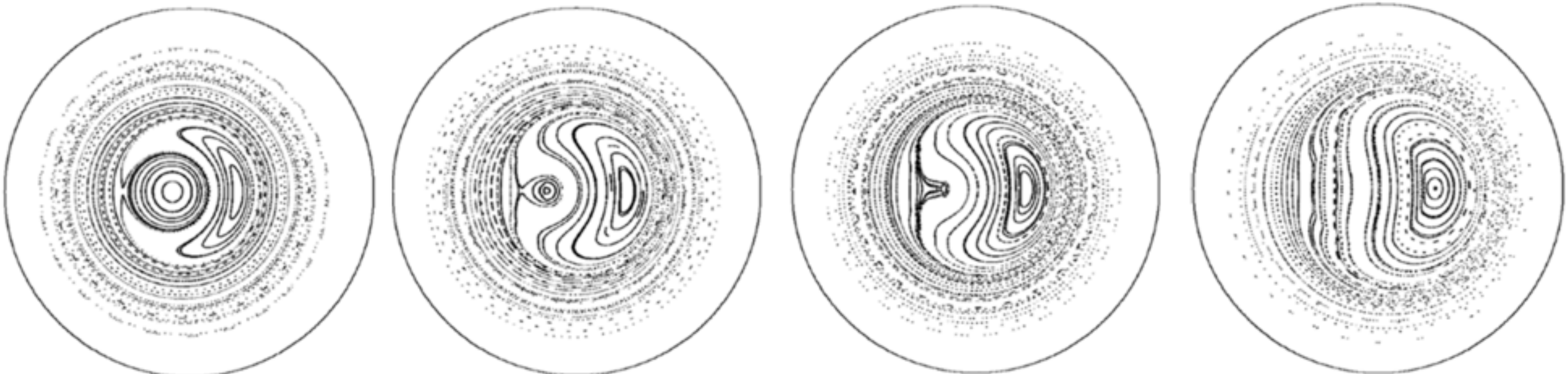
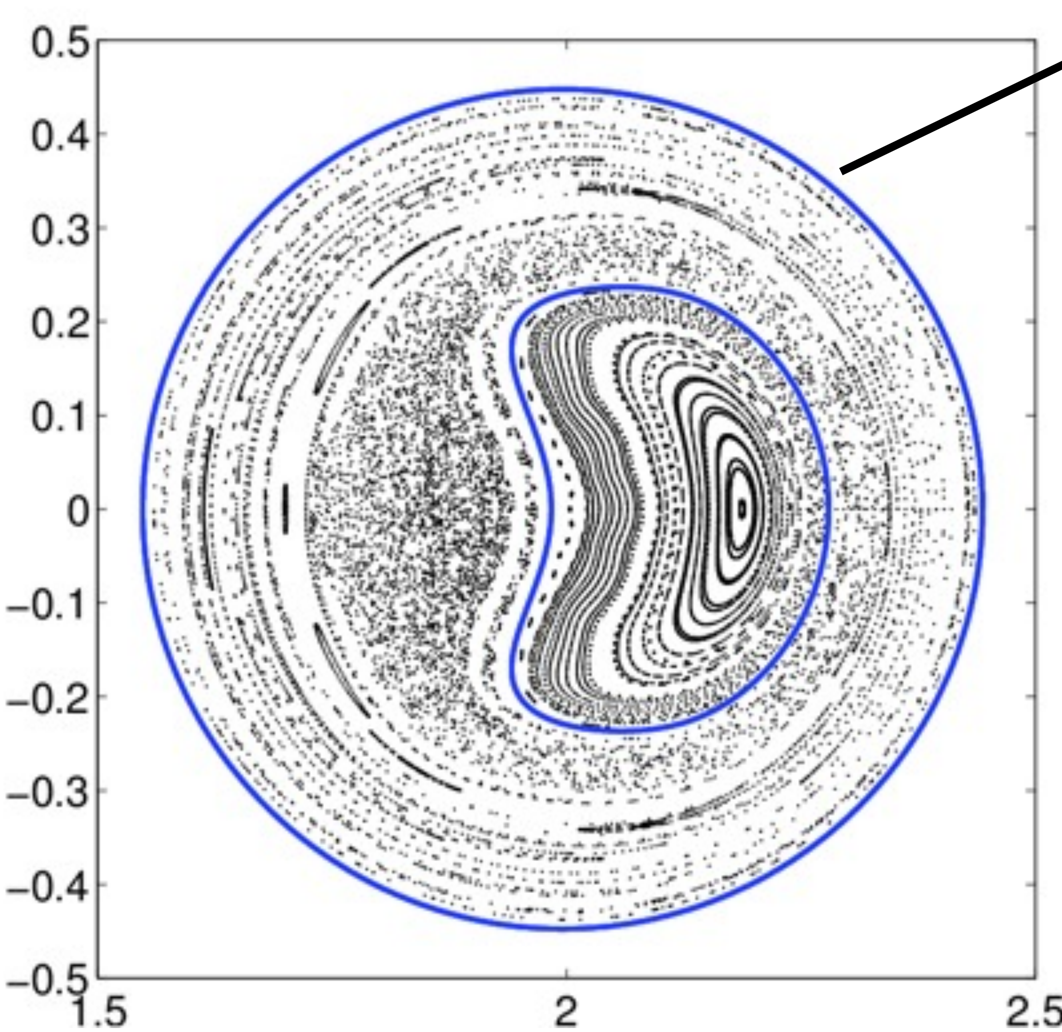
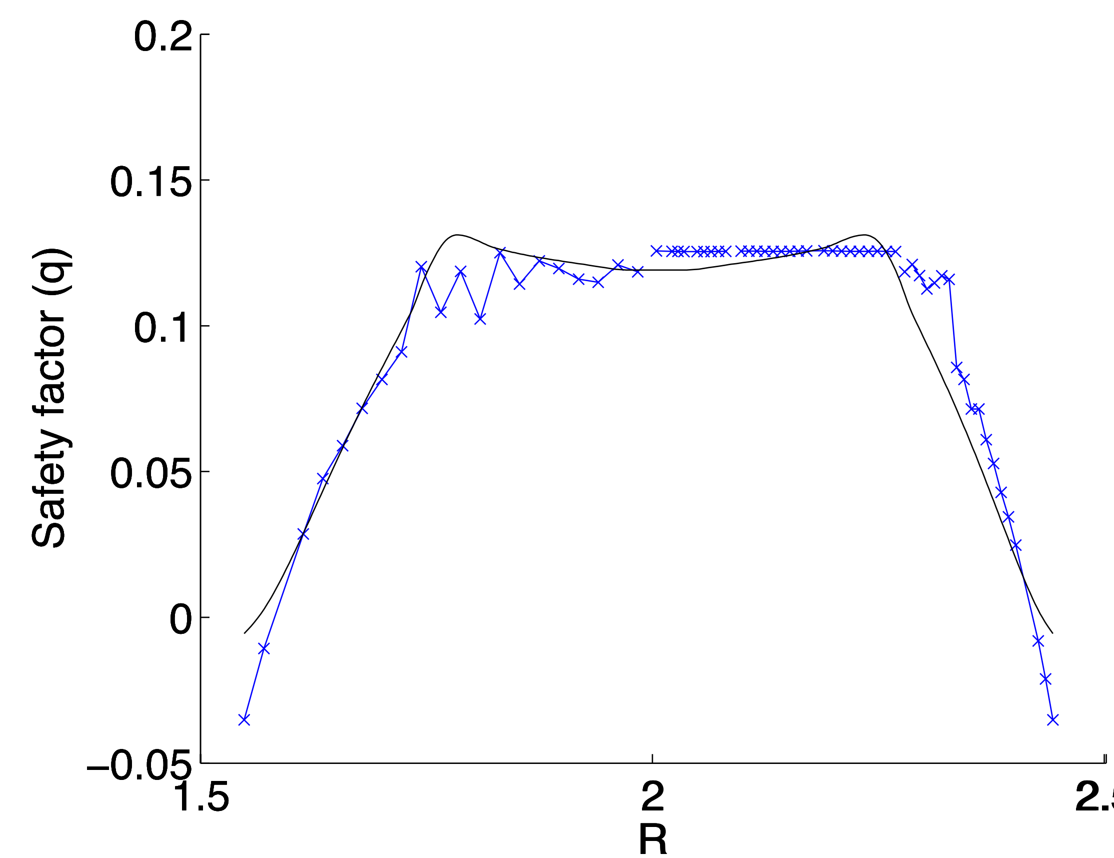
Reconstructed Poincaré plots



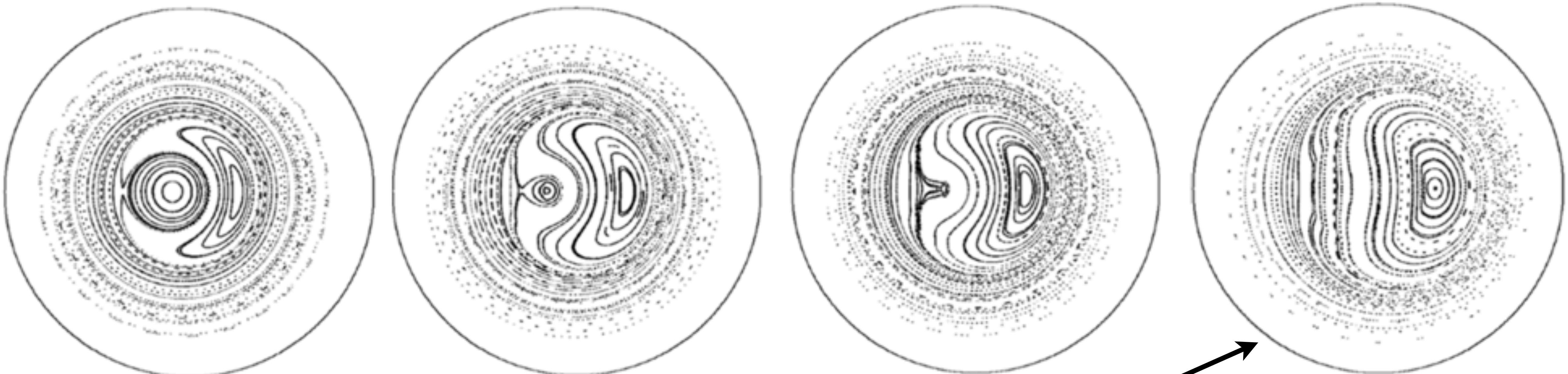


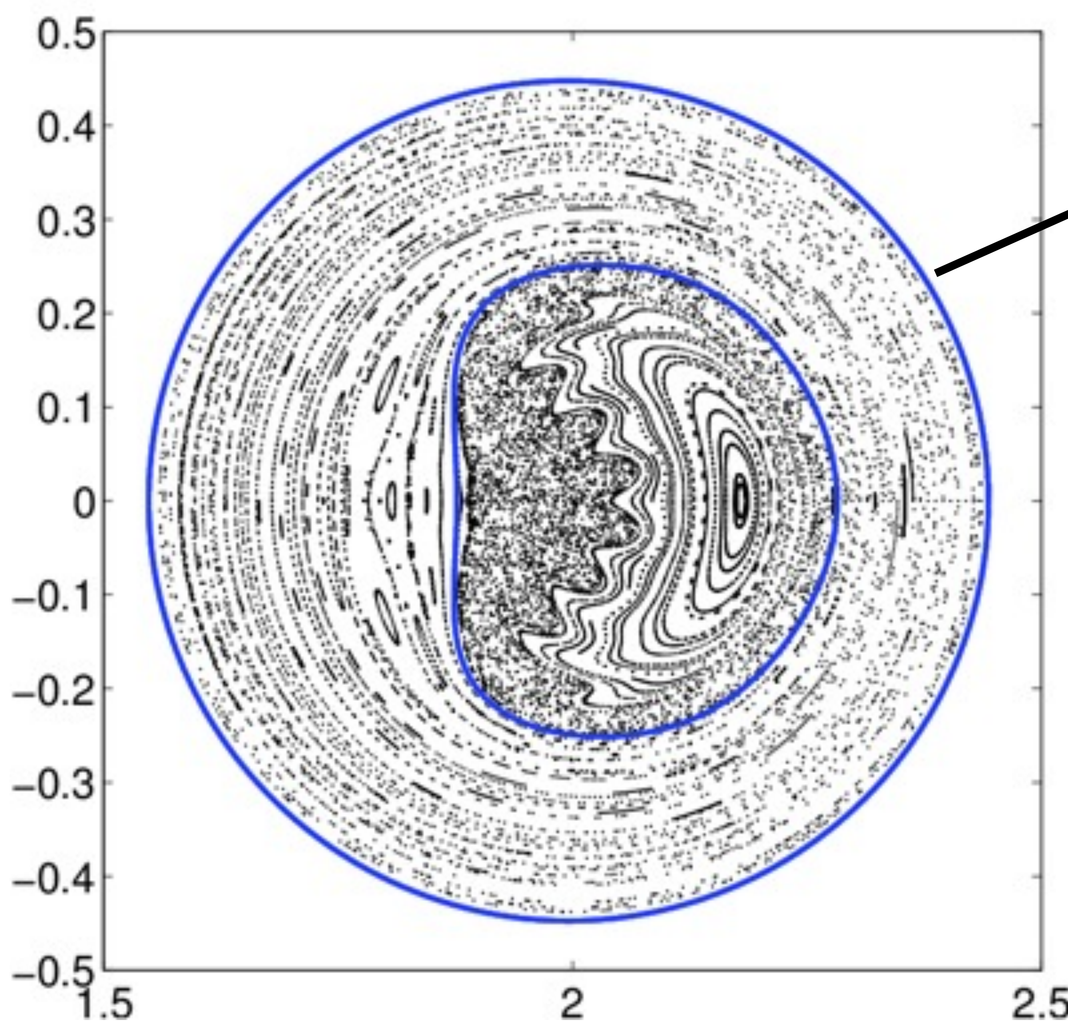
Poincaré section



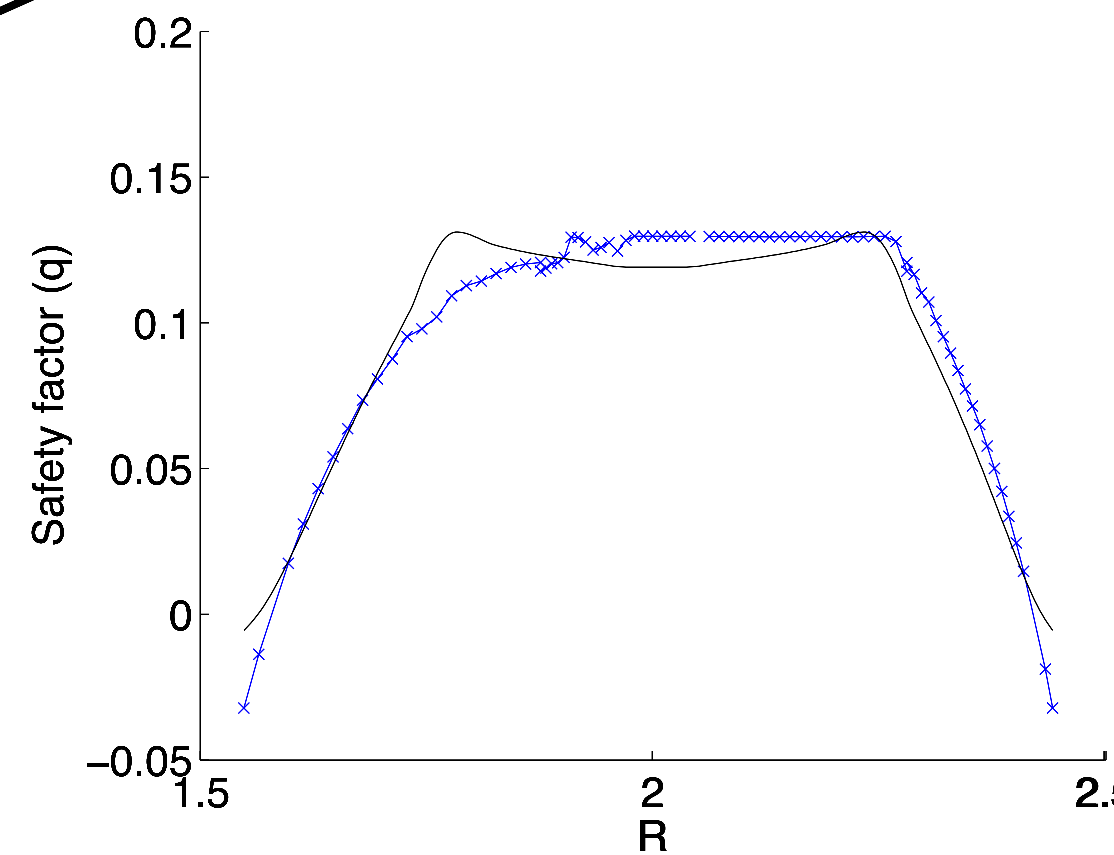
Safety factor profile (q)

Reconstructed Poincaré plots





Poincaré section



Safety factor profile (q)

Conclusions

MRxMHD gives a good qualitative explanation of the high-confinement state in Reversed Field Pinches

With a *minimal* model we reproduced the helical pitch and structure of the Quasi-Single Helicity state in RFP

With MRxMHD we reproduced the second magnetic axis. This is the *first* equilibrium model to be able to reproduce the Double-Axis state.

MRxMHD is a well-formulated model that interpolates between Taylor's theory and ideal MHD

[1] G. Dennis *et al.*, *PRL* **111**, 055003 (2013).

Future Work

Apply the same methodology to 3D structures in tokamaks, in particular, the sawtooth crash

More detailed experimental comparisons with RFX

Considering RFX helical states with pressure

Generalize MRxMHD to include flow