Minimally constrained model of self-organised helical states in RFX

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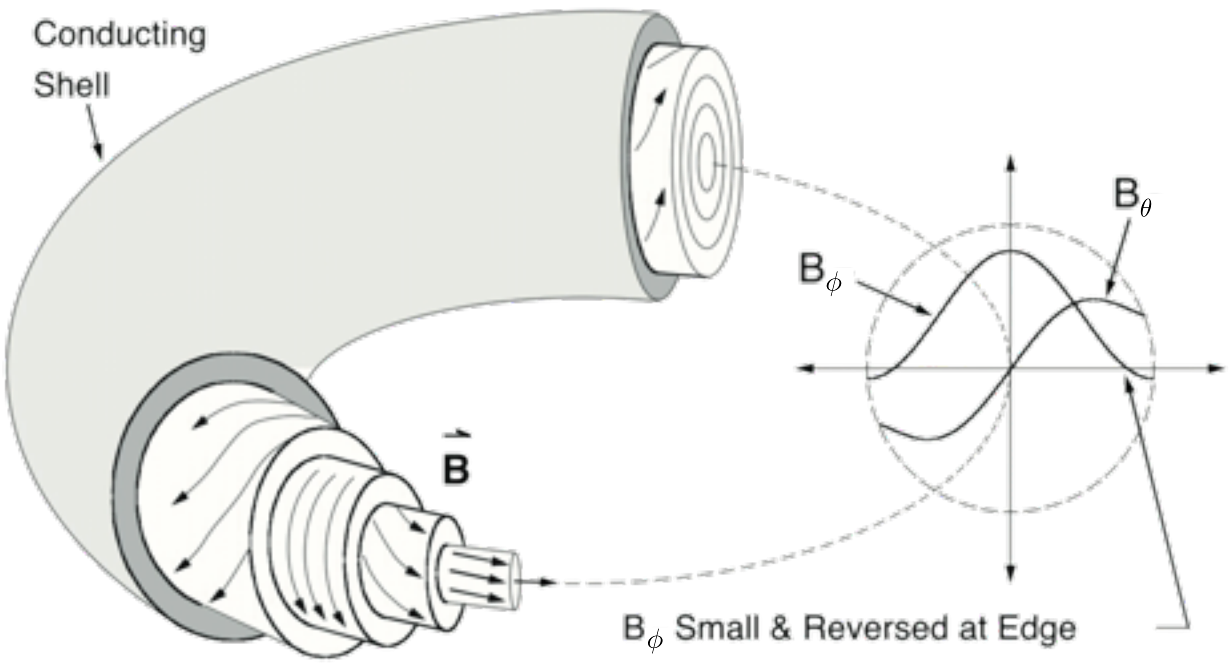
Australian National University

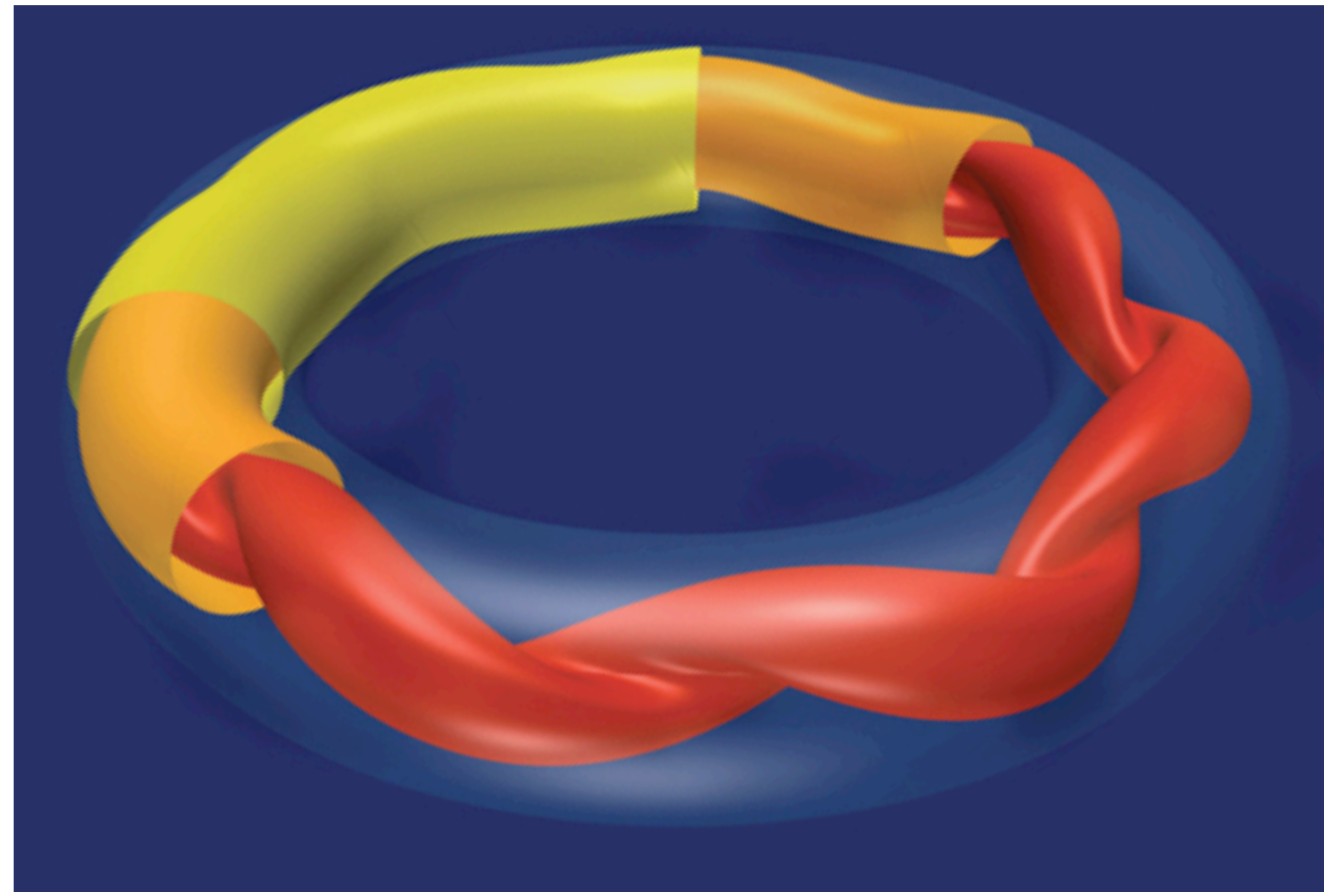
CONSORZIO RFX

PPPL
PRINCETON PLASMA PHYSICS LABORATORY

A self-organized helical state has been observed in RFP experiments



Magnetic Field Structure of the RFP



Limited confinement observed in "traditional" *axisymmetric* RFP states

Better confinement now observed when *helical* state forms in RFX-mod

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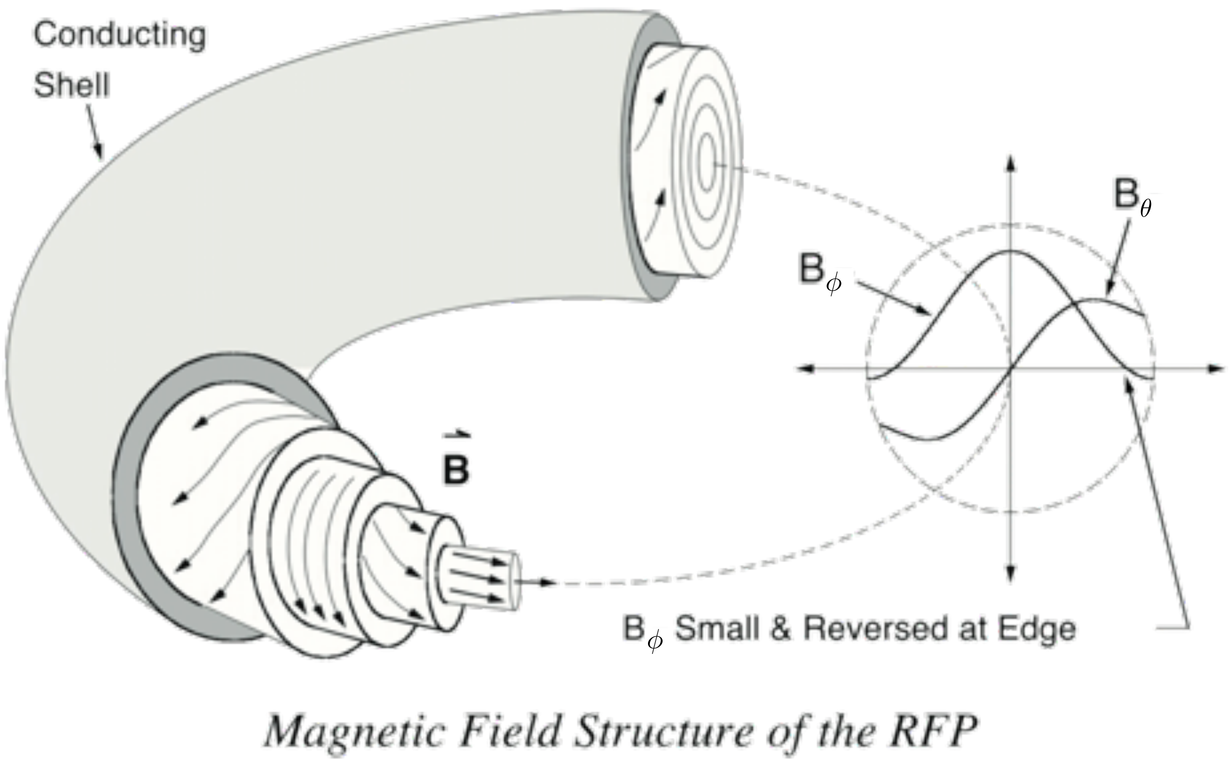

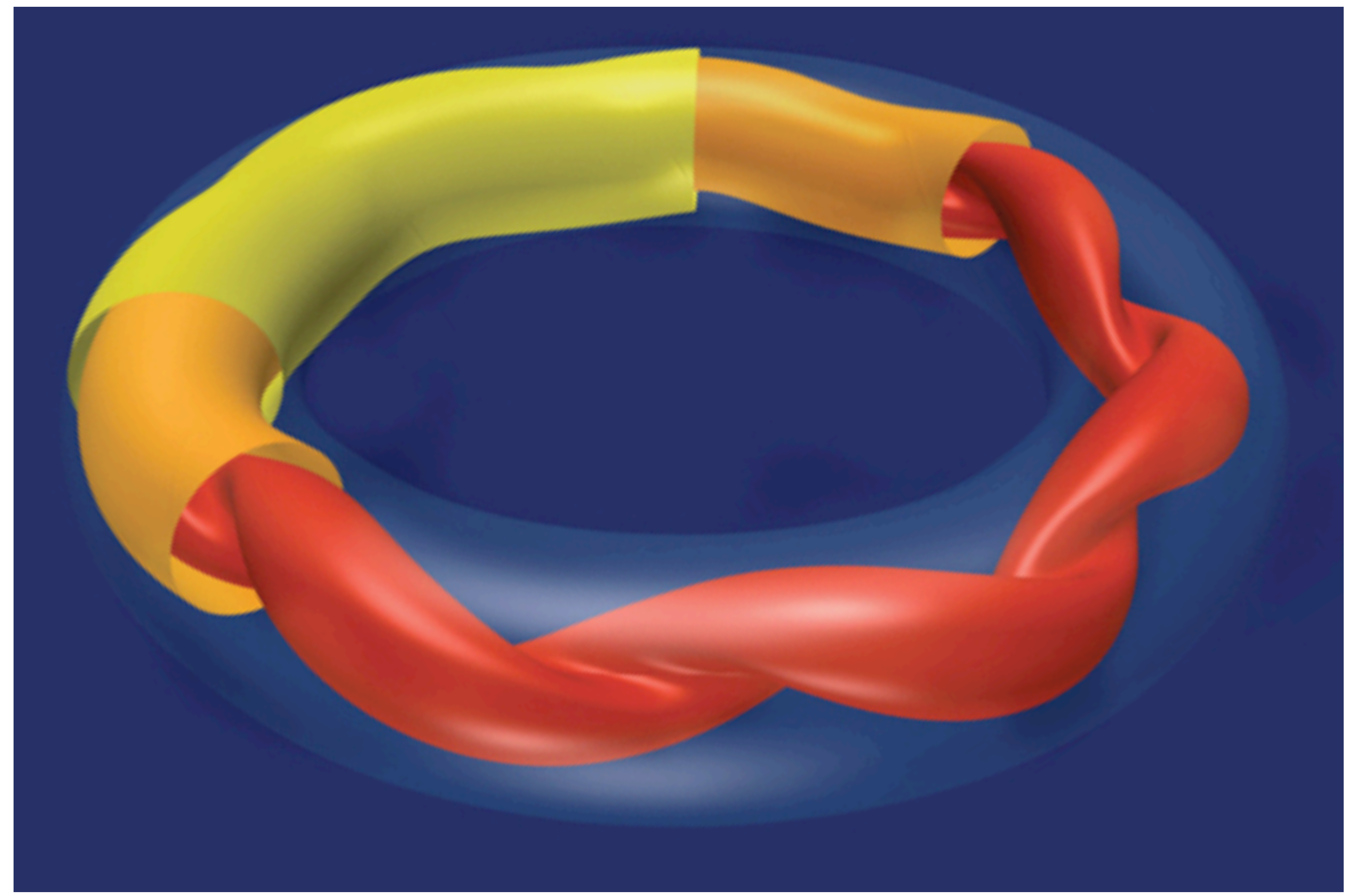

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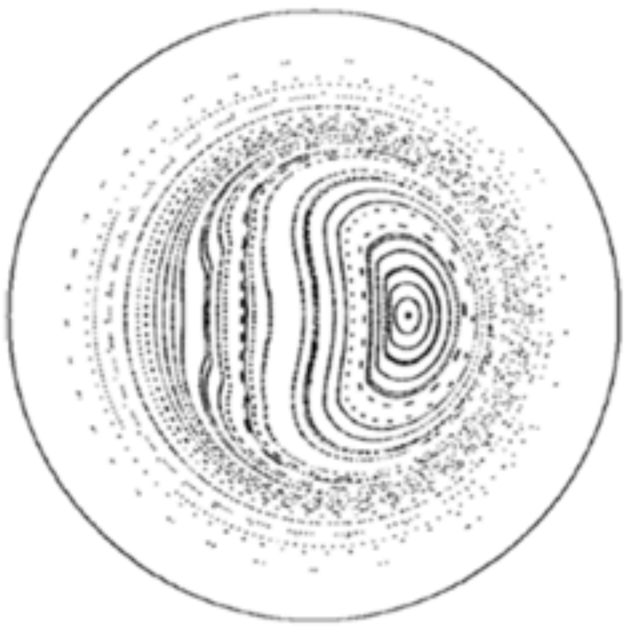
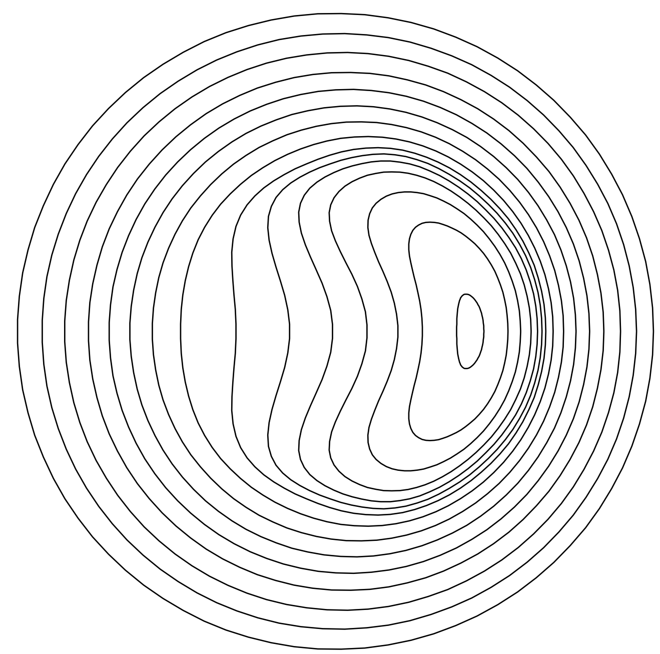
Better confinement now observed when *helical* state forms in RFX-mod

This structure occurs even for an axisymmetric plasma boundary, i.e. it is *self-organized.*

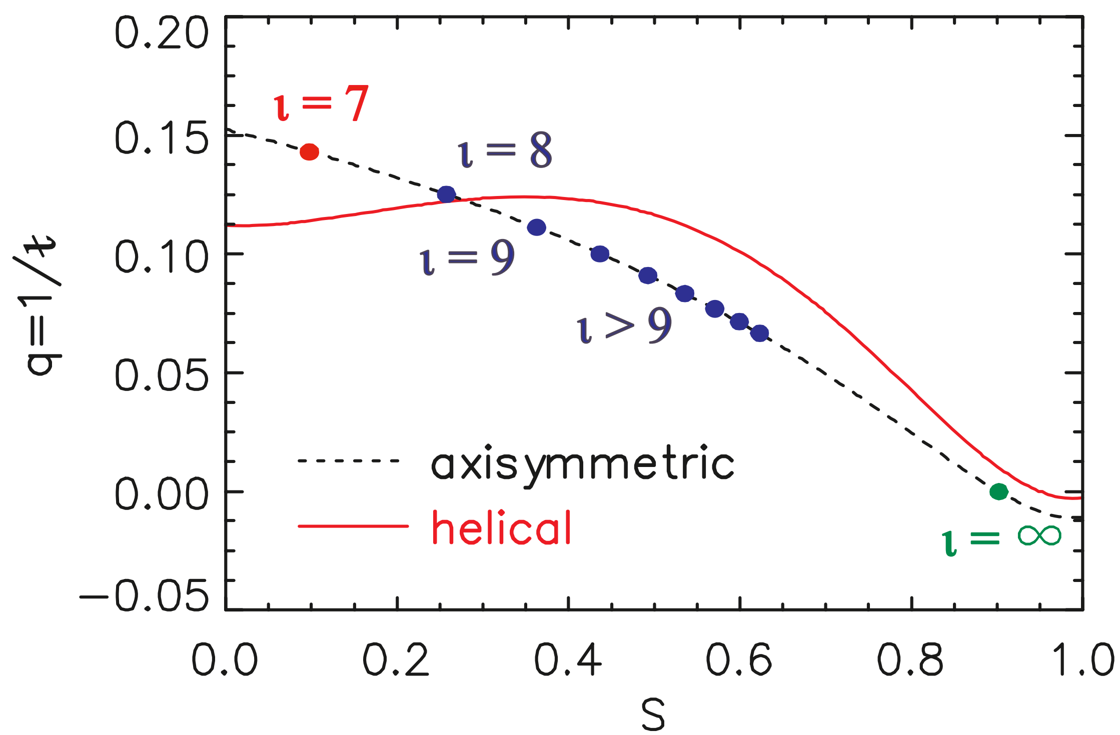
Ideal MHD can model the Single-Helical Axis state



"Experimental" Poincaré plot[1]



Theoretical reconstruction using VMEC[2]



Theoretical safety factor profile[2]

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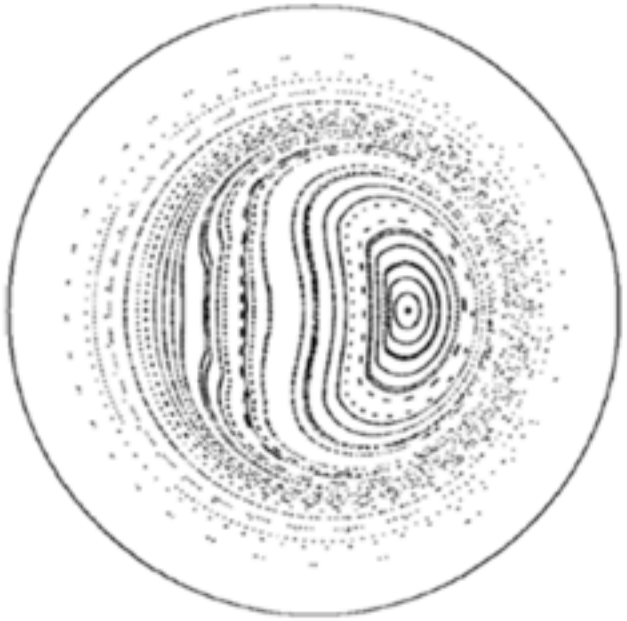

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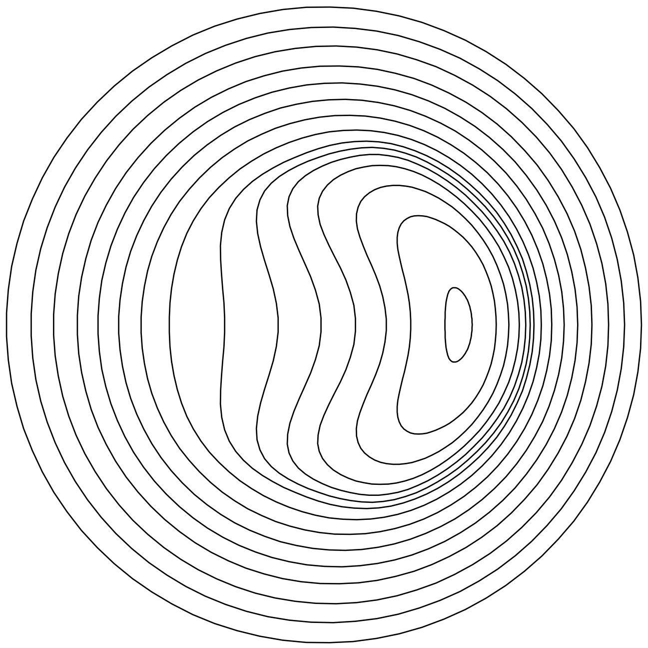

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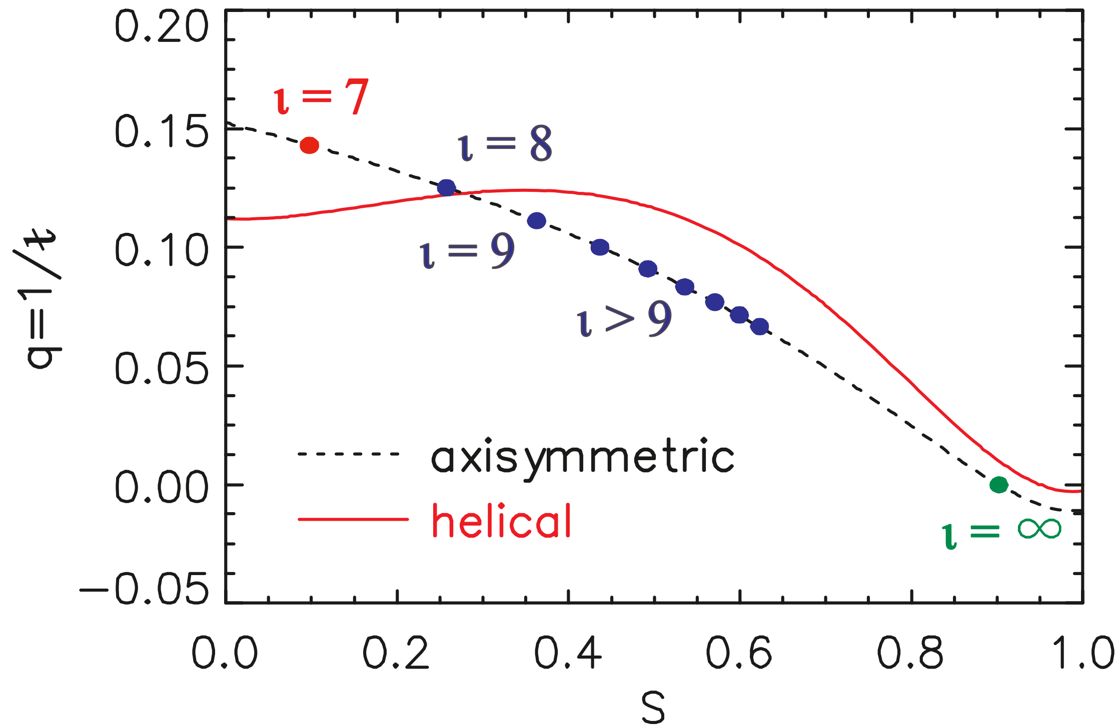

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…but the safety factor profile must be carefully chosen

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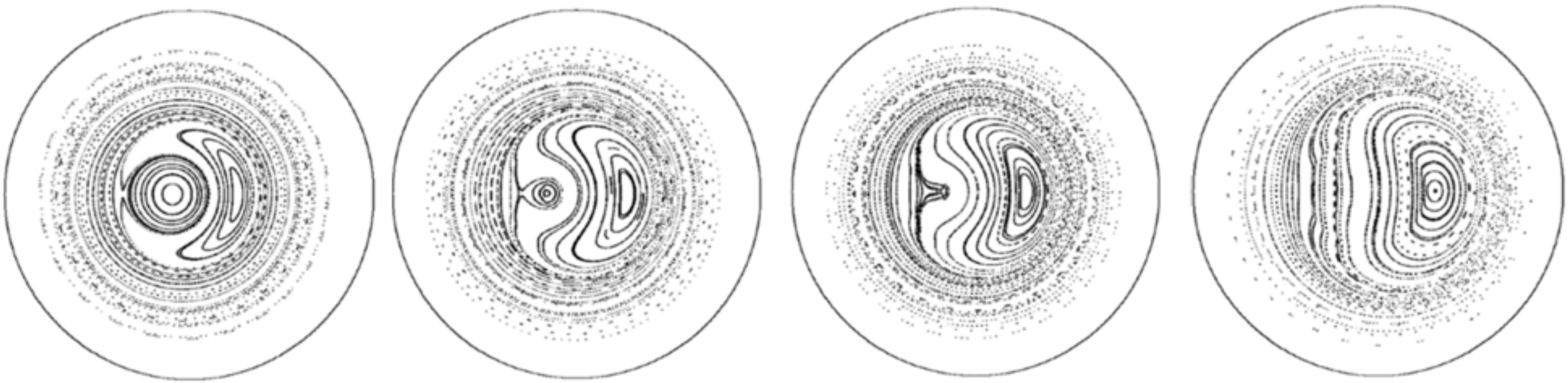
Helical states with non-trivial topology are also observed



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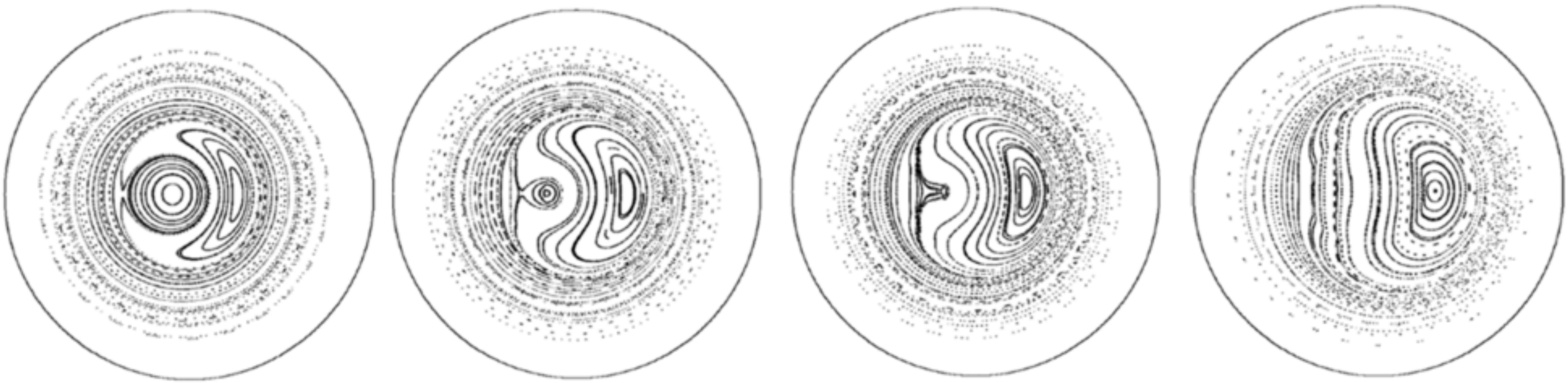

Double-Helical Axis state → Single Helical Axis state

Ideal MHD (with assumed nested flux surfaces) *cannot* model the Double-Helical Axis state.

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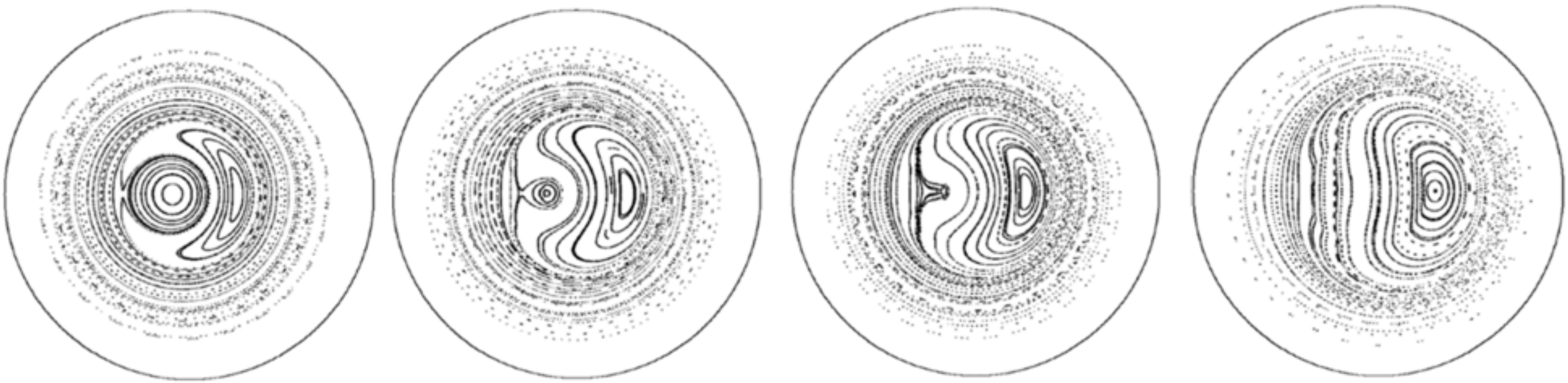

Double-Helical Axis state → Single Helical Axis state

We seek a *minimally constrained* model for all RFX helical states

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Taylor's theory is a good description of axisymmetric Reversed Field Pinches

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Multiple Region Relaxed MHD

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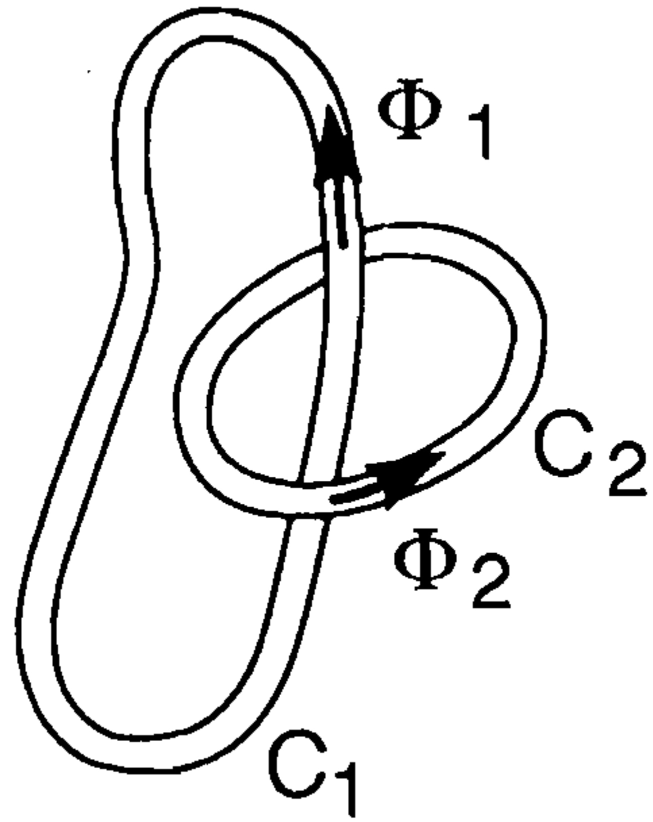

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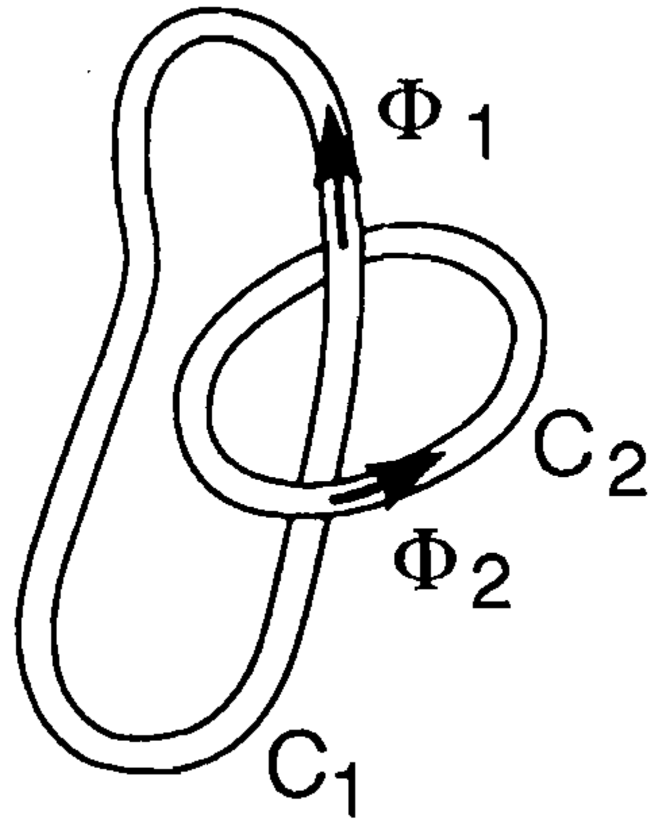

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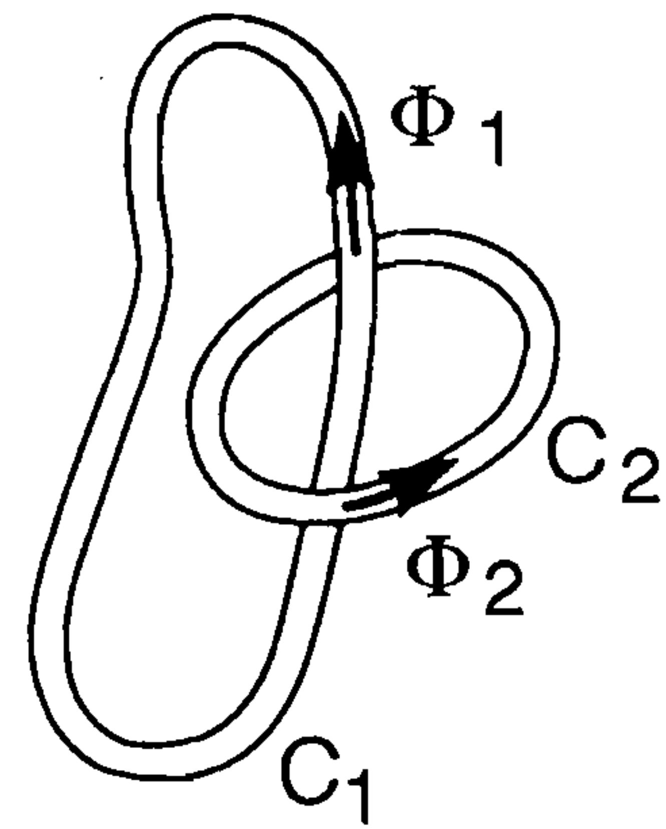

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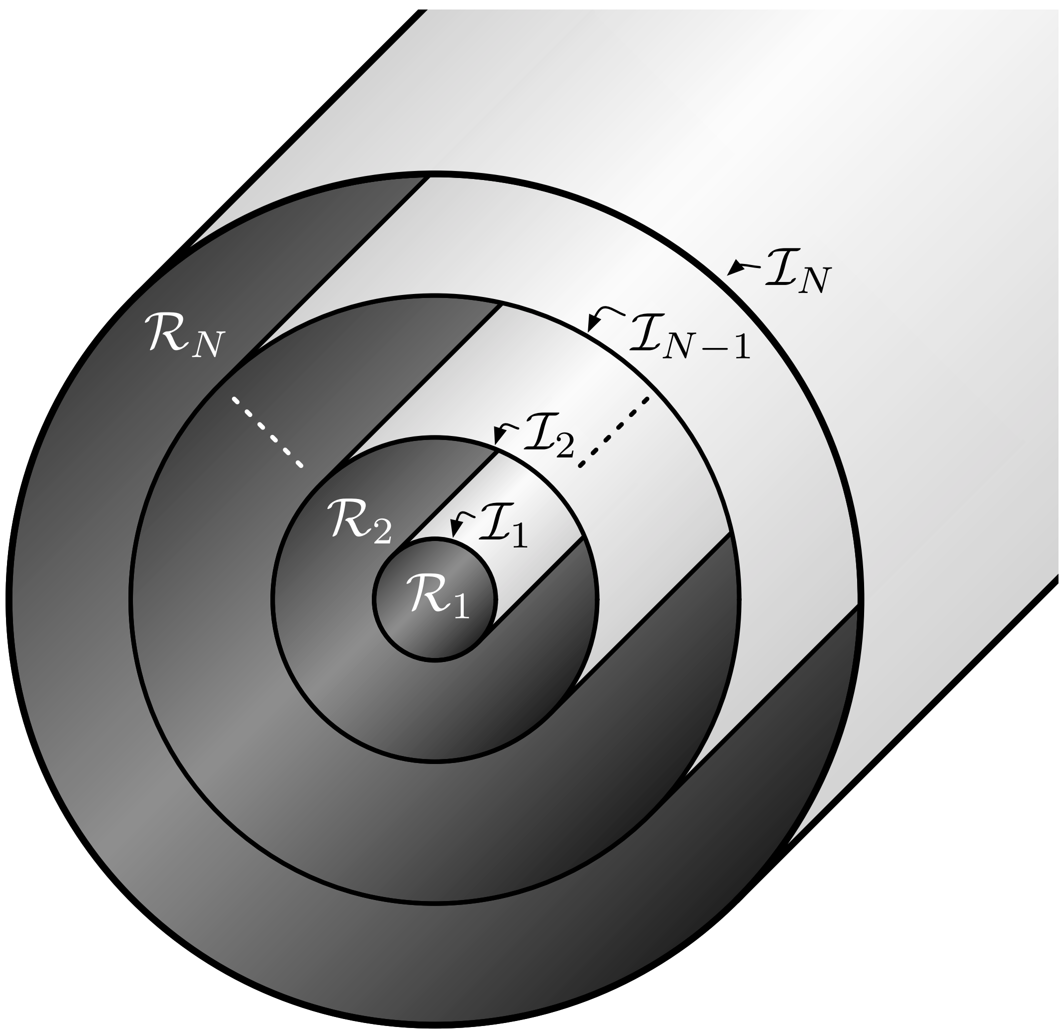
Motivation: with small resistivity, both energy and helicity will decay

$$\dot{H} = \eta \int \mathbf{J} \cdot \mathbf{B} \, dV \sim \eta \sum_k k^1 \mathbf{B}_k^2$$

$$\dot{E} = \eta \int \mathbf{J} \cdot \mathbf{J} \, dV \sim \eta \sum_k k^2 \mathbf{B}_k^2$$

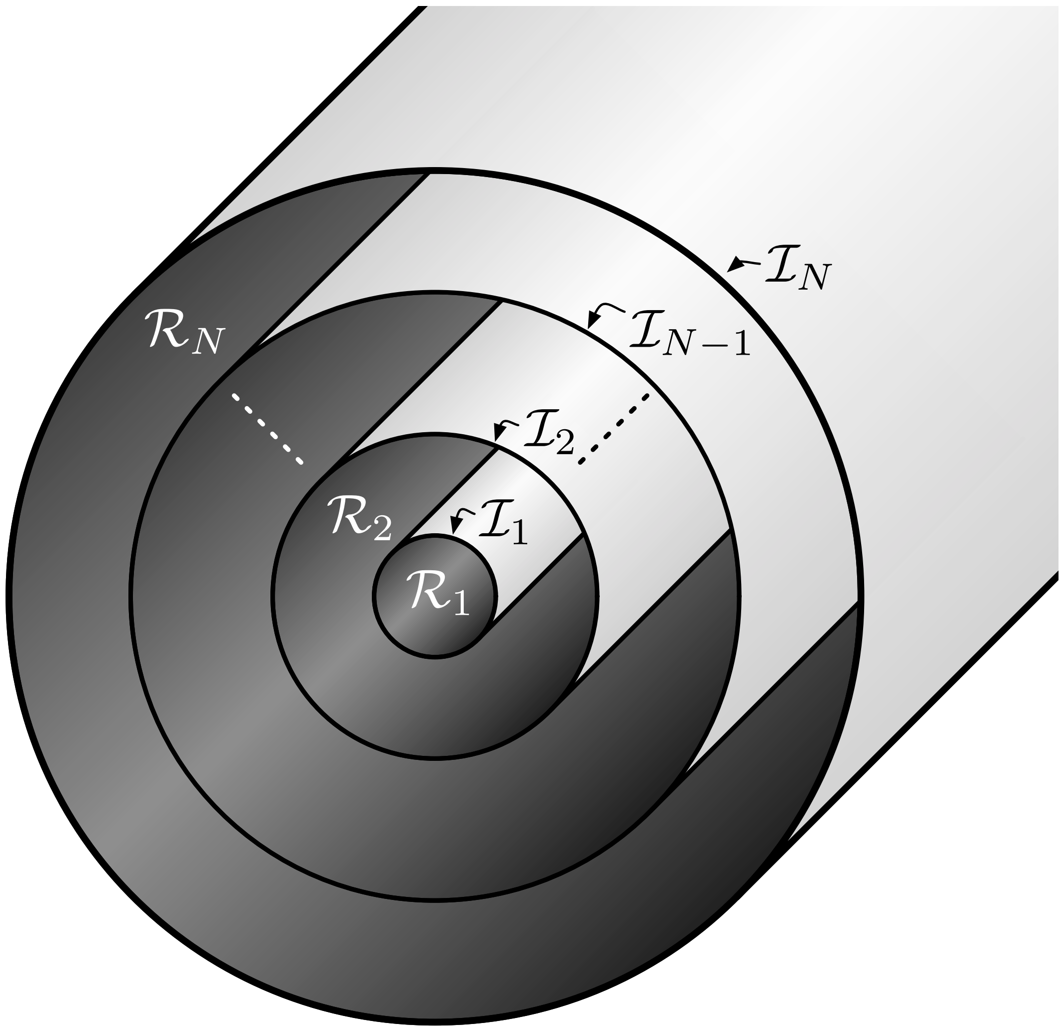
... but energy more quickly (for short length-scale turbulence)

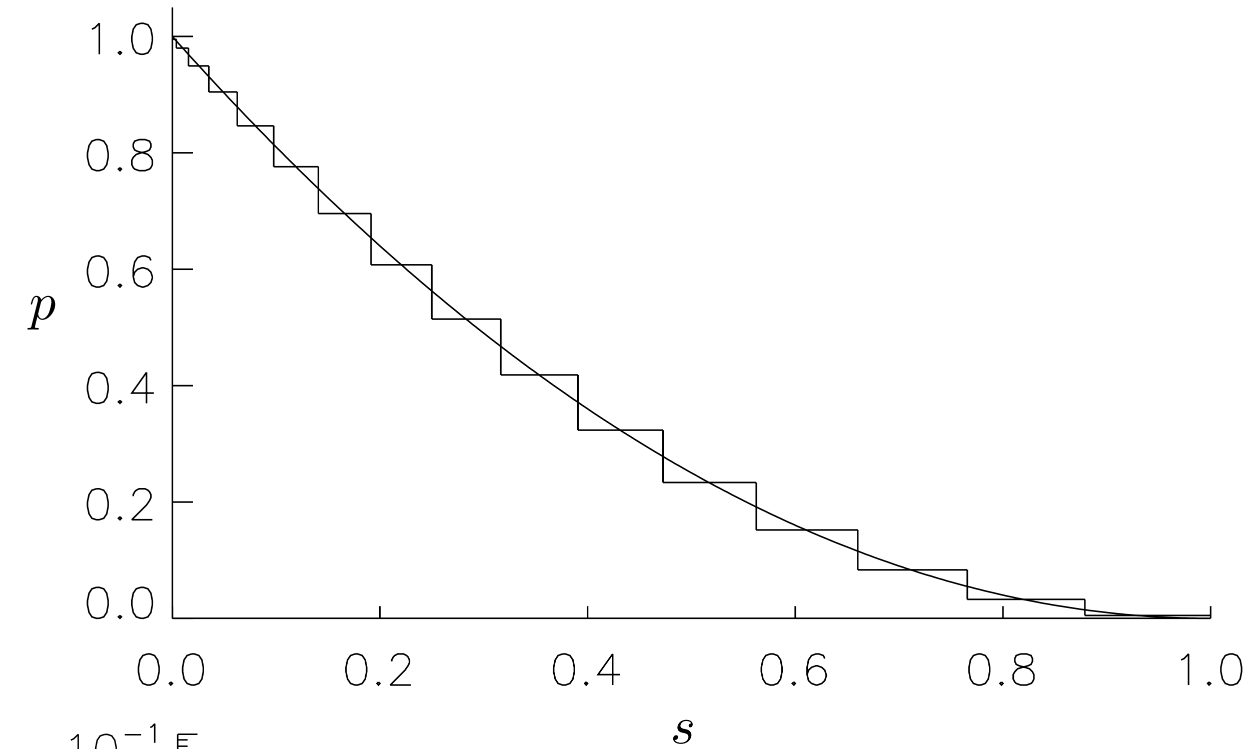
Multiple-Region Relaxed MHD (MRXMHD) extends Taylor Relaxation

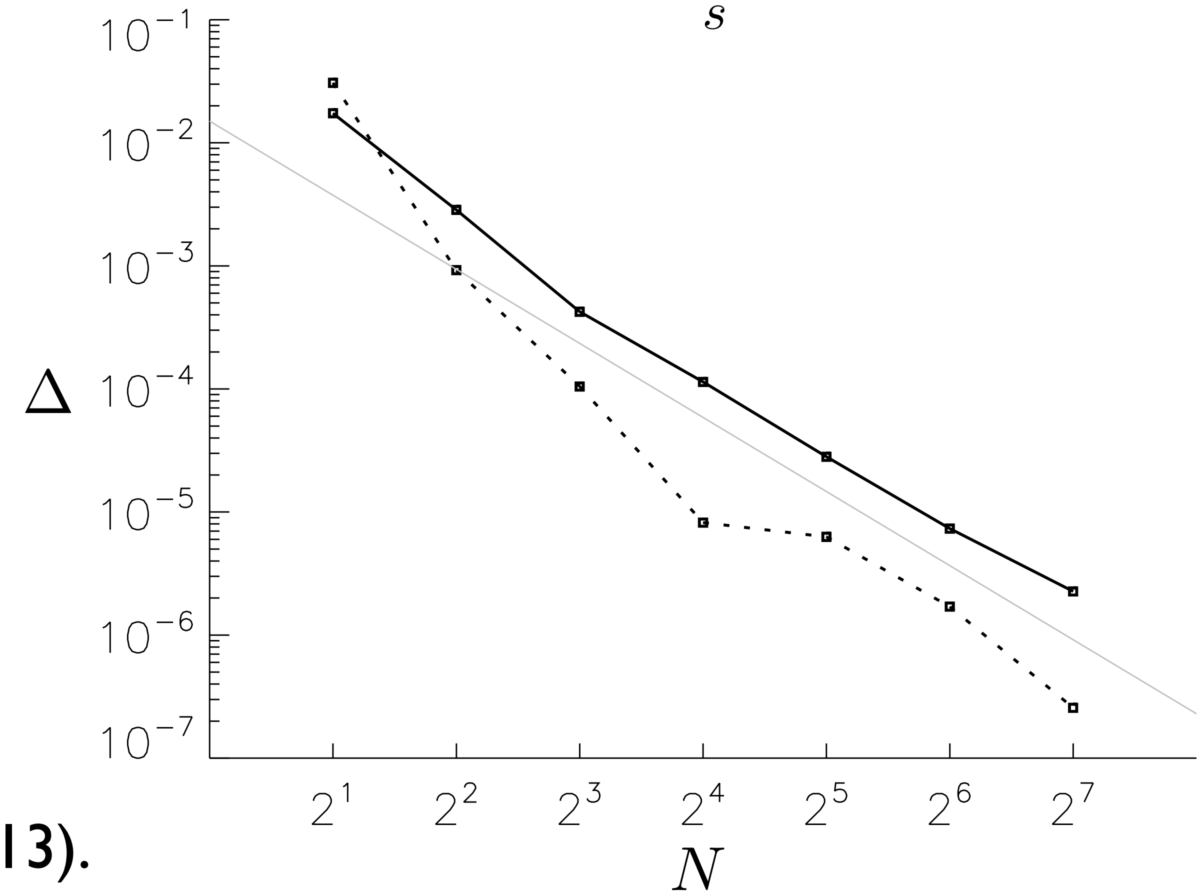


- Relaxed regions $\mathcal{R}_i$, separated by
- nested, ideal, toroidal barrier interfaces $\mathcal{I}_i$, which
- independently undergo Taylor relaxation.
- Magnetic islands and chaos are allowed between the toroidal current sheets
- Each plasma region has constant pressure, creating a piecewise constant pressure profile

Multiple-Region Relaxed MHD (MRXMHD) approaches ideal MHD as $N \to \infty$







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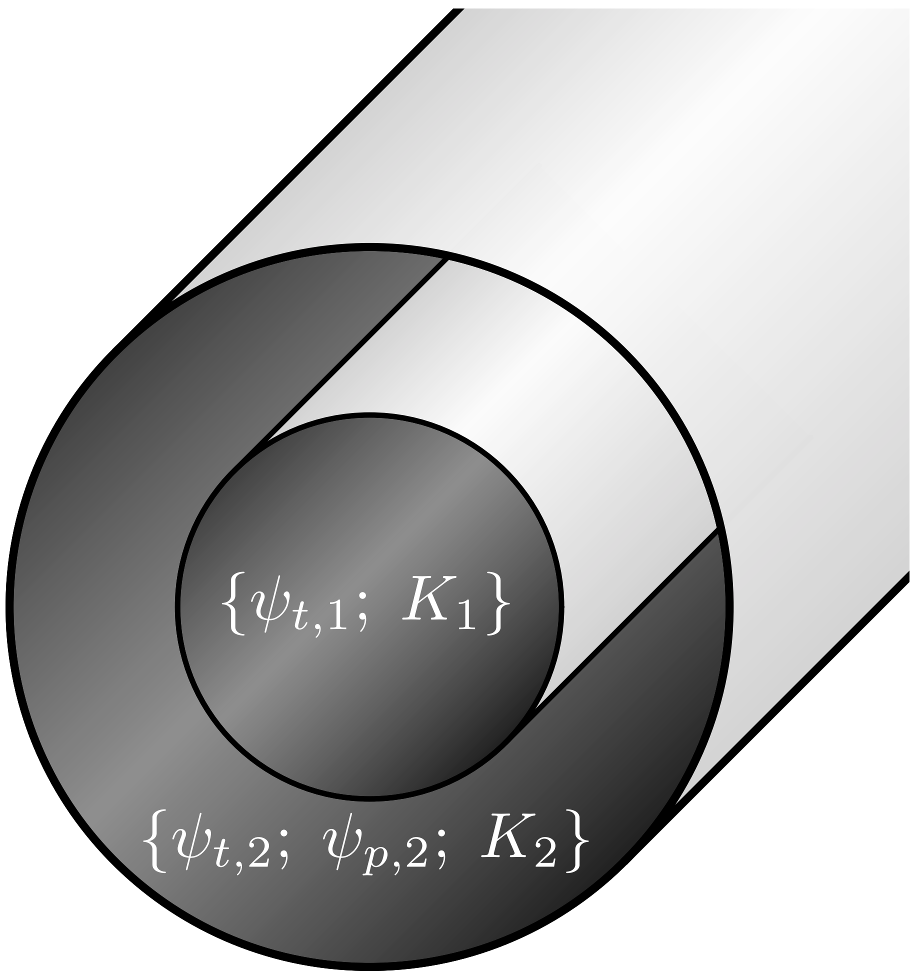
A two-volume MRXMHD model (without pressure) is constrained by 5 parameters



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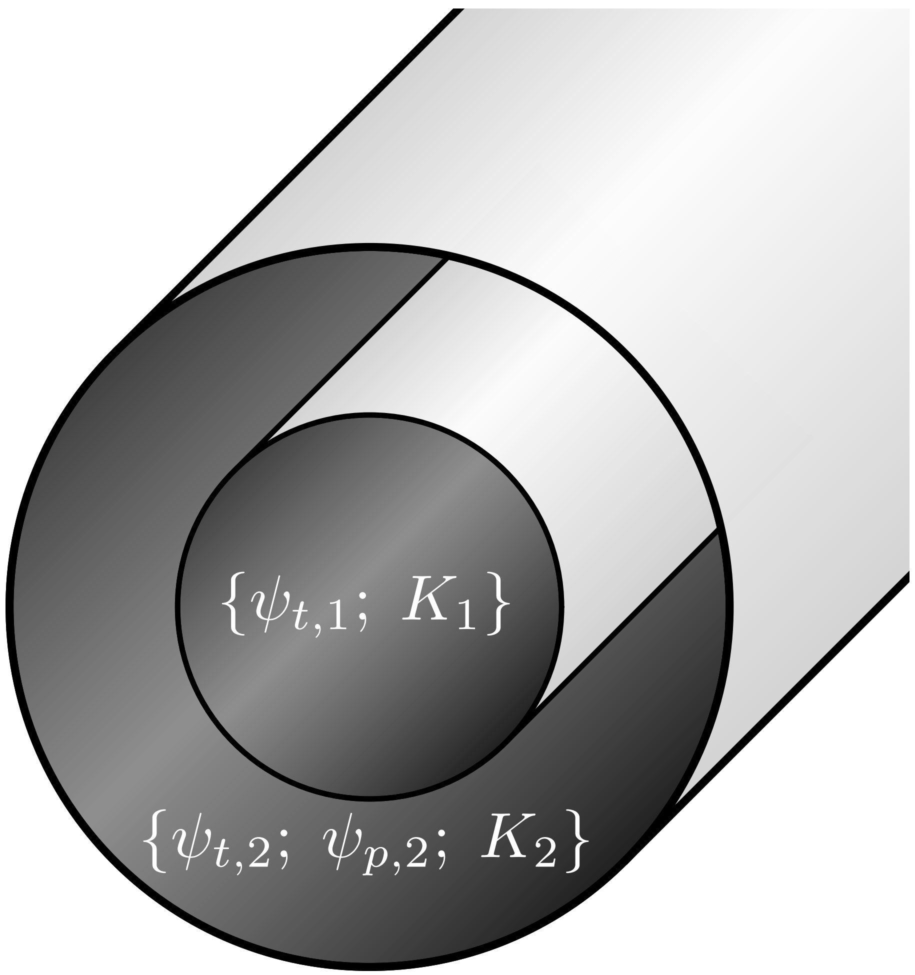

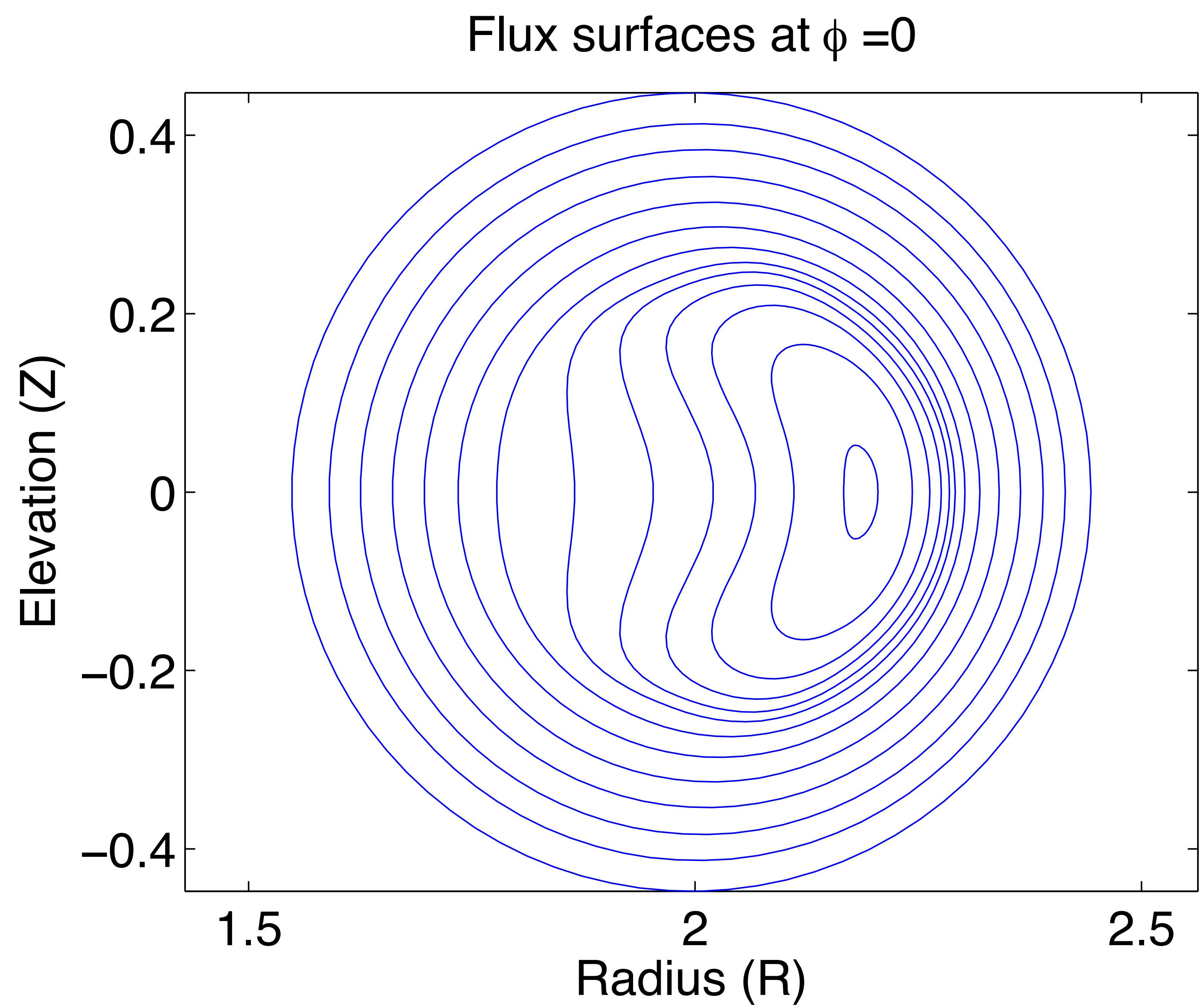

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i.e. examine 1D line in 5D parameter space

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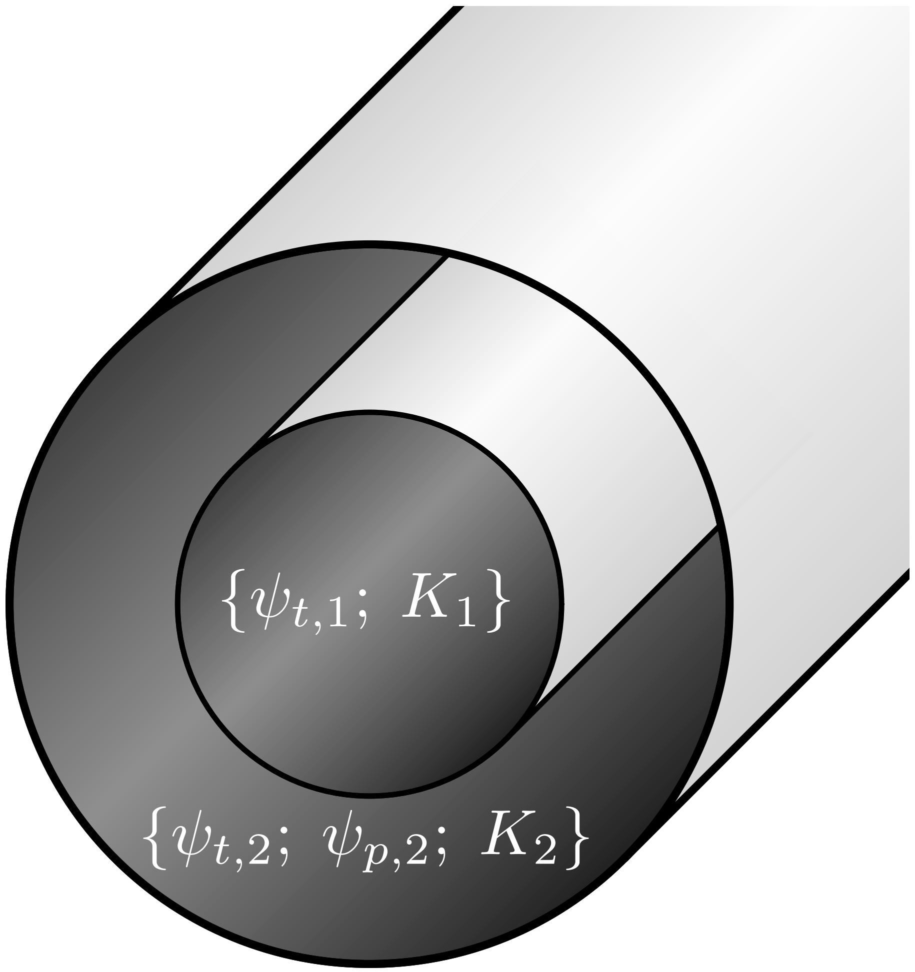

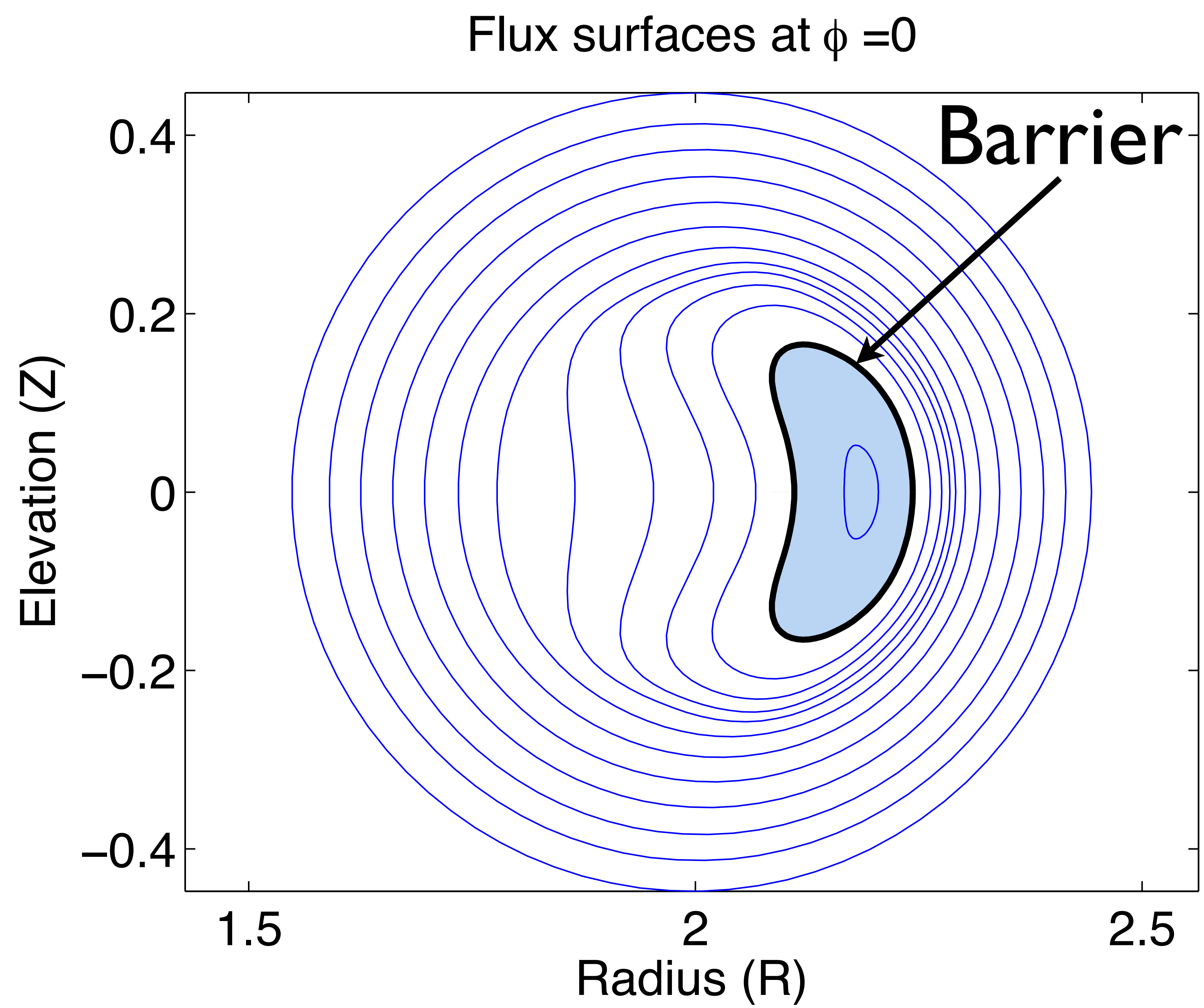

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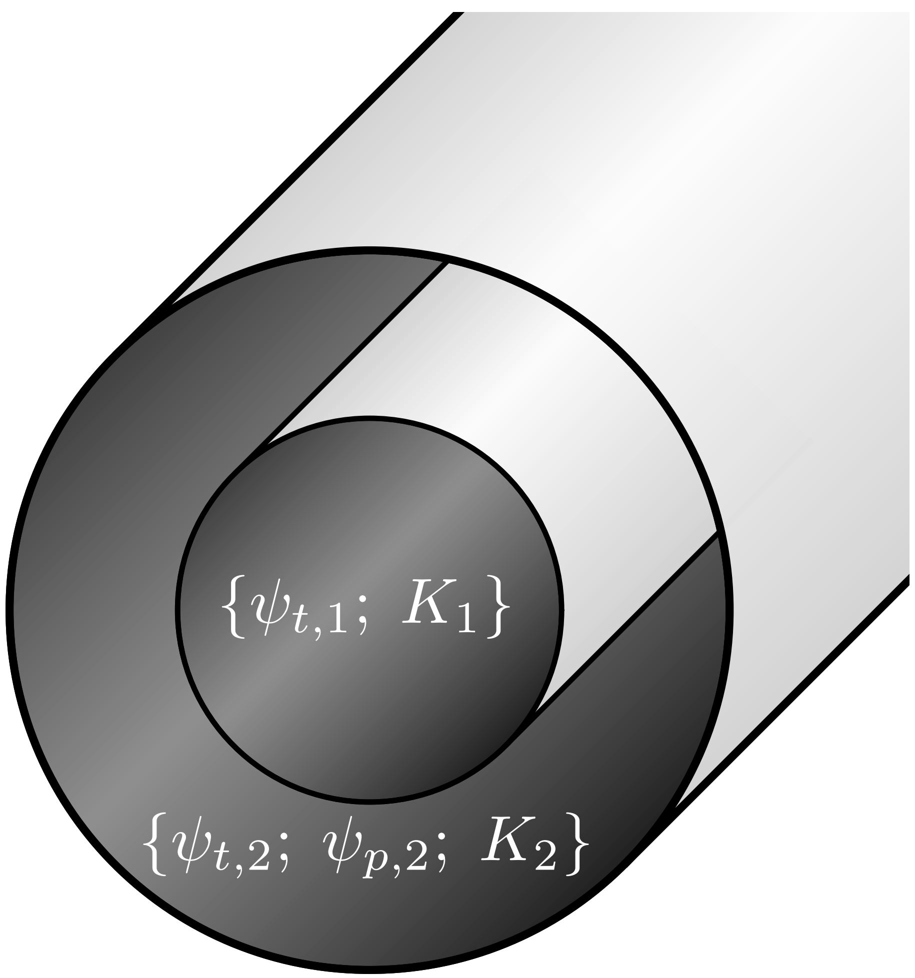

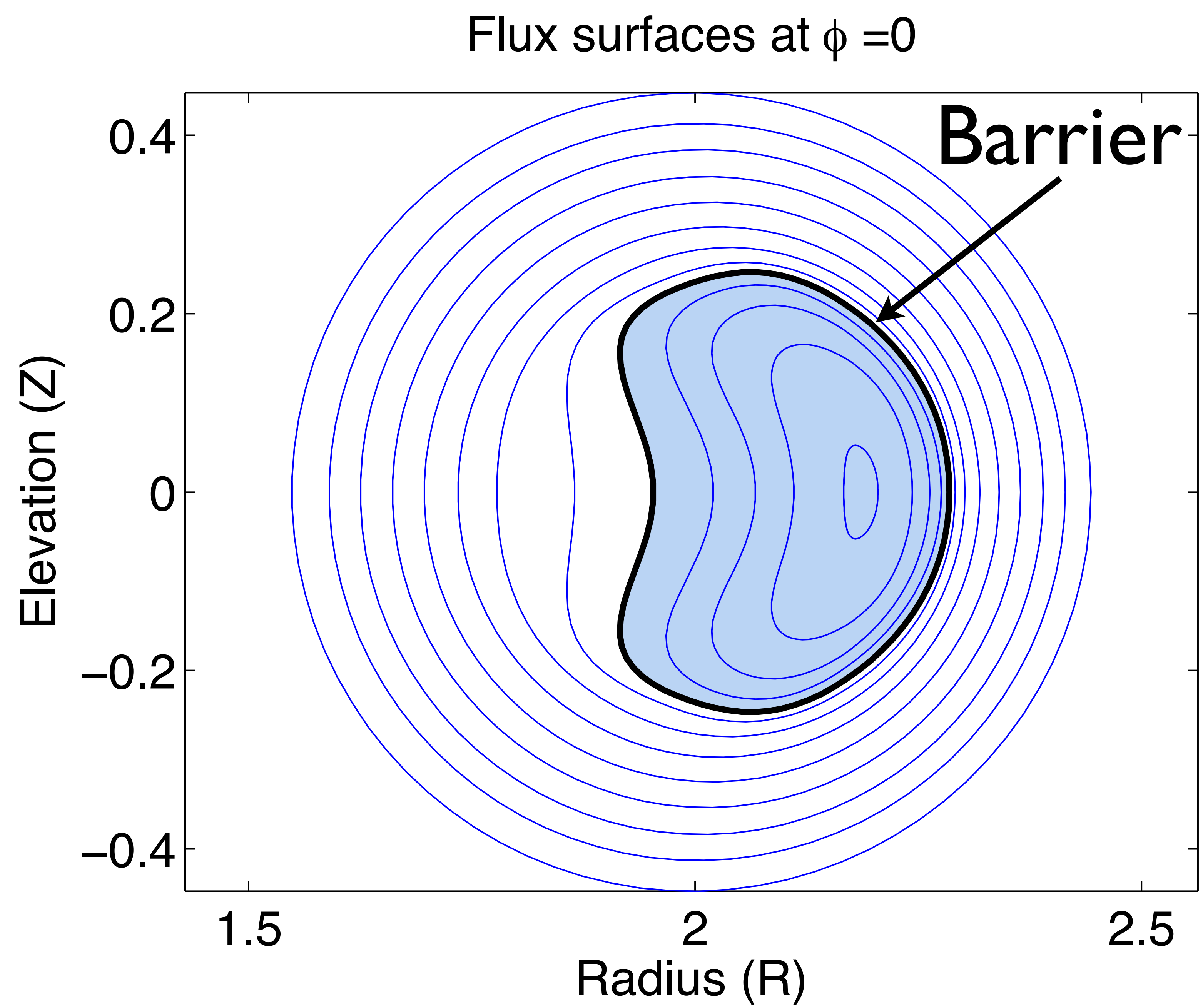

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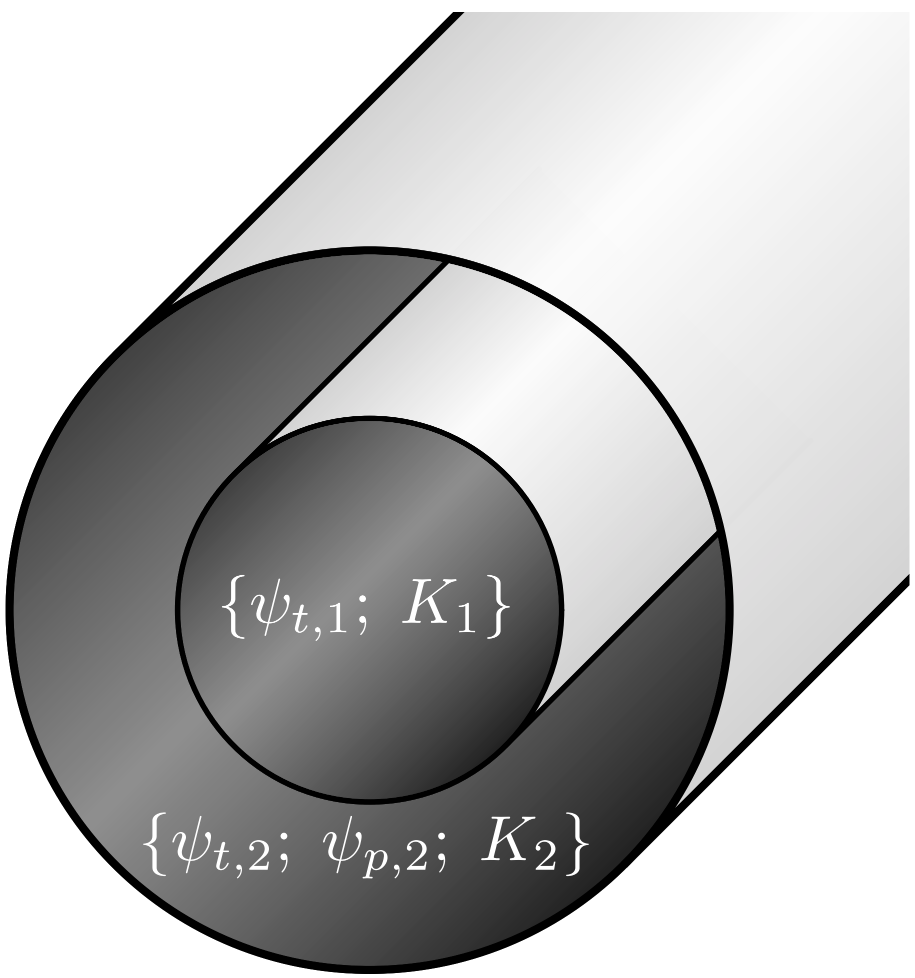

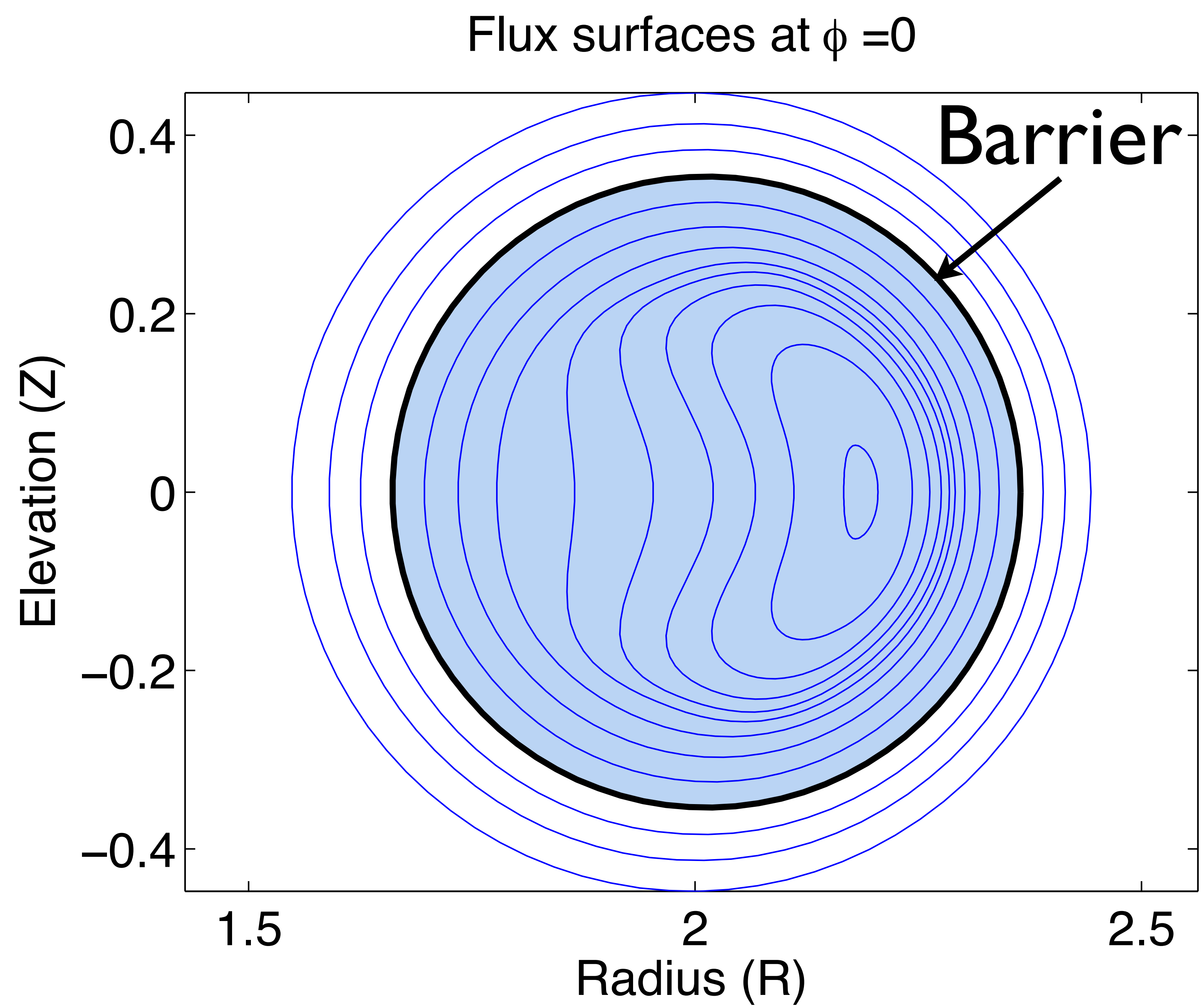

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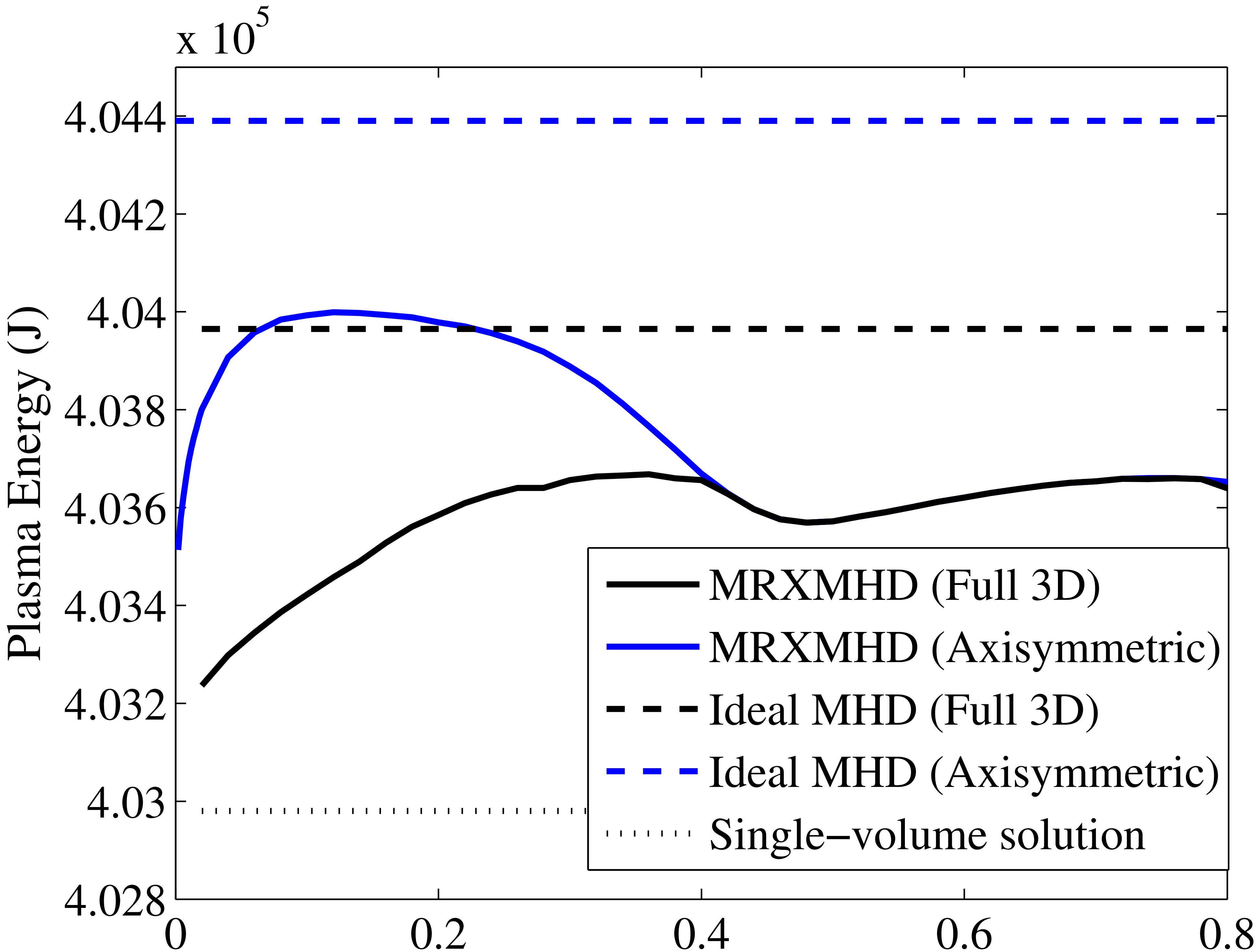

Ideal MHD flux surface chosen as ideal barrier

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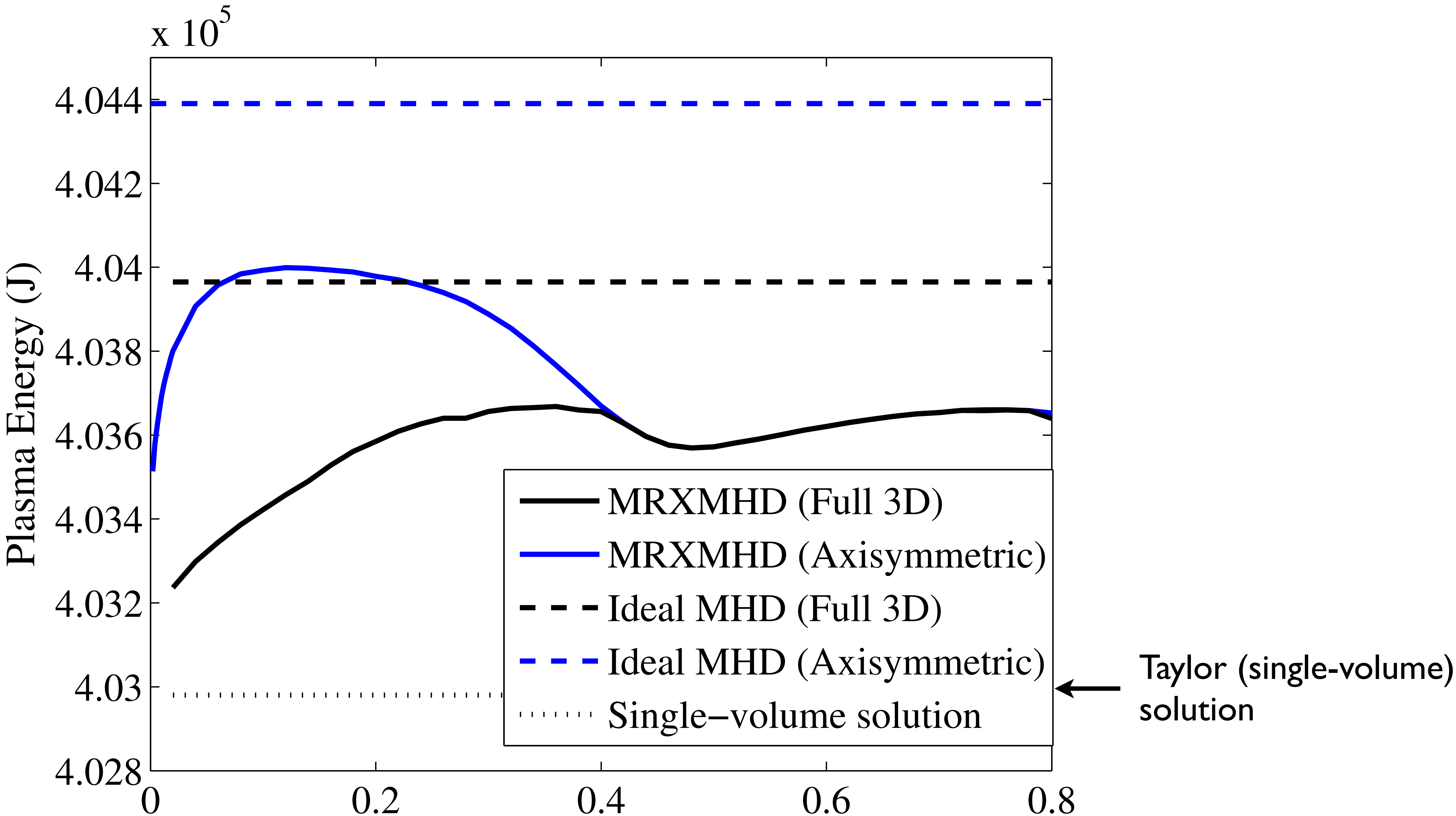

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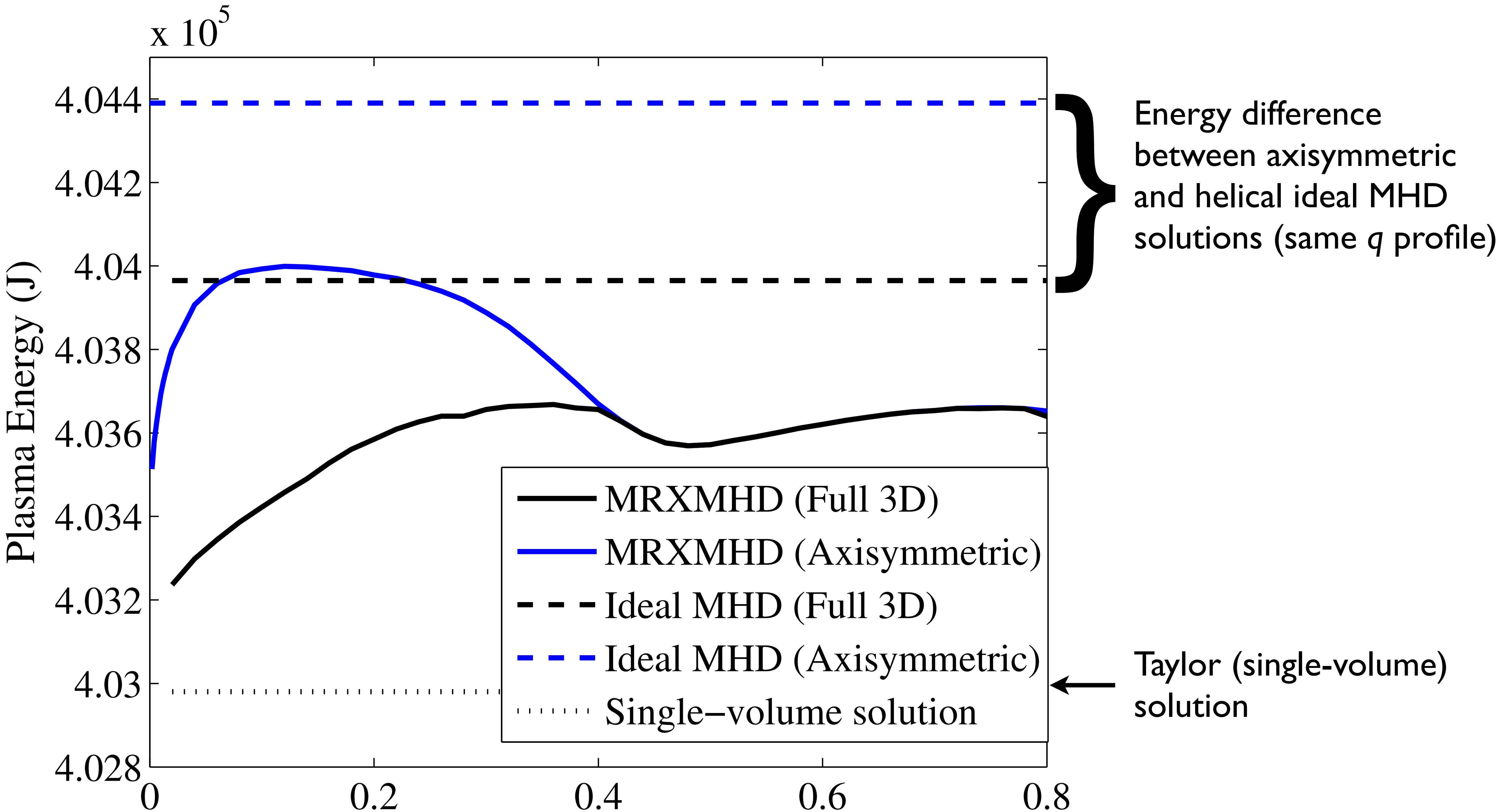

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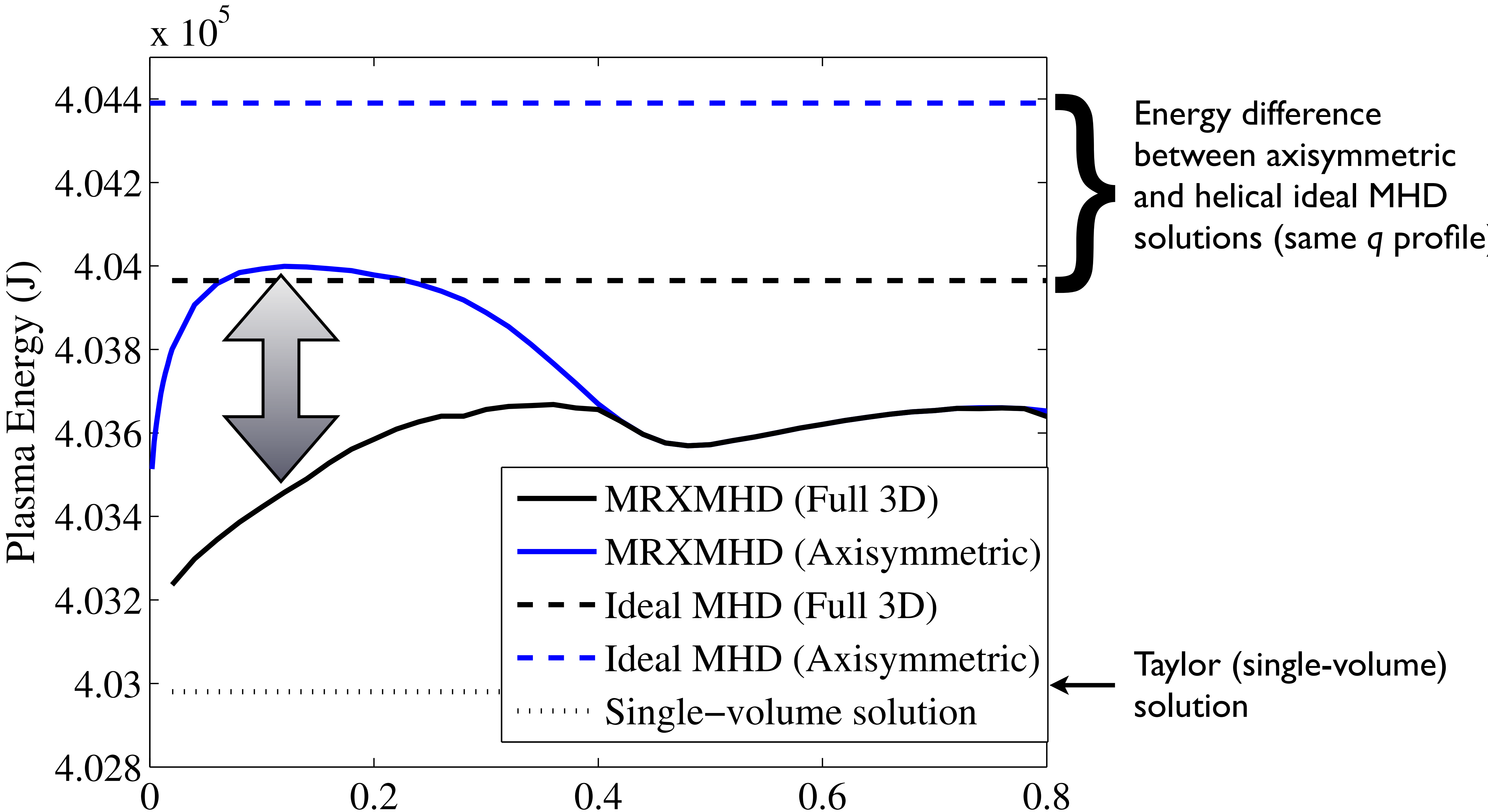

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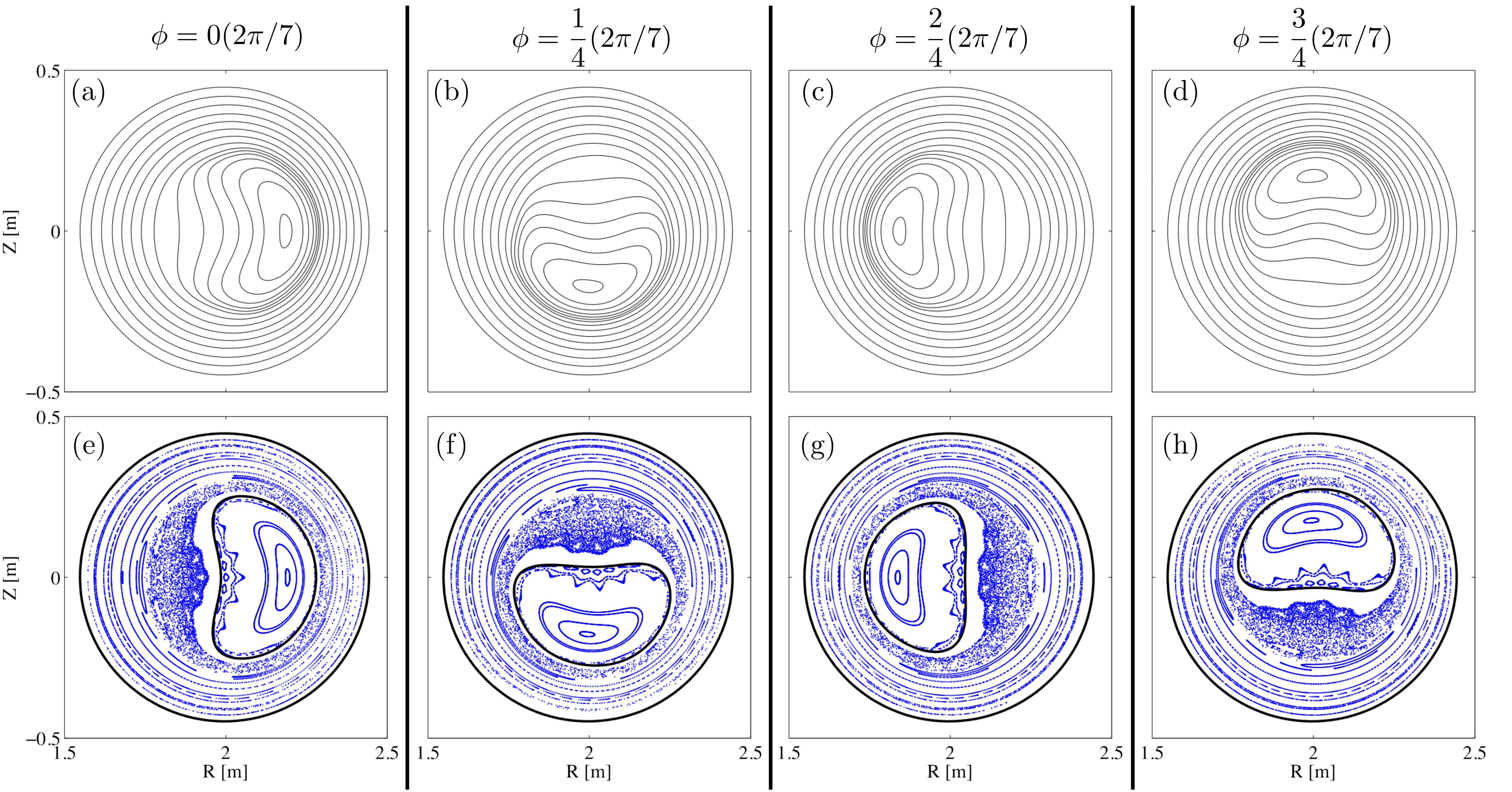
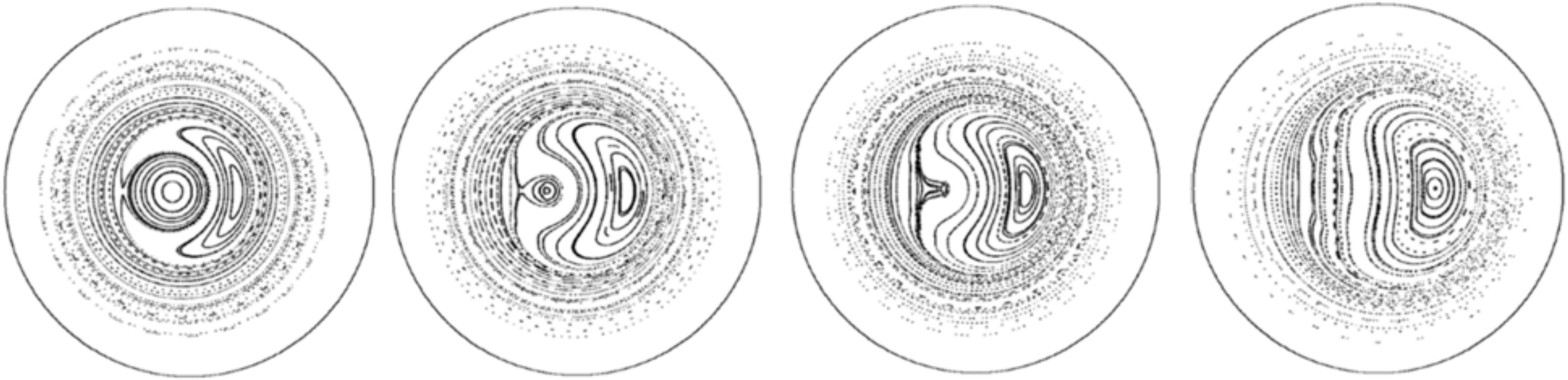
Comparison of VMEC and SPEC RFX-mod equilibria



“Experimental” Poincaré plots



Quasi-single helicity → Single Helical Axis

Top figure source: P. Martin *et al.*, *Nuclear Fusion* **49**, 104019 (2009).

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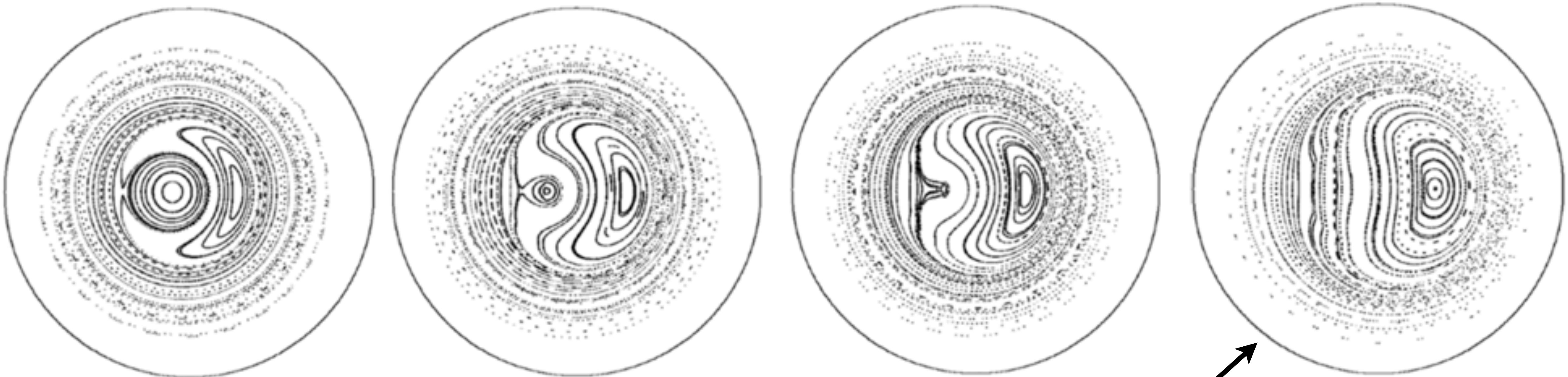

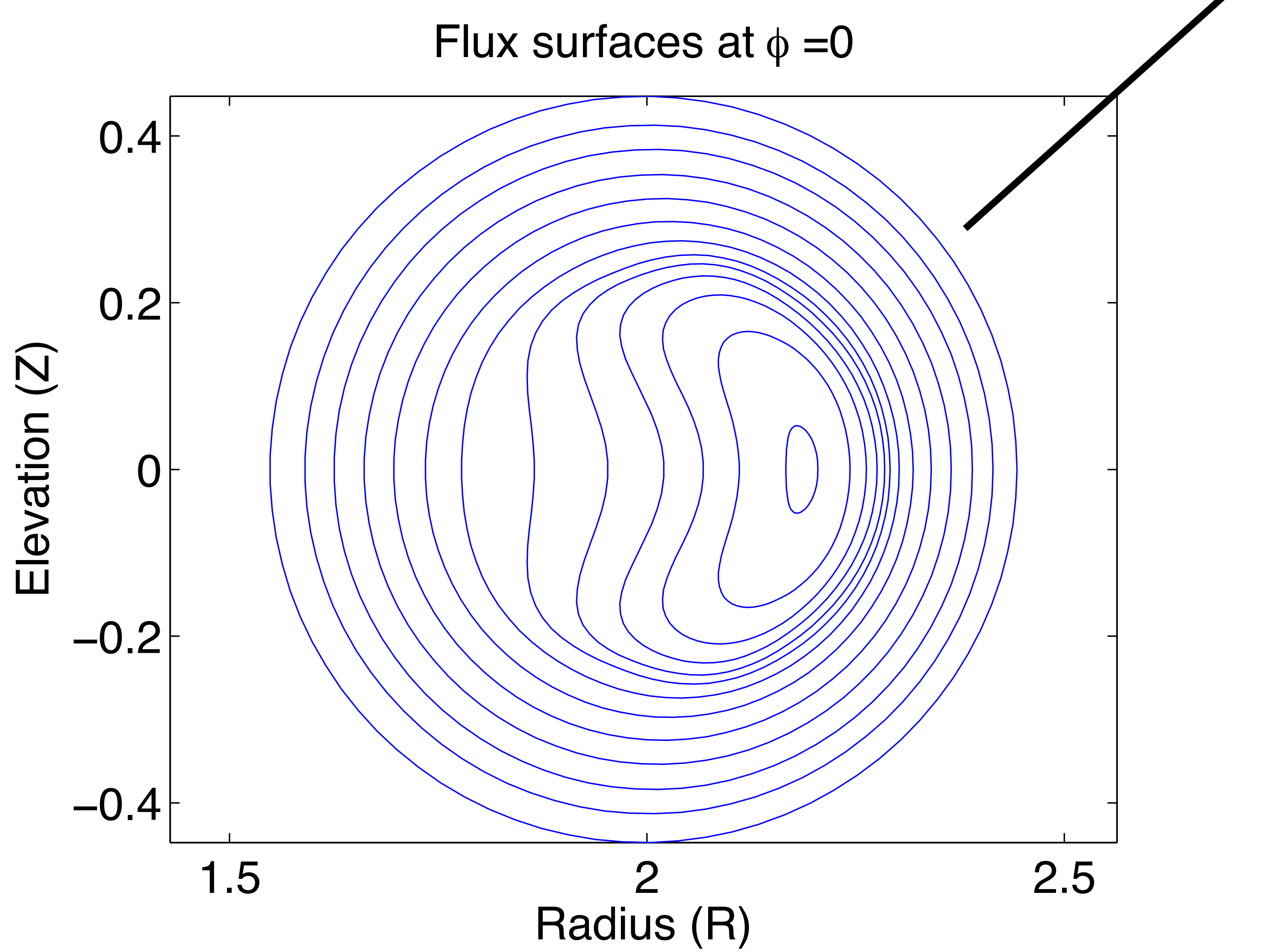

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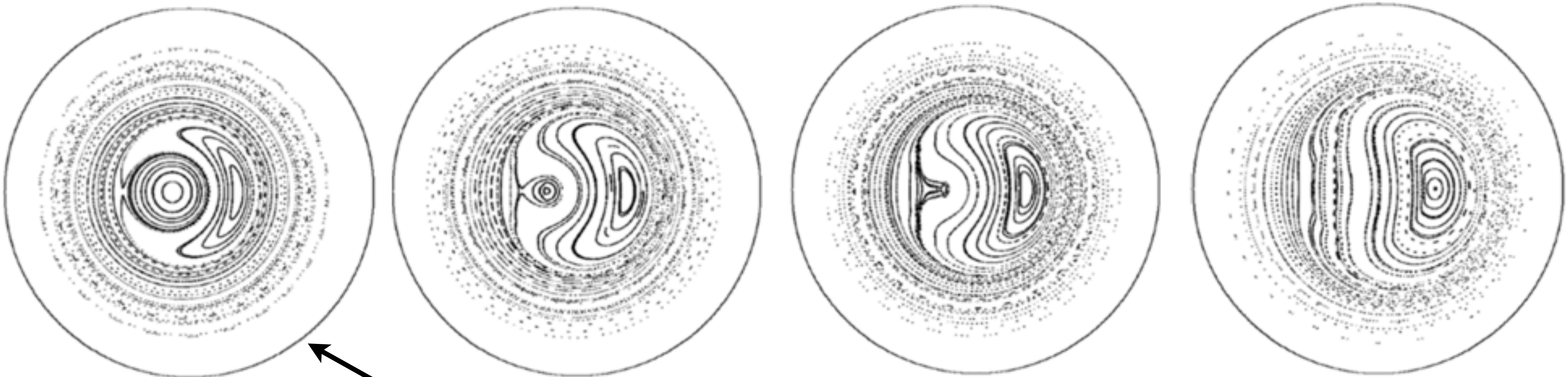

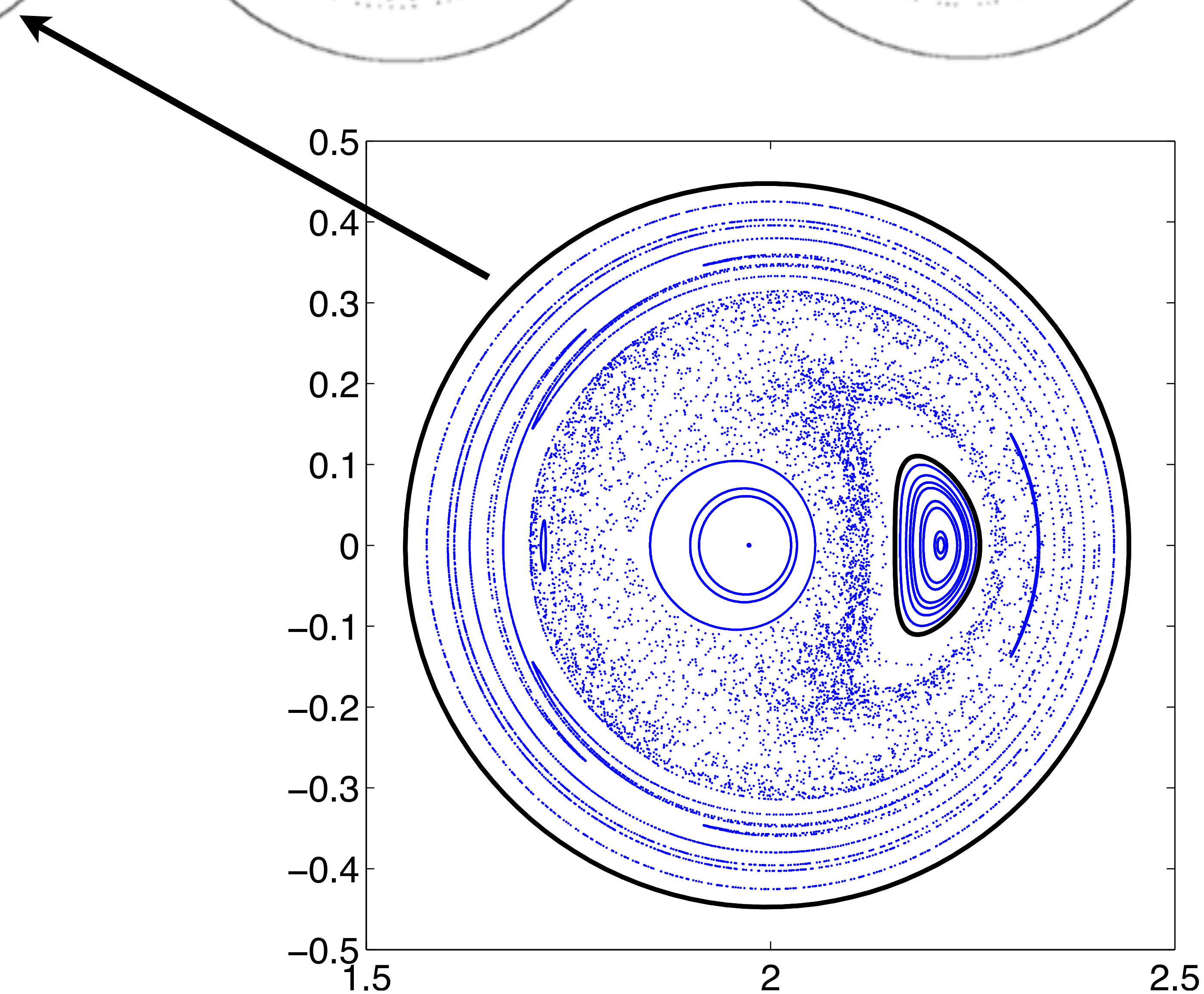

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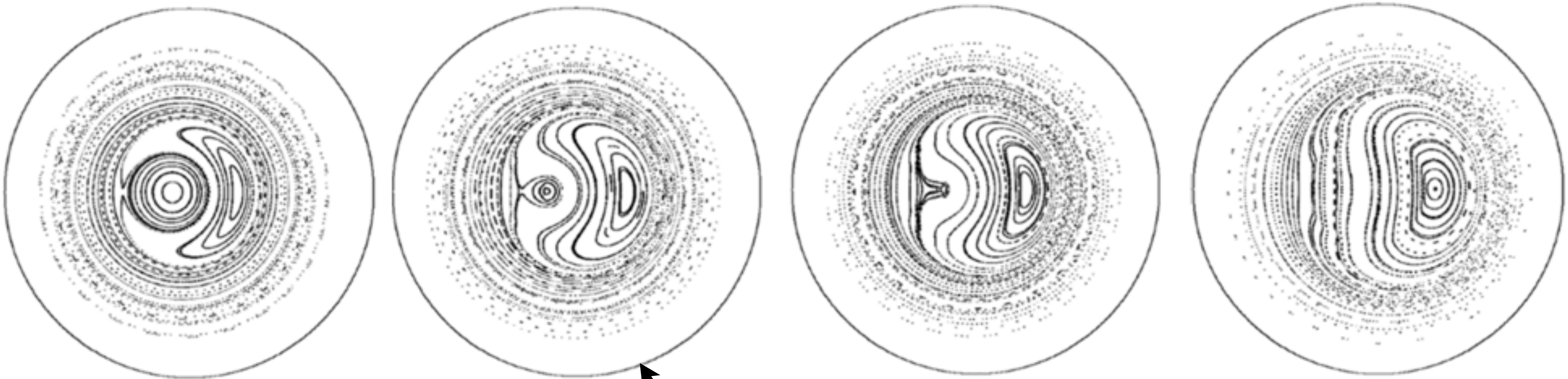

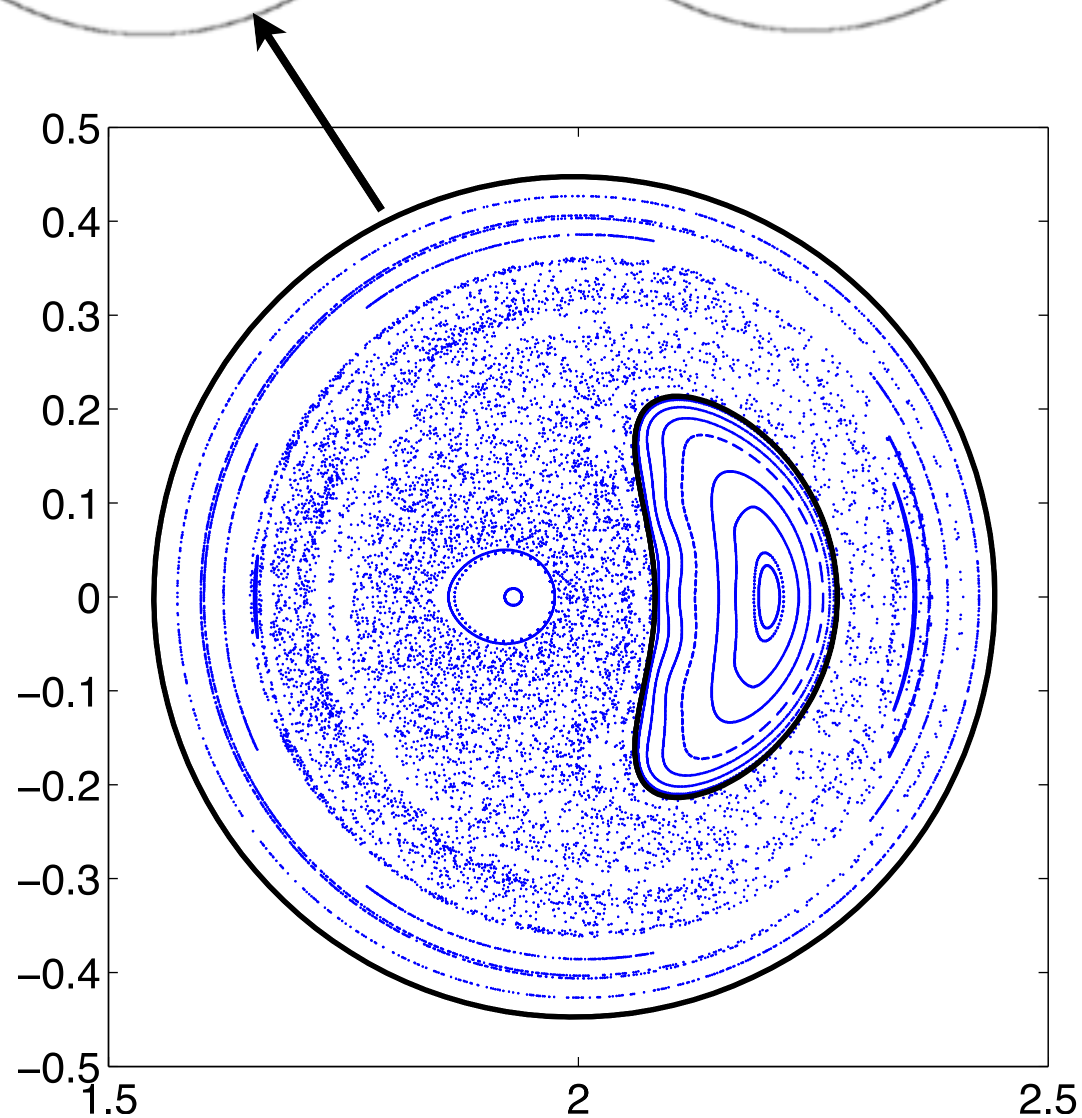

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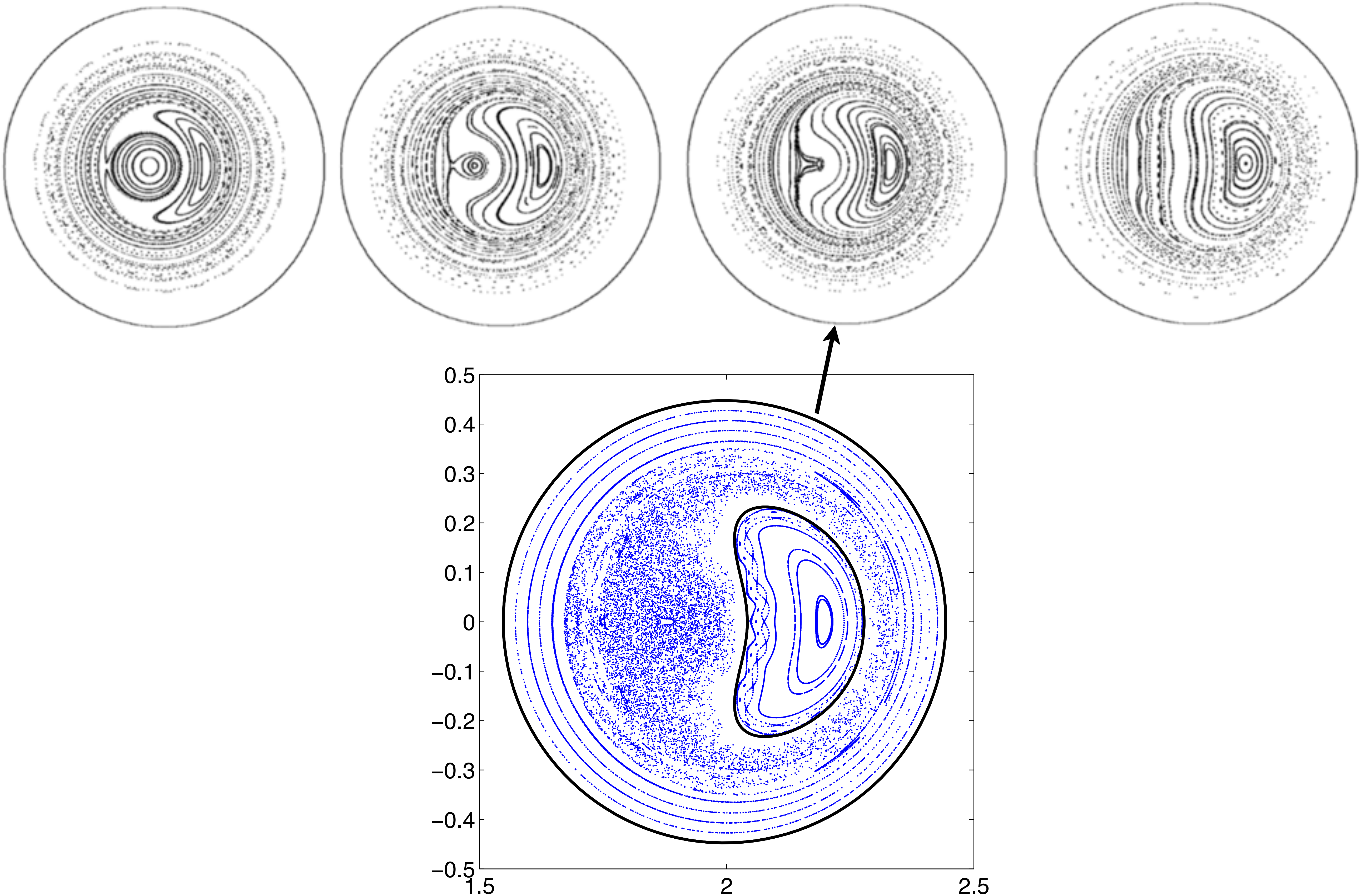

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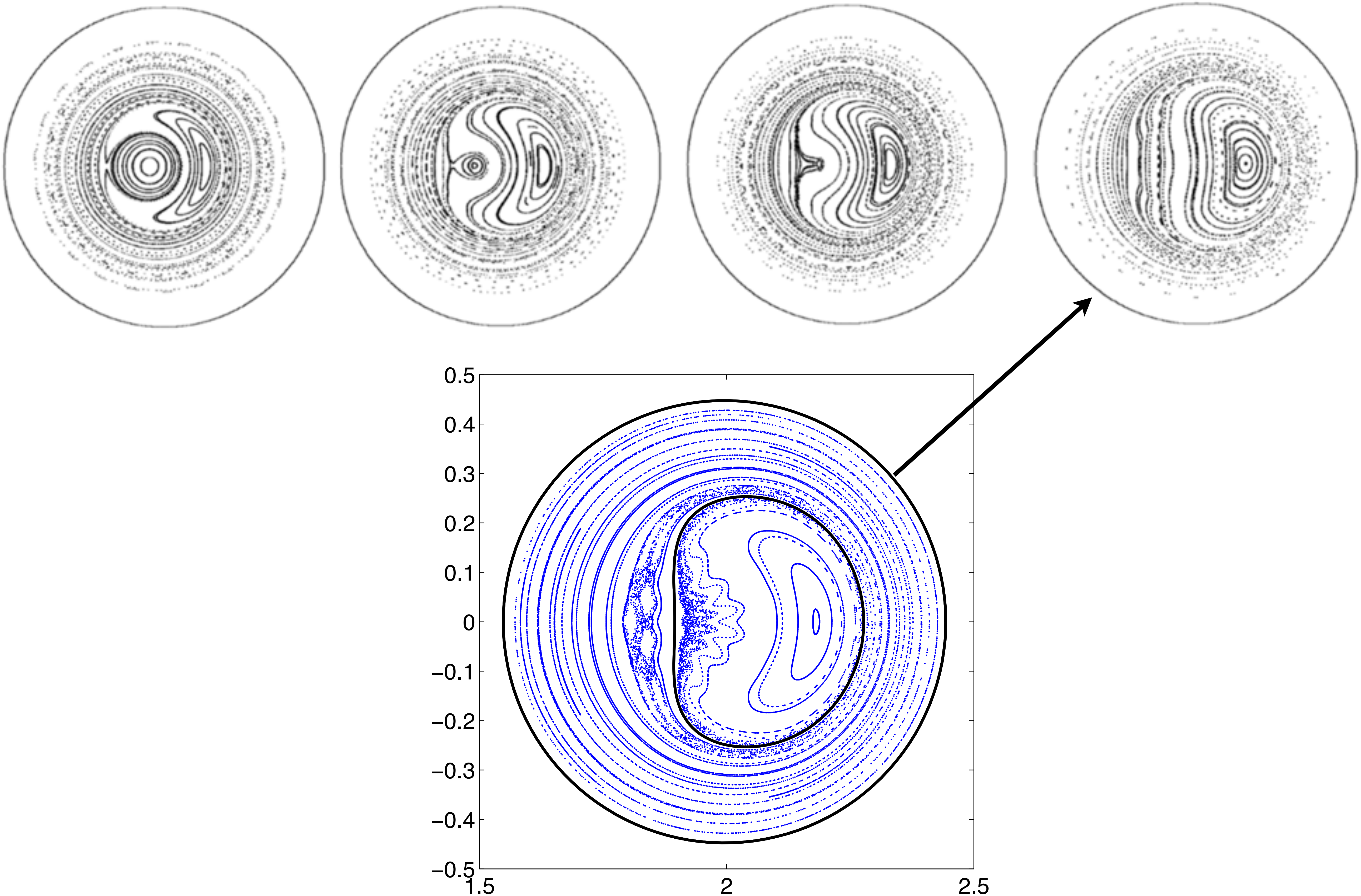

Theoretical Poincaré plots

Conclusions

MRXMHD gives a good qualitative explanation of the high-confinement state in Reversed Field Pinches

With a *minimal* model we reproduced the helical pitch and structure of the Quasi-Single Helicity state in RFP

With MRXMHD we reproduced the second magnetic axis. This is the *first* equilibrium model to be able to reproduce the Double-Axis state.

MRXMHD is a well-formulated model that interpolates between Taylor's theory and ideal MHD

Future Work

More detailed experimental comparisons with RFX

Considering RFX helical states with pressure

Apply the same methodology to 3D structures in tokamaks

Generalize MRXMHD to include flow