Advanced equilibrium models for anisotropy, flow and chaotic fields.

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The purpose of this topical review¹ is to present the state of the art in diagnosis, interpretation and modelling of waves, particles and the magnetic configuration in fusion plasmas. A focus of the review, detailed in this synopsis, is the physics and validation of magnetic configuration, which underpins all confinement, stability and transport physics.

As the effect of fast particles become important enough to modify the macroscopic variables of the plasma, the macroscopic fluid equations for equilibrium need to be modified to encapsulate the effects of pressure anisotropy, particle and heat flow. A recent advance has been the development of EFIT TENSOR², to solve tokamak equilibrium problem with toroidal flow and anisotropy. EFIT TENSOR solves MHD equations with a bi-Maxwellian closure model neglecting poloidal rotation. The code is a modification of the existing force balance solver EFIT++, which is constrained to external magnetics vacuum toroidal field, flux loops, magnetic probes, plasma current, poloidal field coils, safety factor on axis q₀, static and rotational pressure approximations, B components, diamagnetic flux, boundary, equal ψ surfaces, and Motional Stark Effect (MSE). To this set of constraints, EFIT TENSOR adds kinetic constraints of p_{\parallel} , p_{\perp} and a toroidal flow profile. Both EFIT and EFIT TENSOR codes are equilibrium reconstruction codes, which find least squares fit solutions to the data for Grad-Shafranov, and flow and anisotropy modified Grad-Shafranov equations.



Fig. 1: Finite difference radial force balance for profiles produced by TRANSP and EFIT TENSOR, with the inclusion of full order flow and anisotropy for MAST #18696 at 290ms.

To demonstrate the impact of full order flow and anisotropy, Fitzgerald *et al* have compared reconstructions using ideal MHD, and ideal MHD equilibrium with flow and anisotropy for MAST discharge #18696 at 290ms. Figure 1 shows the difference in radial force balance. TRANSP, which uses the rotational pressure assumption, underestimates the plasma pressure contribution either side of the magnetic axis, and the MHD equilibrium with flow and anisotropy is not satisfied away from the magnetic axis. At maximum, the discrepancy is of order 35%, revealing that the effect is significant. A companion code, HELENA-ATF has been written to enable physics studies with anisotropy

and flow, and provide a finely converged equilibrium solution for ongoing stability physics studies. A recent study³ has identified the different components of the toroidal current, and examined the impact of the widely applied approximation $p^* = (p_{\parallel} + p_{\perp})/2$ to anisotropy. This study shows that an isotropic reconstruction can infer a correct p^* , only by getting an incorrect RB_{ϕ} .

We also report on progress in the modelling of fully 3D (non toroidally axis-symmetric) fields with a new physics model, Multiple Relaxed region MHD, or MRxMHD, a generalisation of Taylor's theory, in which the plasma is partitioned into a finite number of nested regions that independently undergo Taylor relaxation. The plasma regions are separated by ideal transport barriers that are also assumed to be magnetic flux surfaces. Recently, we have used the MRxMHD model to develop a minimal model of the RFP QSH regime.⁴ The model comprises two regions which are separated by a transport barrier. An energy minimisation calculation



Fig. 2: Poincare plots computed from an MRxMHD model that describes both double axis (a) and single helical axis (c) states in RFX-mod. The parameter λ is a flux surface label for the barrier position.

reveals that the fully 3D MRxMHD state is the lowest two volume energy state. Poincare sections of the magnetic structure, as shown in Fig. 2, reveal that this supports both double and single helical axis states of the reverse field pinch RFX-mod. These compare well to tomographic inversions of soft x-ray emissivity.

Finally, we report on a new method, based on Bayesian analysis, which unifies the inference of plasma equilibria parameters in a tokamak with the ability to quantify differences between inferred equilibria and Grad-Shafranov (GS) force balance solutions.⁵ At the heart of this technique is the new concept of weak observations, which allows multiple forward models to be associated with a single diagnostic observation. Figure 3 shows expectation values of the



Fig. 3: Expectation values of $J_{\phi}(R, Z)$, $J_{\phi GS}(R, Z)$ and $\Delta J(R, Z)$ inferred for MAST discharge 22254 at 350ms, as calculated from 1800 samples of the posterior, using pickup coils, flux loops, MSE and Rogowski coil data.

toroidal current density inferred from (a) a toroidal current beam model, (b) a Grad-Shafranov constraint, in which J_{ϕ} is computed from Grad-Shafranov from a flux surface, together with fits to the pressure and toroidal flux function, and forward models for magnetics, total plasma current and MSE predictions, and (c) the difference between the two. The difference in J_{ϕ} can give some indication to physical effects neglected in the Grad-Shafranov equation, and/or reflect diagnostic disagreement. In this case the discrepancy is largest at the outboard midplane, and of order of 10%. An ultimate objective is to verify different equilibrium models.

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