An enstrophy minimizing method for 3D MHD Equilibrium with Flow

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A variational technique for finding steady state solutions to the set of ideal MHD equations, with finite velocity terms, is described. In this formulation it is assumed that flows self-organize into minimum enstrophy states and no axisymmetric assumptions are made. As a result the derived equations are applicable to both axisymmetric and 3D systems (stellarators and perturbed tokamaks). The resulting minimized quantity is no longer the energy of the system but rather a linear combination of the flow enstrophy and magnetic energy. The constraints of magneic helicity, flow helicity, cross helicity, and mass are invoked to ensure the extremized states are non-trivial. This avoids the ill-posed problem of minimizing the ideal MHD kinetic energy term subject to such constraints. The resulting set of coupled Beltrami-like equations then define the system with three helicity multipliers. The constraint of flow helicity may then be relaxed, resulting in a similar system of equations (with two helicity multipliers). The analysis presented focuses on the incompressible limit ($\nabla \cdot \vec{v} = 0$) which is relevant to many perturbed tokamaks, stellarators, and reverse field pinches (RFP). The force free limit ($\nabla p = 0$) is directly applicable to the RFP and may be expanded to tokamaks and stellarators (by invoking a stepped pressure). The cylindrical limit is explored to develop connections between the angular momentum and flow vorticity. The physical implications of minimizing this functional are discussed.