

Stepped Pressure Equilibrium Code computes extrema of the multi-region, relaxed MHD energy principle

- The plasma is partitioned into N “relaxed volumes”, separated by “ideal interfaces”.
- Minimize the total energy, subject to the constraints of conserved fluxes and helicity in each region

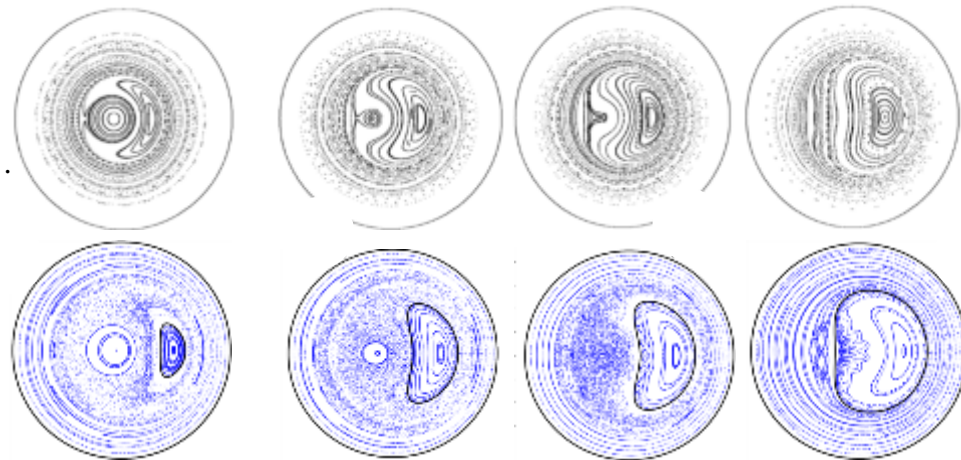
$$F \equiv \sum_{i=1}^N \left[\underbrace{\int_{\mathcal{V}_i} \left(\frac{p}{\gamma-1} + \frac{B^2}{2} \right)}_{\text{energy}} - \frac{\mu_i}{2} \left(\underbrace{\int_{\mathcal{V}_i} \mathbf{A} \cdot \mathbf{B} - K_i}_{\text{helicity}} \right) \right]$$

- In the relaxed volumes, $\nabla \times \mathbf{B} = \mu \mathbf{B}$; and islands, chaotic fields are allowed.
- Across the ideal interfaces, $[[p + B^2/2]] = 0$, and arbitrary pressure profiles are allowed.
- Main publication: [Hudson, Dewar, Dennis, Hole *et al.*, PoP 2012]
- If $N = 1$, obtain a globally-relaxed, Taylor state.
- If $N = 2$, explains helical states in RFP, shown below [Dennis, Hudson, *et al.* PRL, 2013]
- If $N \rightarrow \infty$, recover ideal MHD $\nabla p = \mathbf{j} \times \mathbf{B}$ [Dennis, Hudson *et al.* PoP, 2013]

Overview of RFX-mod results,

P. Martin *et al.*, *NF*, (2009)

Fig.6. . . . transition from a QSH state . . . to a fully developed SHAx state .



Numerical Calculation using Stepped Pressure Equilibrium Code,

G. Dennis *et al.*, *PRL*, (2013)

Topological features correctly reproduced

SPEC: ongoing and future efforts:

- **Theoretical developments**

- a. Included flow into MRxMHD energy functional [Dennis, Hudson, et al. PoP 21:042501, 2014].
- b. Included anisotropy *and* flow into MRxMHD energy functional [Dennis, Hudson, et al. PoP 21:072512, 2014].

- **Numerical developments**

- a. Allow for general geometry (i.e. relax stellarator symmetry “up-down” symmetry constraint); completed!
- b. Free-boundary capability; almost complete;

- **Recent applications**

- a. Computed I/x and δ -function singular currents in ideal equilibria; completed! [Loizu, Hudson et al . 2014]
- b. Benchmarked with non-trivial, semi-analytic equilibrium model; completed! [Loizu, Hudson et al . 2014]

- **Planned activities**

- a. Compute non-linear ideal kink equilibrium in cylindrical geometry; compare with analytical predictions; allow for relaxation and compute 3D equilibrium solution with islands; due 2014;
- b. Compute second variation of energy functional to extend SPEC to calculate stability; almost complete; due 2014;
- c. Include flow, pressure anisotropy into SPEC; due 2015, . . .
- d. Apply to W7X, LHD, . . .; due 2015, . . .