APS-DPP Meeting

Multiphysics/Multiscale Coupling of Microturbulence and MHD Equilibria

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Abstract

1) The effort to obtain a set of “hydromagnetic” equations for a magnetized collisionless plasma started nearly 60 years ago by Chew, Goldberger and Lowe. Many attempts have been made ever since, by Freiman, Davidson, and Kulsrud at PPPL. Here, we will show the derivation of a set of collisionless MHD equations from the gyrokinetic perspective. This set of equations is energy conserving and, in the absence of fluctuations, recovers the usual MHD equilibrium.

2) With this, we then propose to couple

(A) GTS [1] - a global gyrokinetic turbulence code, based on the newly developed electromagnetic capability [2], with
(B) SPEC [3] - an MHD equilibrium code,
for the purpose of self-consistently obtaining a new magnetic configuration which reduces the anomalous transport due to microturbulence.

• The proposed iterative scheme, which requires the two code to "talk to each other," is based on a recent realization [4] that connects the gyrokinetic Vlasov-Maxwell equations with the MHD equilibrium equations via the gyrokinetic vorticity equation and Ohm's law.

[1] W. X. Wang et al., PoP 13, 092505 (2006)
[2] E. A. Startsev et al., Sherwood Conference, NYU, NY (2015)
[3] S. R. Hudson et al., PoP 19, 112502 (2012)
[4] W. W. Lee, Sherwood Conference, NYU, NY (2015).

Darwin Electromagnetic (finite-β) Gyrokinetic Equations

- Original Vlasov Equation $F \equiv F(\mathbf{x}, \mathbf{v}, \mathbf{t})$

$$\frac{\partial F_\alpha}{\partial t} + \mathbf{v} \cdot \frac{\partial F_\alpha}{\partial \mathbf{x}} + \frac{q}{m}\left[\mathbf{E} + \frac{1}{c}\mathbf{v} \times (\mathbf{B}_0 + \delta\mathbf{B})\right] \cdot \frac{\partial F_\alpha}{\partial \mathbf{v}} = 0$$

$$\mathbf{E} = -\nabla\phi - (1/c)\partial\mathbf{A}/\partial t \qquad \delta\mathbf{B} = \nabla \times \mathbf{A}$$

- Using the Lagragian of $L = \frac{1}{2}mv^2 - q\phi + \frac{q}{c}\mathbf{v} \cdot \mathbf{A}$ to obtain

(see, for example, Corben and Stahle, 1966)

$$\frac{\partial F_\alpha}{\partial t} + \mathbf{v} \cdot \frac{\partial F_\alpha}{\partial \mathbf{x}} + \frac{q}{m}\left[-\nabla(\phi - \frac{1}{c}\mathbf{v} \cdot \mathbf{A}) + \frac{1}{c}\mathbf{v} \times \mathbf{B}_0\right] \cdot \frac{\partial F_\alpha}{\partial(\mathbf{v} + q_\alpha\mathbf{A}/m_\alpha c)} = 0$$

- Using $\mathbf{v} \rightarrow \mathbf{v} + \frac{q_\alpha}{m_\alpha c}\mathbf{A}_\perp$

to re-write it as

$$\frac{\partial F_\alpha}{\partial t} + \mathbf{v} \cdot \frac{\partial F_\alpha}{\partial \mathbf{x}} + \frac{q}{m}\left[-\nabla(\phi - \frac{1}{c}\mathbf{v}_\perp \cdot \mathbf{A}_\perp) - \frac{1}{c}\frac{\partial \mathbf{A}_\parallel}{\partial t} + \frac{1}{c}\mathbf{v} \times (\mathbf{B}_0 + \delta\mathbf{B}_\perp)\right] \cdot \frac{\partial F_\alpha}{\partial(\mathbf{v} + q_\alpha\mathbf{A}_\perp/m_\alpha c)} = 0$$

$$F \equiv F(\mathbf{x}, v_\parallel, \mu/B, t)$$

$$\mathbf{v} \approx v_\parallel \mathbf{b} + \frac{c}{B_0}\mathbf{E} \times \mathbf{b}$$

$$\mathbf{E} = -\nabla(\phi - \mathbf{v}_\perp \cdot \mathbf{A}_\perp / c) - (1/c)\partial \mathbf{A}_\parallel / \partial t$$

$$\mathbf{b} = \hat{\mathbf{b}}_0 + \delta \mathbf{B}_\perp / B_0 \qquad \hat{\mathbf{b}}_0 = \mathbf{B}_0 / B_0 \qquad \delta \mathbf{B}_\perp = \nabla \times \mathbf{A}_\parallel$$

- Gyrokinetic Vlasov Equation

$$\frac{\partial F_\alpha}{\partial t} + \left[v_\parallel \mathbf{b} - \frac{c}{B_0}\nabla(\overline{\phi} - \frac{1}{c}\overline{\mathbf{v}_\perp \cdot \mathbf{A}_\perp}) \times \hat{\mathbf{b}}_0 \right] \cdot \frac{\partial F_\alpha}{\partial \mathbf{x}} - \frac{q}{m}\left[\nabla(\overline{\phi} - \frac{1}{c}\overline{\mathbf{v}_\perp \cdot \mathbf{A}_\perp}) \cdot \mathbf{b} + \frac{1}{c}\frac{\partial \overline{A}_\parallel}{\partial t} \right] \frac{\partial F_\alpha}{\partial v_\parallel} = 0$$

$$\nabla^2 \phi + \frac{\omega_{pi}^2}{\Omega_i^2}\nabla_\perp^2 \phi = -4\pi \sum_\alpha q_\alpha \int \overline{F}_\alpha dv_\parallel d\mu$$ -- for $k_\perp^2 \rho_i^2 \ll 1$

$$\nabla^2 \mathbf{A} - \underline{\frac{1}{v_A^2}\frac{\partial^2 \mathbf{A}_\perp}{\partial t^2}} = -\frac{4\pi}{c}\sum_\alpha q_\alpha \int \mathbf{v}\overline{F}_\alpha dv_\parallel d\mu$$ Negligible for $\omega^2 \ll k_\perp^2 v_A^2$

$$\mu = v_\perp^2/2 \qquad \mathbf{v}_p^L = -(mc^2/eB^2)(\partial \nabla_\perp \phi / \partial t) \qquad \mathbf{v}_p^T = -(mc/eB^2)(\partial^2 \mathbf{A}_\perp / \partial^2 t)$$

- Energy Conservation

$$\frac{d}{dt}\left\langle \int (\frac{1}{2}v_\parallel^2 + \mu)(m_e F_e + m_i F_i) dv_\parallel d\mu + \frac{\omega_{pi}^2}{\Omega_i^2}\frac{|\nabla_\perp \Phi|^2}{8\pi} + \frac{|\nabla A_\parallel|^2}{8\pi} \right\rangle_{\mathbf{x}} = 0$$

$$\Phi \equiv \phi - \overline{\mathbf{v}_\perp \cdot \mathbf{A}_\perp}/c$$

- Gyrokinetic Vlasov Equation in General Geometry

[Lee and Qin, PoP (2003), Porazic, PhD thesis (2010)]

$$\frac{\partial F_\alpha}{\partial t} + \frac{d\mathbf{R}}{dt} \cdot \frac{\partial F_\alpha}{\partial \mathbf{R}} + \frac{dv_\parallel}{dt}\frac{\partial F_\alpha}{\partial v_\parallel} = 0$$

$$\frac{d\mathbf{R}}{dt} = v_\parallel \mathbf{b}^* + \frac{v_\perp^2}{2\Omega_{\alpha 0}}\hat{\mathbf{b}}_0 \times \nabla ln B_0 - \frac{c}{B_0}\nabla\bar{\Phi} \times \hat{\mathbf{b}}_0$$

$$\frac{dv_\parallel}{dt} = -\frac{v_\perp^2}{2}\mathbf{b}^* \cdot \nabla ln B_0 - \frac{q_\alpha}{m_\alpha}\left(\mathbf{b}^* \cdot \nabla\bar{\Phi} + \frac{1}{c}\frac{\partial \bar{A}_\parallel}{\partial t}\right)$$

$$\Omega_{\alpha 0} \equiv q_\alpha B_0 / m_\alpha c$$

$$\bar{\Phi} \equiv \bar{\phi} - \overline{\mathbf{v}_\perp \cdot \mathbf{A}_\perp}/c \qquad \overline{\mathbf{v}_\perp \cdot \mathbf{A}_\perp} = -\frac{1}{2\pi}\frac{eB_0}{mc}\int_0^{2\pi}\int_0^{\rho}\delta B_\parallel r dr d\theta$$

$$\mathbf{b}^* \equiv \mathbf{b} + \frac{v_\parallel}{\Omega_{\alpha 0}}\hat{\mathbf{b}}_0 \times (\hat{\mathbf{b}}_0 \cdot \nabla)\hat{\mathbf{b}}_0 \qquad \mathbf{b} = \hat{\mathbf{b}}_0 + \frac{\nabla \times \bar{\mathbf{A}}}{B_0}$$

$$F_\alpha = \sum_{j=1}^{N_\alpha}\delta(\mathbf{R} - \mathbf{R}_{\alpha j})\delta(\mu - \mu_{\alpha j})\delta(v_\parallel - v_{\parallel\alpha j})$$

- Including only the parallel vector potential, Startsev et al. have studied low (m,n) as well as high (m,n) tearing modes [APS 2004, Sherwood 2005] using GTS [Wang et al., PoP 2003].

Gyrokinetic Current Densities

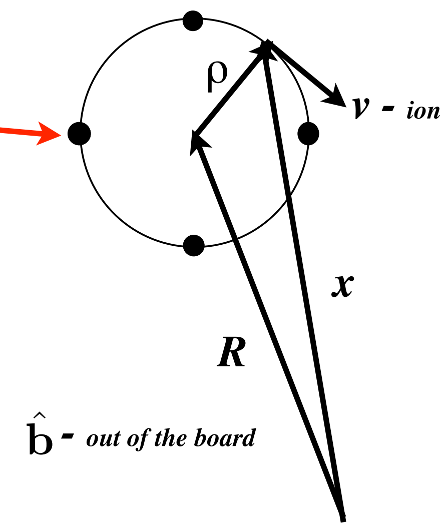
[Qin, Tang, Rewoldt and Lee, PoP **7**, 991 (2000); Lee and Qin, PoP **10**, 3196 (2003).]

1) $k_\perp \rho_i \sim 1$

$$\mathbf{J}_{gc}(\mathbf{x}) = \mathbf{J}_{\parallel gc}(\mathbf{x}) + \boxed{\mathbf{J}^M_{\perp gc}(\mathbf{x})} + \mathbf{J}^d_{\perp gc}(\mathbf{x})$$

$$= \sum_\alpha q_\alpha \langle \int F_{\alpha gc}(\mathbf{R})(\mathbf{v}_\parallel + \mathbf{v}_\perp + \mathbf{v}_d)\delta(\mathbf{R} - \mathbf{x} + \rho)d\mathbf{R}dv_\parallel d\mu\rangle_\varphi$$

$$\mathbf{v}_d = \frac{v_\parallel^2}{\Omega_\alpha}\hat{\mathbf{b}} \times (\hat{\mathbf{b}} \cdot \frac{\partial}{\partial \mathbf{R}})\hat{\mathbf{b}} + \frac{v_\perp^2}{2\Omega_\alpha}\hat{\mathbf{b}} \times \frac{\partial}{\partial \mathbf{R}} lnB$$



FLR calculation

2) $k_\perp \rho_i \ll 1$

$$\mathbf{J}^M_{\perp gc}(\mathbf{x}) = -\sum_\alpha \nabla_\perp \times \frac{c\hat{\mathbf{b}}}{B} p_{\alpha\perp} \qquad p_{\alpha\perp} = m_\alpha \int (v_\perp^2/2)F_{\alpha gc}(\mathbf{x})dv_\parallel d\mu$$

$$\mathbf{J}^d_{\perp gc} = \frac{c}{B}\sum_\alpha \left[p_{\alpha\parallel}(\nabla \times \hat{\mathbf{b}})_\perp + p_{\alpha\perp}\hat{\mathbf{b}} \times (\nabla lnB)\right] \qquad p_{\alpha\parallel} = m_\alpha \int v_\parallel^2 F_{\alpha gc}(\mathbf{x})dv_\parallel d\mu$$

$$\mathbf{J}_{\perp gc} = \mathbf{J}^M_{\perp gc} + \mathbf{J}^d_{\perp gc} \quad = \frac{c}{B}\sum_\alpha \left[\hat{\mathbf{b}} \times \nabla p_{\alpha\perp} + (p_{\alpha\parallel} - p_{\alpha\perp})(\nabla \times \hat{\mathbf{b}})_\perp\right]$$

$$\approx \frac{c}{B}\sum_\alpha \hat{\mathbf{b}} \times \nabla p_{\alpha\perp}$$

$$\boxed{\mathbf{J}_{\perp gc} = \frac{c}{B}\sum_\alpha \hat{\mathbf{b}} \times \nabla p_\alpha}$$

• Gyrokinetic MHD Equations:

1) A reduced set of equations in full toroidal geometry obtained from gyrokinetic Vlasov equation:

-- For $k_\perp^2 \rho_i^2 \ll 1$ $\quad \bar{F} \rightarrow F \quad \bar{\phi} \rightarrow \phi \quad \bar{A}_\parallel \rightarrow A_\parallel \quad \overline{\mathbf{v}_\perp \cdot \mathbf{A}_\perp} \rightarrow 0$

2) Together with gyrokinetic Poisson's equation

$$\frac{\omega_{pi}^2}{\Omega_i^2} \nabla_\perp^2 \phi = -4\pi\rho$$

3) and Ampere's law

$$\nabla^2 A_\parallel = -\frac{4\pi}{c} J_\parallel$$

$$\nabla^2 \mathbf{A}_\perp \underline{- \frac{1}{v_A^2} \frac{\partial^2 \mathbf{A}_\perp}{\partial t^2}} = -\frac{4\pi}{c} \mathbf{J}_\perp$$

Negligible for $\omega^2 \ll k_\perp^2 v_A^2$

4) $\quad \delta \mathbf{B} = \nabla \times \mathbf{A} \qquad \mathbf{B} = \mathbf{B}_0 + \delta \mathbf{B} \qquad \mathbf{b} \equiv \frac{\mathbf{B}}{B}$

• Gyrokinetic MHD Equations (cont.):

-- Pressure Driven Current: $$\mathbf{J}_\perp = \frac{c}{B}\sum_\alpha \mathbf{b}\times\nabla p_\alpha$$

-- Vorticity Equation: $$\left(\frac{\partial}{\partial t} - \frac{c}{B}\nabla\phi\times\mathbf{b}\cdot\nabla\right)\nabla_\perp^2\phi - 4\pi\frac{v_A^2}{c^2}\nabla\cdot(\mathbf{J}_\parallel + \mathbf{J}_\perp) = 0$$

From GK Vlasov Equation

-- Ohm's law: $$E_\parallel \equiv -\frac{1}{c}\frac{\partial A_\parallel}{\partial t} - \mathbf{b}\cdot\nabla\phi = \eta J_\parallel \to 0$$

-- Equation of State: $$\frac{dp_\alpha}{dt} = 0$$

$$\boxed{\frac{v_A}{c} \equiv \frac{\Omega_i}{\omega_{ci}}}$$

-- Normal modes: $$\omega = \pm k_\parallel v_A$$

-- Energy Conservation: $$\frac{\partial}{\partial t}\int\frac{1}{8\pi}\left(|\nabla_\perp\phi|^2 + \frac{v_A^2}{c^2}|\nabla A_\parallel|^2\right)d\mathbf{x} = -\frac{v_A^2}{c^2}\int\mathbf{E}_\perp\cdot\mathbf{J}_\perp d\mathbf{x}$$

$$\mathbf{E}_\perp = -\nabla_\perp\phi$$

- MHD Equilibrium

1. For a given pressure profile, we obtain the pressure driven current from

$$\boxed{\mathbf{J}_\perp = \frac{c}{B}\sum_\alpha \mathbf{b}\times\nabla p_\alpha}$$

2. We then solve the coupled equations of

$$\boxed{\begin{aligned} &\frac{d}{dt}\nabla_\perp^2\phi + \frac{v_A^2}{c}(\mathbf{b}\cdot\nabla)\nabla_\perp^2 A_\parallel - 4\pi\frac{v_A^2}{c^2}\nabla\cdot\mathbf{J}_\perp = 0 \\ &E_\parallel \equiv -\frac{1}{c}\frac{\partial A_\parallel}{\partial t} - \mathbf{b}\cdot\nabla\phi = 0 \end{aligned}}$$

$$\frac{d}{dt} \equiv \frac{\partial}{\partial t} - \frac{c}{B}\nabla\phi\times\mathbf{b}\cdot\nabla$$

3. If we look for a solution for $\phi \to 0$ which, in turn, gives $\dfrac{\partial A_\parallel}{\partial t} \to 0$,

this is then the equilibrium solution that satisfies the quasineutral condition of

$$\boxed{\nabla\cdot(\mathbf{J}_\parallel + \mathbf{J}_\perp) = 0}$$

4. The GK vorticity equation retains all the toroidal physics, different than Strauss' equation [PF 77]

5. Perpendicular current is consisted of both a divergent free diamagnetic current and a magnetic drift current. Only the latter was originally included in Lee and Qin [PoP, 2003].

Reduced MHD Equations vs. Gyrokinetic-MHD Equations

- GK Three-field Equations for $k_\perp \rho_i \ll 1$ w/o geometric simplification [Lee and Qin PP '03]

$$\frac{d}{dt}\nabla_\perp^2\phi + \frac{v_A^2}{c}(\hat{\mathbf{b}}\cdot\nabla)\nabla^2 A_\parallel - \boxed{4\pi\frac{v_A^2}{c^2}\nabla_\perp\cdot\mathbf{J}^d_{\perp gc}} = 0$$

general geometry

$$\frac{1}{c}\frac{\partial A_\parallel}{\partial t} + \mathbf{b}\cdot\nabla\phi = 0.$$

$$\frac{dp_\alpha}{dt} = 0$$

$$\frac{d}{dt} \equiv \frac{\partial}{\partial t} - \frac{c}{B}\nabla\bar{\phi}\times\hat{\mathbf{b}}_0\cdot\nabla$$

$$\mathbf{J}^d_{\perp gc} = \frac{c}{B_0}\sum_\alpha\left[p_\alpha(\nabla\times\hat{\mathbf{b}}_0)_\perp + p_\alpha\hat{\mathbf{b}}_0\times(\nabla lnB_0)\right]$$

- Reduced High-ß Three-Field MHD Equations [Strauss PF '77]

$$\frac{d\nabla_\perp^2\phi}{dt} + \frac{v_A^2}{c}(\hat{\mathbf{b}}\cdot\nabla)\nabla_\perp^2 A_\parallel - \boxed{\frac{2}{R_0}\frac{\partial p}{\partial y}} = 0$$

$$\frac{1}{c}\frac{\partial A_\parallel}{\partial t} + \mathbf{b}\cdot\nabla\phi = 0.$$

simplified geometry

$$\frac{dp}{dt} = 0$$

Based on this interesting property, we propose the following:

1. Use SPEC to give basic magnetic configuration to GTS

2. Use GTS to study microturbulence and to produce perturbed pressure and current

3. Give these information back to SPEC and use SPEC to give a new magnetic configuration to GTS and so on ....

Since we use the nonlinearly modified profiles at every iteration and the equilibrium solutions are supposed to mimic the fluctuation free states, we should expect the system to evolve gradually to a state where fluctuations become less.

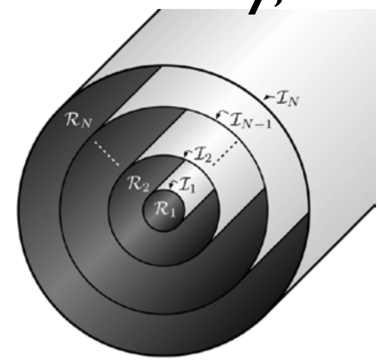
GTS - a global gyrokinetic code with robust capability to simulate turbulence & transport for tokamak experiments

- δf PIC code solving modern GK equation in conservative form

$$\frac{\partial f_a}{\partial t} + \frac{1}{B^*}\nabla_Z \cdot (\dot{\vec{Z}} B^* f_a) = \sum_b C[f_a, f_b]$$

- New, improved weight scheme ensuring phase space imcompressibility
- Full geometry, global simulation (without local ballooning approximation)
  - real space field solvers with field-line-following mesh
  - retains all toroidal modes and full channels of nonlinear energy couplings
  - enable to treat modes with low-n, with finite $k_{\|}$ (e.g., shear flow mode)
- Fully kinetic electrons (both trapped and untrapped electron dynamics)
- Linearized Fokker-Plank operator with particle, momentum and energy conservation for i-i and e-e collisions; Lorentz operator for e-i collisions
- Include neoclassical physics self-consistently in turbulence simulations
  - significant impact on some important transport & confinement issues (bootstrap current, poloidal flow, GAMs and particle transport, etc.)
- Applied to wide experiments for various physics studies:
  NSTX/U, DIII-D, C-MOD, KSTAR and ASDEX-U

MHD equilibrium ≡ constrained, minimum-energy state with given pressure, boundary, . . .



1. SPEC minimizes the global plasma energy, $W \equiv \int_{\mathcal{R}} \left(\frac{p}{\gamma - 1} + \frac{B^2}{2} \right) dv$
   where the pressure $p(\psi)$ is a given function of toroidal flux, ψ.

2. The volume integral is partitioned (and parallelized), $\int_{\mathcal{R}} dv \equiv \sum_{i=1}^{N} \int_{\mathcal{R}_i} dv$

3. The simplest constraints are conserved helicity: $H_i \equiv \int_{\mathcal{V}_i} \mathbf{A} \cdot \mathbf{B}\, dv = H_{i,o} = const.$ in each $\mathcal{R}_i$
   and the "ideal-constraint": $\delta \mathbf{B} = \nabla \times (\boldsymbol{\xi} \times \mathbf{B})$ at each $\mathcal{I}_i$

4. The multi-region, relaxed MHD (MRxMHD) energy functional is
   $$F \equiv \sum_{i=1}^{N} \left[W_i - \frac{\mu_i}{2} (H_i - H_{i,o}) \right]$$ [Hole, et al. JPP, 72:1167, 2006]
   The equilibrium state satisfies $\nabla \times \mathbf{B} = \mu \mathbf{B}$ in each $\mathcal{R}_i$, and $[[p + B^2/2]] = 0$ across each $\mathcal{I}_i$.

5. If $N = 1$, recover globally-relaxed, Taylor state.
   If $N \to \infty$, recover globally-ideal, $\nabla p = \mathbf{j} \times \mathbf{B}$ [Dennis et al. PoP, 20:032509, 2013]
   If N is finite, flat pressure and islands at resonances; pressure jumps at arbitrarily many KAM surfaces.

6. SPEC [Hudson et al. PoP, 19:112502, 2012] is the only equilibrium code that, simultaneously,
   (1) is based on an energy functional, (2) computes magnetic field consistent with given pressure profile,
   (3) accurately computes singular currents in ideal-MHD equilibria [Loizu et al. PoP, 22:022501, 2015],
   (4) allows for partially relaxed fields, magnetic islands and chaos, (5) is parallelized.

Multi-region, relaxed MHD can include pressure anistropy and flow

- two papers on MRxMHD with flow have already been published

PHYSICS OF PLASMAS **21**, 042501 (2014)

Multi-region relaxed magnetohydrodynamics with flow

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We present an extension of the multi-region relaxed magnetohydrodynamics (MRxMHD) equilibrium model that includes plasma flow. This new model is a generalization of Woltjer's

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Multi-region relaxed magnetohydrodynamics with anisotropy and flow

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We present an extension of the multi-region relaxed magnetohydrodynamics (MRxMHD) equilibrium model that includes pressure anisotropy and general plasma flows. This anisotropic

- required modifications to SPEC are minor

Summary

• A set of gyrokinetic MHD equations have been derived and its extension by including high-order moments is underway.

• A white paper entitled "A Multiphysics and Multiscale Coupling of Microturbulence with MHD Equilibria," by Lee, Startsev, Hudson, Wang and Ethier has first been presented to the DoE's Workshop on Integrated Simulations in May this year.

- The purpose is to use GTS and SPEC together in an attempt to minimize turbulence and transport. This approach is based on an iterative procedure, which first "decouple" the transport problem from the equilibrium problem, and then to "couple" them through global parameter exchanges.