Verification of the SPEC code in stellarator geometries

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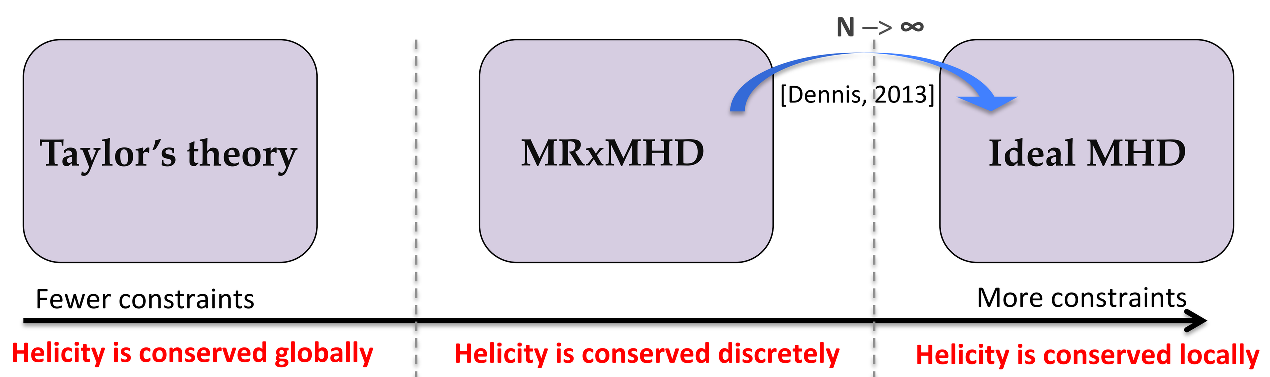
1. Motivation

- **Verification** & **Validation** are the milestones in the path towards predictive code capability [1].
  - Verification answers: *Are we solving the equations right?*
  - Validation answers: *Are we solving the right equations?*
- Fusion research yearns for **fast, robust, and reliable** codes describing 3D MHD equilibria.
- Challenge: intricate combination of magnetic **surfaces**, magnetic **islands**, and **chaos** [2].
- The Stepped Pressure Equilibrium Code (SPEC) was developed to fulfil this nontrivial task [3].
- SPEC was verified in axisymmetry and for slightly perturbed configurations [3-6].
- Here we present the first SPEC calculations of equilibria in stellarator geometries.

2. The SPEC code

- SPEC finds equilibria as extrema of the **M**ultiregion **R**elaxed **MHD** energy functional [7].
- In MRxMHD, discrete topological constraints allow for **partial relaxation**:
  - Plasma is partitioned into N nested volumes, V_v , undergoing Taylor relaxation.
  - Volumes separated by $N-1$ interfaces, ∂V_v , constrained to remain magnetic surfaces.
  - Location and shape of surfaces determined self-consistently by force-balance condition.



- MRxMHD **equilibrium states satisfy**, for $v = 1, \dots N$:

$$\nabla\times\mathbf{B} = \mu_v\mathbf{B} \quad \text{in } V_v$$

$$[[p_v + \frac{B^2}{2}]]_v = 0 \quad \text{in } \partial V_v$$

- SPEC is a **fixed-boundary** code and requires specification of the boundary:

$$R = \sum_{mn} R_{mn}\cos(m\theta - n\varphi) \quad \text{and} \quad Z = \sum_{mn} Z_{mn}\sin(m\theta - n\varphi)$$

- SPEC also needs **two profiles**, e.g. pressure $p(\psi_v)$ and transform $\iota^{\pm}(\psi_v)$.
- **Solution** in terms of the vector potential $\mathbf{A} = A_\theta\nabla\theta + A_\varphi\nabla\varphi$, written in the form

$$A_\alpha(s,\theta,\varphi) = \sum_{m,n,l} A_{\alpha,m,n,l}T_l(s)\cos(m\theta - n\varphi)$$

where $T_l(s)$ are the Chebyshev polynomials.

- **Resolution parameters** are $M_{pol} = max(m)$, $N_{tor} = max(n)$, $L_{rad} = max(l)$.

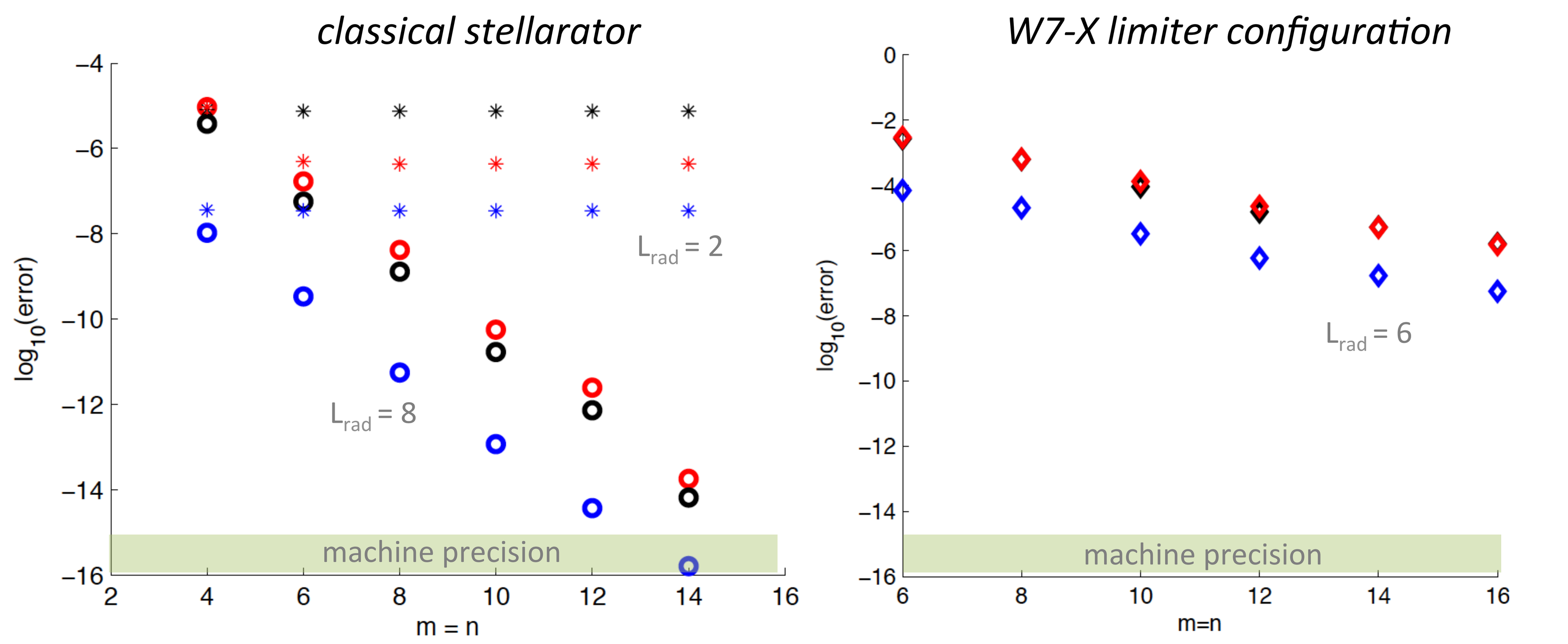
3. Stellarator vacuum fields

- There is a unique magnetic field **B** (up to a scale factor) that satisfies

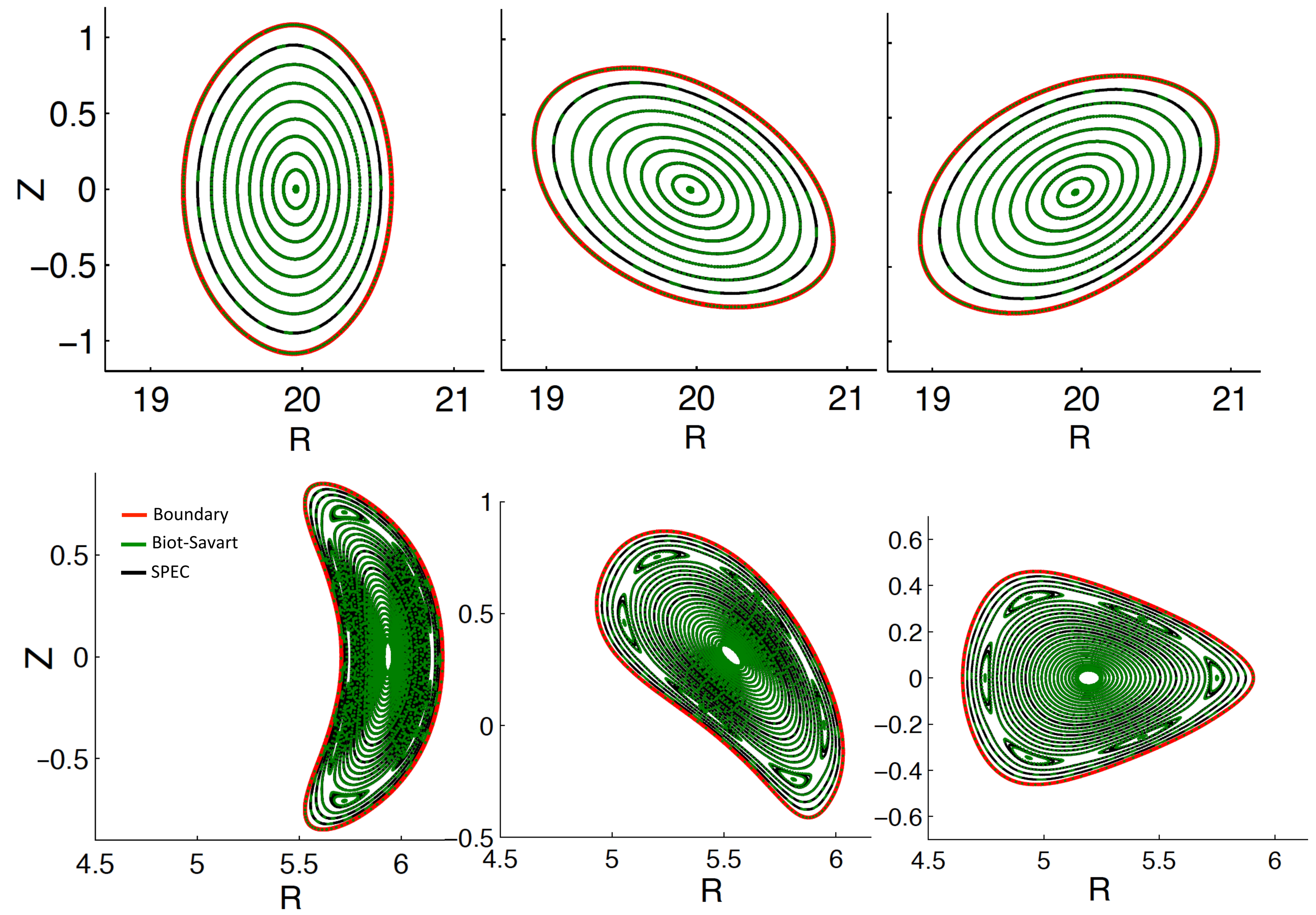
$$\nabla\times\mathbf{B} = 0 \text{ in } V \quad (1)$$
$$\nabla\cdot\mathbf{B} = 0 \text{ in } V \quad (2)$$
$$\mathbf{B}\cdot\hat{\mathbf{n}} = 0 \text{ in } \partial V \quad (3)$$

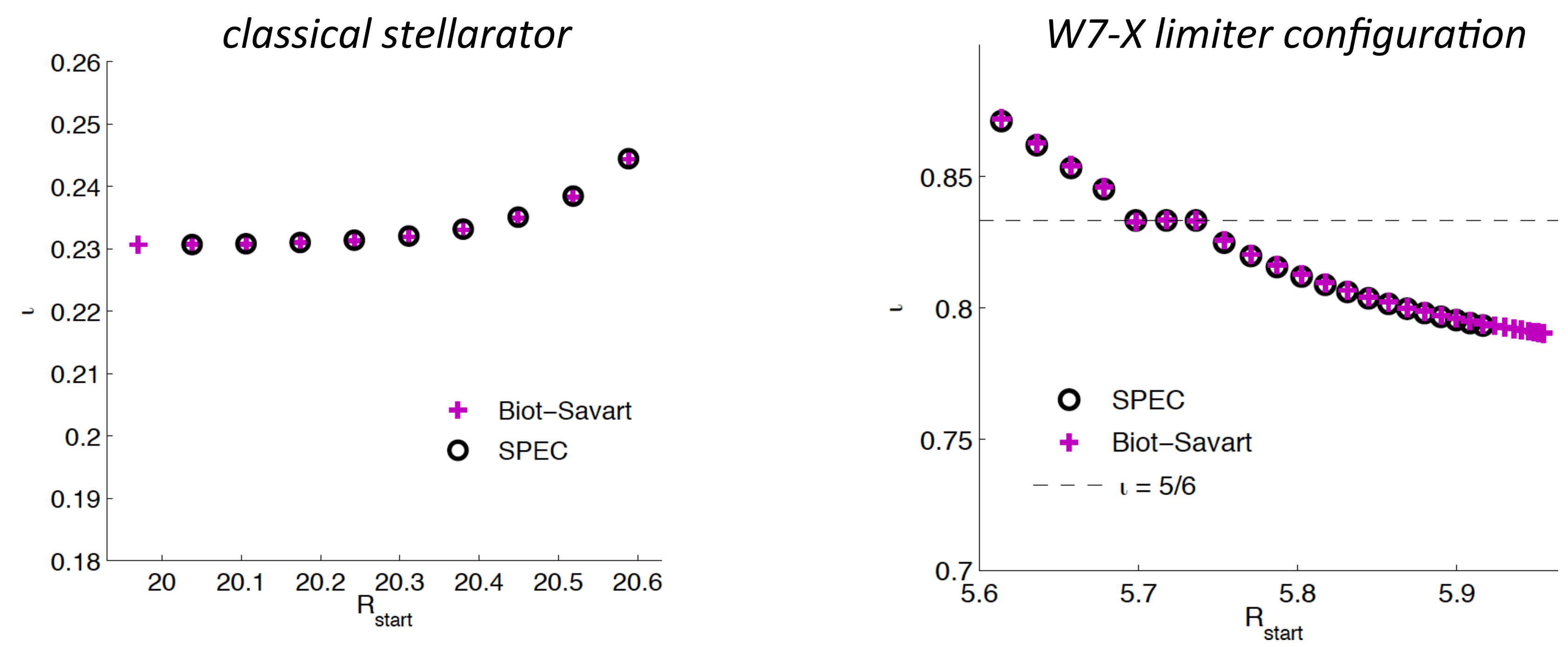
- SPEC can be used to calculate vacuum fields by setting $N = 1$ and $p = 0$, $\mu = 0$.
- Two stellarator configurations are considered: a classical $l = 2$ stellarator and W7-X OP1.1 configuration.
- Boundary harmonics are extracted from field-line-tracing performed on Biot-Savart calculations.
- **VERIFICATION**: numerical proof that Eqs. (1)-(3) are satisfied to arbitrary precision as resolution is increased.
- Equations (2) and (3) are satisfied by construction. For Eq. (1), we define

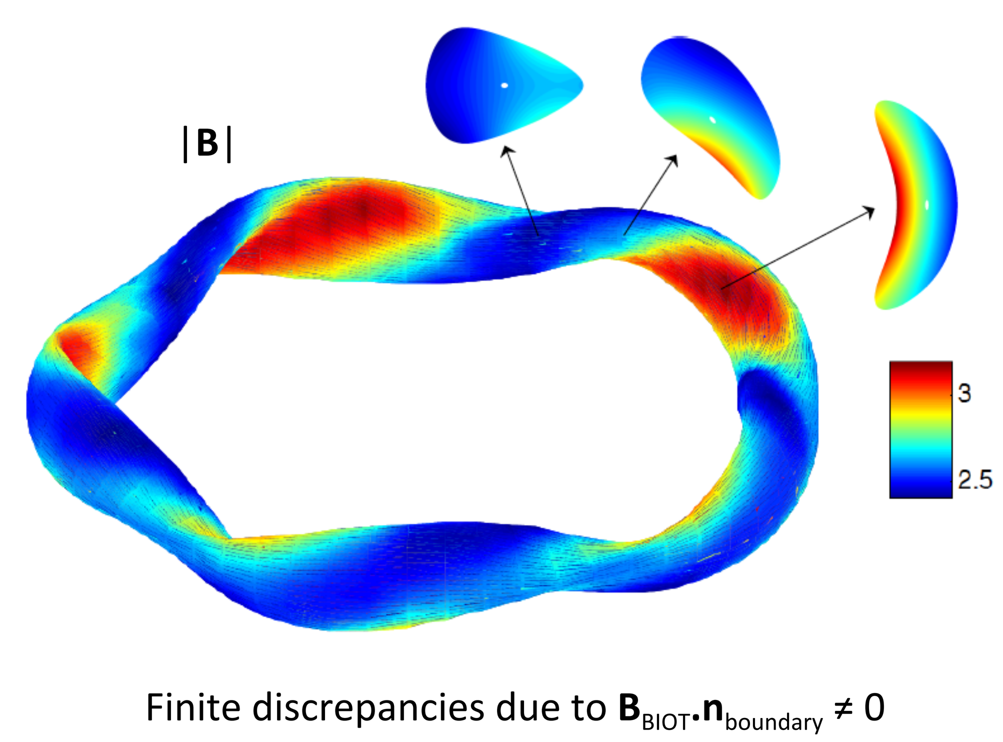
$$\varepsilon_\alpha = \frac{1}{V}\int ds\oint d\theta\oint d\varphi\,\mathcal{J}(s,\theta,\varphi)||(\nabla\times\mathbf{B} - \mu\mathbf{B}).\nabla\alpha||$$

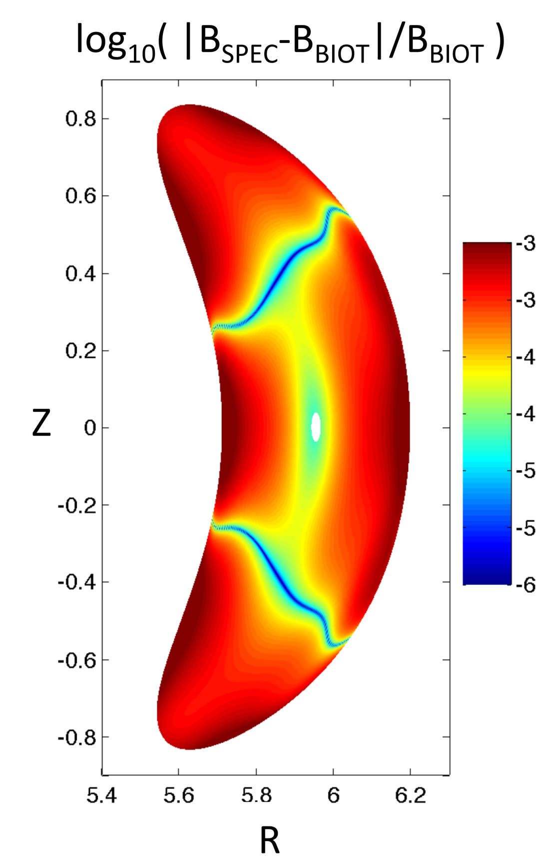


- **BEYOND VERIFICATION**: compare Poincaré plots, rotational transform, and $|\mathbf{B}|$, to Biot-Savart solution.



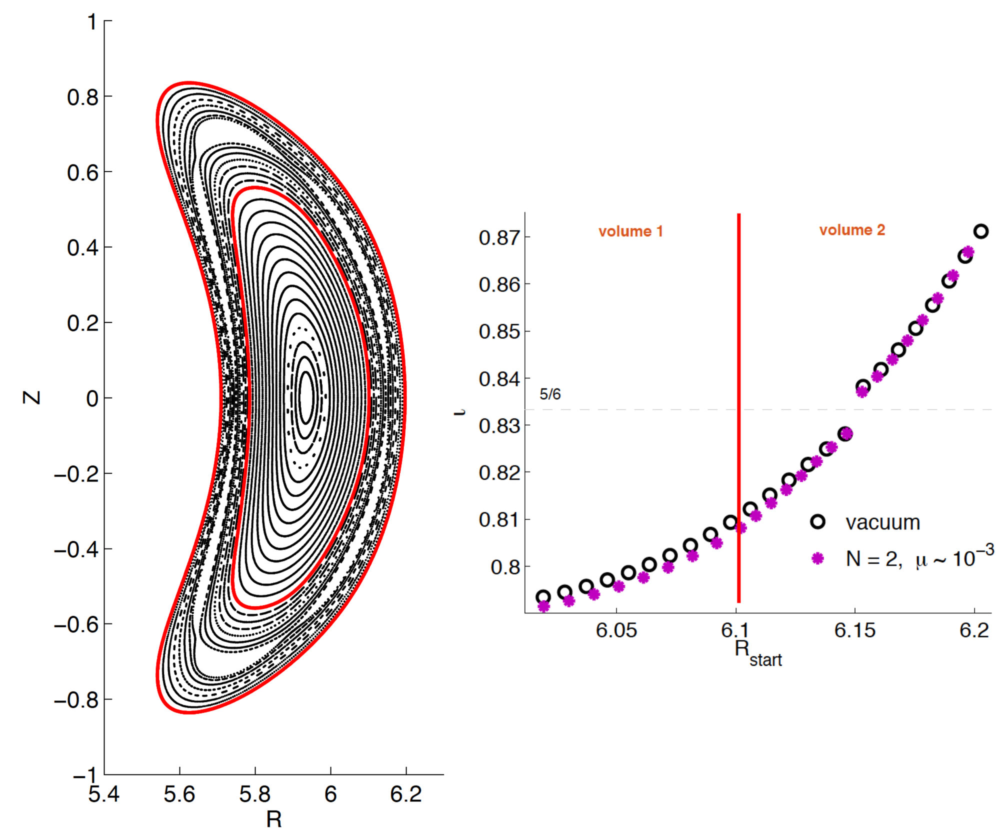






4. Stellarator equilibria with KAM surfaces

- The **KAM theorem** ensures that the most robust surfaces are the most irrational (noble irrationals).
- Run SPEC with $\mathbf{N} = \mathbf{2}$ and a KAM surface with $\iota^{\pm} = \iota_{\text{noble}}$. As input, specify toroidal fluxes and edge transform.



- Force-balance is exact $|f|=10^{-16}$
- Zero-current not guaranteed ($\mu_{1,2} \neq 0$)
- Iterate flux and edge transform to recover the vacuum case ($\mu_1 = \mu_2 = 0$)
- Pressure can be easily added

4. References

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