Equilibrium β-limits in classical stellarators and beyond

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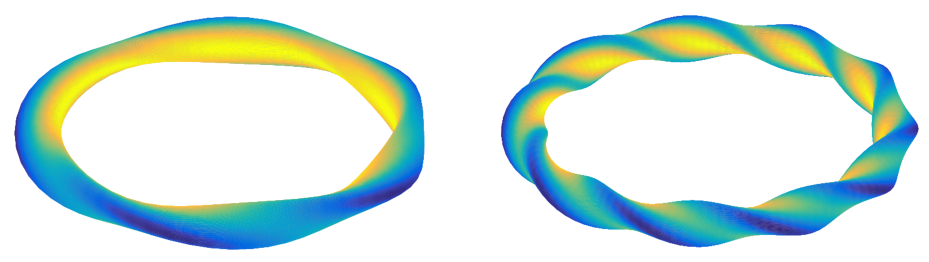
Motivation

- **Maximum achievable** β in stellarators may sometimes be set by the equilibrium and not by its stability [1,2].
- **Magnetic surfaces are not guaranteed to exist** in 3D MHD equilibria without continuous symmetry [3].
  - Vacuum field designed to possess magnetic surfaces [4].
  - Plasma currents potentially degrade magnetic surfaces [2].
- 3D equilibria consist of an intricate combination of **magnetic surfaces**, **magnetic islands**, and **chaos**. Their computation is crucial for confinement, stability, and for the correct interpretation of experimental measurements.
- The **SPEC code** was developed as one possible approach to fulfil this highly non-trivial task [5].
- SPEC finds equilibria using a **variational principle** and allows the plasma to explore energetically favourable reconnection events while constraining pressure and current.
- **Philosophy**: start with a classical stellarator geometry with a simple pressure pedestal and study its β-limit [6].



- **Fixed-boundary**:

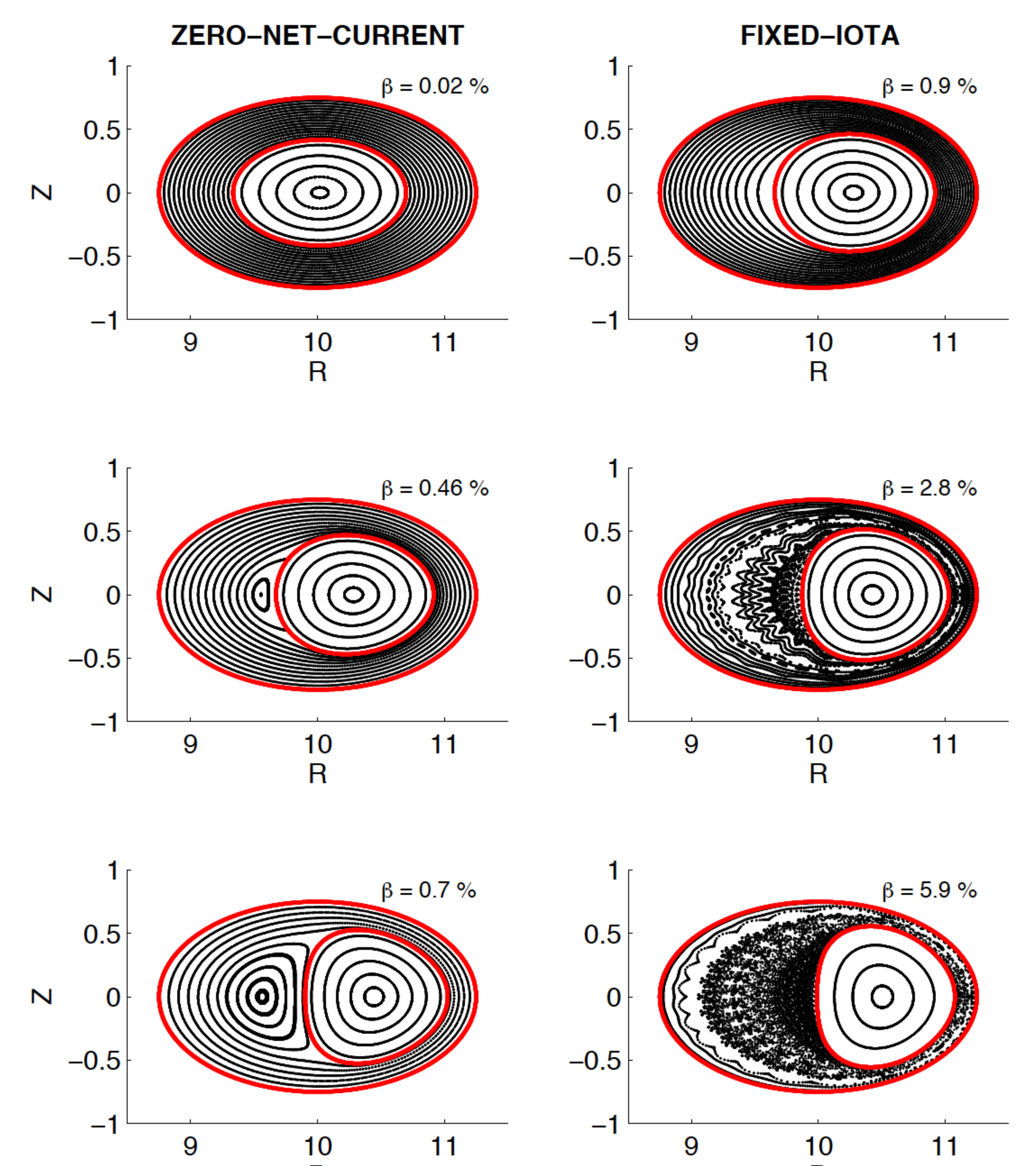
$$R(\theta,\varphi) = R_{00} + \cos\theta + 0.25\cos(\theta - N_p\varphi)$$
$$Z(\theta,\varphi) = -\sin\theta + 0.25\sin(\theta - N_p\varphi)$$

- Simplest model for a **pressure pedestal**:

$$p(\Psi < \Psi_a) = p_0$$
$$p(\Psi > \Psi_a) = 0$$

- Either **zero-net-current** ($I_\varphi = 0$) or **fixed-iota** ($\iota_a = const$).

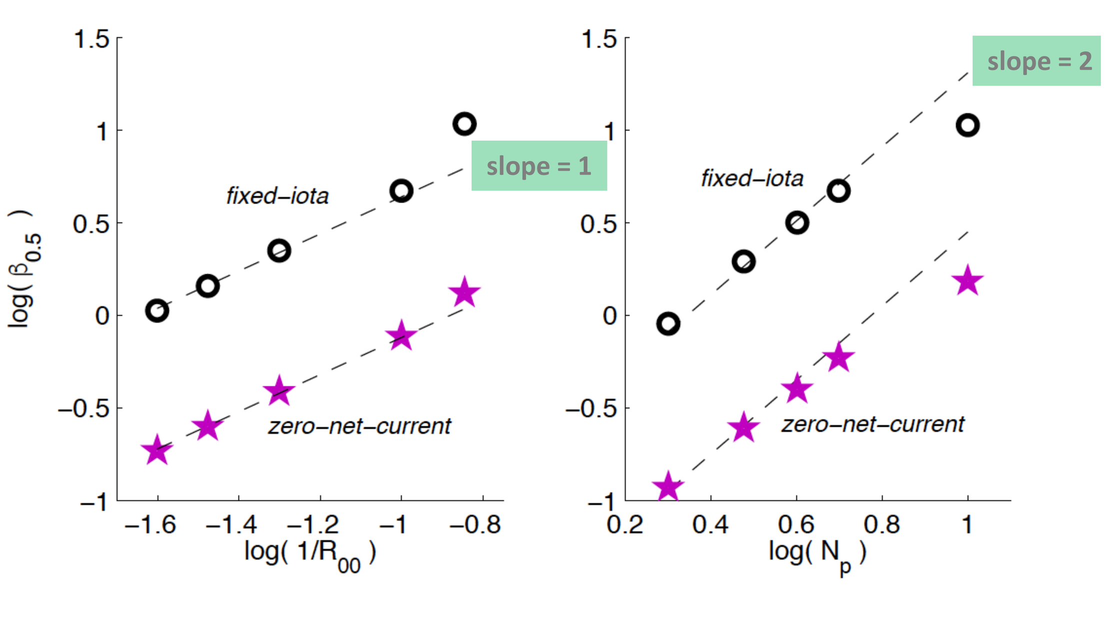
High-β equilibria and Shafranov shift



- Define $\beta_{0.5}$ at which Shafranov shift $= 1/2$ minor radius.
- Theoretical scaling [7],

$$\beta_{0.5} \sim \epsilon\,\iota_v^2 \sim \frac{N_p^2}{R_{00}}$$

confirmed in SPEC calculations.



Ideal β-limit

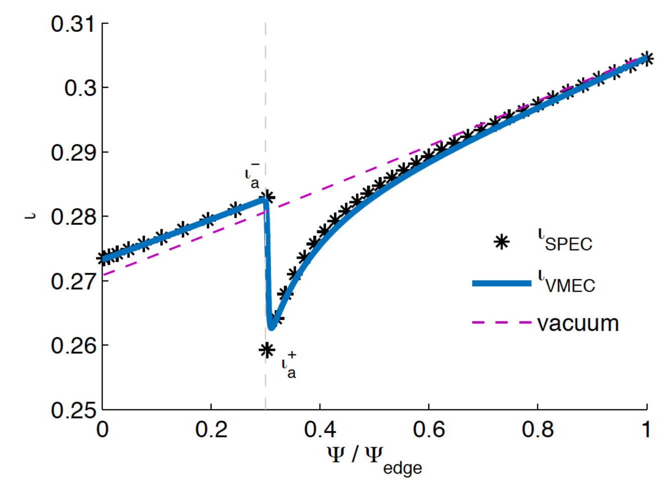
A High-Beta-Stellarator model (HBS) for a classical stellarator [7] was recently developed and predicts how ι_a evolves with plasma β and current I_φ.

$$\iota_a = (\iota_v + \iota_I)(1-\nu^2)^{1/2}$$

where

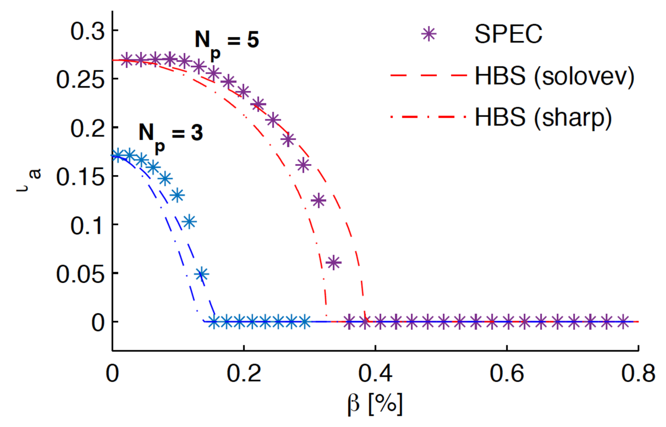
$$\iota_I = \frac{\mu_0 I_\varphi R_0}{2\pi a^2 B_0} \quad \text{and} \quad \nu = \frac{\beta}{\epsilon_a(\iota_v + \iota_I)^2}$$

- Both SPEC and VMEC show that ι_a is indeed reduced at finite β.
- Here $\beta = 0.15\%$ and $I_\varphi = 0$.
- Vertical line indicates $\Psi_a/\Psi_{edge} = 0.3$.



ZERO-NET-CURRENT:

- ι_a scales with β as predicted by HBS.
- Emergence of a separatrix at $\iota_a = 0$ provides ideal β-limit: $\beta_{lim} = \epsilon_a\,\iota_v^2$

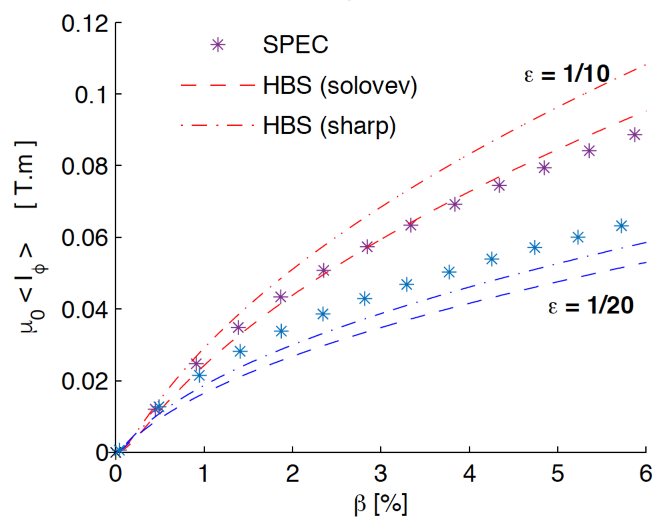


FIXED-IOTA:

- Imposing $\iota_a = \iota_v$ in HBS provides the amount of current required to clamp the rotational transform,

$$\iota_I = \iota_v\left(\sqrt{\frac{1}{2}\left(1+\sqrt{1+4(\beta/\beta_{lim})^2}\right)}-1\right)$$

- There is **no ideal β-limit** (I_φ prevents separatrix to form) but SPEC shows **degradation of magnetic surfaces!**

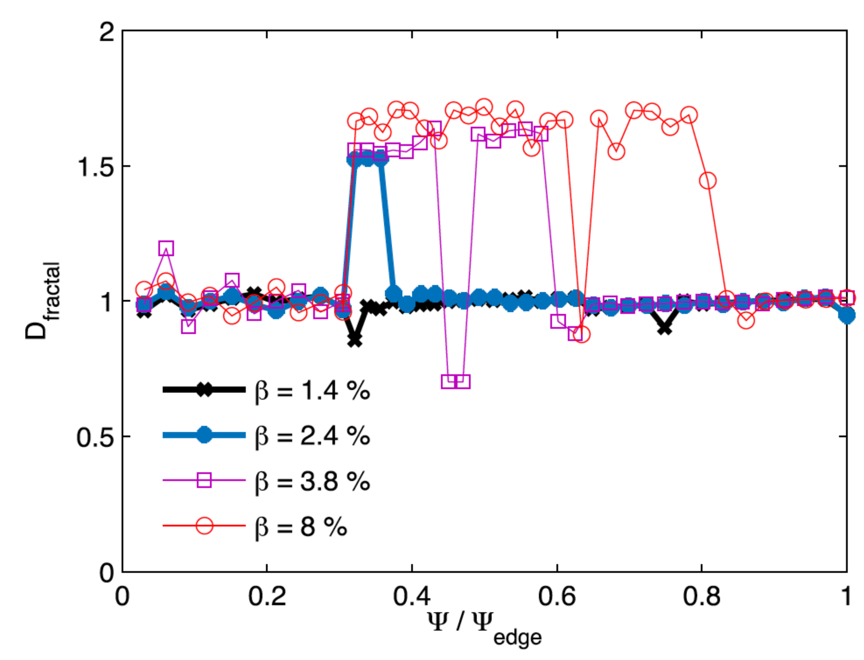


Non-ideal β-limit

We can quantify the emergence of chaos with the **fractal dimension**,

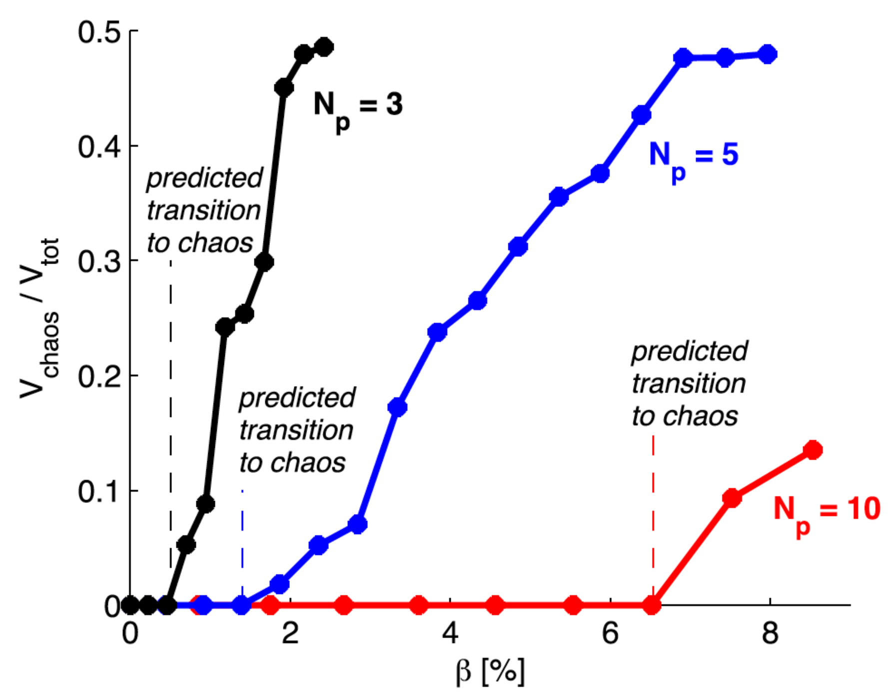
$$D = \lim_{L\to 0}\frac{\log(N)}{\log(L)}$$

of each field line by counting in a Poincaré section the number of boxes N of size L that contain at least a dot. We expect $D = 1$ **for a magnetic surface or an island** and $D > 1$ **for a chaotic field-line**.



- At low β, we observe $D \approx 1$ in the entire volume.
- At sufficiently high β, D jumps to $D_{crit} \approx 1.6$ in regions of increasing volume.
- We can measure the **volume of chaos** using $D(\Psi)$,

$$V_{chaos} = V_{tot}\sum_{i=1}^{n_{lines}}\frac{(\Psi_i - \Psi_{i-1})}{\Psi_{edge}}\mathcal{H}(D(\Psi_i) - D_{crit})$$



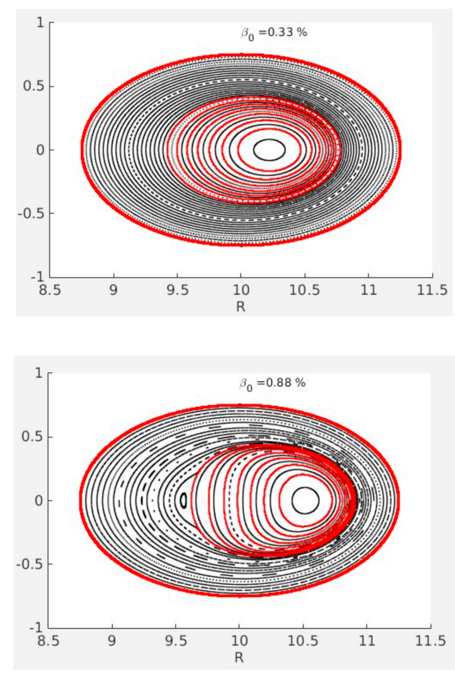
- Assuming that **chaos emerges when** $\iota_I(\beta) \sim \iota_v$, we get from HBS that

$$\beta_{chaos} = \sqrt{12}\epsilon_a\,\iota_v^2$$

which agrees very well with SPEC calculations.

Multivolume equilibria

Classical stellarator β-scan was repeated with SPEC using many volumes each supporting a small pressure step.



Exactly the same results are obtained (β-limit). Indeed the macroscopic equilibrium depends mostly on integral quantities and not so much on the profile details.

W7-X limiter configuration

VACUUM:

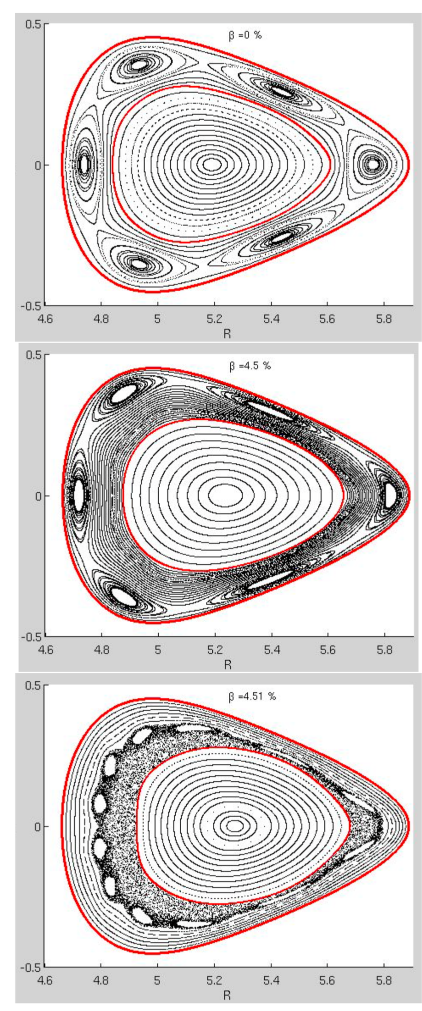
- Boundary extracted from Biot-Savart solution
- 5/6 island chain ($N_p = 5$)

HIGH β, ZERO-CURRENT

- Robust equilibrium and ι_a almost unchanged

HIGH β, FIXED-IOTA

- Small I_φ enough to degrade outer surfaces
- 10/12 island chain



Current-induced healing

SPEC used in newly designed quasi-axisymmetric stellarator with large bootstrap current and $\beta \approx 3\%$ [8].

HIGH-β, ZERO-CURRENT:

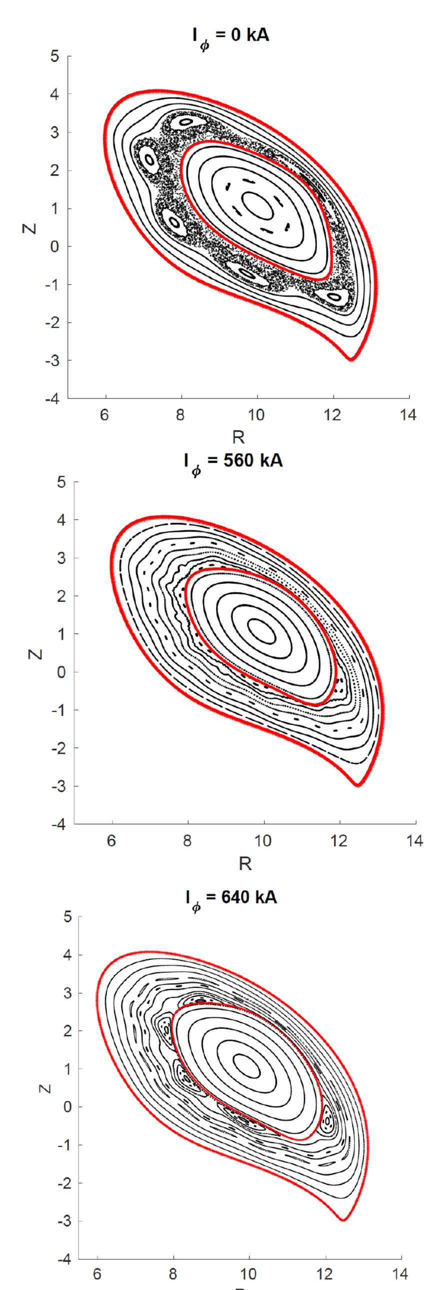
- Boundary extracted from high-β VMEC with $I_\varphi = 0$
- Large island chain opens at $\iota = n/m = 2/6$ ($N_p = 2$)

HEALING:

- Addition of bootstrap-current produces healed equilibria with virtually no islands

DEGRADATION:

- Further increase in current produces new resonances
- Equilibrium appears to be quite sensitive to I_φ



Perspectives

A **free-boundary** version of SPEC is being tested and verified and will allow making predictions that can be validated against experiments.

References

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