# Research projects

S. R. Hudson
*Princeton Plasma Physics Laboratory, PO Box 451, Princeton NJ 08543, USA*
(Dated: June 6, 2016)

Some possible research projects are described.

### A. Gyro-motion across a field discontinuity

An important assumption used for gyro-kinetics is that the equilibrium (background) magnetic field is smoothly varying compared to the length scale of the Larmor radius of the gyro-motion. Many authors have studied gyro-kinetics in ideal-MHD equilibria in stellarator geometry, assuming continuous and smooth pressure and rotational-transform profiles; however, recent work [1] has shown that in three-dimensional (3D) geometry, ideal-MHD equilibria with nested flux-surfaces must have magnetic fields with tangential discontinuities at the rational flux-surfaces in order to eliminate unphysical parallel currents. Most of the work on gyrokinetics in stellarators is thus questionable.

We may consider a simplified Cartesian, slab geometry, with a magnetic field given by $B^z(x,y,z) = 0$, $B^y(x,y,z) = 1$, and

$$B^x(x,y,z) = \begin{cases} f(z) - \frac{1}{2}\Delta\iota, & \text{for } z < 0, \\ f(z) + \frac{1}{2}\Delta\iota, & \text{for } z > 0, \end{cases} \tag{1}$$

for some smooth function $f(z)$. The discontinuity in the tangential field is $\Delta\iota$. (If this field is too simple, we can add some other terms to give appropriate drifts as required.)

Consider a particle that is drifting radially. Away from the discontinuity, the particle motion is accurately described by taking a gyro-average to simplify the numerics, and this is what all major particle codes do. However, the gyro-average is not valid near the discontinuity because the scale length of the variation in the field strength is, being zero, smaller than the gyro-radius. We can, however, follow the exact particle motion, even though this requires more computational effort.

The question is: how does the presence of a discontinuity in the tangential field affect particle motion; can methods that exploit the gyro-average still be used; is there a "correction term" that is required to make the gyro-averaging treatments accurate, does the correction term depend on $\Delta\iota$?

The Larmor radius on one side of the discontinuity will be larger on one side than the other, and I think we will find interesting behaviour in the particle motion near the discontinuity. All numerical codes that follow particle motion in 3D, ideal-MHD equilibria that exploit the gyro-average should carefully treat the particle motion across the current singularities (and accompanying tangential discontinuities) that are inherent in ideal-MHD equilibria.

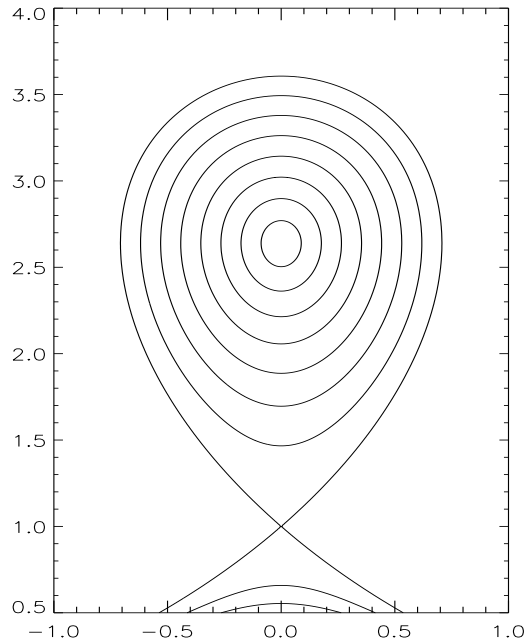

FIG. 1: Flux-surfaces of toy Hamiltonian.

### B. Action-angle coordinates for a simplified tokamak Hamiltonian

Single-null tokamaks have a magnetic field structure similar to

$$H = \frac{1}{2}q^2 + 2p^3 + 3p^2, \tag{2}$$

which is shown in Fig. 1. It would be very useful for a variety of purposes to construct action-angle coordinates for this Hamiltonian, which approximate straight fieldline coordinates for tokamaks. The methods for constructing action-angle coordinates are described in text books on classical mechanics. The only complexity arises is that various elliptic integrals are required. Double-null tokamaks can be similarly approximated by suitably including a $p^4$ term in Eq. (2). With a little thought, it would be possible to also construct simplified Hamiltonians for stellarators, such as the recently-constructed W7-X stellarator that has an island chain as a divertor.

The usefulness of constructing action-angle coordinates for the simplied Hamiltonians is that much of the computational difficulties in treating the separatrix region of tokamaks and stellarators is because of the singularity in straight fieldline coordinates, and by constructing action-angle coordinates for toy Hamiltonians with the same separatrix structure, the coordinate singularity can be treated analytically rather than numerically.

### C. Shafranov shift in knotatrons

In addition to tokamaks and conventional stellarators, there is the recently-discovered *knotatron* [2] class of confinement device (actually re-discovered: knotted confinement devices were first considered theoretically by Solov'jov & Shafranov, 1967 [3]). A knotatron, shown in Fig. 2, is a closed, topologically-toroidal device with the magnetic axis in the shape of a knot; and is a possible, new fusion concept.

By expanding an arbitrary equilibrium about the magnetic axis, analytic expressions for the MHD equilibrium can be derived (see most textbooks on plasma physics, e.g. [4]). Expressions for the Shafranov shift have already been derived for tokamaks and stellarators. What is the expected Shafranov shift in a knotatron? This can be compared to the Shafranov shift for tokamaks and stellarators, and analytic approximations for the equilibrium can later be used to benchmark MHD equilibrium codes in knotted geometry. (The SPEC code [5] has been modified to accomodate knotted geometry, though some work will be required by Dr. S.R. Hudson to update this capability.) Following Solov'jov & Shafranov, 1967 [3], we may derive expressions for the linear, ideal stability etc.

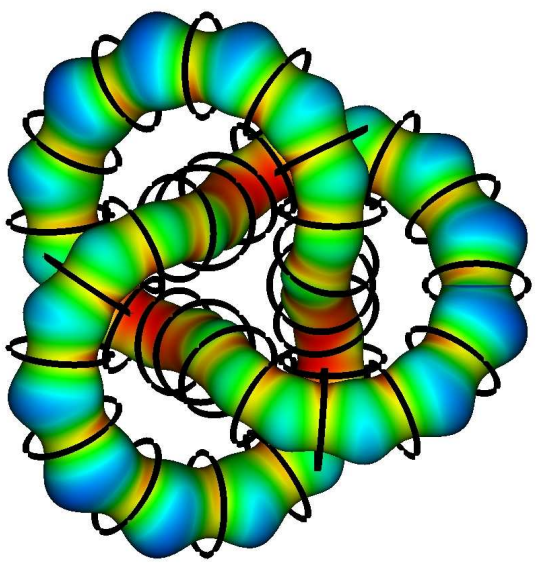

FIG. 2: (color) A trefoil-knotatron: 36 circular, 33cm-radius coils, each with unit current, produce a magnetic field with an axis in the shape of a trefoil knot. The color indicates $|B|$ on a flux-surface.


[1] J Loizu, S. R. Hudson, A. Bhattacharjee, S. Lazerson, and P. Helander. Existence of three-dimensional ideal-mhd equilibria with current sheets. *Phys. Plasmas*, 22:090704, 2015.

[2] S. R. Hudson, E. Startsev, and E. Feibush. A new class of magnetic confinement device in the shape of a knot. *Phys. Plasmas*, 21:010705, 2014.

[3] L. S. Solov'jov and V. D. Shafranov. Closed magnetic configurations for plasma confinement. *Problems in Plasma Theory*, 5, 1967.

[4] R. B. White. *The theory of toroidally confined plasmas.* Imperial College Press, London, 2001.

[5] S. R. Hudson, R. L. Dewar, G Dennis, M. J. Hole, M. McGann, G. von Nessi, and S. Lazerson. Computation of multi-region relaxed magnetohydrodynamic equilibria. *Phys. Plasmas*, 19:112502, 2012.