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Manipulation of islands in a heliac vacuum field

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Abstract

To confine plasma adequately in toroidal magnetic fields for nuclear fusion experiments, it is essential to control the magnetic islands caused by perturbations and non-axisymmetry inherent to stellarators. A method is introduced which enables the perturbation harmonics of the field line Hamiltonian (which determines the size of islands and the degree of chaos) to be approximated quickly. The method is based upon quadratic-flux minimizing surfaces which pass directly through both the X and O points of the island chains. Using a suitable measure of island width, it is possible to utilize standard numerical optimization methods to achieve desirable configurations. A major island chain in the H-1NF heliac vacuum field has been made to both disappear and to reappear with opposite phase. These results have significance for self-healing phenomena and the method is generally applicable to all stellarators and $1\frac{1}{2}$-dimensional Hamiltonian dynamical systems.

1. Introduction

For the toroidal confinement of plasma, the existence of good flux surfaces (nested invariant tori of the field line flow) is essential [1]. Such surfaces are guaranteed only for configurations possessing an ignorable coordinate, such as the ideal tokamak. For systems without an ignorable coordinate, such as the stellarator or tokamak with field ripple, the complicated geometry implies that in general, there will exist islands and chaotic regions, though the KAM theorem [2,3] gives reason to believe that some flux surfaces will remain. It is reasonable to expect that to have good surfaces at non-zero plasma pressure, we must begin with nested flux surfaces for the vacuum field, as suggested in Ref. [4]. However, it has been observed that islands present in the vacuum field may "self-heal" as the plasma pressure is increased [5,6]. With this in mind, we might actually *prefer* the existence of islands of a specified phase in the vacuum field. Ultimately, we would like to control where islands will occur, in both the vacuum and low pressure plasma, and their phase and magnitude. If this information can be determined quickly, and a numerical parameter reflecting the desirability of a certain configuration is introduced, then computer routines that construct numerical derivatives in coil current configuration parameter space may rapidly determine how to alter the currents to achieve the desired island structure. The Cary-

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Hanson method [7,8] was introduced with such aims in mind. A measure of island width (namely Greene's residue [9]) that is reasonably quick to compute, and goes to zero if the island vanishes, was used in computer optimization routines to find vacuum configurations that reduce the size of specified islands.

In this paper, we suggest a new, more efficient method by which island width may be quickly approximated. At the heart of the approach are the quadratic-flux minimizing surfaces introduced by Dewar et al. [10] and studied further by Hudson and Dewar [11]. These surfaces, being constructed from a family of periodic pseudo field lines (including the true closed field lines with given rotational transform), pass directly through the X and O points of island chains on Poincaré cross sections. If we regard the actual field as the superposition of a perturbing field and an integrable field, the maximum component of the magnetic field normal to these surfaces is directly related to the amplitude of the resonant perturbation. If also the shear of the nested system is known, this amplitude determines the width of the island. The method is theoretically similar to an averaging procedure which identifies the amplitude of the perturbation to an integrable field [12,13], but in implementation it is similar to a more recent technique that uses only information obtained by integrating around a periodic curve [14]. Our technique is efficient computationally, flexible and applicable to all stellarator devices. It enables current configurations to be found that eliminate island chains, as well as configurations in which islands of a particular type and phase may be preset to specified size and radial location in the vacuum configuration.

In Section 2 we explain the construction of quadratic-flux minimizing surfaces and introduce periodic pseudo orbits and the action gradient defined on periodic curves. The action gradient is intimately associated with islands and will be used as a measure of the island widths. In Section 3 it is explained how stellarator symmetry prevents island chains from rotating as the current configuration is altered because periodic orbits, either stable or unstable, are always found on symmetry lines. In Section 4 the method of manipulating magnetic islands is explained and applied to the vacuum field of the heliac H-1NF [15]. The results indicate magnetic islands may be manipulated by variation of the coil current configuration, in this application the vertical field coil currents. Section 5 discusses how the method may be applied in a more general approach, its relevance to self-healing phenomena and a comparison with the Cary–Hanson technique.

2. Periodic pseudo orbits

In considering which are the most suitable coordinates for analyzing magnetic fields with some flux surfaces and some islands, Dewar et al. [10] consider surfaces that minimize the functional

$$\varphi_2 = \int_{\Gamma} \frac{B_n^2}{2C_n}\, \mathrm{d}\sigma, \tag{1}$$

where $A_n \equiv \boldsymbol{A}\cdot\boldsymbol{n}$, with $\boldsymbol{n}$ the unit normal to the trial surface Γ, and where $\boldsymbol{C}$ is an auxiliary divergence-free field everywhere transverse to the surface Γ. The choice of $\boldsymbol{C}$ is arbitrary, but as the resulting surface depends upon $\boldsymbol{C}$ it should be chosen with care. Typically we use $\boldsymbol{C} = \nabla\theta \times \nabla\zeta$ in some toroidal coordinates (ρ, θ, ζ), where ρ is a radial coordinate, θ a poloidal angle and ζ some toroidal angle. With this choice $\boldsymbol{C}$ is both parallel to the radial basis vector $\boldsymbol{e}_\rho \equiv \partial_\rho \boldsymbol{r}$ and related to the action gradient defined on periodic curves [11]. The arbitrariness of $\boldsymbol{C}$ is associated with the arbitrariness of the underlying coordinates in which the quadratic-flux minimizing surfaces are to be constructed. Preferably, straight field line coordinates for an assumed nearby nested magnetic field are used though such a choice is not essential for the construction of the surfaces.

On allowing the surface Γ to vary, Dewar et al. [10] obtain the Euler–Lagrange equation required to make $\delta\varphi_2 = 0$,

$$\boldsymbol{B}_\nu \cdot \nabla\nu = 0, \tag{2}$$

where $\boldsymbol{B}_\nu \equiv \boldsymbol{B} - \nu\boldsymbol{C}$ and $\nu \equiv B_n/C_n$. We call $\boldsymbol{B}_\nu$ the pseudo field and ν the action gradient for surface s. Eq. (2) shows that ν is constant along a pseudo field line and that the solution surfaces, with rational rotational transform, are comprised of a family of periodic pseudo field lines.

The action defined on curves $\theta = \theta_0(\zeta)$, where the curves are periodic and lie on surfaces of constant s, is written $S = \oint \boldsymbol{A}\cdot \mathrm{d}\boldsymbol{l}$ [16]. Considering the difference

in action between a reference periodic pseudo orbit $\theta_0(\zeta)$ and an arbitrary periodic pseudo orbit $\theta_1(\zeta)$, we write

$$S - S_0 = \oint_{\partial S} (\nabla \times \boldsymbol{B}) \cdot \mathrm{d}\boldsymbol{l} = \int_S \boldsymbol{B} \cdot \mathrm{d}\sigma. \tag{3}$$

Here S represents the surface bounded by the periodic curves $\theta_1(\zeta)$ and $\theta_0(\zeta)$, and ∂S represents the boundary of the surface, which is just the curves θ_1 and θ_0 again but with the directions of the line integral taken in alternate directions. Varying the boundary curve to $\theta_1(\zeta) + \delta\theta(\zeta)$, we derive the action gradient

$$\delta S = \int_{\theta_1} \frac{\delta S}{\delta \theta} \delta\theta \, \mathrm{d}\zeta = \int_{\theta_1} \frac{\boldsymbol{B} \cdot \boldsymbol{n}}{\boldsymbol{n} \cdot \nabla\theta \times \nabla\zeta} \delta\theta \, \mathrm{d}\zeta. \tag{4}$$

We now recognize ν as the action gradient $\delta S/\delta\theta$ on the periodic curves that comprise the quadratic-flux minimizing surface and that the quadratic-flux minimization principle is essentially the minimization of the square of the action gradient over the (θ, ζ) domain.

We restrict attention to rational quadratic-flux minimizing surfaces, these being surfaces upon which the rotational transform of the pseudo magnetic field lines is a rational value $\iota = n/m$ and periodic pseudo orbits satisfy $(\rho_{j+m}, \theta_{j+m}) = (\rho_j, \theta_j + 2\pi n)$ where (ρ_j, θ_j) label successive points for the pseudo Poincaré map $P_\nu : (\rho, \theta) \rightarrow (\rho_1, \theta_1)$, defined on the cross section plane $\phi = 0$, by following the pseudo magnetic field $\boldsymbol{B}_\nu$ for some given ν (constant along a pseudo field line). To locate the real periodic orbits we set $\nu = 0$ and search, using Broyden's method [17] in the two dimensions (ρ, θ), for the fixed points of the mth iteration of the pseudo Poincaré map P_ν^m, which with $\nu = 0$ is identical to the Poincaré map for the real magnetic field. For computational efficiency a good starting guess is given. Rather than specify ν and search for the periodic pseudo orbit as just described, a more robust method to determine all periodic pseudo orbits is instead to specify θ and to search in the two dimensions (ρ, ν) for a fixed point of P_ν^m. The action gradient vanishes on all good flux surfaces, which reduces the two-dimensional search to a one-dimensional search [11] in this case. In this manner the function $\nu(\theta)$ is determined on each quadratic-flux minimizing surface. In the construction of the full periodic surface, a periodic pseudo orbit is required at every poloidal angle.

The intersection of a quadratic-flux minimizing surface with the symmetry plane $\phi = 0$ is shown as the dashed curve in Fig. 1, along with a Poincaré plot of the magnetic field in H-1NF (see Section 4) and a coordinate grid of the background toroidal coordinates (ρ, θ, ϕ) used to construct the surface (see Section 3). Due to stellarator symmetry, the Poincaré plot on the symmetry plane $\phi = 0$ is up–down symmetric about the line $z = 0$ (see Section 3), and the significant island chain (5, 3) has an O point on the outward symmetry line (also discussed in Section 3). The coordinate grid is shown as it implicitly defines the choice of $\boldsymbol{C}$ used in Eq. (1). Importantly, the coordinate cross section is up–down symmetric on the symmetry plane. Periodic quadratic-flux minimizing surfaces necessarily pass directly through both the X points and O points of island chains, and this is clearly indicated. The few parameters specifying the background coordinate cross section are chosen by hand to match the inner nested flux surfaces, but this crude fit is not generally reliable for all regions of the Poincaré cross section.

The function $\nu(\theta)$ for this surface is shown as the solid line in Fig. 2. Consistent with the existence of $2m$ X or O points are the $2m$ zeros of ν, as will be discussed in Section 3.

3. Stellarator symmetry

Systems with a continuous symmetry, either asymmetric [1] or helically symmetric [18] are guaranteed to possess good flux surfaces. For real stellarators however, both cylindrical and helical symmetry are broken and from the outset islands are expected to occur even in vacuum magnetic fields. Yet stellarators are designed to have a discrete symmetry known as the *stellarator symmetry*, which for example ensures that Poincaré sections of the magnetic field on a symmetry plane will be up–down symmetric. A system possesses stellarator symmetry if a choice of the $\phi = 0$ plane exists such that the total current density satisfies

$$J^R(R, -\phi, -z) = -J^R(R, \phi, z), \tag{5}$$

$$J^\phi(R, -\phi, -z) = J^\phi(R, \phi, z), \tag{6}$$

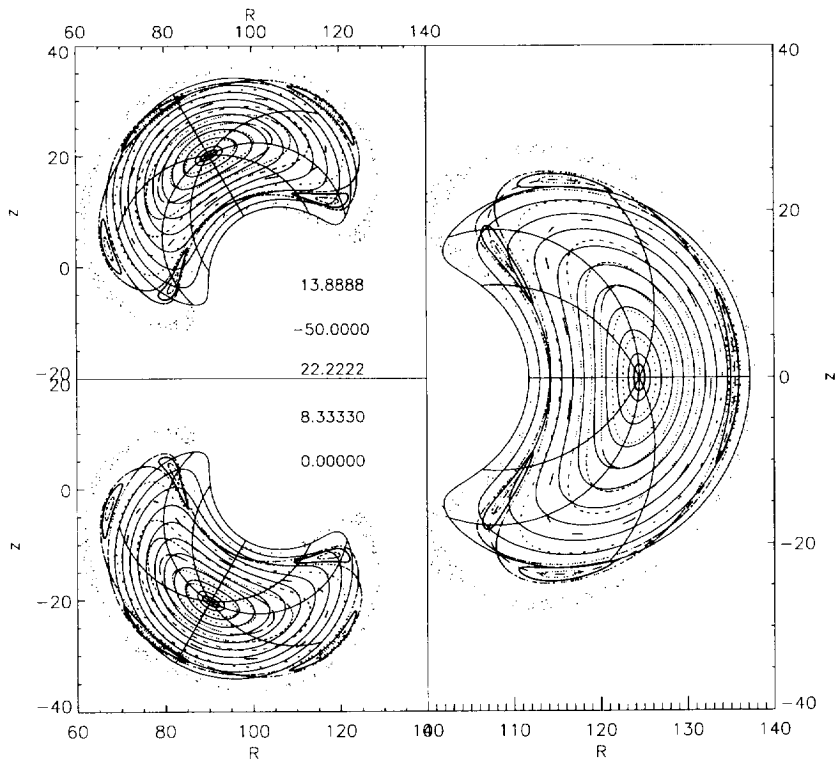

Fig. 1. Poincaré plot (dots) of the standard magnetic field in H-1NF shown on three toroidal planes ($\phi = 0$ on right, $\phi = 2\pi/9$ on above left, and $\phi = 4\pi/9$ on bottom left). Also shown on each section are the helically toroidal coordinate grid (solid line) and the (5, 3) quadratic-flux minimizing surface on the $\phi = 0$ section (dashed curve).

$$J^z(R, -\phi, -z) = J^z(R, \phi, z), \tag{7}$$

where J^R, J^ϕ and J^z are the contravariant components of the current in cylindrical coordinates. The equation $\nabla \times \boldsymbol{B} = \mu_0 \boldsymbol{J}$ implies that the magnetic field $\boldsymbol{B}$ will also satisfy the symmetry provided $\boldsymbol{B} \to 0$ at infinity, and the equation defining field line flow, using the cylindrical angle ϕ as the field line parameter, $d\boldsymbol{r} = R\widehat{\boldsymbol{B}}d\phi$, has the property that if $(R(\phi), z(\phi))$ is a field line, then so is $(R(-\phi), -z(-\phi))$. This is the analogue of time reversal symmetry in Hamiltonian dynamical systems, ϕ playing the role of time. The plane $\phi = 0$ is a symmetry plane and the line $\phi = 0, z = 0$ is a symmetry line. We see that if a periodic trajectory exists, then the reflection of its Poincaré section in the $z = 0$ plane will also correspond to a periodic trajectory. The Poincaré–Birkhoff theorem [3] ensures that after periodic surfaces have been destroyed by perturbation, there will remain at least one X trajectory and at least one O trajectory. Assuming there is only one of each (the typical case) requires each periodic trajectory to coincide with its own reflection. This is enough to show that for any given periodicity, there must be a symmetric periodic orbit on the symmetry line [19].

If the background toroidal coordinate system is up–down symmetric on a symmetry plane and $\theta = 0$ corresponds to the symmetry line, the guaranteed existence of the periodic orbit on this line (and the fact that $B^R(R, 0, 0) = 0$) ensures that $\nu(0) = 0$. We call this phase locking of the action gradient, and this is responsible for the numerically observed locked phase property of the magnetic islands [5,6]. We expect that the function $\nu(\theta)$ is something like a sine curve, as it is periodic and equal to zero for both the X and O points. The periodic orbit on the symmetry line may either be an X point or an O point, as this detail depends on the sign of the gradient of the action gradient on the symmetry line, $\mathrm{sgn}[\nu'(0)]$, and the sign of the shear of the underlying integrable system.

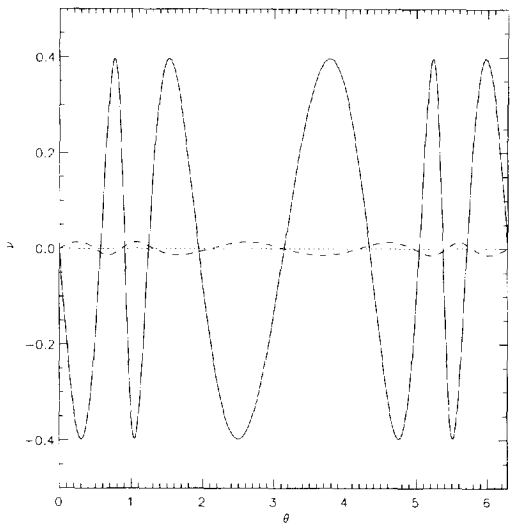

Fig. 2. Behaviour of ν with θ for the $(5,3)$ surface for the various current configurations. The solid line is the ν obtained for the configuration displayed in Fig. 1, the dashed line corresponds to the configuration shown in Fig. 3 and the dotted line corresponds to the configuration shown in Fig. 3. The phase locking of ν is clearly indicated as all three lines pass through zero for $\theta = 0$.

The amplitude of ν is not determined by symmetry but by the size of the supposed perturbation and how well the background coordinate system reconstructs straight field line coordinates for an integrable field. We may estimate the island widths by recording the maximum component of the magnetic field normal to the quadratic-flux minimizing surface with the shear of the pre-supposed underlying integrable system using the expression

$$\Delta\rho = 2\sqrt{\frac{\max(|B_n|)}{m|\iota'|}}. \tag{8}$$

The maximum normal magnetic field is obtained during the construction of the quadratic-flux minimizing surfaces and the shear of the underlying integrable system may be estimated from field line tracing methods. In this application, it is not required to determine these quantities precisely as we may use a different indicator of island width. We simply use the maximum value of $|\nu|$ on a surface to indicate the size of the island chain.

The phase locking of the action gradient implies that the angular location of $\max(|\nu|)$, θ^*, cannot rotate, though it may vary slightly. Once θ^* has been located for a particular current configuration, we may consider it fixed as we make small variations of the currents. Shown in Fig. 2 is the behaviour of ν constructed using the $(m,n) = (5,3)$ quadratic-flux minimizing surface for the three configurations displayed in this article. For each configuration, $\nu(0) = 0$ (which confirms the phase locking of the action gradient) and $\theta^* \approx 0.31$.

Starting with the standard configuration shown in Fig. 1 we define a measure, ν^*, of the size of the (m,n) island and its phase to be the value of ν at this angle, $\nu(\theta^*)$. Though we have not estimated the shear, and so cannot estimate island widths, we may utilize the fact that the (m,n) island has vanished if ν^* is zero, and the phase of the island chain is related to $\mathrm{sgn}(\nu^*)$.

4. Application to H-1NF heliac

The ideas discussed in the previous sections will be applied to the H-1NF heliac in a new method that enables subtle control of the vacuum magnetic islands. H-1NF is a stellarator type toroidal plasma confinement device in operation at the Plasma Research Laboratory, Australian National University [15,20]. The geometry of the device is periodic in toroidal angle with periodicity length $2\pi/3$ and the stellarator symmetry is present in design (though there are small symmetry breaking errors). We denote the current configuration in H-1NF as a ratio between the currents in the toroidal field coils (of which there are 36), the central ring conductor, the inner vertical field coils of radius 200 cm (of which there are 2), the outer vertical field coils of radius 72 cm (of which there are 2), and the helical winding currents (of which there are 4) as $I_{\mathrm{tfc}}/I_{\mathrm{cr}}/I_{\mathrm{iv}}/I_{\mathrm{ov}}/I_{\mathrm{hw}}$. The standard configuration shown in Fig. 1 is 13.88/−50.00/8.33/22.22/0.00. Setting the current in all the toroidal field coils to the same value maintains the period-3 nature of H-1NF. With the currents in the upper outer and inner vertical coils equal to those in the lower outer and inner coils respectively, the stellarator symmetry of H-1NF is maintained. The symmetries are independent of current configuration provided these constraints are respected.

For the background coordinates, we use helically toroidal coordinates chosen with the axis closely coinciding with the magnetic axis. This enables the periodicity of orbits to be determined by introducing poloidal and toroidal angles θ and ζ respectively. Also, if we are to use $\boldsymbol{C} = \nabla\theta \times \nabla\zeta$, then by requiring $\boldsymbol{C}\cdot\boldsymbol{n}$ to be

nowhere zero on a quadratic-flux minimizing surface we require the surfaces of constant ρ to be nowhere transverse to the magnetic field. This requirement is generally satisfied by toroidal coordinates. The estimation of the resonant perturbation amplitude, ν^*, however is more sensitive to the choice of coordinates, as they should be close to straight field line coordinates for a supposed underlying nested field. A coordinate system is chosen that has a bean-shaped cross section that reflects the shape of magnetic surfaces in H-1NF. This cross section rotates in the negative θ direction with periodicity 3, to match the periodicity of H-1NF, and is centered on the magnetic axis (which is located in cylindrical coordinates). Three different cross sections of this coordinate system are plotted with the Poincaré plots in Fig. 1 which shows both the bean shaped cross section and the rotation.

For H-1NF, we modify the definition of the pseudo Poincaré map P_ν to be the map from $\phi = 0$ to $\phi = 2\pi/3$ obtained by following the pseudo field $\boldsymbol{B}_\nu$, and (m, n) periodic orbits are those that satisfy $(\rho_{j+m}, \theta_{j+m}) = (\rho_j, \theta_j + 2\pi n)$. The rotational transform in helically toroidal coordinates is $\iota_h = 3n/m$, and in a non-rotating, right handed toroidal coordinate system the rotational transform is $\iota = \iota_h - 3$. Fig. 1 shows a large magnetic island. We will show that magnetic islands may be manipulated by variation of the vertical field coil currents, as suggested by Ref. [21]. In the following discussion, the toroidal field current, the central ring current and the helical winding current are considered constant. It is these currents which are dominant in determining the rotational transform on axes and thus in this application the overall rotational transform profile varies marginally. We consider the function

$$\Lambda = (\nu^* - \nu_0^*)^2, \tag{9}$$

where ν_0^* is to be set, indicating the desired island size and phase (magnitude and sign of ν_0^* respectively). From Fig. 2, we observe $\nu^* \approx -0.4$ and $\theta^* = 0.31$. To firstly remove this island chain, we set $\nu_0^* = 0$. The configuration that minimizes Λ, and thus minimizes the amplitude of the resonance harmonic producing the island, is determined by a numerical search in the two-dimensional, vertical field coil current space. The subroutine E04JAF from the NAG library was used. For each trial configuration used to evaluate the partial derivatives, the magnetic axis and the periodic pseudo orbit each take about two or three iterations of the first return or mth return map respectively. By varying the vertical fields, it was possible to reduce Λ by several orders of magnitude. To confirm that the configuration was as desired, a Poincaré plot of the minimizing magnetic field is shown in Fig. 3 and the full quadratic-flux minimizing surface was constructed. Here we see the $(5, 3)$ island chain has become negligible in width. Importantly the $(5, 3)$ surface is still present, so that the $(5, 3)$ island chain has not been eliminated by simply shifting the rotational transform profile away from this resonance. The action gradient (shown in Fig. 2 as the dotted curve) is very close to zero, indicating that the family of periodic pseudo orbits is degenerate in action and the quadratic-flux minimizing surface has reduced to the periodic flux surface.

The island chain may be recreated, but with the X and O points having swapped their location, by setting $\nu_0^* = 0.4$ (equal in magnitude but opposite in sign to that observed for the initial configuration) and again minimizing Λ. A Poincaré section of the optimized configuration is shown in Fig. 3 where we see the $(5, 3)$ island chain has changed phase by 180°. The quadratic-flux minimizing surface passes through both the X and O points of the islands. The behaviour of ν is shown as the dashed curve in Fig. 2. In this instance, ν^* was not close to the set value ν_0^*. It is at least positive, and so the Λ minimization has swapped the phase of the island. The size of the island chain indicated by the Poincaré plot is comparable to the size of the initial island chain, so perhaps the local shear of the underlying magnetic shear has also been reduced, compensating for the lower estimate of the resonant perturbation amplitude. A better estimate of the resonance amplitude is provided by the maximum normal component of the magnetic field to the quadratic-flux minimizing surfaces.

We observe that for this configuration region, the vertical field coils provide a means to "fine-tune" the configuration. Such results are interesting in association with self-healing of islands. This indicates the self-healing of magnetic islands may result from a geometrical mechanism as suggested by Mikhailov and Shafranov [21]. Such results also indicate the flexibility of the stellarator design.

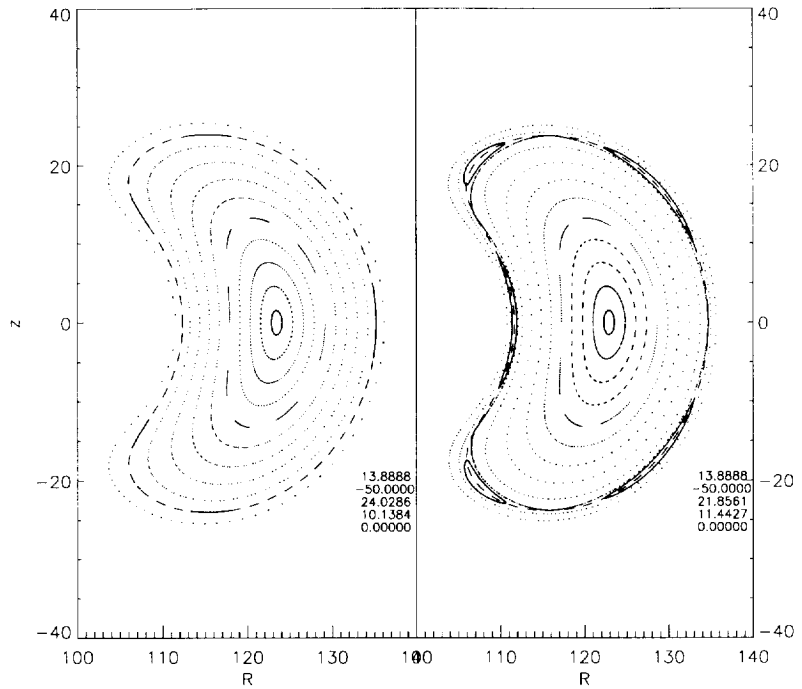

Fig. 3. Poincaré plots for the configurations that reduce the (5, 3) island chain (shown on left) and the configuration that produces a significant (5, 3) island chain with the phase flipped (shown on right). The (5, 3) quadratic-flux minimizing surface is shown for each configuration as the dashed curve. The configurations are similar to that shown in Fig. 1, with alterations only to the vertical field currents.

5. Discussion

In this paper, only one island chain was considered and no attention has been given to the radial position of the island. The shear of the system has not been examined, and no estimate of the shear of the assumed underlying integrable system is provided. This paper has only utilized the resonant perturbation amplitude and phase in its analysis and control of magnetic islands, and relied on the assumption that the rotational transform profile of the configuration is not altered significantly. This assumption is justified if only small variations are made to the vertical field coil currents only. A more general approach is to extend the definition of Λ given in Eq. (9) to consider more island chains, and to allow all the currents to vary in a five-dimensional minimization. In this case, the rotational transform may vary quite significantly and a contribution to Λ, perhaps of the form $(\rho^* - \rho_0^*)^4$ for each island chain, should be made. Here ρ_0^* is the preferred radial location of the periodic orbit at θ^*, and ρ^* is the actual location of the periodic orbit for a particular trial configuration. An exponent of 4 strongly restricts the configuration to ensure the periodic orbit is near the chosen value. A coordinate-independent measure of the resonance amplitude is provided by using the maximum normal component of the magnetic field to the quadratic-flux minimizing surface. If several island chains are to be examined, then a good estimate of the shear of the somewhat arbitrary underlying integrable system is provided by using a few low order rational periodic orbits as these are likely to be the least displaced with resonant perturbations. The islands considered in this Letter are quite low order, as these are cheaper to evaluate computationally, requiring less iterations of the Poincaré map and are likely to the biggest islands present. H-1NF is a low shear device, and since low order resonances are to be avoided, in a more complete analysis we would be forced to work with higher order rationals. For practical purposes, it may be required to increase the volume occupied by nested flux surfaces by focussing attention upon island chains that arise near the edge of the plasma region [8].

The Cary–Hanson technique requires the real periodic orbit to be located and the tangent mapping approximated at that point. A similar number of iterations of the Poincaré map is required to locate the real periodic orbit as for the pseudo periodic orbit, so the techniques will be comparable in the location of orbits. The Cary–Hanson method needs to evaluate the tangent mapping further, so requires more computation after the orbit is located. In our method, some computational overheads are required prior to the location of the periodic orbits, namely the construction of Fig. 2 to determine θ^*. This value may vary slightly as the configuration is varied about some average position, but this has little effect on the success of the method. In practice, the zeros and maxima of $|\nu|$ for any given surface vary very slightly as shown in Fig. 2. The extra calculations required in updating the background coordinate system as the current configuration is varied is not essential as the magnetic axis varies marginally and thus may be omitted.

Controlling the size and phase of islands in the vacuum field of H-1NF enables one to set vacuum currents which oppose the growth of islands caused by nonzero plasma pressure. This is an important aspect of adequately confining the plasma. Experimental verification of the island manipulations in H-1NF is possible using the techniques described in Ref. [20]. It would be interesting to examine the results of the HINT [5] code to the magnetic configurations obtained in this paper.

6. Conclusion

By working in a coordinate system that is tied to the invariant periodic sets associated with chaotic island structure (the periodic orbits), a natural parameter of island size is derived. This parameter is essentially the component of the magnetic field normal to the coordinate surface that passes though both the X and O points. It provides a quick, robust and accurate method of estimating both the phase and size of islands of arbitrarily high order. Such estimations have enabled optimization routines to manipulate vacuum magnetic islands enabling suitable current configurations to be obtained.

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