A minimally constrained model of self-organized helical states in reversed-field pinches

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We show that the self-organized single-helical-axis (SHAx) and double-helical-axis (DAx) states in reversed field pinches can be reproduced in a minimally constrained equilibrium model using only five parameters. This is a significant reduction on previous representations of the SHAx which have required an infinite number of constraints. The DAx state, which has a non-trivial topology, has not been previously reproduced using an equilibrium model that preserves this topological structure. We show that both states are a consequence of transport barrier formation in the plasma core, in agreement with experimental results.

A major goal of the theory of complex physical systems is to find relatively simple organizing principles that operate when systems are strongly driven. A famous early example of such a universal principle is the Taylor relaxation principle [1], which postulates that a plasma tends to minimize its total magnetic energy subject only to the constraints of conservation of global magnetic flux and global magnetic helicity. This principle has been successful in describing the classical behavior of the core region of Reversed Field Pinch (RFP) experiments which contained many magnetohydrodynamic (MHD) modes resonating on different plasma layers. These modes formed overlapping magnetic islands and resulted in a chaotic field region, extending over most of the plasma volume [2]. The consequent destruction of magnetic surfaces led to modest confinement, and was thought to prevent fusion power development with the RFP.

This classical paradigm of the RFP as a chaotic plasma with modest confinement properties has been challenged in recent years with the observation of the high-confinement quasi-single-helicity (QSH) regime [3, 4]. The transition to the QSH regime occurs as the plasma current is increased (>1MA), and a single dominant helical mode arises spontaneously. A second (helical) magnetic axis forms associated with this helical mode and this state is known as the double-axis state (DAx) [5]. As the current is increased further a topological change in this magnetic configuration is observed: the main magnetic axis and the second axis of the DAx state merge, forming a helical plasma column despite the axisymmetric plasma boundary. This is the single-helical-axis (SHAx) state [6] which has recently been observed in RFX-mod [7, 8] and is associated with strong electron transport barriers and significantly improved plasma confinement.

As the DAx and SHAx states are formed by a self-organized process, they should be describable in terms of a small number of parameters. Taylor's theory was successful in describing the classical chaotic regime in the core of the RFP with only two parameters, however it is unable to describe the self-organized states in the QSH regime because although it has a helical solution for sufficiently high magnetic helicity [1], the helical pitch of this solution is opposite to that of the observed QSH states [3].

The SHAx state in the QSH regime has been reproduced using the ideal MHD equilibrium framework assuming continuously nested magnetic flux surfaces [9] (see Figure 1(a)–(d)). The continuously nested flux surface assumption typically used with ideal MHD requires the specification of the enclosed toroidal and poloidal fluxes as a function of the magnetic flux surface. These continuous flux functions are an infinite number of constraints on the plasma equilibrium, and are therefore not a natural description of the self-organized QSH regime. The continuously nested flux surface assumption also prevents the description of non-trivial magnetic structure such as islands and chaotic regions. These constraints prevent this equilibrium framework from describing the DAx state, which has two magnetic axes. This Letter presents the results of a generalization of Taylor's theory which describes both the SHAx and DAx states in the QSH regime with a minimum number of free parameters. Both states are naturally reproduced as a result of a single transport barrier in the core of the plasma. This is in agreement with experimental observations of an electron transport barrier surrounding the core of the plasma in the SHAx state [8].

A stable plasma equilibrium is a constrained minimum of the plasma energy

$$W = \int \left(\frac{\mathbf{B}^2}{2\mu_0} + \frac{p}{\gamma - 1} \right) d^3x, \tag{1}$$

where $\mathbf{B}$ is the magnetic field, μ_0 is the permeability of free space, p is the plasma pressure and γ is the ratio of specific heats. The plasma states over which W is minimized must be constrained to avoid the trivial $\mathbf{B} = 0$ solution. The traditional approach of ideal MHD is to con-
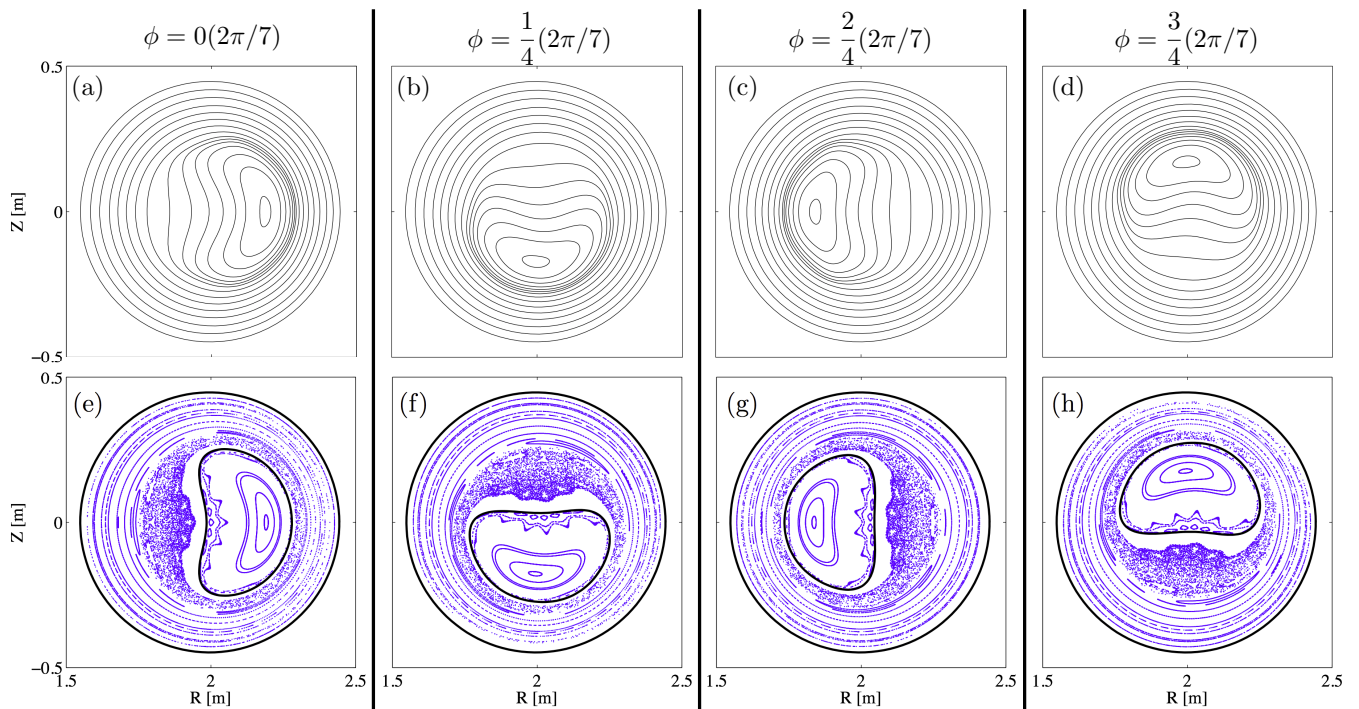
FIG. 1. Comparison of the ideal MHD representation of the SHAx state in RFX-mod and the minimal model (MRXMHD) of this state presented in this work. Figures (a)–(d) show the (poloidal) magnetic flux contours of the ideal MHD plasma equilibrium at equally spaced toroidal angles covering one period of the helical solution. Figures (e)–(h) show Poincaré plots of the minimal model at the same toroidal locations as (a)–(d). The thick black lines mark the location of the transport barrier separating the two plasma volumes. The minimal model corresponds to the $s = 0.3$ configuration of Figure 3.
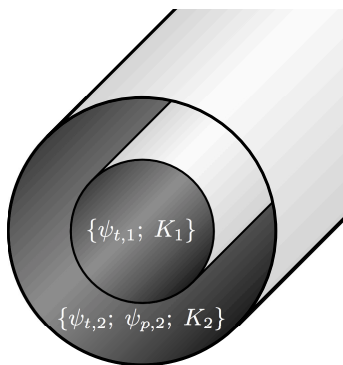

FIG. 2. Five constraints are needed to specify the two-volume MRXMHD plasma equilibrium: the toroidal flux in each volume, $\psi_{t,i}$; the poloidal flux in the outer volume, $\psi_{p,i}$; and the magnetic helicities in each volume, K_i.

sider only states with nested magnetic flux surfaces with the enclosed toroidal and poloidal fluxes specified as a function of the magnetic flux surface. This Letter considers a wider class of plasma equilibria by relaxing the *continuously* specified constraints of the traditional equilibrium framework to a finite number of *discrete* constraints. We apply the MRXMHD framework [10, 11], which is a generalization of Taylor's relaxation theory in which the

plasma is partitioned into a finite number of nested regions $\mathcal{R}_i$ that independently undergo Taylor-relaxation. The plasma regions are separated by ideal transport barriers $\mathcal{I}_i$ that are also assumed to be magnetic flux surfaces (the two-volume case is illustrated in Figure 2). In the MRXMHD framework plasma equilibria are obtained by minimizing (1) subject to discrete constraints on the enclosed magnetic fluxes, magnetic helicity and thermodynamic quantities in each plasma region [12]. The magnetic helicity is a topological constraint related to the Gauss linking number of flux tubes, which is the most preserved of the ideal MHD invariants in the presence of small amounts of resistivity [1, 13]. Taylor's relaxation theory preserves the magnetic helicity globally throughout the entire plasma and can be physically interpreted as the idea that a weakly resistive plasma will evolve to minimize the plasma energy, but the magnetic field cannot untangle itself. The MRXMHD framework extends this idea to include a number of transport barriers partitioning the plasma and preventing complete reconnection. In the MRXMHD framework the magnetic topology within each plasma region is completely free; only the ideal transport barriers are constrained to be magnetic flux surfaces.

In this Letter we obtain a minimal model of the RFP QSH regime by taking a traditional equilibrium with as-

sumed nested flux surfaces and reducing the constraints. As the ideal transport barriers in MRXMHD are also magnetic flux surfaces, we select a number of flux surfaces (zero or one in this Letter) of the ideal MHD equilibrium to act as those barriers and compute the magnetic flux and helicity constraints associated with each region from the ideal MHD solution. We then self-consistently solve for the plasma equilibrium that minimizes the plasma energy subject to these constraints. This procedure cannot increase the plasma energy relative to the ideal MHD equilibrium because the minimum energy is taken over a superset of plasma states due to the looser constraints. As the number of transport barriers in the MRXMHD solution increases, the solution will approach the ideal MHD solution [14], but the number of parameters defining these solutions increases with the number of transport barriers and therefore a natural description of the QSH regime should have the minimum number of ideal transport barriers. We show in this Letter that only a single transport barrier is needed to reproduce the QSH regime. This is the minimal extension of Taylor's relaxation theory.

The ideal MHD state chosen as the reference solution in this Letter (see Figure 1(a)–(d)) is a representation of the QSH state in RFX-mod obtained by Terranova *et al.* [9] (Figure 2 of [9]). This solution has an axisymmetric circular cross-section with major radius 2m, and minor radius 0.448m. As the QSH regime is a high-current regime the effect of pressure can be negligible, and this is the case for the ideal MHD equilibrium presented in Figure 1(a)–(d). Zero pressure has been assumed in this ideal MHD equilibrium and will be assumed in the remainder of this Letter.

The smallest number of constraints possible in the MRXMHD model is when the entire plasma is taken as a single volume without any transport barriers partitioning it. In this case only two constraints specify the equilibrium: the enclosed toroidal flux ψ_t and the magnetic helicity K. This single plasma-region case is Taylor's relaxation theory [1]. The solution for this configuration is axisymmetric because the magnetic helicity is 40% below the bifurcation point where the solution becomes helical. At least one transport barrier is therefore required to reproduce the QSH regime, which increases the number of constraints to five (see Figure 2): the toroidal fluxes in each volume, the helicity in each volume, and the poloidal flux in the outer volume. The poloidal flux is an additional constraint required in the outer annular-toroidal region due to its different topology compared to the inner toroidal region.

To model the QSH regime with a single transport barrier a flux surface of the ideal MHD equilibrium must be chosen to act as that barrier. We perform a parameter scan over the possible choices parameterized by s, the normalized poloidal flux enclosed by the transport barrier ($0 \leq s \leq 1$). Figure 3 depicts the minimum en-
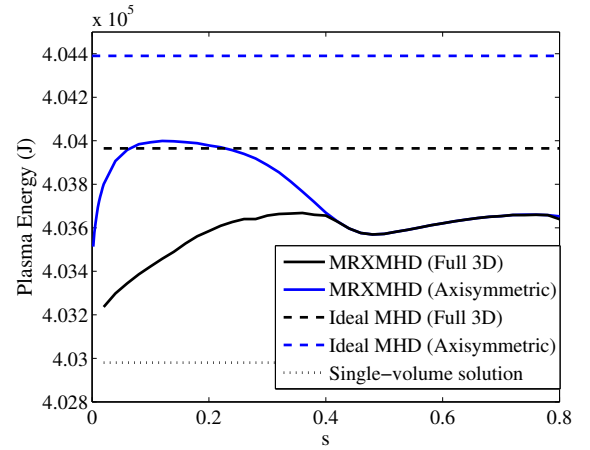

FIG. 3. Plot of the plasma energy for different plasma equilibria as a function of the magnetic flux surface chosen as the transport barrier in the MRXMHD model.

ergy for each of these configurations (as computed by the Stepped Pressure Equilibrium Code [15]) and compares them to the single volume solution (no transport barrier; Taylor's relaxation theory) and the continuously nested flux surface solution (ideal MHD). Also plotted are the minimum energies obtained for the same configurations but artificially forcing the solution to be axisymmetric. As the ideal MHD equilibria have a superset of the constraints of the MRXMHD equilibria, the energies of the ideal MHD are an upper bound for the corresponding MRXMHD equilibria. The single-volume solution has fewer constraints than the other equilibria and is a lower bound for the energies of the other solutions. As $s \to 0$ the two-volume solutions approach the energy of the single-volume solution as the transport barrier contracts to a point. The trend towards this behavior can be seen near $s = 0$ in Figure 3, however the transition is rapid in s because $s \sim r^2$ near the origin.

In Figure 3, for low s the difference between the energies of the single transport barrier solutions and the corresponding solutions with enforced axisymmetry indicates that a non-axisymmetric solution develops associated with a transport barrier in the core region. This non-axisymmetric structure is helical in nature as shown in the Poincaré plots in Figure 1(e)–(h), which have the same qualitative structure as the ideal MHD solution with continuous flux surfaces in Figure 1(a)–(d) with the exception of additional topological structure such as islands and chaotic regions that cannot be represented in the ideal MHD solution. The similarity between these two figures demonstrates that only a single transport barrier is required to reproduce the self-organized SHAx state. This is the first time such a nontrivial magnetic topology has been reproduced non-perturbatively within a plasma equilibrium description.

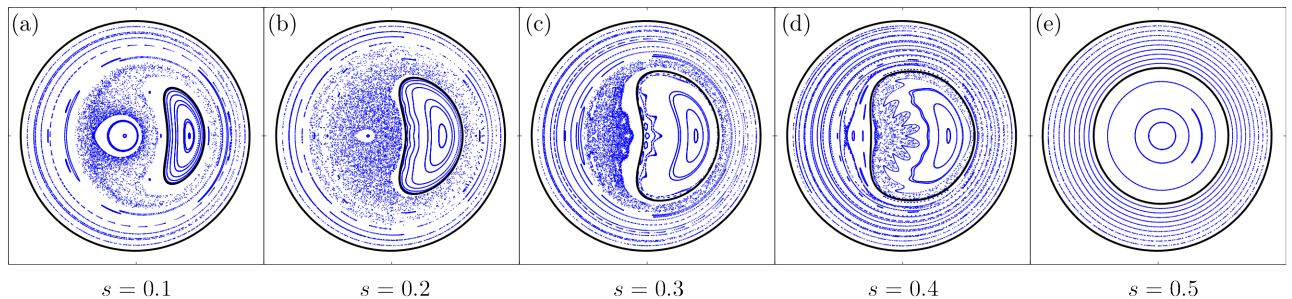Figure 4 illustrates Poincaré plots for a range of trans-

FIG. 4. Poincaré plots for single-barrier MRXMHD equilibria for different values of s, the ideal MHD flux surface chosen to act as the transport barrier. These figures have the same scale as in Figure 1.

port barrier locations demonstrating that we can reproduce DAx-like solutions in (a) and (b) as well as the SHAx-like solutions in (c) and (d). The DAx-like solutions are in good agreement with reconstructed Poincaré plots from the MST device [4] and RFX-mod [16]. As the transport barrier leaves the plasma core and approaches the edge, the solution becomes mostly axisymmetric [see Figure 4(e)] with some small island structure. This suggests that both the QSH regime is correlated to the formation of a transport barrier near the plasma core. The existence of a transport barrier near the plasma core is supported by experimental measurements in RFX-mod [8].

Experimental measurements in RFX-mod [17, 18] show the temperature profile to be relatively flat in the core. This has previously been considered [8, 18] as evidence for small amounts of chaos in the plasma core and the interpretation of flux surfaces in the core as ghost surfaces [19]. Alternatively as the plasma density is also relatively flat in the core [17], together these observations suggest that the pressure may also be constant in the core. Although zero pressure has been assumed in this Letter, the MRXMHD model predicts a constant pressure in each plasma region [10], which is consistent with these observations. A more detailed comparison with experiment will be necessary to distinguish these two models of the plasma core.

This Letter has demonstrated a minimal model that is able to qualitatively reproduce the magnetic structure of both the self-organized SHAx and DAx states in the QSH regime of RFPs. Previous recreations of the SHAx state have required an infinite number of constraints to parameterize the model; the model presented in this Letter has only five: the enclosed toroidal fluxes and helicities in the inner and outer volumes, and the enclosed poloidal flux in the outer volume. Fewer constraints are not possible as there is no model with 3 or 4 constraints, and Taylor's relaxation theory which has 2 constraints cannot reproduce the QSH regime of RFPs.

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