1 stepped pressure equilibrium code : cb02aa

Contents

1 stepped pressure equilibrium code : cb02aa **1**

1.0.1 purpose and overview . . . 1

1.1 field magnitude . . . 1

1.2 interface force balance . . . 1

1.3 energy functional gradient . . . 1

1.4 construction of surface potential . . . 2

1.0.1 purpose and overview

1. Constructs field magnitude on interfaces and writes surface potential to file. Planned redundancy;

1.1 field magnitude

1. The contravariant components of $\mathbf{B}$ are obtained from $\mathbf{B} = \nabla \times (A_\theta \nabla\theta + A_\zeta \nabla\zeta)$, giving

$$\begin{array}{rcl} B^s & = & (\quad \partial_\theta A_\zeta \quad - \quad \partial_\zeta A_\theta \quad) \quad / \quad \sqrt{g}, \\ B^\theta & = & \qquad\qquad - \quad \partial_s A_\zeta \qquad\quad / \quad \sqrt{g}, \\ B^\zeta & = & \quad \partial_s A_\theta \qquad\qquad\qquad / \quad \sqrt{g}. \end{array} \tag{1}$$

2. On the interfaces, $B^s = 0$ by constraint.

3. The covariant components of $\mathbf{B} = B_s \nabla s + B_\theta \nabla\theta + B_\zeta \nabla\zeta$ are determined using the lowering metrics, g_{ij},

$$B_s = B^s g_{ss} + B^\theta g_{s\theta} + B^\zeta g_{s\zeta}, \tag{2}$$

$$B_\theta = B^s g_{s\theta} + B^\theta g_{\theta\theta} + B^\zeta g_{\theta\zeta}, \tag{3}$$

$$B_\zeta = B^s g_{s\zeta} + B^\theta g_{\theta\zeta} + B^\zeta g_{\zeta\zeta}. \tag{4}$$

1.2 interface force balance

1. The force balance condition depends on the quantity $[[p + B^2/2]]$, where the pressure is given $p = P/V^\gamma$ where P is an adiabatic constant, and the field magnitude is given $B^2 = B^\theta B_\theta + B^\zeta B_\zeta$.

2. A fast Fourier transform of $[[p + B^2/2]]$ provides a discrete set of constraints.

3. If angle constraints are also required, a fast Fourier transform of the spectral width extremizing condition, $I \equiv R_\theta X + Z_\theta Y$, is constructed. If `Wcb02aa.eq.T`, then the FFT of I is compared to the "analytic" construction of the I_j as computed by double angle formulae.

1.3 energy functional gradient

1. Consider

$$\delta F = -\int_{\mathcal{I}} [[p + B^2/2]] \; \boldsymbol{\xi} \cdot \mathbf{dS} = -\iint [[p + B^2/2]] \; \boldsymbol{\xi} \cdot \mathbf{e}_\theta \times \mathbf{e}_\zeta \; d\theta d\zeta. \tag{5}$$

   (a) `Lgeometry.eq.1` Cartesian geometry:

$$\mathbf{x} = \theta \, \hat{\mathbf{i}} + \zeta \, \hat{\mathbf{j}} + R \, \hat{\mathbf{k}} \tag{6}$$

$$\mathbf{e}_\theta \times \mathbf{e}_\zeta = \hat{\mathbf{i}} \times \hat{\mathbf{j}} + R_\zeta \, \hat{\mathbf{i}} \times \hat{\mathbf{k}} + R_\theta \, \hat{\mathbf{k}} \times \hat{\mathbf{j}} \tag{7}$$

$$\delta\mathbf{x} \cdot \mathbf{e}_\theta \times \mathbf{e}_\zeta = \delta R \tag{8}$$

$$\frac{\partial F}{\partial R_j} = [[p + B^2/2]]_j \tag{9}$$

(b) `Lgeometry.eq.2` cylindrical:

$$\mathbf{x} = R\cos\theta\,\hat{\mathbf{i}} + R\sin\theta\,\hat{\mathbf{j}} + \zeta\,\hat{\mathbf{k}} \tag{10}$$

$$\mathbf{e}_\theta \times \mathbf{e}_\zeta = -RZ_\theta\hat{R} + (Z_\theta R_\zeta - R_\theta Z_\zeta)\hat{\phi} + RR_\theta\hat{z} \tag{11}$$

$$\delta\mathbf{x}\cdot\mathbf{e}_\theta \times \mathbf{e}_\zeta = \delta R\; R \tag{12}$$

$$\frac{\partial F}{\partial R_j} = -\left([[p + B^2/2]]\; R\right)_j \tag{13}$$

(c) `Lgeometry.eq.5` cylindrical:

$$\mathbf{x} = R\,\hat{\mathbf{i}} + Z\,\hat{\mathbf{j}} + \zeta\,\hat{\mathbf{k}} \tag{14}$$

$$\mathbf{e}_\theta \times \mathbf{e}_\zeta = Z_\theta\hat{\mathbf{i}} - R_\theta\hat{\mathbf{j}} \tag{15}$$

$$\delta\mathbf{x}\cdot\mathbf{e}_\theta \times \mathbf{e}_\zeta = +\delta R\; Z_\theta - \delta Z\; R_\theta \tag{16}$$

$$\frac{\partial F}{\partial R_j} = +\left([[p + B^2/2]]\; Z_\theta\right)_j \tag{17}$$

$$\frac{\partial F}{\partial Z_j} = -\left([[p + B^2/2]]\; R_\theta\right)_j . \tag{18}$$

(d) `Lgeometry.eq.6` toroidal geometry:

$$\mathbf{x} = R\,\hat{\mathbf{R}} + Z\,\hat{\mathbf{k}} \tag{19}$$

$$\mathbf{e}_\theta \times \mathbf{e}_\zeta = -RZ_\theta\hat{R} + (Z_\theta R_\zeta - R_\theta Z_\zeta)\hat{\phi} + RR_\theta\hat{z} \tag{20}$$

$$\delta\mathbf{x}\cdot\mathbf{e}_\theta \times \mathbf{e}_\zeta = -\delta R\; R\; Z_\theta + \delta Z\; R\; R_\theta \tag{21}$$

$$\frac{\partial F}{\partial R_j} = -\left([[p + B^2/2]]\; R\; Z_\theta\right)_j \tag{22}$$

$$\frac{\partial F}{\partial Z_j} = +\left([[p + B^2/2]]\; R\; R_\theta\right)_j . \tag{23}$$

1.4 construction of surface potential

1. From $\nabla\times\mathbf{B} = \mu\mathbf{B}$, and $B^s \equiv \mathbf{B}\cdot\nabla s = 0$ on the interfaces, we obtain $B_\theta = \partial_\theta f$ and $B_\zeta = \partial_\zeta f$ where

$$f = I\theta + G\zeta + \sum_j f_j \sin(m_j\theta - n\zeta). \tag{24}$$

2. The I, G and f_j are constructed from a fast Fourier transform of B_θ and B_ζ.

3. The information required for the pressure-jump Hamiltonian analysis of interface force balance is written to file, named `ext.pjh:xxxx:i`, where the `xxxx` labels volume and `i` selects the inner, `i=0`, or outer, `i=1`, adjacent interface of that volume. With the convention that volume $\mathcal{V}_l$ is bounded by the $\mathcal{I}_{l-1}$ and $\mathcal{I}_l$ interfaces, we have, for example,

   - `ext.pjh:0001:1` contains the surface potential on the inner side of the $l = 1$ interface (which is computed using the field in $\mathcal{V}_1$), and
   - `ext.pjh:0002:0` contains the surface potential on the outer side of the $l = 1$ interface (which is computed using the field in $\mathcal{V}_2$).

4. This `ext.pjh:xxxx:i` file has the following form:

```
Lgeometry                    ! integer
I  G  pl  ql  pr  qr  iota   ! real real integer integer integer integer real
mn                           ! integer
m_1  n_1  f_1  R_1  Z_1      !
m_2  n_2  f_2  R_2  Z_2      !
 .    .    .    .    .       !
m_j  n_j  f_j  R_j  Z_j      ! integer integer real real real
```

5. The I, G, m_j, n_j and f_j for $j = 1$,`mn` match Eq.(24), is the total number of Fourier harmonics.

6. The rotational transform of the interface is given by $\iota = (p_l + \gamma p_r)/(q_l + \gamma q_r)$, where $\gamma = (1 + \sqrt{5})/2 = 1.6180339\ldots$ is the golden mean.

7. The geometry of the interface is described by the $R = \sum_j R_j \cos(m_j\theta - n_j\zeta)$ and $Z = \sum_j Z_j \sin(m_j\theta - n_j\zeta)$ according to

   - if `Lgeometry.eq.1`, Cartesian: $\mathbf{x} = \theta\,\hat{\mathbf{i}} + \zeta\,\hat{\mathbf{j}} + R(s,\theta,\zeta)\,\hat{\mathbf{k}}$,
   - if `Lgeometry.eq.6`, toroidal: $\mathbf{x} = R(s,\theta,\zeta)\hat{\mathbf{R}} + Z(s,\theta,\zeta)\hat{\mathbf{k}}$, where $\hat{\mathbf{R}} \equiv \cos\zeta\,\hat{\mathbf{i}} + \sin\zeta\,\hat{\mathbf{j}}$.

cb02aa.h last modified on 2012-09-19 ;