ig00aa

briefly

[called by: mp00ac, dforce.] [calls: coords.]

contents

1.1 enclosed currents

1. In the vacuum region, the enclosed currents are given by either surface integrals of the current density or line integrals of the magnetic field,

$$\int_{\mathcal{S}} \mathbf{j} \cdot d\mathbf{s} = \int_{\partial \mathcal{S}} \mathbf{B} \cdot d\mathbf{l}, \tag{1}$$

   and line integrals are usually easier to compute than surface integrals . . .

2. The magnetic field is given by the curl of the magnetic vector potential, as described in e.g. bfield.

3. The toroidal, plasma current is obtained by taking a "poloidal" loop, $d\mathbf{l} = \mathbf{e}_\theta \, d\theta$, on the plasma boundary, where $B^s = 0$, to obtain

$$I \equiv \int_0^{2\pi} \mathbf{B} \cdot \mathbf{e}_\theta \, d\theta = \int_0^{2\pi} \left(-\partial_s A_\zeta \; \bar{g}_{\theta\theta} + \partial_s A_\theta \; \bar{g}_{\theta\zeta} \right) \, d\theta, \tag{2}$$

   where $\bar{g}_{\mu\nu} \equiv g_{\mu\nu}/\sqrt{g}$.

4. The poloidal, "linking" current through the torus is obtained by taking a "toroidal" loop, $d\mathbf{l} = \mathbf{e}_\zeta \, d\zeta$, on the plasma boundary to obtain

$$G \equiv \int_0^{2\pi} \mathbf{B} \cdot \mathbf{e}_\zeta \, d\zeta = \int_0^{2\pi} \left(-\partial_s A_\zeta \; \bar{g}_{\theta\zeta} + \partial_s A_\theta \; \bar{g}_{\zeta\zeta} \right) \, d\zeta. \tag{3}$$

1.2 "Fourier integration"

1. Using $f \equiv -\partial_s A_\zeta \; \bar{g}_{\theta\theta} + \partial_s A_\theta \; \bar{g}_{\theta\zeta}$, the integral for the plasma current is

$$I = \sum_i{}' f_i \cos(n_i \zeta) 2\pi, \tag{4}$$

   where $\sum'$ includes only the $m_i = 0$ harmonics.

2. Using $g \equiv -\partial_s A_\zeta \; \bar{g}_{\theta\zeta} + \partial_s A_\theta \; \bar{g}_{\zeta\zeta}$, the integral for the linking current is

$$G = \sum_i{}' g_i \cos(m_i \zeta) 2\pi, \tag{5}$$

   where $\sum'$ includes only the $n_i = 0$ harmonics.

3. The plasma current, Eqn.(4), should be independent of ζ, and the linking current, Eqn.(5), should be independent of θ. (Perhaps this can be proved analytically; in any case it should be confirmed numerically.)