1 stepped pressure equilibrium code : ir00aa

Contents

1.1 outline

1. Locates invariant, irrational surfaces (i.e. KAM surfaces) of the magnetic field, and writes input for pressure-jump Hamiltonian analysis code.

1.1.1 construction of invariant surface

2. The method for constructing an irrational surface is described in [Hudson, 2004] [1].

3. A trial curve, represented by a discrete set of points (s_i, θ_i),

$$\begin{array}{rcccl} s_i & = & & & \sum_{m=0}^{M} s_m \cos(m\alpha_i), \\ \theta_i & = & \alpha & + & \sum_{m=1}^{M} \theta_m \sin(m\alpha_i), \end{array} \tag{1}$$

where α is a straight field line angle, and α_i are regularly spaced, and $M \equiv$`irrMpol`, is mapped under the field line flow $(s, \theta) \mapsto (\tilde{s}, \tilde{\theta})$, with the field (and tangent map) given by `bf00ac`, and compared to the rigidly shifted curve,

$$\begin{array}{rcccl} \bar{s} & = & & & \sum s_m \cos(m\bar{\alpha}), \\ \bar{\theta} & = & \bar{\alpha} & + & \sum \theta_m \sin(m\bar{\alpha}), \end{array} \tag{2}$$

where $\bar{\alpha} = \alpha + 2\pi\,\iota$.

4. The 'function' we seek a zero of is the error vector, given $\mathbf{e} = (\bar{s} - \tilde{s}, \bar{\theta} - \tilde{\theta})^T$. The correction to the harmonics, s_m and θ_m, that describe the curve are obtained using a Newton method, for which the derivative matrix is required:

$$\begin{array}{rcccl} \partial(\bar{s} - \tilde{s})/\partial s_m & = & \cos(m\bar{\alpha}) & - & \cos(m\alpha)\partial\tilde{s}/\partial s, \\ \partial(\bar{s} - \tilde{s})/\partial \theta_m & = & & - & \sin(m\alpha)\partial\tilde{s}/\partial \theta, \\ \partial(\bar{\theta} - \tilde{\theta})/\partial s_m & = & & - & \cos(m\alpha)\partial\tilde{\theta}/\partial s, \\ \partial(\bar{\theta} - \tilde{\theta})/\partial \theta_m & = & \sin(m\bar{\alpha}) & - & \sin(m\alpha)\partial\tilde{\theta}/\partial \theta, \end{array} \tag{3}$$

where the $\partial\tilde{x}/\partial y$ are elements of the tangent map.

5. Given an initial guess for the irrational invariant *curve*, the routine `invarianterr` is called iteratively (maximum iterations given by `Mirrits`) until the root mean square error is less than `irrtol`. Corrections are calculated by singular value decomposition inversion (see `singvalues`) of the derivative matrix.

6. (There is an option to solve sum-of-squares using NAG routine `E04GYF`. This routine does not perform as well as the SVD routine above. I think this is probably due to integration errors, which may confuse the NAG routine.)

7. NOTE: If the root mean square error tolerance is greater than `irrtol`, a Fourier representation of the invariant *surface* will **not** be constructed.

8. The invariant *surface* is obtained by allowing the invariant *curve* to 'flow' along the field. Note: the surface thus constructed is initially obtained on a regular (α, ζ) grid, where α labels field lines. To obtain a regular grid in (θ_s, ζ), where θ_s is the straight-field line angle consistent with ζ, it is required to interpolate. It is (probably) more accurate to interpolate to get the background coordinates (s, θ, ζ), which probably vary slightly, and then perform the (exact) transformation to cylindrical coordinates, (R, ϕ, Z).

9. The contravariant field is transformed to cylindrical coordinates via

$$\begin{pmatrix} B^R \\ B^\phi \\ B^Z \end{pmatrix} = \begin{pmatrix} R_s & R_\theta & R_\zeta \\ 0 & 0 & -1 \\ Z_s & Z_\theta & Z_\zeta \end{pmatrix} \begin{pmatrix} B^s \\ B^\theta \\ B^\zeta \end{pmatrix}. \tag{4}$$

Recall that $B_\phi = R^2 B^\phi$.

10. The surface is Fourier decomposed in the straight field line angle, with harmonics idenified by `irrim(1:irrmn)` and `irrin(1:irrmn)`, which are constructed in `readin`.

11. The surface geometry harmonics are stored in the global arrays `irrsurf%Rbc(:)`, `irrsurf%Zbs(:)`.

12. The vector transformation to straight-field-line coordinates should exploit fast Fourier transforms.

13. The covariant field is transformed

$$\begin{pmatrix} B_s \\ B_\theta \\ B_\zeta \end{pmatrix} = \begin{pmatrix} R_s & 0 & Z_s \\ R_{\theta_s} & 0 & Z_{\theta_s} \\ R_\zeta & -1 & Z_\zeta \end{pmatrix} \begin{pmatrix} B_R \\ B_\phi \\ B_Z \end{pmatrix}. \tag{5}$$

14. The normal field is

$$\mathbf{B} \cdot \mathbf{e}_{\theta_s} \times \mathbf{e}_\zeta = Z_{\theta_s} R B_R + (Z_{\theta_s} R_\zeta - R_{\theta_s} Z_\zeta) B_\phi R^{-1} - R_{\theta_s} R B_z. \tag{6}$$

15. The transform is given

$$\iota = \frac{B^{\theta_s}}{B^\zeta} = \frac{B_{\theta_s} g_{\zeta\zeta} - B_\zeta g_{\theta_s \zeta}}{-B_{\theta_s} g_{\theta_s \zeta} + B_\zeta g_{\zeta\zeta}}. \tag{7}$$

16. The surface potential f, where $B_{\theta_s} = \partial_{\theta_s} f$ and $B_\zeta = \partial_\zeta f$, is calculated and written to `ext.irr.**.pj`.

1.1.2 input file for pressure-jump Hamiltonian analysis code

17. The information required for the pressure-jump Hamiltonian analysis is written to `ext.p1:q1:p2:q2:M:N.pjh`, where the transform of the irrational surface is given by $\iota = (p_1 + \gamma p_2)/(q_1 + \gamma q_2)$, where $\gamma = (1 + \sqrt{5})/2$ is the golden mean, and M and N describe the poloidal and toroidal Fourier resolution.

18. The format of this file is as follows:

```
Mpol Ntor Nfp
Itor Gpol p1 q1 p2 q2 iota
m n fmn Rmn Zmn
. . ... ... ...
```

19. The tangential magnetic field on the surface is given by a surface potential, $f = I\theta + G\zeta + \sum f_{m,n} \sin(m\theta - n\zeta)$, via $B_\theta = \partial_\theta f$ and $B_\zeta = \partial_\zeta f$.

20. The surface geometry is given by $R = \sum R_{m,n} \cos(m\theta - n\zeta)$ and $Z = \sum Z_{m,n} \sin(m\theta - n\zeta)$.

1.1.3 calculation of enclosed flux

21. The toroidal flux between the irrational surface and the inner interface is calculated as a double integral using the NAG routine `E01BEF`.

22. The toroidal flux is given by $\psi \equiv \int_{\mathcal{S}} \mathbf{B} \cdot d\mathbf{s}$, where $\mathcal{S}$ is the surface on the $\zeta = 0$ plane bounded by the $l-1$ interface and the irrational surface, $\bar{s} \equiv \bar{s}(\theta)$, and $\mathbf{B} = \nabla \times (A_\theta \nabla\theta + A_\zeta \nabla\zeta)$.

$$\psi = \int_0^{2\pi} d\theta \int_{s_{l-1}}^{\bar{s}} ds\ \partial_s A_\theta. \tag{8}$$

23. (The toroidal flux between the outer interface and the irrational interface should also be constructed for debugging and accuracy confirmation.)

24. The accuracy of the calculation is controlled by the input parameter `absacc`.

ir00aa.h last modified on 2012-12-18 ;

[1] S. R. Hudson. Destruction of invariant surfaces and magnetic coordinates for perturbed magnetic fields. *Phys. Plasmas*, 11(2):677, 2004.