1 stepped pressure equilibrium code : method

Contents

1.1 outline

1. Describes some of the numerical tasks required to compute a stepped-pressure equilibrium [1] solution.

1.2 introduction

1. Consider a toroidal volume consisting of nested subvolumes. In each subvolume it is required to compute the magnetic field (as described below).
2. Then, it will be required to adjust the boundary of each subvolume in order to achieve global force balance (this problem is not considered here).

1.3 solving the Beltrami equation in a toroid

1. Assume that a smooth toroidal surface, $\mathbf{x}(\theta, \zeta)$, is given.
2. Compute the field that satisfies

$$\nabla \times \mathbf{B} = \mu \mathbf{B}, \tag{1}$$

   for a given μ, in the toroidal volume $\mathcal{V}$ with boundary $\partial\mathcal{V} \equiv \mathbf{x}$.
3. The field is tangential to the boundary, $\mathbf{B} \cdot \mathbf{n} = 0$.
4. The enclosed toroidal flux is given, $\int_{\mathcal{S}} \mathbf{B} \cdot d\mathbf{s} = \psi_t$.

1.4 solving the Beltrami equation in a toroidal annulus

1. Assume that smooth toroidal surfaces, $\mathbf{x}_{inn}(\theta, \zeta)$ and $\mathbf{x}_{out}(\theta, \zeta)$, are given, and that $\mathbf{x}_{inn}$ is inside $\mathbf{x}_{out}$.
2. Compute the field that satisfies

$$\nabla \times \mathbf{B} = \mu \mathbf{B}, \tag{2}$$

   for a given μ, in the toroidal volume $\mathcal{V}$ with inner boundary $\partial\mathcal{V} \equiv \mathbf{x}_{inn}$ and outer boundary boundary $\partial\mathcal{V} \equiv \mathbf{x}_{out}$.
3. The field is tangential to both boundaries, $\mathbf{B} \cdot \mathbf{n} = 0$.
4. The enclosed toroidal flux is given, $\int_{\mathcal{S}} \mathbf{B} \cdot d\mathbf{s} = \psi_t$.
5. The enclosed poloidal flux is given, $\int_{\mathcal{S}} \mathbf{B} \cdot d\mathbf{s} = \psi_p$.

1.5 solving Laplace's equation in a toroidal annulus

1. Assume that smooth toroidal surfaces, $\mathbf{x}_{inn}(\theta,\zeta)$ and $\mathbf{x}_{out}(\theta,\zeta)$, are given, and that $\mathbf{x}_{inn}$ is inside $\mathbf{x}_{out}$.
2. Compute the field that satisfies
$$\nabla \times \mathbf{B} = 0, \tag{3}$$
in the toroidal volume $\mathcal{V}$ with inner boundary $\partial\mathcal{V} \equiv \mathbf{x}_{inn}$ and outer boundary boundary $\partial\mathcal{V} \equiv \mathbf{x}_{out}$.
3. The field is tangential to the inner boundary, $\mathbf{B}\cdot\mathbf{n} = 0$.
4. The field is *not* tangential to the outer boundary but instead the quantity $\mathbf{B}\cdot\mathbf{n} = b(\theta,\zeta)$ is assumed given.
5. The current linking the toroidal annulus is given, $\int_{\mathcal{S}} \mathbf{j}\cdot d\mathbf{s} = I$.
6. The current inside the inner interface is given, $\int_{\mathcal{S}} \mathbf{j}\cdot d\mathbf{s} = G$.

1.6 required output

For each of the problems described in Sec.1.3, Sec.1.4 and Sec.1.5, . . .

1. Usually, during the iterative scheme required to solve for the non-linear plasma equilibrium (which is not discussed here), the only place that $\mathbf{B}$ is required is on the boundary. It is only on completion (for post-diagnostics) is $\mathbf{B}$ required on the internal volume.
2. However, it is sometimes required to compute the following integrals:
$$\text{magnetic energy} \equiv \int_{\mathcal{V}} \mathbf{B}\cdot\mathbf{B}\, dv, \tag{4}$$
$$\text{magnetic helicity} \equiv \int_{\mathcal{V}} \mathbf{A}\cdot\mathbf{B}\, dv \tag{5}$$
where $\mathbf{A}$ is the magnetic vector potential, $\mathbf{B} = \nabla\times\mathbf{A}$. In this case, it is required to compute the field, $\mathbf{B}$, that minimizes the magnetic energy subject to the constraint of conserved magnetic helicity.
3. In either case, it is required to know how the required output (either $\mathbf{B}$ on the boundary, or $\int \mathbf{B}\cdot\mathbf{B}dv$ and $\int \mathbf{A}\cdot\mathbf{B}dv$) changes when either (i) μ, (ii) the enclosed poloidal flux, or (iii) the geometry of the boundary changes. This information is required for an efficient iterative scheme that adjusts the boundary of each domain in order to obtain the global plasma equilibrium.

method.h last modified on 2012-11-09 ;

[1] S. R. Hudson, R. L. Dewar, G Dennis, M. J. Hole, M. McGann, G. von Nessi, and S. Lazerson. Computation of multi-region relaxed magnetohydrodynamic equilibria. *Phys. Plasmas*, 2012. `http://w3.pppl.gov/~shudson/Papers/Published/HDDHMcGvNL_12.pdf`.