1 stepped pressure equilibrium code : pq00aa

Contents

1.1 Brief

1. Locates periodic orbits and their residues
2. This routine uses a simple trajectory following method to locate fieldlines that satisfy

$$\begin{aligned} s(\zeta+2\pi q) &= s(\zeta) &(1)\\ \theta(\zeta+2\pi q) &= \theta(\zeta)+2\pi p, &(2)\end{aligned}$$

   where the integers p and q are given on input.

3. The tangent map, M, is exploited. This is defined by

$$\bar{s}(s+\delta s,\theta+\delta\theta)=\bar{s}(s,\theta)+M\cdot(\delta s,\delta\theta)^T\,, \tag{3}$$

   where $\bar{s}\equiv s(\zeta+2\pi q)$ and $\bar{\theta}\equiv\theta(\zeta+2\pi q)$.

4. The field and the tangent map are provided by `bf00aa`.
5. The residue is related to the tangent map, and provides information regarding the stability of the orbit [1].
6. More details of the algorithm are supplied in [2]. To locate high-order periodic orbits in strongly chaotic fields, Lagrangian integration will usually be preferred [3].

1.1.1 input parameters

1. `odetol` : o.d.e. integration accuracy provided to NAG routine `D02BJF`;
2. `Mpqits` : maximum iterations allowed in search;
3. `Lpqsym` : controls whether search is constrained along symmetry line, $\theta=0$;
4. `pqs, pqt` : initial guess for the (s,θ); only relevant if `Lpqsym=0`;

1.1.2 symmetry line

1. The default is to assume stellarator symmetry, and only periodic orbits along $\theta=0$ line are located.
2. In this case, there is only 1 degree-of-freedom in the numerical search, which is the radial location, s.
3. The condition that the trajectory is periodic is

$$\theta(2\pi q)=\theta(0)+2\pi p, \tag{4}$$

   which must be satisfied to within `q*odetol`.

4. Note that stellarator symmetric orbits need only be followed *half* of the full periodicity distance, so that the periodicity condition reduces to

$$\theta(\pi q)=\theta(0)+\pi p. \tag{5}$$

   This is under construction.

1.1.3 algorithm details

1. A standard Newton search is performed to locate periodic orbits.

$$\left(\begin{array}{c} \bar{s}(s+\delta s, \theta+\delta\theta) \\ \bar{\theta}(s+\delta s, \theta+\delta\theta) \end{array}\right) = \left(\begin{array}{c} \bar{s}(s,\theta) \\ \bar{\theta}(s,\theta) \end{array}\right) + \left(\begin{array}{ccc} \partial_s \bar{s} & , & \partial_\theta \bar{s} \\ \partial_s \bar{\theta} & , & \partial_\theta \bar{\theta} \end{array}\right) \left(\begin{array}{c} \delta s \\ \delta\theta \end{array}\right) = \left(\begin{array}{c} s+\delta s \\ \theta+\delta\theta+2\pi p \end{array}\right) \tag{6}$$

2. If the Newton correction takes the radial guess outside [`low,upp`], which define the computational boundary, the new radial guess is random.

1.1.4 constructing unstable manifold

1. Given the tangent map at the unstable periodic orbit, all that is required to identify the unstable manifold is to compute the eigenvectors. The NAG routine `F02EBF` is employed for this.

pq00aa.h last modified on 2015-09-09 ;

[1] J. M. Greene. A method for determining a stochastic transition. *J. Math. Phys.*, 20(6):1183, 1979.

[2] S. R. Hudson. Destruction of invariant surfaces and magnetic coordinates for perturbed magnetic fields. *Phys. Plasmas*, 11(2):677, 2004.

[3] S. R. Hudson. Calculation of cantori for Hamiltonian flows. *Phys. Rev. E.*, 74:056203, 2006.