

1 stepped pressure equilibrium code : sc00ab

1. Enforces spectral condensation constraint: Z is determined as a function of R .
2. The representation of a given toroidal surface by Fourier harmonics is not unique. Any Fourier representation implies a particular angle parametrization, and different choices of angles results in a different Fourier representation. This freedom may be exploited to obtain a preferred angle parameterization.
3. A given toroidal surface may be described by $R(\theta, \zeta) = \sum_i R_i \cos \alpha_i$ and $Z(\theta, \zeta) = \sum_i Z_i \sin \alpha_i$, where $\alpha_i \equiv m_i \theta - n_i \zeta$. Intuitively, better numerical accuracy will be obtained at a given Fourier resolution the angle parameterization can be adjusted to condense the Fourier spectrum. Consider the spectral width,

$$M = \frac{1}{2} \sum_i c_i (R_i^2 + Z_i^2), \quad (1)$$

where c_i are constants, e.g. $c_i = m_i^p + n_i^q$ for integers p and q . As the angle parameterization is defined by the Fourier representation (and vice versa), the task is to vary the R_i and Z_i in order to minimize M ;

4. The variation of the R_i and Z_i must be restricted by the constraint that the surface geometry does not change: the geometry of the surfaces will be adjusted in order to satisfy force balance. To minimize M we restrict attention to *tangential* variations of the form

$$\delta R = R_\theta \delta u, \quad (2)$$

$$\delta Z = Z_\theta \delta u, \quad (3)$$

where δu is an arbitrary function.

5. The variation in the Fourier harmonics is given

$$\delta R_i = \iint d\theta d\zeta R_\theta \delta u \cos \alpha_i, \quad (4)$$

$$\delta Z_i = \iint d\theta d\zeta Z_\theta \delta u \sin \alpha_i. \quad (5)$$

6. The first variation in M is given

$$\delta M = \iint d\theta d\zeta (R_\theta X + Z_\theta Y) \delta u, \quad (6)$$

where $X \equiv \sum_i c_i R_i \cos \alpha_i$ and $Y \equiv \sum_i c_i Z_i \sin \alpha_i$. The Euler-Lagrange equation for the minimization of the spectral width, M , with respect to arbitrary variations, δu , is $I \equiv R_\theta X + Z_\theta Y = 0$.

7. The condition $I \equiv R_\theta X + Z_\theta Y = 0$ allows Z to be expressed as a function of R . Expanding the Fourier summation, we find

$$I = \sum_{j,k} [(-m_j)R_j \sin \alpha_j c_k R_k \cos \alpha_k + (+m_j)Z_j \cos \alpha_j c_k Z_k \sin \alpha_k] \quad (7)$$

$$= \sum_{j,k} \frac{m_j c_k}{2} \{-R_j R_k [\sin(\alpha_j - \alpha_k) + \sin(\alpha_j + \alpha_k)] + Z_j Z_k [\sin(\alpha_k - \alpha_j) + \sin(\alpha_k + \alpha_j)]\}. \quad (8)$$

8. The Z_j may be determined (numerically) by setting the Fourier harmonics, I_i , to zero, where $I = \sum_i I_i \sin \alpha_i$ and the R_j are assumed given.

9. Question: Is the above correct?

10. Question: Does a solution for the Z_j always exist, for any given $R(\theta, \zeta) = \sum_i R_i \cos \alpha_i$?

11. Question: Is this solution unique?

12. Question: Are there any constraints on the R_i ?

13. Question: Is there a class of variations in $R(\theta, \zeta)$ that amount to tangential variations in the surface geometry?

1.0.1 Some double angle formulae for convenience

1.

$$2 \sin \alpha \sin \beta = \cos(\alpha - \beta) - \cos(\alpha + \beta), \quad (9)$$

$$2 \sin \alpha \cos \beta = \sin(\alpha - \beta) + \sin(\alpha + \beta), \quad (10)$$

$$2 \cos \alpha \cos \beta = \cos(\alpha - \beta) + \cos(\alpha + \beta), \quad (11)$$

sc00ab.h last modified on 2012-05-08 ;
