Drift Waves - Experiment vs Theory

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- Overview
- Simple theory
- Comparisons of experiment and theory
 - Collisional drift waves (Hendel et 1968)
 - Ion temperature gradient mode (Sen et al 1992)
 - Drift wave control (Klinger et al 2001)
- Open questions / work in progress

Overview of Drift Waves

- "Drift" refers to the diamagnetic drifts expected with any density gradients, e.g. $v_{de} = (T_e/eB)(dn/dr)/n = (T_e/eB) 1/L_n$ (note these are not fluid drifts !)
- Thus drift waves were considered "universal" instabilities in any magnetically confined plasma with a density gradient
- Drift waves are electrostatic in low β plasmas, with fluctuations in n, φ, and T (and not B or I, at least in simplest picture)
- The relatively long wavelength $\lambda \ge \rho_i$ made them candidates for explaining "anomalous diffusion" across magnetic fields ($D \propto \lambda^2 \omega$)
- Nearly coherent drift waves are usually seen in linear experiments, and drift wave turbulence is usually seen in toroidal plasmas (see "Turbulence")
- Drift wave turbulence is thought to be the dominant source of anomalous transport in tokamaks and stellarators (e.g. "TE mode", "ITG mode" etc)

History

- Bohm suggested that anomalous diffusion in arc discharges could be caused by ExB fluctuations (1949)
- Theory showed drift wave instability in inhomogeneous low β plasmas, both collisional and collisionless (1955-1965)
- Experiments in linear Q-(quiescent) machines identified various types of drift waves and compared with linear theory (1965-present)
- Linear drift wave theory developed and applied to toroidal geometry (1970-80)
- Non-linear drift wave theory developed to explain observed turbulence and transport in tokamaks (1980 present)
- Detailed comparisons of drift wave measurements in laboratory experiments with non-linear theory (present)
- Detailed comparisons of drift wave turbulence measurements with non-linear drift wave theory (present)

Some Drift Wave References

(665 papers found under category of "drift waves" in Web of Science, ≈ 90% theory)

General:

Chen, "The Leakage Problem in Fusion Reactors", Scientific American (1967) p. 76 Motley, Q-Machines (1975) Chen, Plasma Physics and Controlled Fusion, 2nd Ed (1984) Goldston and Rutherford, Introduction to Plasma Physics (1995)

Theory:

Horton, "Drift Waves and Transport", Rev. Mod. Phys. p. 735 (1999) Liewer, comparison of measurements and theory, Nucl. Fusion p. 543 (1985) Tang, "Microinstability Theory in Tokamaks", Nucl. Fusion p. 1089 (1978)

Experiment (somewhat Princeton-centric subset):

Hendel, Chu, Politzer, Phys. Fluids p. 2426 (1968) - collisional drift waves in Q-1 Politzer, Phys. Fluids p. 2410 (1971) - collisionless drift waves in Q-1 Jassby, Phys. Fluids p. 1590 (1972)- velocity shear instabilities in Q-3 Ellis and Motley, Phys Fluids p. 582 (1974) - current driven drift waves in Q-3 Okabayashi et al, Nucl. Fusion p. 497 (1977) - drift waves in FM-1 spherator Dixon et al, Plasma Physics p. 225 (1978) - trapped electron mode in CLM Pecseli et al, PPCF p.1173 (1983) - drift wave turbulence in low beta Q-machine Scamozzino et al, PRL p. 1729 (1986) - curvature driven trapped particle mode in CLM Plaut et al, Phys. Fluids B p .293 (1989) - trapped ion mode in CLM Kauschke, et al, Phys. Fluids B, p. 38 (1990) - coherent drift waves at low B in low pressure arc Chen et al, Phys. Fluids B p. 512 (1992) - slab ITG mode in CLM Greaves, Chen, Sen, PPCF p. 1253 (1992) - slab ITG mode in CLM Tham and Sen, Phys. Rev. A p. R4520 (1992) - feedback stabilization of ITG in CLM Song, Sen, Chen, Phys. Fluids p. 4341 (1993) - ITG transport in CLM Chen and Sen, PoP p. 3063 (1995) mixed slab-toroidal ITG in CLM Tinkle and Sen, PoP p. 4287 (1996) quasicoherent structures in ITG in CLM Sen et al, PPCF p. A333 (1997) - basic studies of fluctuations, transport, and control in CLM Chiu and Sen, PRL p. 5503 (1999) - nonlinear dynamics of ExB mode in CLM Schroder, Klinger et al, PRL p. 5711 (2001) - control of drift waves in MIRABELLE

Simple Theory of Drift Waves

[Chen, Plasma Physics and Controlled Fusion, 2nd Ed, p. 218 (1984)]

- Assume density gradient \perp B, perturbation with k₁ and k₁ and no electron resistivity or diffusion of ions (no instability, just drift waves)
- If ion perturbation moves outward, positive charge causes electrons to flow along field line until e $\partial \phi / T_e \approx \partial n/n$ (from Boltzman relation for parallel electron pressure balance mn($\partial v_{II}/\partial t$) = enE_{II} $\partial p_e/\partial t$ with m=0)
- Then $\delta \phi$ is in phase with δn , so ExB drift is: $v_{1x} = i k_v \phi_1/B (v_{1x} is 90^\circ \text{ out of phase})$
- Then density perturbation is: $\partial n_1 / \partial t = -v_{1x} \partial n_0 / \partial x = (-ik_y \phi_1 / B) \partial n_0 / \partial x$





using Boltzman relation and assuming $n_1 \propto exp[i(k_yy-\omega t)]$ $-i\omega n_1 = -ik_y [T_e (n_1/n)/eB]\partial n_0/\partial x$

$$= 0.00 \text{ m/k}_{y} = (T_{e}/eB)1/L_{n}$$

phase velocity = v_{de}

Collisional Drift Wave Instability

- Without resistivity, radial EXB drift and ion density fluctuation are 90° out of phase, perturbation propagates ⊥ ∇n (like water wave) => no radial transport
- With resistivity there is a delay in electron response (extra term in parallel momentum balance), hence phase lag of φ with respect to n => radial velocity outward where plasma is shifted outward net growth => instability and transport
- Parallel electron motion with resistivity is determined by e-i collision frequency v_{ei} :

 $mn(\partial v_{\parallel}/\partial t) = enE_{\parallel} - \partial p_e/\partial t - nmv_{ei}v_{\parallel} = 0$ (for drift waves with $\omega \ll \omega_{ci}$)

• Result is dispersion relation depending on v_{ei} [Chen p. 222]

$$\omega \approx \omega_* + i v_{ei} [k_v/k_{II}]^2 [\omega_*^2/\omega_{ce}\Omega_{ci}]$$
, where $\omega_* = k_v v_{de}$

• Electron collisions always causes drift waves to grow, although relatively slowly except when $v_{ei} > k_{II}v_e$ (collisonal plasma)



Varieties of Drift Waves

• Common features:

pressure gradient \perp B (density or temperature) finite wavelength along B (usually k_{II} << k_{\perp}) electron and ion motions different (2 - fluid)

- Some variations:
 - collisional drift wave (electron resistivity)
 - collisionless drift wave (electron-wave resonance)
 - current driven drift wave (includes δT_e)
 - rotation shear (Kelvin-Helmholtz instability)
 - B-field curvature (resistive ballooning mode)
 - ion and electron temperature gradient (ITG, ETG)
 - trapped electron and ion instabilities (toroidal)
 - drift-Alfven waves (finite magnetic perturbations)

Collisional Drift Waves in Q-1 (PPPL)

Hendel, Chu, Politzer, Phys. Fluids **11**, 2426 (1968) Hendel, Coppi, Perkins, Politzer, PRL **18**, p. 439 (1967) <u>Q-machines</u>, Motley (1975)



Steady state ion beam source, no currents or flows $n_e = 5 \times 10^{10} - 5 \times 10^{12} \text{ cm}^{-3}$ (fully ionized K⁺ or Cs⁺ plasma) L $\approx 128 \text{ cm}, a \approx 1.5 \text{ cm}, B \approx 2-7 \text{ kG}, \rho_i \approx 0.1 \text{ cm}, \beta \le 10^{-6}$ $T_e \approx T_i \approx 0.25 \text{ eV}$ (2800° K \approx W plate temperature)

Observation of Drift Waves



- Near-coherent mode ≈ few kHz localized in high ∇n region
- Avoid high ∇T mode near edge
- eδφ/T ≈ δn/n as expected from Boltzman relationship
 - δ n leads δφ, as expected from linear theory ($\psi \approx 45^{\circ}$)

Theory for this Experiment

- Assume uniform $T_e = T_i$, no rotation, neglect ion motion along B and electron inertia => linearized 2 fluid equations (5 eq's for n_e , n_i , $v_{e\perp}$, $v_{i\perp}$, and parallel electron motion)
- Assume $k_{II} \ll k_v$ and $k_x \gg 1/L_n$ (i.e. 1-D "slab" geometry)
- Ion collisions and ion Larmor radius added through "viscosity" $\mu_{\perp} \approx nT_i v_{ii}/4\Omega_i^2 \propto \rho_i^2 v_{ii} \implies$ ion diffusion across B ($\propto B^{-2}$)
- Solution for drift wave in limit of $\omega_i \ll \omega_r$ (Motley p. 51)

 $\omega_r \approx \omega_*/(1 + 2b)$ where $b = k_v^2 \rho_i^2/2$ (FLR parameter)

$$\omega_{i} \approx \frac{b\omega_{r}(\omega_{r} + \omega_{\star}) - v_{||} v_{|i|} b^{2}/2}{v_{||}(1 + 2b) + b^{2} v_{|i|}/4}$$

2 where $v_{||} = k_{||}^2 T/m_e v_{ie}$ is - rate of electron diffusion over distance $\lambda_{||} = 1/k_{||}$

Physical Picture of Instability

- Wave stabilization occurs when rate of ion diffusion over λ_{\perp} exceeds rate of electron diffusion over $\lambda_{\rm II}$, i.e. $\nu_{\rm II}$ < ν_{\perp}



Unstable when: $B/k_v \ge (n/T)^{1/2} / M_i^{3/8}$

or $k_v \rho_i \approx 0.5$ (usual for drift waves)

Higher B destabilizing due to lower μ_{\perp} !

<== stability for potassium plasma n=10¹¹ cm⁻³, T=2800 °K, $L_n \approx 1$ cm

Perpendicular Phase Velocity



- For collisional drift wave $v_p \approx v_{de}/2 \approx cT_e/eB (1/2L_n)$
- Measured v_d by phase delay between 2 probes
- Corrected for measured ExB velocity for each m
- Good agreement (±30%) for all cases, including some increase with T

Mode Scaling with B Field



- Density and temperature held constant while B field varied
- $\lambda_{II} \approx 2L$ (set by column length)
- "m" measured by probe phases
- Observed m's near maxima of calculated growth rate vs. m (----- slab model with Bx1.5)
- Mode amplitude not predicted by theory (nonlinear state)
- $\omega \approx 1/2k_{\perp}v_{de}$ (within $\approx x2$)

Stability Threshold and Wavelength



- Threshold scales with n^{1/2} at fixed T as expected
- Coefficient of fit within ≈ 50% of (slab) theory with x1.5 B

• λ_r and λ_{θ} decrease with B as expected from $k_{\perp} \rho_i > 0.5$

Ion Mass Dependence



- Vary m_i by varying ratio of K to Cs ions
- Phase velocity does not depend on m_i as expected from v_{de}
- Consistent with scaling M^{3/8} from ion viscosity, but not inconsistent with M^{1/2} (ρ_i effect only) or M^{1/3} (resistivity only)

Limitations of this Comparison

- The only wave diagnostic was probes, which can perturb the plasma to some extent (acknowledged in paper)
- Slab model calculation applied to cylindrical geometry, maybe causing critical field for stability off by x1.5
- Linear theory is applied to a steady-state "saturated" instability, mode amplitudes not really understood (see also Hinton and Horton, PF 14, 116, 1971)
- Various effects ignored in theory, e.g. electron temperature fluctuations, radial diffusion and heat conduction
- Significance of classical ion 'viscosity" in damping is debatable (how big is effect of ion diffusivity ?)

Ion Temperature Gradient Mode (ITG)

- Thought to be dominant cause of anomalous transport in hot tokamaks (ion heating => "L-mode scaling")
- Physical nature of ITG is similar to ion acoustic wave driven unstable by large ∇T_i (Cowley, PF '91 p. 2769):
 - any compression of ions along B produces a density increase, ambipolar parallel electric field, and (assuming k_v≠0) a perpendicular E_v
 - with a large radial ∇T_i, the radial drift v_{rad} = E_y x B can move cold ions into region of high density, lowering pressure ("negative compressibility"), leading to further compression of ions, more cooling, etc
 - Instability when $\nabla T_i/T_i$ is above some threshold
 - Frequency $\omega \approx k_v V_{di}$ in ion direction (opposite cdw)
 - Perpendicular wavelength $k_v \rho_s \approx 1$ (similar to cdw)

ITG Modes in the CLM (Columbia)

Sen, Chen, Mauel, PRL 66, 429 (1991) Chen, Sen, Migliuolo, Phys. Fluids B 4, 512 (1992) Parker and Sen, PoP 9, 3440 (2002)



Steady state, no currents, no applied electric fields $n_e \approx 10^8 \cdot 10^9 \text{ cm}^{-3}$ (fully ionized Hydrogen plasma) $L \approx 150 \text{ cm}, a \approx 2.7 \text{ cm}, B \approx 1 \text{ kG}, \beta \le 10^{-6}$ $T_e \approx 5 \cdot 10 \text{ eV}, T_i = 5 \cdot 15 \text{ eV}, \rho_i/a \approx 0.1 \cdot 0.2$

Creation of ITG Mode in CLM

- Theory showed that ITG is dominantly affected by ∇T_{ill} not $\nabla T_{i\perp}$ (Mathey and Sen, PF p. 725, 1992)
- Parallel ion heating created by acceleration of ions from source region by ≈ 1 cm tungsten mesh electrode



- ion temperature measured by gridded energy analyzer
- instability measured by I_{sat}
- density and T_e measured with Langmuir probes

Observation of ITG Mode

ITG

120

140

80

100



- T_{iii} at r/a < 1 cm increased with bias voltage
 - mode increases with larger ∇T_{iii}
 - n, T_e and T_{il} fairly constant vs. volt
 - => ITG mode ?

Threshold and Mode Number

• Theoretical evaluation of threshold vs. mode number for slab ITG for CLM (Mathey and Sen, PF 1990)

$$\begin{split} L_{\text{Till}} &\equiv T_{\text{ill}} / \nabla T_{\text{ill}} = 2.2 \text{ cm (m=1 ITG)} \\ L_{\text{Till}} &\equiv T_{\text{ill}} / \nabla T_{\text{ill}} = 4 \text{ cm (m=2 ITG)} \end{split}$$

- "ExB" mode has m=1, always present => not ITG
- Measured ITG mode number and threshold:

m=2 and $< L_{Till} > \approx 2.4$ cm (averaged over mode)

=> Theory close to experimental results (within x2)

ITG Mode Frequency

- Theoretically $\omega(ITG) \approx -k_y V_{di}$ (should go in ion drift direction)
- Observed ω (lab)= ω (ITG) + m_{ITG} ω _{ExB} (Doppler effect of ExB)
- Observed ExB mode at f=65 kHz in electron diamagnetic dir.

=> calculated Doppler shift = 2(65)kHz = 130 kHz

• Observed frequency of m=2 ITG feature at f= 116 kHz

=> inferred ITG frequency = - 14 kHz in ion direction

=> theoretical ω (ITG) \approx - 10 kHz in ion direction

• Pretty close agreement with theory (within a factor-of-two)

Parallel Wavelength

• Parallel wavelength measured with 4 Langmuir probes

 $\lambda_{II} \text{ (m=1)} \approx 750 \text{ cm (flute-like mode)}$ $\lambda_{II} \text{ (m=2)} \approx 270 \text{ cm (drift-like mode)} \approx 2L$



=> agrees with theory, which predicts m=2 and its λ_{II} to be ITG unstable

Effect of End Plate Bias



FIG. 8. Mode dependence on the fraction of heated ions. (a)-(d) density fluctuation spectra for increasingly larger fraction of heated ions (10 kHz per division).

- Normally ion transit freq.
 ≈ wave frequency => weak drive for ITG
- Biasing end plate positive confines ions and should increase drive of ITG
- Mode amplitude behaves qualitatively as expected

Radial Mode Structure



FIG. 11. Experimentally observed radial mode structures with the profiles of Fig. 5(b).



FIG. 10. Shooting code results of the radial mode structure (m = 2) with the profiles of Fig. 5(b). The linear growth rate γ and the real frequency ω , are also shown.

- More general theory was developed for non-local ITG (but still in slab model)
- Radial mode structure and stability calculated using the experimental profiles
- Observed m=2 mode structure similar to calculated ITG, but observed mode peaks farther outward than expected (by $\approx \rho_i$)

Subsequent Studies of ITG in CLM

- RF ion heating method (Greaves, Chen, Sen PPCF 1992)
 - ion distribution function more Maxwellian
 - better λ_{II} measurement ($\lambda_{II} \approx 2-4$ L)
 - better identification of threshold ($\eta_i \approx 1.4$)
 - better check of frequency vs ITG theory =>
 - disagreement with linear theory for mode number !



 Several other papers on ITG transport, "mixed slab-toroidal ITG" (with magnetic mirrors to simulate toroidal curvature), measuring linear growth rate with feedback control, and non-linear mode coupling (see Reference list)

Non-linear Simulation of ITG in CLM

[Parker, Sen PoP p. 3440 (2002)]

- 3-D nonlinear electrostatic gyrokinetic simulation of CLM
 - assumes adiabatic electrons and zeros-out zonal flows (not seen in CLM)
 - uses 65,000 particles in 32 x 32 x 16 box with timestep $\Delta t = 20/\Omega_{I}$
 - physical dimensions of box are 32 $\rho_i x$ 32 $\rho_i x$ 1200 ρ_i (6 cm x 6 cm x 240 cm)
 - assumes initial condition $T_{iII}(0)$ [= $T_{i\perp}=T_e$], $L_{T0}=5 \rho_i$, mode width w = 4 ρ_i



- Simulation shows most unstable ITG mode is m=4, but m=1-3 modes grow to amplitudes ≥ that for m=4
- Does not agree with dominant m=2 seen in experiment in non-linearly saturated state !

Limitations of This Comparison

- Dependence of mode amplitude on mesh and end plate bias only qualitatively consistent with (linear) theory (fluctuation level not discussed)
- Reason for ITG mode peaking outside of maximum ∇T_{ill} not well understood (non-linear effect ?)
- Dominance of m=2 mode in experiment is not explained by theory (could be due to sheared EXB rotation)

=> non-linear drift wave theory was not successful in explaining experiment !

Drift Wave Control

Schroder, Klinger et al, Phys. Rev. Lett. **86**, p. 5511 (2001) Klinger et al, Phys. Plasmas **8**, p. 1961 (2001) Klinger et al, PPCF **39**, B145 (1997)



Fig. 1. The triple plasma device MIRABELLE.

Steady state, no currents, hot cathode discharge $n_e \approx 2x10^{10} \text{ cm}^{-3} \text{ Ar}^+ \text{ plasma}, T_e \approx 1-3 \text{ eV}, T_i < 0.1 \text{ eV}$ $L \approx 140 \text{ cm}, a \approx 15 \text{ cm}, B \approx 0.4-1 \text{ kG}, \beta \le 10^{-5}$

Experiment and Theory

Experiment:



Applies rotating δE field to plasma

Theory :

$$\frac{\partial}{\partial t}\nabla_{\perp}^{2}\phi + \vec{V}_{\mathbf{E}\times\mathbf{B}}\cdot\nabla\nabla_{\perp}^{2}\phi = \nabla_{\parallel}J_{\parallel} + \mu_{w}\nabla_{\perp}^{4}\phi, \quad (1)$$

$$\frac{\partial}{\partial t}n + \vec{V}_{\mathbf{E}\times\mathbf{B}} \cdot \nabla(N_0 + n) = \nabla_{\parallel}J_{\parallel} + \mu_n \nabla_{\perp}^2 n, \quad (2)$$

Hasagawa-Wakatani 2-D model

Comparison of Experiment and Theory







Limitations of this Comparison

- No verification of Boltzman relation for drift waves
- No verification of perpendicular drift wave scale length
- No verification of parallel drift wave scale length
- No comparison with drift wave frequency ($\omega \approx \Omega_c \approx 15 \text{ kHz }$?)
- No comparison with threshold for linear drift wave instability
- Theory done for 2-D not 3-D as for collisional drift wave
- Parameters used in model seem arbitrary ("the parameters are chosen to establish a saturated turbulent state similar to the experimentally observed one")

Open Question #1

Can nonlinear plasma theory explain the drift waves observed in laboratory plasmas ?

Answer so far: no !

Open Questions / Work in Progress

- How does DW turbulence arise from linear instability ?
 - increase in Re and 3-wave coupling in CSDX (Burin et al, APS '02)
 - transition through dynamical chaos Grulke and Klinger NJP p. 67.1 (2002)
 - Iow dimensional chaos / attractors (Klinger and Piel, Phys. Fluids 1992)
- How can drift waves be controlled (reduced) ?
 - feedback stabilization of single mode in linear machine (Chiu, Sen PRL '99)
 - driving selected drift mode to reduce DW turbulence (Schroder PRL 2001)
 - chaos control Klinger et al, PoP p. 1961 (2001)
 - control of velocity shear in CSDX (Crocker et al, APS '02)
 - autosynchronization method Gravier et al, Eur. Phys. J. D, p. 451 (2000)
- What do DW look like in toroidal geometry ?
 - coherent structures in TEDDI torus Grulke et al, PPCF p. 525 (2001)
 - coherent structures in Blaumann torus Riccardi. Fredriksen PoP p. 199 (2001)
 - drift waves in RF driven plasma in torus Ferreira et al, PoP p. 3567 (2000)
 - finite beta DW in tokamak Fredrickson and Bellan, Ph. Fluids p. 1866 (1985)
 - Helimak (cylindrically symmetric torus) planned for Texas (Luckhardt, APS '02)

More Open Questions...

- What is interaction between DW and zonal flow ?
 - oscillating flow in DIII-D (McKee et al, APS '02)
 - oscillating zonal flow in TEXT (Schoch, APS '02)
- Are there any other types of DW or related waves ?
 - ion drift waves in Kr plasma Klose et al, Cont. Plasma Phys. p. 467 (2001)
 - very low B case with $k_{\perp}\rho_s$ > 1 Kauschke et al, Phys. Fluids p. 38 (1990)
 - DW in weakly ionized plasma Sosenso et al, J. Pl. Phys. p. 157 (2000)
 - lower hybrid drift waves Carter et al, Phys. Plasmas 9, 3272 (2002)
 - ion cyclotron drift waves Yamada/Hendel, Phys. Fluids 21, 1555 (1978)
 - convective cells Pecseli et al, PPCF p. 837 (1985)
 - drift-Alfven waves on LAPD Penano et al, PoP p. 144 (2000)

• What are drift waves like in:

- single species plasma ?
- dusty plasma ?
- space plasma ?
- industrial plasma ?

Recent Experiments on Drift Waves

