Homework 2

APAM 4990 (2010)

We have discussed earlier that the Vlasov-Poisson system,

$$\frac{dF}{dt} \equiv \frac{\partial F_\alpha}{\partial t} + v\frac{\partial F_\alpha}{\partial x} - \frac{q_\alpha}{m_\alpha}\frac{\partial \phi}{\partial x}\frac{\partial F_\alpha}{\partial v} = 0$$

and

$$\frac{\partial^2 \phi}{\partial x^2} = -4\pi e \int (F_i - F_e)\, dv,$$

are number density, momentum and energy conserving, where α denotes species. Show that

$$\frac{d}{dt}\left\langle \frac{m_e}{2}\int v^2 F_e dv + \frac{m_i}{2}\int v^2 F_i dv + \frac{1}{8\pi}\left|\frac{\partial \phi}{\partial x}\right|^2 \right\rangle_x = 0,$$

where

$$< \cdots >_x \equiv \int dx/L$$

is the spatial average. Remember that you need to invoke the Vlasov equation again for the energy exchange term. Also derive the conservation of entropy,

$$S \equiv -F \ln F,$$

starting from

$$\frac{dS}{dt} = 0,$$

and

$$\left\langle \int \frac{dS}{dt} dv \right\rangle_x = 0.$$