Homework 3

APAM 4990 (2010)

As shown earlier, by letting

$$F = F_0 + \delta f,$$

we obtain

$$\frac{d\delta f}{dt} = -\frac{q}{T} v \frac{\partial \phi}{\partial x} F_0, \tag{1}$$

where q is the charge, T is the temperature, ϕ is the potential and

$$\frac{d}{dt} \equiv \frac{\partial}{\partial t} + v\frac{\partial}{\partial x} - \frac{q}{m}\frac{\partial \phi}{\partial x}\frac{\partial}{\partial v} = 0.$$

The corresponding Poisson's equation becomes

$$\frac{\partial^2 \phi}{\partial x^2} = -4\pi e \int (\delta f_i - \delta f_e)\, dv. \tag{2}$$

Through linearization, this set of equations gives us the the plasma waves and the ion acoustic waves for the Maxwellian F_0, which you are familiar with.

1. For $\partial/\partial t \ll v\partial/\partial x$, i.e., $\omega \ll kv$, show that

$$\delta f = -\frac{q}{T}\phi F_0, \tag{3}$$

by using the ordering argument that $\omega \sim \phi \sim o(\epsilon)$, where ϵ is a smallness parameter. Equation (3) is the so-called adiabatic response, since it is independent of frequency.

2. Now by assuming that a part of δf is adiabatic and letting

$$F = F_0 - \frac{q}{T}\phi F_0 + \delta h,$$

show that

$$\frac{d\delta h}{dt} = \frac{q}{T}\frac{\partial \phi}{\partial t} F_0, \tag{4}$$

and the corresponding Poisson's equation becomes

$$\frac{\partial^2 \phi}{\partial x^2} - \frac{\phi}{\lambda_{De}^2} = -4\pi e \int (\delta f_i - \delta h_e)\, dv. \tag{5}$$

3. For $k^2\lambda_{De}^2 \ll 1$, the first term on the LHS of Eq. (5) can be neglected and the resulting equation describes a quasineutral system. Show that Eqs. (1), (4), and (5) support only ion acoustic waves.

4. For

$$w = \delta f/F$$

and

$$w^{NA} = \delta h/F,$$

particle pushing for the $j-th$ can be accomplished by using

$$\frac{dx_j}{dt} = v_j$$

$$\frac{dv_j}{dt} = -\frac{q}{m} \left.\frac{\partial \phi}{\partial x}\right|_{x_j}$$

$$\frac{dw_j}{dt} = -(1-w_j)\frac{q}{T}v_j \left.\frac{\partial \phi}{\partial x}\right|_{x_j},$$

and

$$\frac{dw_j^{NA}}{dt} = (1-w_j^{NA})\frac{q}{T} \left.\frac{\partial \phi}{\partial t}\right|_{x_j}.$$

While $\partial\phi/\partial x$ is given by Eq. (2), show that $\partial\phi/\partial t$ can be calculated by

$$\frac{\partial^2}{\partial x^2}\frac{\partial \phi}{\partial t} = 4\pi e \frac{\partial}{\partial x}\int v(\delta f_i - \delta h_e)dv. \tag{6}$$

Note that the electron current on the RHS is usually the dominant term.