Chapter 8

Conclusions

8.1 Moment Closures in One Dimension

A general linear closure for the hierarchy of moment equations of the stochastic oscillator problem is introduced here, based on the Landau-fluid approach to obtaining fluid moment closures for collisionless plasmas (Hammett and Perkins 1990). The n closure coefficients for the n-moment system are computed using conditions obtained by matching the response function for the closed moment system to the exact response function q times in the $\omega \to 0$ limit and $n - q$ times in the $\omega \to \infty$ limit. Results concerning the form of this closure are easily expressed for closures written in terms of polynomial moments where the polynomials are orthogonal with respect to the underlying distribution.

There is strong numerical evidence that the linear response of the closed moment system converges to the exact response for any choice of closure with increasing numbers of moments n, as long as there is some dissipation in the closure for the highest moment and the underlying distribution function is sufficiently smooth (i.e., can be represented as a Gaussian times a few low order Hermite polynomials). Convergence can be accelerated with a judicious choice of the closure dissipation, and is faster if q scales with n. However, if the underlying distribution function contains small scale features or sharp discontinuities (so its expansion in terms of Hermite polynomials requires high order polynomials), then convergence may be very slow, and a very large number of moments would be required. In some examples with sharp features in the distribution function, there was no convergence with n unless q also increased proportional to n. The poles of the approximate response functions were found to lie in the lower-half complex plane for a large number of choices of n and q.

The linear response to a potential perturbation given by the one-dimensional Vlasov equation has been shown to be equivalent to the stochastic oscillator problem. The fluid moment closure problem in this case is therefore equivalent to a statistical moment closure problem. It has also been shown that the fluid moments of the Vlasov equation are equivalent to Hermite polynomial moments, which in turn are equivalent to coefficients in the Hermite series that has traditionally been used in simulations of the Vlasov equation (Armstrong et al. 1970).

A model nonlinear problem, the plasma echo, was considered as a simple nonlinear test of the closure. The plasma echo is essentially the second-order component of the nonlinear response expanded in the limit of small perturbations. The second-order response has a simple exact solution (4.3) in the limit where the self-consistent potential is dropped. This exact solution was used to gauge the weakly nonlinear performance of moment systems with closure.

An essential factor in resolving the echo is having a large enough number of moments. (See Figs. 4.4 and 4.6.) The results, summarized in Fig. 4.6, show that modeling the second-order response for interaction time $t_2 - t_1$ requires $n \sim [(t_2 - t_1)k_1 v_t]^2$ moments. A very large number of moments would therefore be required to model a large time separation $t_2 - t_1$. Physically the large number of moments are required because the echo involves nonlinear interactions that un-phase-mix very fine scales in velocity space, which require many moments or Hermite polynomials to be fully represented. In the plasma echo problem, a simple model of the rate of decay of perturbations due to a finite collision rate ν indicates that $n \sim (kv_t/\nu)^{2/3}$ moments are sufficient to resolve all second-order effects.

This physics of the echo problem appears to be related to the problem pointed out by Mattor (1992) that the Landau-fluid equations miss certain nonlinear effects in some weak-turbulence regimes (or at least would require a very large number of moments to accurately represent them). However, we believe it reasonable that the Landau-fluid approximations will usually be adequate in strong turbulence regimes where the decorrelation times τ_c are relatively short, i.e., if the effective $t_2 - t_1 \sim \tau_c$ is relatively small or the effective $\nu \sim 1/\tau_c$ is relatively large. [Hammett et al. (1993) and Dorland (1993) discuss these issues in greater detail.] This picture is supported by comparisons between Landau-fluid simulations and particle simulations of nonlinear slab 3D ITG turbulence (Parker et al. 1994), which found fairly good agreement in the turbulent heat diffusivity χ_i, though there were $\sim 40\%$ differences in the fluctuation level of $|\phi|$. More work on this would be useful, and detailed Landau-fluid/particle comparisons in toroidal geometry have recently begun but the results are not yet clear.

In summary, the picture which emerges from our work is that, while the Landau-fluid closure approximations might be expected to work well in strong-turbulence regimes where decorrelation rates are short and the dominant saturation mechanisms are $\mathbf{E}\times\mathbf{B}$ transfers from unstable modes to Landau-damped modes at high k, one should be aware that the approximations break down (or require a large number of moments to converge) in cases such as the plasma echo or certain weak-turbulence regimes (Mattor 1992), where fine-scale structures in velocity space are important in the nonlinear physics.

8.2 Subgrid-Scale Closures

A new parameterized hyperviscosity, Eqs. (6.19) and (6.21), was derived by analyzing the eddy viscosity calculated from simulations with a range of parameters and resolutions. Simulations using the parameterized hyperviscosity perform somewhat better than those using hyperviscosity with fixed power, and significantly better than those using no extra damping term or a Smagorinsky-type eddy viscosity. Accurate results are obtained provided the cutoff wave number k_c is approximately four or more times greater than the characteristic wave number k_{av} of the advecting velocity. At lower resolutions, the parameterization fails to accurately model the eddy viscosity.

Because this hyperviscosity is based on physics generic to most drift-wave turbulence, application to more complete models is straightforward. This hyperviscosity is very useful computationally, since reducing resolution requirements by even a factor of 2 reduces the computational requirements by a factor of 8 in two dimensions, or 16 in three dimensions.

Heuristic arguments concerning the effect of velocity shear on elongated eddies were used to extend the 2D parameterized hyperviscosity of Eqs. (6.19) and (6.21) to applications in three dimensions where eddies are highly elongated along the additional dimension. The 3D hyperviscosity was tested in a comprehensive toroidal gyrofluid turbulence code (Beer 1995). Simulations of the Numerical Tokamak Project Test Case set of parameters indicates that the new heuristically derived parallel damping term can reduced the required parallel resolution by a factor of 2 in some cases. A set of high-gradient edge-like parameters were simulated to demonstrate that there are conditions under which the saturated spectrum shifts to significantly longer wavelengths ($k_\theta \rho_i \sim 0.03$). These nonlinear simulations could not have been performed without the use of the subgrid damping term proposed here.

8.3 Future Directions

The results in three dimensions are still somewhat preliminary in nature. Ideally one would like to compute the transfer rates in the 3D simulation to see if the magnitude of the transfer in parallel wave-number space compared to the perpendicular transfer corresponds to the heuristic predictions made in Section 7.1. The NTP test case results seem to indicate that the transfer of fluctuation energy to high parallel wave number may be more important physically than the transfers in perpendicular wave-number space. Measurements of the eddy damping due to transfer in both directions could provide useful insights into the nature of drift-wave turbulence saturation.

It would be interesting to apply the techniques of Chapter 2 to the more general problem of toroidal-drift phase mixing that has been successfully modeled with 6 fluid moment equations (Beer and Hammett 1996). One could imagine writing down the general toroidal Landau-fluid equations for the arbitrary ${v_\parallel}^n {v_\perp}^m$ moment (or in the appropriate orthogonal polynomials in $v_\parallel$ and $v_\perp$) and modifying a code to solve these equations for general n and m. Such a direct nonlinear turbulent test of the convergence with the number of moments evolved would add to the justification of the Landau-fluid approach to simulating drift-wave turbulence. This could compliment related work of ongoing comparisons of particle and Landau-fluid simulations (Parker et al. 1994).

Our approximations for both Landau damping fluid closures and sub-grid turbulence closures focus on the net transfer from resolved scales (in velocity or space) to unresolved scales. There are cases however where inverse-transfers can occur from unresolved scales to resolved scales. One might wonder if the closure approximations could be extended in some way to to represent such inverse-transfers, perhaps as noise source terms in resolved scales.

A fairly simple parameterization of the subgrid dissipation was constructed in this thesis. As it stands, the parameterized hyperviscosity yields significant improvements for numerical simulations of drift-wave turbulence. Recently, renormalization-group techniques have been successfully used to predict the subgrid transfer in the inverse cascade range of 2D turbulence (Chekhlov et al. 1994) and improved closure models have successfully predicted the nonlinear transfer rates for anisotropic 2D drift-wave turbulence (Bowman and Krommes 1997; Hu et al. 1997). It is possible that use of one of these theories may lead to more accurate estimates of the subgrid dissipation.