Supplementary Material: Physics of Anomalous Resistivity of Lower Hybrid Drift Waves

Jongsoo Yoo,[1,*] Jonathan Ng,[2] Hantao Ji,[1,3] Sayak Bose,[1] Aaron Goodman,[4] Andrew Alt,[3] Li-Jen Chen,[5] Peiyun Shi,[1] and Masaaki Yamada[1]

[1] *Princeton Plasma Physics Laboratory, Princeton, New Jersey 08542, USA*
[2] *Department of Astronomy, University of Maryland, College Park, Maryland 20742, USA*
[3] *Department of Astrophysical Sciences, Princeton University, New Jersey 08544, USA*
[4] *Department of Mechanical and Aerospace Engineering, Princeton University, New Jersey 08544, USA*
[5] *NASA Goddard Space Flight Center, Greenbelt, Maryland 20771, USA*

Unlike other electrostatic waves, quasi-electrostatic lower hybrid (ES-LHDW) has a capability of generating density fluctuations (δn_e) almost in phase with fluctuations in the electric field (δE_Y). By examining each term of the electron momentum equation, we will try to understand why δn_e and δE_Y are almost in phase.

Figure 1 shows the geometry of the local theory for lower hybrid drift waves. The subscript 0 indicates equilibrium quantities. The model is in the ion rest frame and electrons have velocity ($\mathbf{u}_{\mathrm{e}0}$) on the x-z plane. The equilibrium magnetic field is along the z direction and the density gradient direction is along the y direction. The wave vector ($\mathbf{k}$) lies on the x-z plane due to our assumption of negligible k_y. Thus, our theoretical model is local and valid only when the wavelength of the LHDW is much smaller than the thickness of the current sheet in the y direction [1].

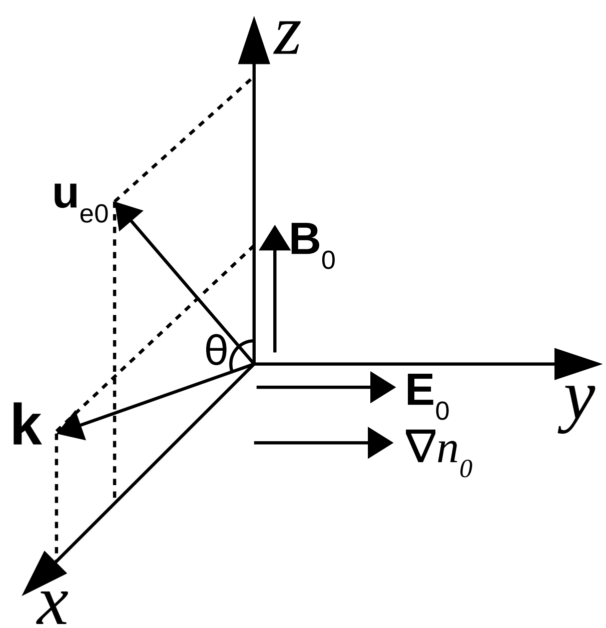

FIG. 1. Geometry of the local theory. We are working in the ion rest frame with the z direction toward the equilibrium magnetic field ($\mathbf{B}_0$) and the y direction along the density gradient direction. Due to the force balance, the equilibrium electric field $\mathbf{E}_0$ is also along the y direction. The equilibrium electron flow velocity $\mathbf{u}_{\mathrm{e}0}$ and wave vector $\mathbf{k}$ reside on the x-z plane. The angle between $\mathbf{k}$ and $\mathbf{B}_0$ is given by θ.

* jyoo@pppl.gov

The ion force balance along the y direction give us

$$en_0 E_0 = T_{\rm i0}\frac{dn_0}{dy} = \epsilon n_0 T_{\rm i0}, \tag{1}$$

where n_0 is the equilibrium density, $T_{\rm i0}$ is the equilibrium ion temperature, and $\epsilon = (dn_0/dy)/n_0$ is the inverse of the density gradient scale. The zeroth order electron momentum equation gives us

$$-en_0(E_0 - u_{{\rm e0}x}B_0) = T_{\rm e0}\frac{dn_0}{dy}, \tag{2}$$

where $u_{{\rm e0}x}$ is the x component of the equilibrium electron flow velocity and $T_{\rm e0}$ is the equilibrium electron temperature. Then, the equilibrium electric field is

$$E_0 = \frac{T_{\rm i0}}{T_{\rm e0}+T_{\rm i0}} u_{{\rm e0}x}B_0. \tag{3}$$

The inverse of the gradient scale is given by

$$\epsilon = \frac{e u_{{\rm e0}x}B_0}{T_{\rm e0}+T_{\rm i0}}. \tag{4}$$

The first order electron momentum equation is given by

$$i m_{\rm e} n_0\left(\omega - \mathbf{k}\cdot\mathbf{u}_{\rm e0}\right)\mathbf{u}_{\rm e1} = i\mathbf{k}\cdot\mathbf{P}_{\rm e1} + en_0(\mathbf{E}_1 + \mathbf{u}_{\rm e1}\times\mathbf{B}_0 + \mathbf{u}_{\rm e0}\times\mathbf{B}_1) + e(\mathbf{E}_0 + \mathbf{u}_{\rm e0}\times\mathbf{B}_0)n_{\rm e1} - \mathbf{R}_{\rm e1}, \tag{5}$$

where $\mathbf{P}_{\rm e1}$ is the perturbed electron pressure tensor and $\mathbf{R}_{\rm e1}$ is the perturbed resistivity. We will work with the x component of the electron momentum equation. The x component of the first-order electron momentum equation is [2]

$$i m_{\rm e} n_0\left(\omega - \mathbf{k}\cdot\mathbf{u}_{\rm e0}\right)u_{{\rm e1}x} = ik_\perp(n_0 T_{\rm e1}^\perp + T_{\rm e0}n_{\rm e1}) + en_0(E_{1x} + B_0 u_{{\rm e1}y} - u_{{\rm e0}z}B_{1y}) - R_{{\rm e1}x}, \tag{6}$$

where $T_{\rm e1}^\perp$ is the perturbed perpendicular electron temperature and $k_\perp$ is the perpendicular (x) component of $\mathbf{k}$. With 6, $n_{\rm e1}$ can be expressed as

$$\frac{n_{\rm e1}}{n_0} = \frac{ie}{k_\perp T_{\rm e0}}E_{1x} + \frac{m_{\rm e}\left(\omega - \mathbf{k}\cdot\mathbf{u}_{\rm e0}\right)}{k_\perp T_{\rm e0}}u_{{\rm e1}x} + \left[\frac{ieB_0}{k_\perp T_{\rm e0}}u_{{\rm e1}y} - \frac{ieu_{{\rm e0}z}}{k_\perp T_{\rm e0}}B_{1y}\right] - \frac{T_{\rm e1}^\perp}{T_{\rm e0}} - \frac{iR_{{\rm e1}x}}{k_\perp n_0 T_{\rm e0}}. \tag{7}$$

Based on the origin of each term, we will call 5 terms on the right hand side (RHS) the electric field, inertial, Lorentz force, temperature, and resistivity term, respectively.

By using the linear relation for the mode with the maximum growth rate, each term on RSH can be expressed in terms of $\delta E_{\rm Y}$ with a complex coefficient. After defining each coefficient as A_1 to A_5, we have $n_{\rm e1}/n_0 = \sum_{n=1}^{5} A_i \delta E_{\rm Y}$. By examining each complex coefficient, we can figure out which term contributes to anomalous resistivity. To have anomalous resistivity, the sum of 5 complex coefficients needs to have a sizable real component; if it has a dominant imaginary component, two fluctuations become out of phase (either 90° or 270°) such that after averaging over the wave period the anomalous effect disappears. Thus, among those 5 complex coefficients, we need to identify which coefficient has a sizable real component.

Among these 5 terms, the Lorentz force term is the key for anomalous resistivity; as demonstrated in Fig. 2, only the Lorentz force (LF, green arrow) term has a sizable real component. The temperature term ($T_{\rm e1}$, black arrow) also contributes to anomalous resistivity by reducing the imaginary component, which comes mostly from the electric field (E_{1x}, red arrow) term. As indicated by the blue arrow ($\sum A_i$), the theory expects that the phase difference between $\delta n_{\rm e}$ and $\delta E_{\rm Y}$ is 30°, which is verified by laboratory measurements.

This means that the positive correlation between $\delta n_{\rm e}$ and $\delta E_{\rm Y}$ is caused by compression by the Lorentz force. To be more specific, it is the $(ieB_0/k_\perp T_{\rm e0})u_{{\rm e1}y}$ term; the magnitude of the $u_{{\rm e1}y}$ term is larger than the B_{1y} term by more than two orders of magnitude, as ES-LHDW generates a relatively weak perturbation in the magnetic field.

We need to understand why the $u_{{\rm e1}y}$ term can be out of phase with E_{1x}. Ordinarily, $u_{{\rm e1}y}$ is in phase with E_{1x}, since it is driven by $\mathbf{E}\times\mathbf{B}$. For LHDWs, however, there is additional terms in the y component of the electron momentum equation associated with E_0 and B_0. The y component of the Eqn. 5 is

$$i m_{\rm e} n_{\rm e0}\left(\omega - \mathbf{k}\cdot\mathbf{u}_0\right)u_{1y} = en_{\rm e0}(E_{1y} - B_0 u_{{\rm e1}x} - u_{{\rm e0}x}B_{1z} + u_{{\rm e0}z}B_{1x}) + e(E_0 - u_{{\rm e0}x}B_0)n_{\rm e1}. \tag{8}$$

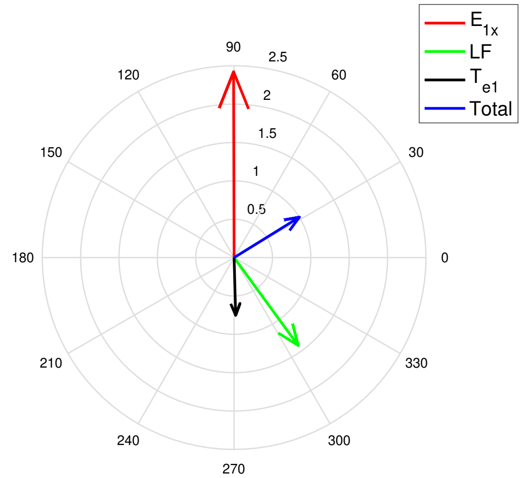

FIG. 2. Demonstration of the complex coefficient in the complex plane. Theory expects the phase difference between $\delta n_{\rm e}$ and $\delta E_{\rm Y}$ is 30°, as shown in the blue arrow, which is the sum of 5 complex coefficients. The electric field term (red) has a dominant positive imaginary component, while the temperature term (black) has a dominant negative imaginary component. Contributions from both inertia and resistivity terms to the total coefficient (blue) is negligible. The Lorentz force (LZ) term is the only term that has a significant real component, which drives anomalous resistivity.

When ω has no imaginary component and $n_{\rm e1}$ is out of phase with $\mathbf{E}_1$, it is hard to balance the imaginary part from the $n_{\rm e1}$ term unless $E_0 - u_{\rm e0x} B_0 = -T_{\rm e0} u_{\rm e0x} B_0/(T_{\rm e0} + T_{\rm i0})$ is negligible; the inertial term on the left hand side is typically small. Thus, it is natural that when there is enough free energy source ($u_{\rm e0x}$), ω has a sizable imaginary part and $n_{\rm e1}$ has a sizable real part to balance this equation. Thus, an unstable (growing) ES-LHDW mode is closely related to the physics of anomalous resistivity.

When $B_0 \ll 1$, this argument becomes irrelevant; this is related to stable ES-LHDW in low $\beta_{\rm e}$ plasma. It is also true that $T_{\rm i} \gg T_{\rm i}$ is unfavorable to the growth rate of ES-LHDW. For high β plasmas, the mode becomes EM-LHDW and the stability of the EM-LHDW is usually dominated by the parallel force balance. We will discuss the dynamics of LHDW in more detail in a future publication.

[1] H. Ji, R. Kulsrud, W. Fox, and M. Yamada, An obliquely propagating electromagnetic drift instability in the lower hybrid frequency range, J. Geophys. Res. **110**, A08212 (2005).

[2] J. Yoo, Y. Hu, J.-Y. Ji, H. Ji, M. Yamada, A. Goodman, K. Bergstedt, W. Fox, and A. Alt, Effects of coulomb collisions on lower hybrid drift waves inside a laboratory reconnection current sheet, Physics of Plasmas **29**, 022109 (2022), https://doi.org/10.1063/5.0052555.