Negative central current and $q_0 < 1$ equilibria in tokamaks

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A solution for toroidal equilibria is given for the situation with a negative current ($j(0) < 0$) in the center of the plasma column (with a positive overall current). It includes (a) a central core region with simply nested magnetic surfaces and a negative total current, (b) an $m = 1$, $n = 0$ magnetic island with a positive current density, and (c) an outer region with the conventional magnetic surfaces and positive current density. The same solution, applied for $q(0) < 1$ in tokamaks explains the existence of a stationary phase between internal relaxations in tokamaks with both central pressure below and above the ideal Bussac limit. The theory gives a classification of relaxation regimes in tokamaks (consistent with observations).

The case of MHD equilibrium in tokamaks with a negative current density near the magnetic axis and with a positive total current is rather intriguing as, since the early 60s, it was believed theoretically that such equilibria are not possible [1]. The issue recently attracted considerable attention because of Joint European Torus (JET) experiments [2] with an off axis non-inductive current drive, generating an electromotive force, which may drive a negative current in the plasma core.

Recently the theory of equilibrium was revised in conjunction with JET experiments. It was shown in [3] that within the theory [1], the negative current equilibria may exist but would require a hollow plasma pressure and that even in this case, the topology of magnetic surfaces will be disturbed by the $m > 1$ islands near the inversion surface for poloidal mangetic field. This result has been extended by G. Hammett [4] to arbitrary shaped configurations with nested magnetic surfaces. These results are in agreement with the lack of sufficiently reversed internal currents in the JET experiments, even when electron cyclotron current drive (ECCD) was applied in a central "current hole" region.

Although not conclusive, still there are indications from equilibrium reconstruction [5] that some negative current may be present in the JET plasma core in contrast with existing MHD equilibrium theory. In the present paper, we make the next step in the theory revision and present the $m = 1, n = 0$ solution (m, n are poloidal and toroidal wave numbers) which shows that in a different topological situation with a magnetic island the negative current toroidal equilibria can exist in a quasi-stationary manner.

We also extended our theory to the $q(0) < 1$ situation in tokamaks, where a stationary $m = 1$, $n = 1$ internal kink was believed to be unstable according to MHD theory. While there were numerous experimental measurements on the Tokamak Fusion Test Reactor (TFTR) [6] indicating local central pressure far exceeding the ideal stability limit [7], there was no convincing explanation of experimental stability of this internal kink mode in such a plasma[6, 8]. Moreover, even for the "ideally stable" plasma, the stationary phase between internal collapses remains unexplained.

Here, we apply the same negative current solution for analysis of the helical deformation and show that both ideally "unstable" and "stable" profiles can be consistent with a quasi-stationary state in the form of an island-held equilibrium. This correlates well with a frequently observed in TFTR [9] stationary $m = 1$, $n = 1$ perturbation (in an otherwise apparently grossly "unstable" situation). In fact, such equilibria should emerge after each internal collapse as the only stable state for the plasma.

Just before the submission of this paper, we learned that independent numerical studies, partially overlapping with our results for $m = 1, n = 0$ case, have been performed [10] and submitted for publication. These studies are consistent with our theory and extend the consideration of the $j(0) < 0$ case to the shaped plasma.

Leaving the exact calculations to numerical simulations (e.g., similar to those in [10]), we consider the toroidal equilibrium in the Shafranov approximation of circular magnetic surfaces shifted by $\xi(a)$. The equation for ξ has the form

$$(aB_\theta^2\xi')' = -\frac{aB_\theta^2 - 2a^2\bar{p}'}{R}, \tag{1}$$

where a, R are the minor and major radii of the magnetic surfaces, and θ is the poloidal angle (with $\theta = 0$ on the low field side), B_θ is the poloidal magnetic field, $\bar{p}(a) \equiv 4\pi 10^{-7} p(a)$ is the (normalized) plasma pressure. Its solution gives the Shafranov shift

$$\xi'(a) = -\frac{a}{R}\left(\beta_j + \frac{l_i}{2}\right), \tag{2}$$

$$\beta_j(a) \equiv 2\frac{\langle\bar{p}\rangle - \bar{p}(a)}{B_\theta^2(a)}, \tag{3}$$

$$l_i(a) \equiv \frac{\langle B_\theta^2(a)\rangle}{B_\theta^2(a)}. \tag{4}$$

We are interested in the case when $B_\theta(a)$ is changing direction

$$B_\theta(a) = B'_\theta x, \quad x \equiv a - a_i \tag{5}$$

inside the plasma. Because $B_\theta(a_i) = 0$, there is no solution for $\xi' \propto 1/x^2$ near the inverse point $a = a_i$.

In fact, the behavior of ξ' near the inverse point is similar to the tearing mode perturbations with $m > 1$. Therefore, an island maintained equilibrium [11] can be expected at the inversion surface.

Near the resonance $a = a_i$ the configuration is 2-dimensional and can be described by a simple exact solution assuming a uniform current density inside the island and its vicinity (which is the leading approximation). We convert this solution into an approximate form, which explicitly gives the shape of magnetic surfaces

$$x(\chi,\theta) \equiv \bar{x}\left(\sqrt{1+\frac{\bar{x}}{3a_i}} + \frac{w^2\cos\theta}{4a_i}\right), \qquad (6)$$

$$\bar{x} \equiv \chi\sqrt{1-\frac{w^2}{\chi^2}\sin^2\frac{\theta}{2}}, \qquad (7)$$

$$B_\theta = j_i\frac{a^2-a_i^2}{2a} - j_i\frac{w^2\cos\theta}{4a_i}. \qquad (8)$$

Here, χ is the flux coordinate, which is constant along the field lines, w is the half-width of the island, and j_i is the current density inside the island.

The matching conditions with the core (index 'c') and outer ('e') regions at $|\chi| > w$ are

$$\xi(a_{c,e}) = 2\left\langle x(\chi_{c,e},\theta)\cos\theta\right\rangle, \qquad (9)$$

$$\xi'_x(a_{c,e}) = 2\left\langle x'_\chi(\chi_{c,e},\theta)\cos\theta\right\rangle\frac{d\chi}{dx_{c,e}}, \qquad (10)$$

where the matching points a_c, a_e are determined by

$$a_{c,e} = a_i + x_{c,e}, \qquad (11)$$

$$x_{c,e} \equiv \left\langle x(\chi_{c,e},\theta)\right\rangle, \qquad (12)$$

and$\langle\ldots\rangle$ means averaging over θ. For small islands the half-width is given by

$$\frac{w^2}{4} = -[x^2\xi'(a_i+x)]_{x\to 0}. \qquad (13)$$

For the torodial equilibrium ($m = 1, n = 0$) it has an explicit form

$$w_{1,0}^2 = 4\frac{a}{R\mu'^2}\left(\mu^2\beta_j + \mu^2\frac{l_i}{2}\right)_{a_i}, \quad \mu \equiv \frac{1}{q} = \frac{RB_\theta}{aB_{tor}} \qquad (14)$$

where B_{tor} is the toroidal magnetic field.

The above solution describes the "island" held equilibrium (IHEq) which can withstand the ballooning force acting on the core in the direction of the island (Fig. 1a). It does not contain any rapidly evolving currents (Fig. 1b) and, thus, can be quasi-stationary on the resistive time scale. This is in contrast to the "Waelbroeck ribbon" (Fig. 1 c) held equilibria [12, 13], which have a spiked current inside the ribbon and, thus, evolves at the fast reconnection time $\tau_{rec} \simeq c_A/(R\mu'\rho_s)$ in a high temperature plasma (c_A is the Alfven speed, ρ_s is the ion sound larmor radius). The plasma pressure profile (Fig. 1d) is relatively flat within the core if compared with the external pressure profile but still can be monotonic.

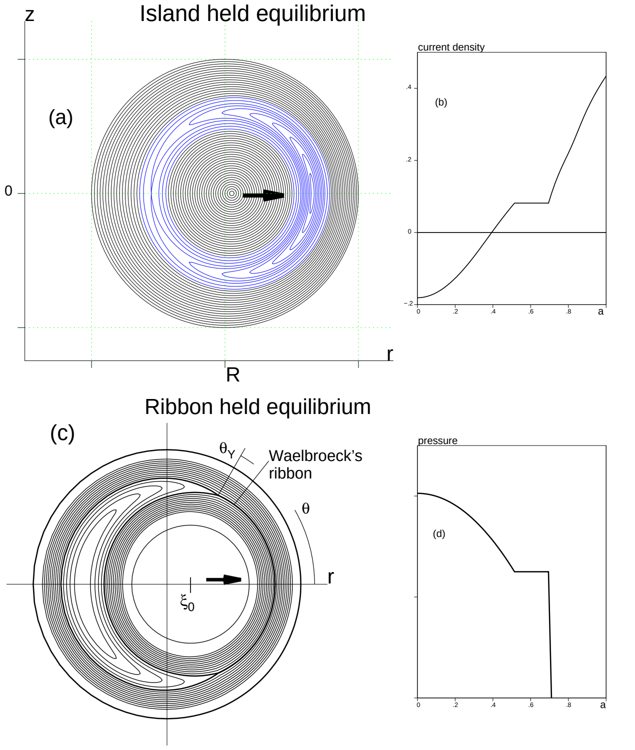

Fig.1. (a) "Island", and (b) "Ribbon" held equilibria. (c) Current density and (d) pressure profiles for IHEq. Arrows show the direction of the ballooning force acting on the core.

Now, we consider the $m = 1$ $n = 1$ perturbations in tokamaks. There is a well-known analogy between the toroidal $n = 0$ case and the helical $n = 1$ equilibria, when the central stability factor $q(0) = 1/\mu(0) < 1$ and $\mu(a_i) = 1$. All the previous theory remains the same if B_θ is replaced by $B_\theta^* \equiv (q-1)B_\theta$, $j(a)$ by $j^* \equiv 2B_{tor}/R - j(a)$ and μ by $1-\mu$.

For the $m = 1, n = 1$ equilibrium in tokamaks, expression (13) can be reduced near the island to

$$\xi' = -\frac{\mu_i'^2\lambda_H(a)\xi_0}{(\mu-1)^2} \simeq -\frac{\lambda_H(a)\xi_0}{x^2}, \qquad (15)$$

$$\lambda_H \equiv \frac{ax^2}{R^2(\mu-1)^2}\left[\frac{c}{1-c}{\beta_j}^2 - \frac{13}{16}\left(l_i - \frac{1}{2}\right)\right] \qquad (16)$$

using the Bussac internal mode theory, where ξ_0 is the displacement of the core center. The parameter c describes the coupling with the $m = 2$ mode as explained in Ref. [7]. The width of the island is given by

$$w_{1,1}^2 = 4\lambda_H(a_i)\xi_0. \qquad (17)$$

The presence of ξ_0 in the island width makes a fundamental distinction between the toroidal $n/m = 0/1$ case and helical $n/m = 1/1$ cases. In the toroidal case ξ' is given by the right hand side of Eq. (2), while in the helical case it is an operator, proportional to ξ_0. In fact,

ξ_0 can be calculated analytically from Eq. (15) using representation $\mu_{c,e} - 1 \simeq \mu_i'(1+\alpha_{c,e}x)$

$$\alpha_{c,e} \equiv \frac{\mu''_{c,e}}{2\mu'} = -\frac{j'R}{q'B_{tor}} - \frac{3}{2}, \qquad (18)$$

which gives for displacement

$$\frac{\xi_e}{\xi_0} = \lambda_H \left(\frac{1}{x} - 2\alpha_e \ln \left| \frac{1+\alpha_e x}{\alpha_e x} \right| + \frac{\alpha_e}{1+\alpha_e x} \right), \qquad (19)$$

$$\frac{\xi_c}{\xi_0} = 1$$
$$-\lambda_H \left(-\frac{1}{x} + 2\alpha_c \ln \left| \frac{1+\alpha_c x}{\alpha_c x} \right| - \frac{\alpha_c}{1+\alpha_c x} \right). \qquad (20)$$

Matching them through the island

$$\xi_e(w) = -\xi_c(-w) \qquad (21)$$

leads to an expression determining the width of the island in the form

$$-\frac{\alpha_c \ln(\alpha_c w) + \alpha_e \ln(\alpha_e w)}{\alpha_c + \alpha_e} = \frac{1}{2} + \frac{1}{2\lambda_H(\alpha_c+\alpha_e)} =$$
$$\frac{1}{2} + \frac{R^2 q'^2}{2(\alpha_c+\alpha_e)a_i \left[\frac{c}{1-c}\beta_j{}^2 - \frac{13}{16}\left(l_i - \frac{1}{2}\right)\right]}. \qquad (22)$$

(We substituted μ' at the $q = 1$ surface by the more conventional in practice q').

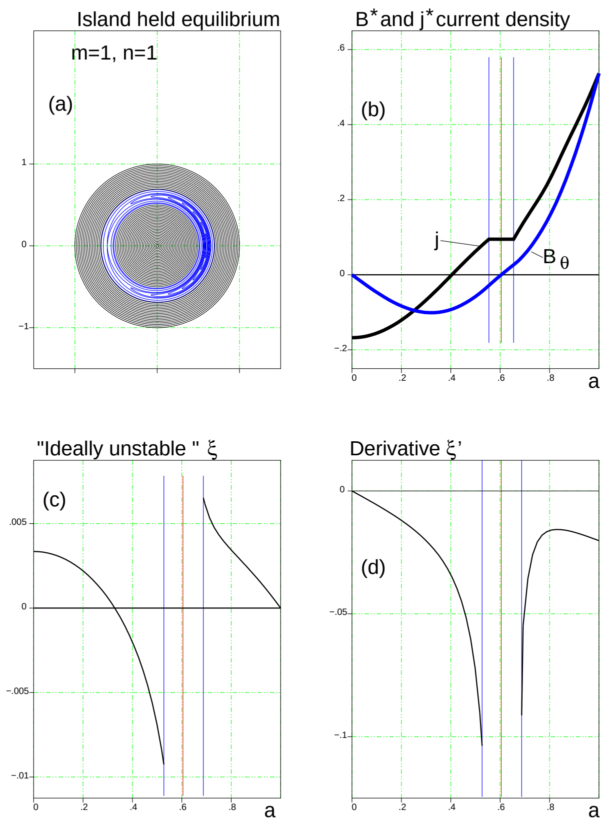

Fig. 2. (a) IHEq for an "ideally unstable" profiles, (b) Kadomtsev's current density j^* and B^* profiles, (c) $m = 1$ perturbation ξ, and (d) its radial derivative ξ'.

For typical situation for relaxations $\alpha_e a_i \gg 1 \gg \alpha_c a_i$

$$\sqrt{e}\alpha_e w = e^{-\frac{1}{2\alpha_e \lambda_H}}. \qquad (23)$$

Two situation are possible with the $m/n = 1/1$ mode: (a) $\lambda_H > 0$, $\xi_0 > 0$, when the plasma is "ideally unstable", and (b) $\lambda_H < 0$, $\xi_0 < 0$, when the plasma is "ideally stable". In both cases, the quasi-stationary equilibria can exist if Eq.(22) allows for a solution for w.

Fig. 2 shows the numerical solution for $\lambda_H > 0$ when the plasma displacement being positive in the center intersects the axis. The island serves as a barrier between the central core and external region, thus, keeping this otherwise unstable configuration from collapse.

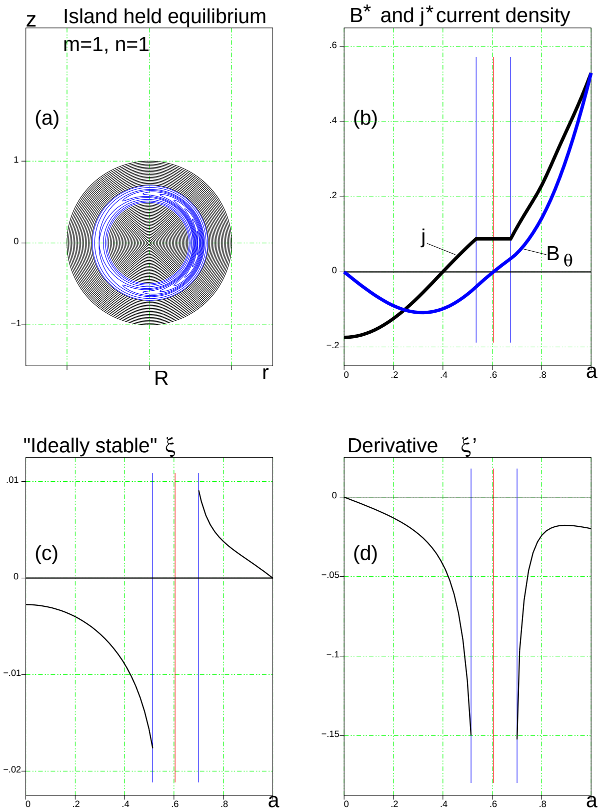

Fig. 3. (a) IHEq for an "idealy stable" profiles, (b) j^* and B^*, (c) ξ , and (d) ξ'.

Fig. 3 shows the island held equilibrium in the case $\lambda_H < 0$, when the plasma is "ideally stable". In this case $\xi(0) < 0$, which in the "stable" situation creates a force opposite to displacement. This force is balanced by the island.

The "marginally stable" situation with $\alpha_e\lambda_H \simeq 0$ is inconsistent with the presented equilibrium solution.

Summarizing, our solution for the island held equilibria for both the negative toroidal current and the stationary $m/n = 1/1$ mode in tokamaks seems to resolve a fundamental theoretical problem related to the existence of a quasi-stationary state in two situations when only fast evolving configurations were known. In this regard, it allows for the negative core current density in

JET current drive experiments as well as explaining for the first time the stationary phase between internal collapses (sawtooth oscillations and internal disruptions) in tokamaks.

Moreover, the equation (20) shows that in the "ideally unstable" situation, typical for high performance machines, the achievable shear q' at the $q = 1$ surface is proportional to β_j, while not preventing the following collapse. This explains, in particular, a high level of $1-q(0)$ observed in TFTR (with peaked temperature and density profiles, and $\beta_j \simeq 1 - 1.5$) compared to the other machines. This fact and explanation given here may have important implications for plasma stability control in the large tokamak experiments.

Although we leave for separate studies the further development and comparison with experiments of the present theory, it is possible to emphasize the new elements of this approach. In tokamak plasmas there are several time scales relevant to internal relaxations. The slowest is the magnetic diffusion time τ_σ, responsible for evolution of the magnetic configuration, e.g., q', then, there is the energy confinement time τ_E (responsible for evolution of β_j), the Rutherford regime time scale $\tau_\sigma \lambda_H / a_i$, the fast collisionless reconnection time $\tau_{rec} \simeq c_A / |R\mu' \rho_s|$ and the fastest ideal MHD time $\tau_{MHD} \simeq c_A / |R\mu' a_i|$ (essentially irrelevant).

So far, the stationary phase between relaxations was interpreted either as a "stabilized" ideal or reconnection mode or, at best, as a transitional Rutherford regime. The present theory has found a state, which exists at the slowest time scale τ_σ and is affected by the plasma heating, having τ_E as the time scale. It is essentially separated from the MHD stability effects and their times scales. At the same time, if the island disappears during the evolution, the system undergoes a collapse. In the case of an "ideally stable" plasma it should go into a transition phase, related to the Rutherford regime, and then to the Kadomtsev (typically "resistive") reconnection. In the high performance plasma, there is not such a transitional phase and the system should go into a fast (collisionless) reconnection.

It is rather obvious, that the equilibria obtained by this theory are stable (for perturbations with the same helicity) in the ideal MHD approximation due to magnetic flux conservation inside the magnetic island.

The present theory also gives some insight of the entire process. Right after each collapse, the system is in a marginal state with $q \simeq 1$ everywhere in the core. While being pushed to the $q(0) < 1$ situation by the temperature profile, the system is testing numerous equilibrium states, until our IHEq will emerge from the noise. After this it goes into a quasi-stationary evolution until the next relaxation occurs, and the process repeats itself.

On the resistive and heating time scales different evolutions are possible, We leave the appropriate analysis and generalization of the present theory on the evolving situation for future work.

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