Equilibrium reconstruction in eddy current environment in tokamaks[1]

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Abstract

In middle and small size tokamaks the eddy currents in the passive structures affect both the interpretation of magnetic signals and the plasma equilibrium. Complicated and essentially unpredictable current paths of eddies add a considerable problem to equilibrium reconstruction, one of the widely used tools for tokamak diagnostics.

The presented theory introduces the response functions of magnetic signals as a necessary element of magnetic reconstruction. The appropriate system of equilibrium reconstruction equations, which uses the time history of the discharge and signals, mitigates uncertainties associated with the eddy currents and allows for a proper weighting of equations. The theory gives a guidance for calibrating the magnetic diagnostics in tokamaks for purposes of equilibrium reconstruction as well as a method of recovering the response functions from measurements.

Contents

Equilibrium reconstruction ≃ GSh equation+magnetic diagnostics

The problem is to find a RHS of GSh equation

$$\begin{aligned} \Delta^*\bar{\Psi} &\equiv \frac{\partial^2\bar{\Psi}}{\partial r^2} - \frac{1}{r}\frac{\partial\bar{\Psi}}{\partial r} + \frac{\partial^2\bar{\Psi}}{\partial z^2} = -J(r,\bar{\Psi}), \\ J(r,\bar{\Psi}) &\equiv T(\bar{\Psi}) + r^2 P(\bar{\Psi}), \quad T \equiv \bar{F}\frac{d\bar{F}}{d\bar{\Psi}}, \quad P \equiv \frac{d\bar{p}}{d\bar{\Psi}}r^2, \end{aligned} \tag{1.1}$$

which leads to a solution consistent with magnetic measurements. A vector $\vec{j}^{pl}$ of adjustable parameters is introduced by the expansion

$$\begin{aligned} J(r,\bar{\Psi}) &\equiv J(r,\bar{\Psi}) + \sum_{k=0}^{N_T} J_k^T j^k(\bar{\Psi}) + r^2 \sum_{k=0}^{N_P} J_k^P j^k(\bar{\Psi}), \\ \vec{j}^{pl} &\equiv \{\vec{J}^T, \vec{J}^P\}. \end{aligned} \tag{1.2}$$

At every iteration, equations for the vector of signals $\vec{S}$ are linear

$$\vec{S} = \mathsf{A}^{pl}(\Psi)\vec{j}^{pl} + \mathsf{A}^{PFC}\vec{I}^{PFC} + \mathsf{A}^{eddy}(\vec{I}^{eddy}_{pl} + \vec{I}^{eddy}_{PFC}). \tag{1.3}$$

Conventional schemes consider model, independent on time matrixes A

Eddy currents affect both magnetic signals and plasma equilibrium,
$\boldsymbol{\Psi = \Psi_{pl} + \Psi_{PFC} + \Psi_{eddy}}$

The effect of eddies on interpretation of signals is the dominant one because all the sensors are mounted on conducted surfaces.

There are 3 issues associated with the eddy currents:

1. *Elimination of contribution of eddy currents generated by the PFCoils into signals.*
2. *Elimination of contribution of eddy currents generated by the plasma.*
3. *The effect of eddies on equilibrium*

All issues are resolvable and require a special calibration of tokamaks

Tokamaks is $\simeq$ a linear system with time independent magnetic properties.

In the absence of plasma the signal $S_i^{PFC,k}$ from each poloidal field coil (PFC) with the index k can be written as

$$S_i^{PFC,k}(t) = \int_0^t s_i^k(t-\tau)\frac{dI_k^{PFC}(\tau)}{d\tau}d\tau, \tag{2.1}$$

where it is assumed that $I_k^{PFC}(t)|_{t<0} = 0$.

Here, $s_i^k(t)$ is the response function, i.e., the time dependence of the signal S_i^k generated by a normalized step function current

$$I_k^{PFC}(t) = H(t) \equiv \begin{cases} 0 & t<0 \\ 1 & t>0 \end{cases}, \quad s_i^k(t) = \int_0^t s_i^k(t-\tau)\frac{dH(\tau)}{d\tau}d\tau, \tag{2.2}$$

where $H(t)$ is the Heaviside function.

Heaviside step function is impossible to generate on the real machine

A special numerical code (`c2e`) was written for reconstruction of response functions

Calibration of s_i^k is determined by the numerical code uniquely

Asymptotic values of response functions determine the direct contribution of PFCoils into signal

$$S_i^{PFC,k} = \bar{s}_i^k I_k^{PGC}, \quad \bar{s}_i^k \equiv s_i^k(t)|_{t\to\infty}. \tag{2.3}$$

Given $s_i^k(t)$ the eddy current contribution is determined by

$$S_{PFC,i}^{eddy,k}(t) = \int_0^t [s_i^k(t-\tau) - \bar{s}_i^k] \frac{dI_k^{PFC}(\tau)}{d\tau} d\tau, \tag{2.4}$$

Comparison of the uncalibrated measured $\bar{s}_i^k$ and the numerically calculated signal $\bar{s}^k_{Comp,i}$ from the same current gives the calibration factor α_i of the sensors

$$\bar{s}^k_{Comp,i} = \alpha_i \bar{s}_i^k. \tag{2.5}$$

Inconsistencies in diagnostics can be revealed by this technique: $\alpha_i^k = \alpha_i$

Eddy currents in a conductor are determined by the time history of $B_{\perp}^{ext}$

In tokamaks, the most essential conducted surfaces are quasi-toroidal and the eddy currents are determined by external poloidal flux $\Psi^{ext}(l,t)$.

If the magnetic flux of the plasma can be represented in terms of fluxes from the coils

$$\Psi^{pl}(l,t) = \sum_k^{N_{PFC}} M_k(t)\psi^{PFC,k}(l), \quad \Psi^{pl,j}(l,t) = \sum_k^{N_{PFC}} M_k^j(t)\psi^{PFC,k}(l), \tag{3.1}$$

where $\psi^{PFC,k}(l)$ flux distribution from a coil with a unit current, then the contribution of the plasma generated eddies can be calculated with the same set of response functions

$$S_{pl,i}^{eddy} = \sum_k^{N_{PFC}} \int_0^t [s_i^k(t-\tau) - \bar{s}_i^k] \frac{dM_k(\tau)}{d\tau} d\tau, \tag{3.2}$$

The accuracy of representation (3.1) can be analyzed numerically

Time history should be included into equilibrium reconstruction

Using response functions the Equilibrium Reconstruction Equations should be written as

$$\vec{S} = \mathsf{S}^{pl}\vec{j}^{pl} + \sum_{k}^{N_{PFC}} \int_0^t [\mathsf{S}(t-\tau) - \overline{\mathsf{S}}] \frac{d\mathsf{M}(\tau)\vec{j}^{pl}}{d\tau} d\tau + \int_0^t \mathsf{S}(t-\tau)\frac{d\vec{S}_I^{PFC}(\tau)}{d\tau} d\tau, \tag{4.1}$$

where matrix $\mathsf{M}(t) = \mathsf{M}(\Psi, t)$ *is calculated at every iteration.*

$\vec{S}_I^{PFC}$ *here are the measured* $\vec{I}^{PFC}$ *by the method used for obtaining response functions*

ERE does not contain eddy currents. Its accuracy is based on accuracy of measurements

$$\vec{S} = \mathsf{A}^{pl}\vec{j}^{pl} + \mathsf{A}^{PFC}\vec{I}^{PFC} + \mathsf{A}^{eddy}(\vec{I}_{pl}^{eddy} + \vec{I}_{PFC}^{eddy}). \tag{4.2}$$

Potentially ERE can be used for revealing deviations from the GSh equations

Effect of eddies on equilibrium can be determined separately, using any plausible model

Flux loop signals in a conventional form

$$\vec{\Psi} = \mathsf{A}^{pl}\vec{j}^{pl} + \mathsf{A}^{PFC}\vec{I}^{PFC} + \mathsf{A}^{eddy}\vec{I}^{eddy} \tag{4.3}$$

allows to determine the eddy currents, because everything else is already determined.

New EREs properly separate signals from eddy current effects on equilibrium

Weights of all conditions can be justifies either by measurements or by straightforward calculations.

Time history of magnetic signals gives an opportunity for better reconstruction

Two effects of eddy currents, i.e., on the signals and on equilibrium, can be separated rigorously.

1. *Eddy current contribution can be eliminated from the signals despite most probes a situated typically in their proximity.*
2. *Weights of the signals in the set of reconstruction equations can be justifiably assigned based on accuracy of measurements.*
3. *Effect of eddy currents on equilibrium can be localized to the extend that their simplified (e.g., axisymmetric) model could be appropriate.*

The theory gives an instructional guide for magnetic calibration of tokamaks

It is the turn for machines to prepare the necessary data