

# Where is the edge in toroidal plasmas ?<sup>1</sup>

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# Abstract

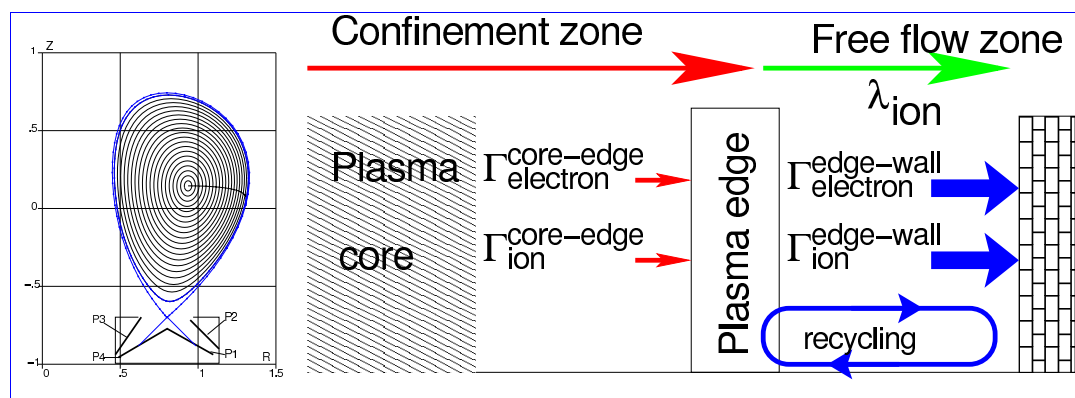
*It is believed that by definition the plasma edge is the separatrix which separates the confinement zone from the convection dominated plasma periphery. Another belief is that the H-mode has a miraculous "edge transport barrier" providing a steep temperature pedestal in front of the last closed magnetic surface.*

*DIII-D experiments with Resonant Magnetic Perturbations undermine both of these religious beliefs. In addition, the recent discovery of unconditional instability of the so-called Wall Touching Kink Modes (and their edge version Takahashi Kink Modes) gives a basis for an emerging self-consistent understanding of the plasma edge and its MHD activity.*

*The current sharing effect between the plasma surface and the wall, which was missed by MHD theory for more that 50 years, has been revealed as a key factor in destabilizing the plasma edge.*

## 1 What is the plasma edge

**Plasma edge separates the confinement zone from "free" flow**



*The plasma edge (probably two of them) is located approximately at one mean free path*

$$\lambda_{D,m} = 121 \frac{T_{keV}^2}{n_{20}} \quad (1.1)$$

*from the plasma facing surface.*

**Separatrix by definition seems to be the plasma edge**

# The “edge transport barrier”

This notion was introduced at the time of the discovery of the H-mode regime on Asdex in the early 80s

*The “transport barrier” provides a steep temperature pedestal in front of the last closed magnetic surface.*

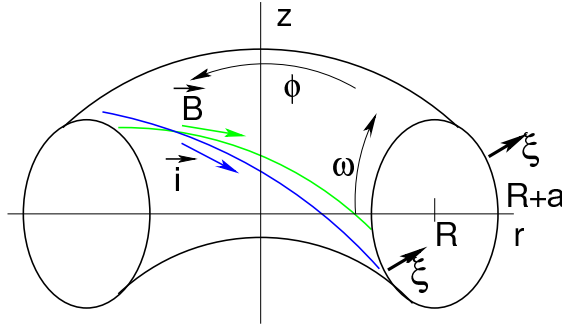
**Apparently obvious, the concept of the “edge transport barrier” contains many hidden inconsistencies**

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## 2 The basics of the kink mode 1/1

Deformation of the plasma surface generates the surface current



$$\begin{aligned} r &= R - \rho \cos \omega, \\ z &= \rho \sin \omega, \\ \rho &= a + \xi(\omega, \varphi), \\ q &= \frac{a B_\varphi}{R B_\omega}, \end{aligned} \quad (2.1)$$

Its value is determined by the condition

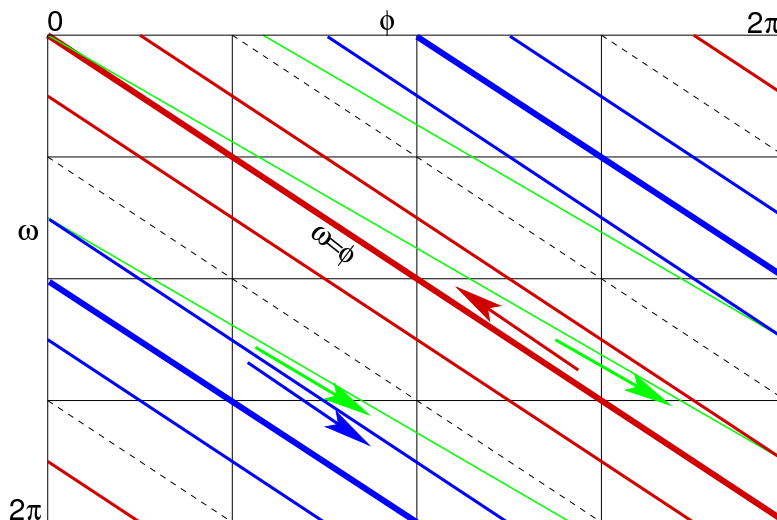
$$\vec{B} \cdot \nabla \rho = 0. \quad (2.2)$$

If the plasma core is deformed in accordance with equilibrium conditions the only force acting on the plasma is the electromagnetic pressure

$$p_{j \times B} \nabla \rho = \vec{i} \times \vec{B}. \quad (2.3)$$

For  $q > 1$  pressure  $p_{j \times B}$  suppresses the perturbation

## Surface currents are periodic



Green is for  $\vec{B}$ -lines.

Direction of the surface current:

Blue lines are for current along  $B_\varphi$

Red is for currents opposite to  $B_\varphi$

Surface currents will be related to experimental measurements of  $I_{pl}(\varphi)$  and the toroidal peaking factor (TPF)

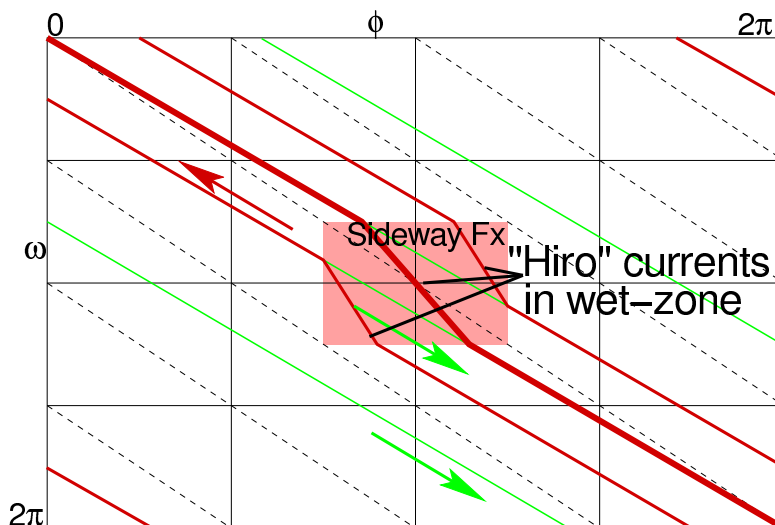
$$\mu_0 \vec{i} = -\xi_{11} \frac{2B_\varphi}{R} \cos(\omega - \varphi) \mathbf{e}_\varphi - \frac{1}{R} \xi_{11} \frac{2aB_\varphi}{R^2} \sin(\omega - \varphi) \mathbf{e}_\omega. \quad (2.4)$$

If  $q > 1$  the kink mode 1/1 is stable (Shafranov, 1952)

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## 3 Wall Touching Kink Mode (WTKM)



The presence of the wet-zone makes the stable plasma unstable.

Electromagnetic pressure is applied to the in-vessel conductors

$$\vec{i}(\omega, \varphi) = -\frac{1}{a} I'_{\omega} \vec{e}_{\varphi} + \frac{1}{R} I'_{\varphi} \vec{e}_{\omega}, \quad (3.1)$$

**“Hiro” currents are generated by the plasma perturbation.**

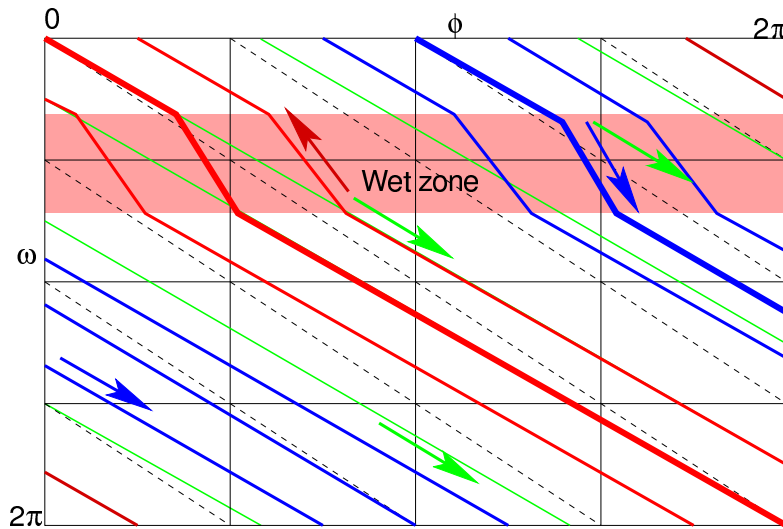
**They are not the halo-currents**

The sideways force can be generated

$$F_x = \pi I_{pl} B_{\varphi} (1 - q) \xi_{11}, \quad (3.2)$$

# Linearly unstable WTKM

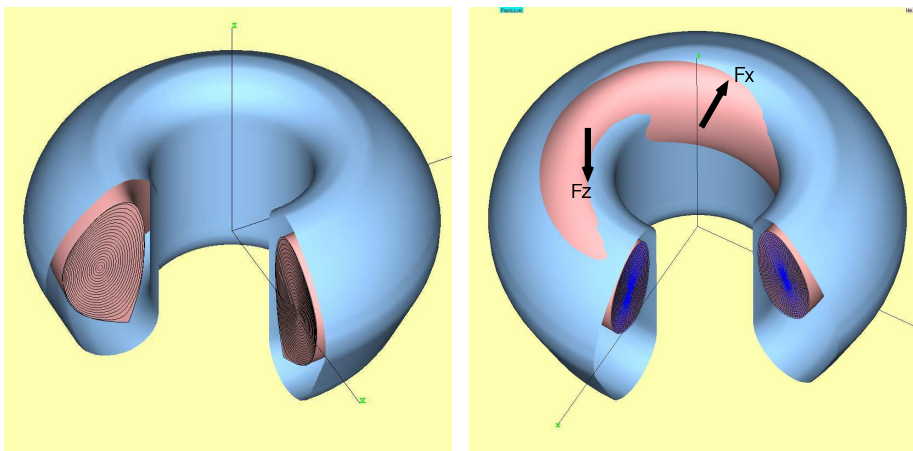
Always unstable as soon as the plasma touches the wall, WTKM becomes unstable during the vertical disruption events (VDE)



At the nonlinear stage the mode leads to a disbalance in plasma current measurements at different azimuths  $\varphi$  (TPF)

## Forces to the vessel

The force is applied to the wet-spot and is directed toward the plasma



Both thermal and current quench make the vertical equilibrium field

$$B_{z,ext} = -\frac{0.1I_{pl}}{R} \left( \ln \frac{8R}{a} + \beta_j + \frac{l_i}{2} - \frac{3}{2} \right), \quad \beta_j \rightarrow 0 \quad (3.3)$$

becomes excessive and pushes plasma inward, leading to the sideways force.

The trajectory of the plasma during VDEs depends on equilibrium

# Theory of WTKM (uniform current)

For arbitrary  $m, n$  and perturbation of the form

$$\rho = a + \Re \left( \sum_{m,n} \xi_{mn} e^{im\omega - in\varphi} \right), \quad (3.4)$$

the surface currents are determined by the perturbed equilibrium theory (Zakharov, Sov.J. Plasma Physics, (1981), Reviews of Plasma Phys. v.11)

$$\begin{aligned} \vec{i} &= \nabla I(\omega, \varphi) \times \vec{e}_n = -\frac{1}{a} I'_\omega \vec{e}_\varphi + \frac{1}{R} I'_\varphi \vec{e}_\omega, \\ i(\omega, \varphi) &\equiv \Re \left( \sum_{m,n} i_{nm,\varphi} e^{im\omega - in\varphi} \right), \\ \mu_0 i_{nm,\varphi} &= \xi_{nm} \left[ (m-1)j - \frac{2nB_\varphi}{R} \right] e^{im\omega - in\varphi} \end{aligned} \quad (3.5)$$

The electromagnetic pressure is given (for a uniform current density) by

$$p = 2I_{pl}B_\omega \Re \left( \sum_{nm} \xi_{nm} \frac{nq - m + 1}{m} (m - nq) e^{im\omega - in\varphi} \right), \quad (3.6)$$

**Together with electro-dynamics of the vacuum vessel they are sufficient for simulation of disruptions**

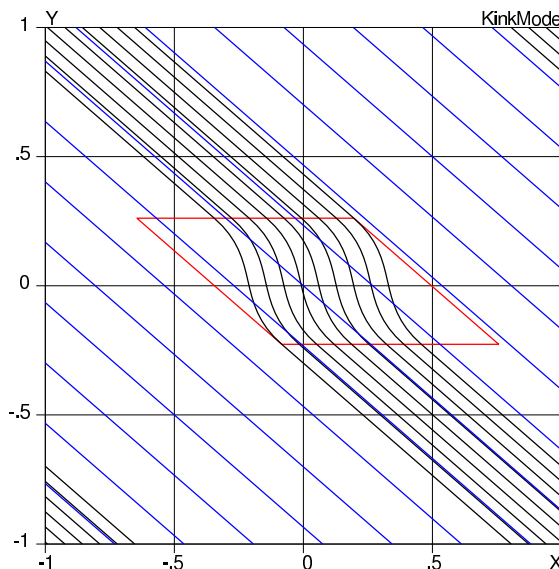


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## A model example

**Eq.(3.6) converts surface current into plasma displacement**



$$\begin{aligned} \vec{i} &= \nabla I(\omega, \varphi) \times \vec{e}_n, \\ I(\omega, \varphi) &= I(x), \\ x &= x(\omega, \varphi), \\ \vec{i} &= -\frac{I'}{a} x'_\omega \vec{e}_\varphi + \frac{I'}{R} x'_\varphi \vec{e}_\omega \end{aligned} \quad (3.7)$$

The figure shows the current flow lines  $x = \text{const}$  of the halo currents

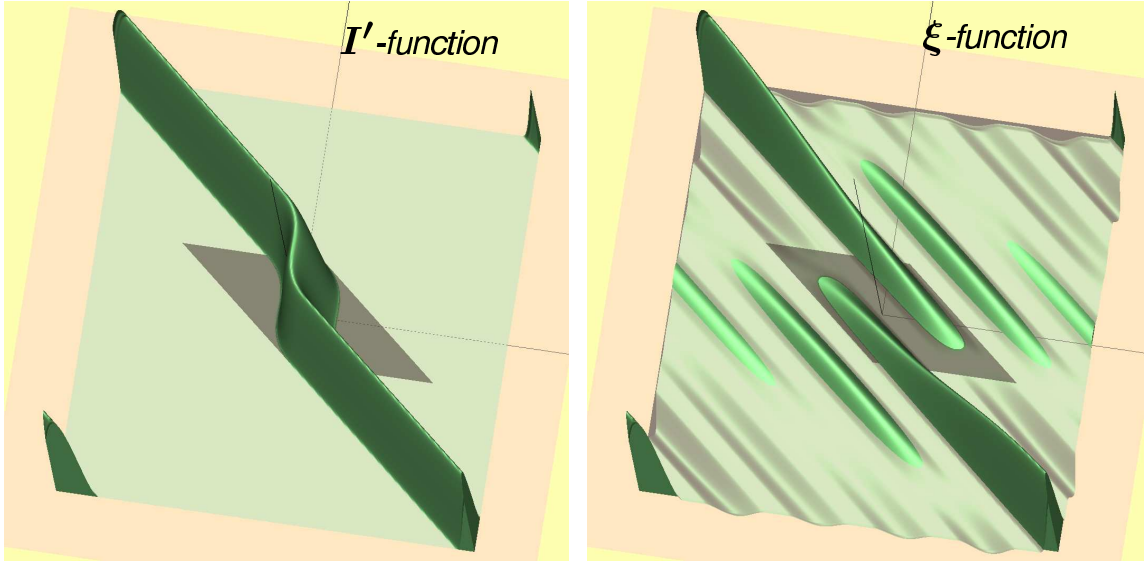


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# Surface current and displacement

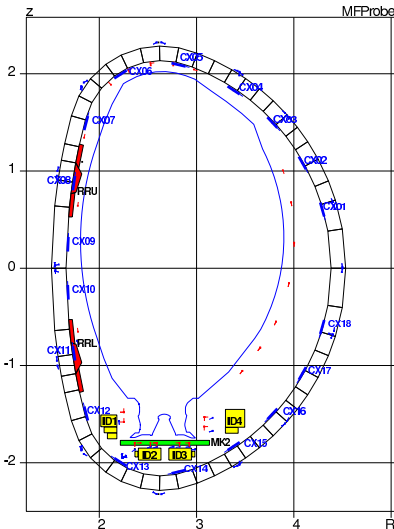
Eq.(3.6) converts the surface current into plasma displacement



Even this hand-prescribed marginally stable plasma perturbation reveals that

**The shape of the surface displacement is consistent with the “Hiro” current supply into the wet-zone**

## Comparison with JET data



ESC numerical model of JET vacuum vessel.

$$I_{3,7} = \sum_{i=1}^{i \leq 18} B_i^{3,7} L^i - \sum_{k=1}^{k \leq 4} I_{Dk} T^k - I_{MK2} - I_{RRU} - I_{RRL}, \quad (3.8)$$

$$I_{RRU} = \frac{V_{RRU}}{Res_{RRU}}, \quad I_{RRL} = \frac{V_{RRL}}{Res_{RRL}}.$$

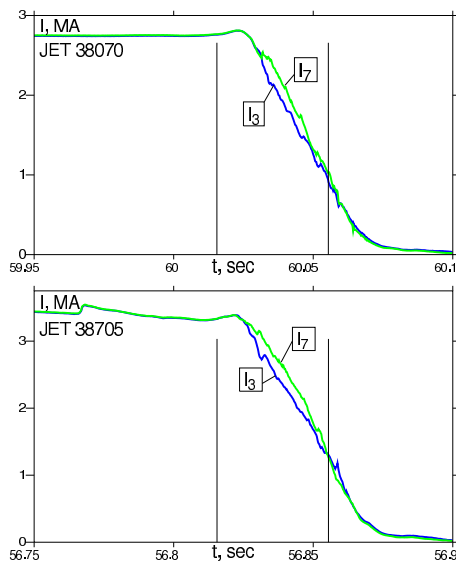
Here,  $B_i^{3,7}$  are the probe signals, and  $V_{RRU}, V_{RRL}$  are voltages along the restraining rings, and  $Res_{RRU}, Res_{RRL}$  are their resistances.

$$Z_{3,7} = \frac{1}{I_{3,7}} \sum_{i=1}^{i \leq 18} \left( B_{3,7}^i - \sum_{k=1}^{k \leq 4} B_{Dk}^i I_D^k - B_{RRU}^i I_{RRU} - B_{RRL}^i I_{RRL} \right) L^i z^i, \quad (3.9)$$

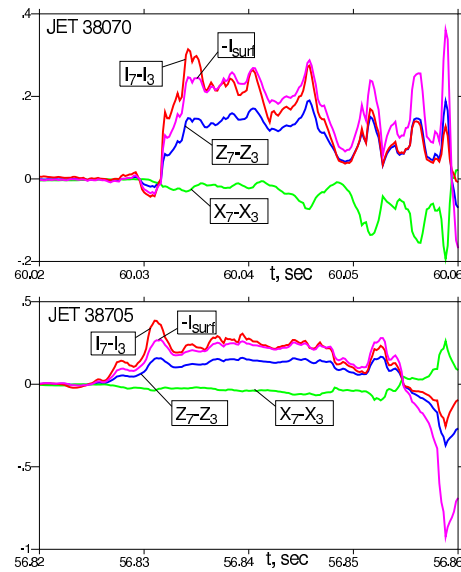
$$R_{3,7} - R_0 = \frac{1}{I_{3,7}} \sum_{i=1}^{i \leq 18} \left( B_{3,7}^i - \sum_{k=1}^{k \leq 4} B_{Dk}^i I_D^k - B_{RRU}^i I_{RRU} - B_{RRL}^i I_{RRL} \right) L^i \frac{R^i - R_0^2}{2R_0}.$$



# WTKM gives right asymmetry in $I_{pl}$



(a)



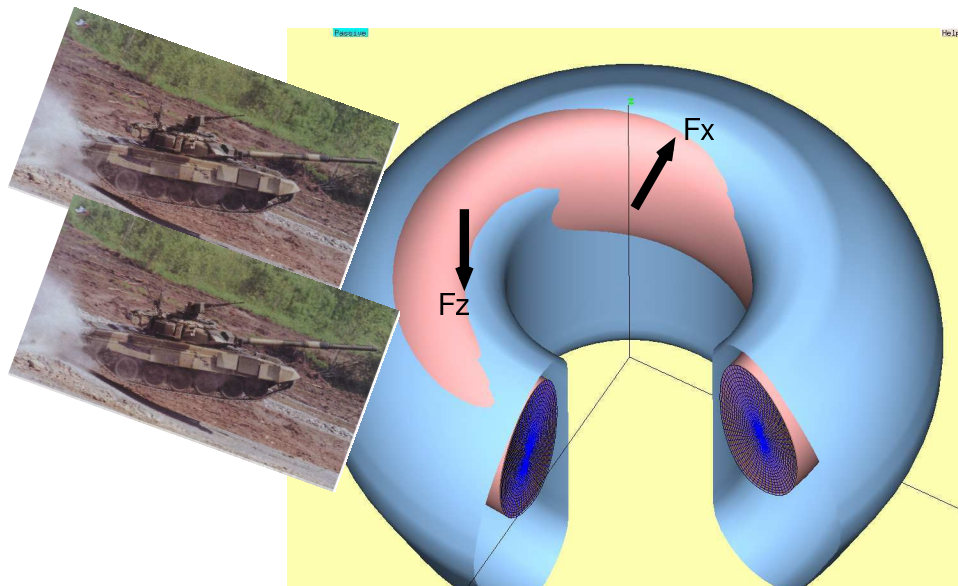
(b)

(a) Plasma currents  $I_{3,7}$  (3.8) in octants 3,7 on JET during the disruptions. (b)  $Z_7 - Z_3$  and  $R_7 - R_3$  (3.9),  $I_7 - I_3$  and its prediction  $-I_{surf}$  from the present theory.

$$I_{surf} \equiv 2ai_\varphi = -a\xi_{11}\frac{4B_\varphi}{R_0\mu_0}, \quad \xi_{11} \simeq \frac{Z_7 - Z_3}{2}, \quad a \simeq Z_{top} - Z_7. \quad (3.10)$$

## Sideway force is a concern for ITER

The sideway force impulse  $\simeq 2MN \cdot sec$  in ITER is equivalent to the impulse of two 50 tonne tanks T-90S at the speed of 72 km/hour.



**This example illustrates the significance of the effect missed in MHD for 57 years**

It is not like the second order correction to the guiding center equations "leading" to the paleoclassical transport

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## 4 Takahashi Kink Modes (TKM)

**TKMs are the Wall Touching Kink Modes, associated with the Scrape Off Layer Currents in the quasi-stationary plasmas**

*The current sharing effect can be easily included into MHD formalism (for both WTKM and TKM)*

$$W_{WTKM} \equiv \underbrace{W_{MHD}}_{\text{conventional}} + \underbrace{\frac{\lambda}{2} \int_{\text{wet-zone}} \vec{\xi}_n \cdot (\vec{i} \times \vec{B}) dS}_{\text{virtual work against the wet surface}}. \quad (4.1)$$

*If the plasma test perturbation satisfies the equilibrium conditions, the  $W_{TKM}$  is reduced to*

$$W_{WTKM,eq} = -\frac{1}{2} \int_{\text{free plasma surface}}^* \vec{\xi}_n \cdot (\vec{i} \times \vec{B}) dS. \quad (4.2)$$

*In the presence of the wet-zone (with an electric contact between the plasma and the wall) it is always possible to create a test perturbation with*

$$\vec{i} \parallel \vec{B}, \quad W_{WTKM} = 0. \quad (4.3)$$

**In MHD, the existence of a marginally stable test perturbation is equivalent to the ideal instability.**

# Energy principle for WTKM

TKM are WTKM is a universal feature of toroidal plasmas

The formalism of the energy principle is applicable for both tokamaks and stellarators

In any toroidal configuration, the TKM becomes unstable as soon as the plasma touches the conducting surface.

## Scrape Off Layer Currents

Edge plasma is always in electric contact with the plasma facing components. TKM display themselves through the SOLC.

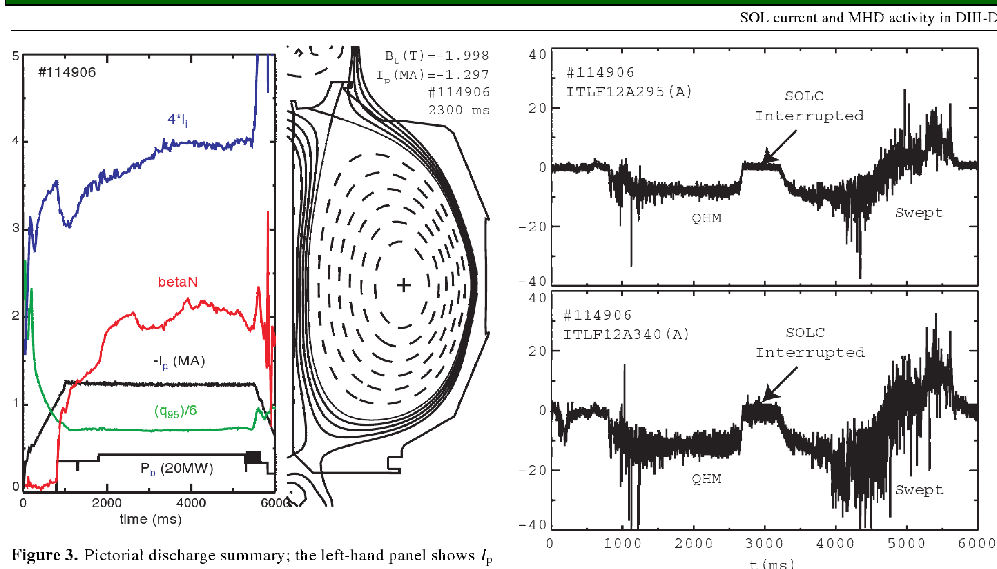


Figure 3. Pictorial discharge summary; the left-hand panel shows  $I_p$  in units of megaamperes,  $P_{tot}$  in units of 20 MW,  $q_{95}$  divided by 6,  $\beta_N$ , and the nominal no-wall limit (here, 4 li). The right-hand panel shows the plasma boundary and four exterior flux surfaces in the

Figure 4. Signals from tile current sensors in tile ring #12A in the discharge shown in the previous figure. It has a period of OHM over

Hiro Takahashi and Eric Fredrickson (NF,2004) have found a link between SOLC and MHD activity on DIII-D

# TKM are always IDEALLY unstable

## The “non-ideal” effects have little effect on TKMs stability

*In his pioneering analysis of the current sharing effect, Richard Fitzpatrick (Phys. Plasmas, 2007) has made an incorrect analogy between SOLC and resistive walls, assuming the Ohm's law for Hiro currents in the form*

$$\vec{i} \equiv \nabla I \times \mathbf{n}, \quad I = \gamma \tau_p \psi. \quad (4.4)$$

*Valid for a solid walls, for a plasma it should be written as*

$$\vec{i} \equiv \nabla I \times \mathbf{n}, \quad I = \gamma \tau_p (\psi + \vec{\xi} \cdot \nabla \Psi). \quad (4.5)$$

*In contrast to the resistive wall eddy currents, which are stabilizing, the Hiro currents are the reason of instability with the growth rate*

$$\gamma \gg \frac{1}{\tau_p} \quad (4.6)$$

*and are insensitive to the Ohm's law.*

**The plasma edge density and recycling, rather than resistivity, are the key factors for TKM behavior and amplitude.**

## TKM and the edge MHD activity

### All edge MHD activity most probably is related to the TKM

#### 1. The LiWF regime, *with negligible Hiro currents.*

*Low recycling, low plasma density, no ELMs, no blobs, perfectly stable plasma edge. Exactly like in DIII-D experiments*

#### 2. Edge harmonic oscillations *at low-collisionality H-mode, QHM, with Hiro currents limited by the ion-saturation current.*

#### 3. The ELMy H-mode, *metastable regime with limited Hiro currents and reduced recycling. Probably the resistive effects leads to relaxations.*

#### 4. Blobs in L-mode, *unstable regime with unlimited Hiro currents and large recycling.*

#### 5. High density disruptions, *Feed back between plasma core and the Hiro currents.*

**The plasma edge density explicitly enters into tokamak MHD stability through the TKM.**

# Extention of Energy Principle for TKM

*The plasma core is the same as in conventional MHD theory*

$$\rho \frac{\partial \mathbf{v}}{\partial t} = \kappa(\mathbf{v}, \vec{\xi}, \vec{\widetilde{B}}, \tilde{p}), \quad \nabla \times \vec{\widetilde{B}} = 0, \quad (4.7)$$

*Across the free plasma boundary,  $B_n$*

$$\mathbf{n} \cdot (\vec{\widetilde{B}}_v - \vec{\widetilde{B}}_{pl}) = 0 \quad (4.8)$$

*and the total pressure*

$$\begin{aligned} \vec{\widetilde{B}} \cdot \vec{\widetilde{B}}_{pl} - \vec{\widetilde{B}} \cdot \vec{\widetilde{B}}_v &= \mu_0 \mathbf{n} \cdot (\mathbf{i} \times \mathbf{B}) \\ &= \gamma \mu_0 p (\nabla \cdot \vec{\xi}) + \xi_n \frac{\partial}{\partial n} \left( \frac{B_v^2 - B_{pl}^2}{2} - \mu_0 p \right) \end{aligned} \quad (4.9)$$

*are continuous. The potential energy acquires an additional surface term*

$$\begin{aligned} W_{WTKM} &\equiv W_{MHD} + \frac{\lambda}{2} \int_{wet-zone} \vec{\xi}_n \cdot (\mathbf{i} \times \vec{\widetilde{B}}) dS \\ &= W_{MHD} - \frac{\lambda}{2\mu_0} \int_{wet-zone} \vec{\widetilde{B}} \cdot (\vec{\widetilde{B}}_e - \vec{\widetilde{B}}_i) \xi_n dS. \end{aligned} \quad (4.10)$$



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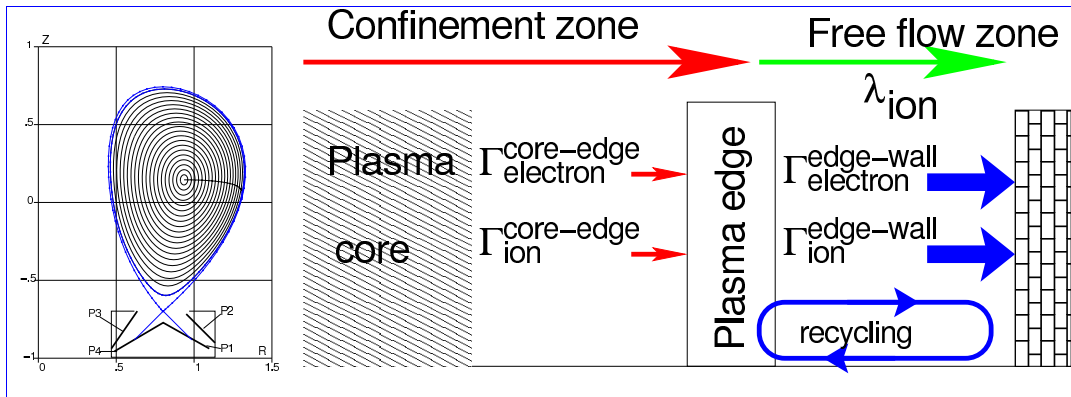
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# 5 What is the plasma edge

Analysis comes from LiWF, which requires recycling  $R \ll 1$ , i.e.

$$\Gamma_{ion}^{core-edge} \simeq \Gamma_{ion}^{edge-wall}, \quad \Gamma_{electron}^{core-edge} \simeq \Gamma_{electron}^{edge-wall} \quad (5.1)$$

*Lithium PFC satisfy, at the very least, the condition of low recycling.*



*The plasma edge, understood as a transition zone from diffusive transport to a convective one, is located approximately at one mean free path*

$$\lambda_{D,m} = 121 \frac{T_{keV}^2}{n_{20}} \quad (5.2)$$



*from the plasma facing surface.*

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## $T_{edge}$ is a boundary condition

The edge plasma temperature is determined self-consistently by the particle fluxes (Krasheninnikov)

*Across the last mean free path,  $\lambda_D$ , in front of PFC surface*

$$\lambda_{D,m} = 121 \frac{T_{keV}^2}{n_{20}} \quad (5.3)$$

*the energy is carried out by the moving particles*

$$\frac{5}{2} \Gamma_{electron}^{edge-wall} T_e^{edge} = \int_V P_e dV, \quad \frac{5}{2} \Gamma_{ion}^{edge-wall} T_i^{edge} = \int_V P_i dV \quad (5.4)$$

The transport plasma properties near the edge do not affect  $T_{edge}$

*In LiWF*

$$\Gamma_{electron,ion}^{edge-wall} \simeq \Gamma_{electron,ion}^{core-edge}, \quad \rightarrow T_{edge} \simeq T_{core}$$

*For edge temperature  $T_{e,i} \simeq 1$  keV (low collisionality H-mode) the mean free path  $\lambda_D$  is very long  $\simeq km$ 's*

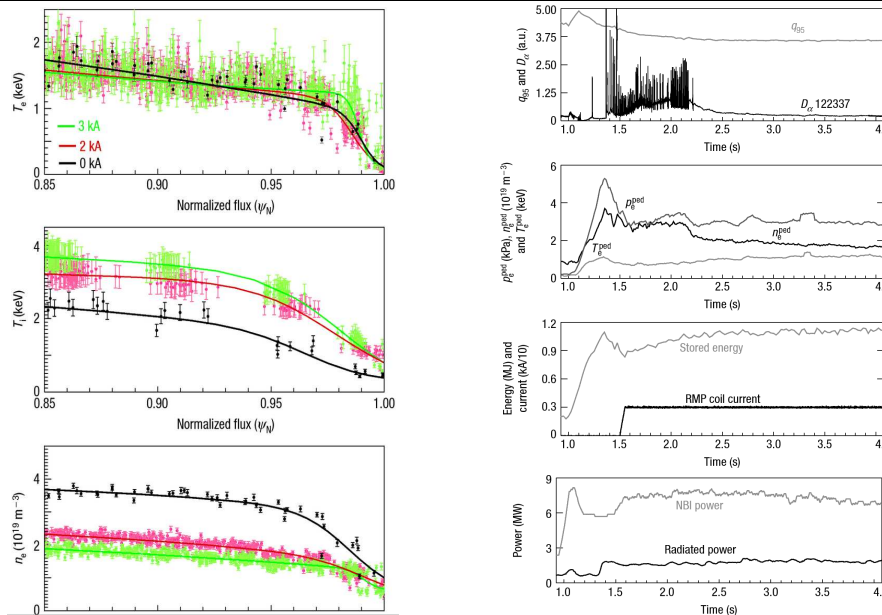


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# DIII-D made crucial input to LiWF

Resonance Magnetic Perturbation experiments have confirmed our, LiWF, views. The pedestal  $T_{edge}$  is not affected by RMP.



0 kA, 2 kA, 3 kA  $I_{RMP-coil}$  T.Evans at al., Nature physics 2, p.419, (2006)

**There is no confinement in the “edge transport barrier” zone**



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## The Tedge width is controlled by TKM

The density profile IS affected by RMP leading to reduction in TKM

The TKM instability exists with and without RMP and maintains the destruction of magnetic surfaces near the separatrix.

Unstable TKM are responsible for the position of the temperature pedestal inside the separatrix.

High density and high recycling, which are the suppliers of the Hiro current carriers are key factors in MHD activity of the edge plasma.

Reduction of density by RMP perturbations mitigates TKM and makes the  $T_e$  pedestal more sharp in DIII-D experiments, contrary to any transport theory predictions.

**Having no TKM in mind, the existing boundary physics is far from the reality.**

**Understanding of TKM is a key to understanding how edge plasma density affects stability (Greenwald, blobs, ELMs, EHOs) (as well as how perfect is the LiWF concept)**



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## 6 Summary.

**The answer to the title question is indeed very simple: the plasma edge is located at the tip of the temperature pedestal. TKMs are responsible for its location.**

*In addition to remarkable DIII-D experiments, in order to get this answer it was required to shake the religious beliefs (a) of transport theory in the “edge transport barrier”, (b) of MHD theory in its ability to explain all MHD effects by manipulations with ellipticity, triangularity, current and pressure profiles, and (c) of existing edge plasma physics into its relevance to the reality.*

- 1. In tokamaks the plasma is always in the electric contact with the conducting in-vessel structures, the fact, ignored by MHD theory for more than 50 years.*
- 2. The present theory of WTKM and TKM introduces the current sharing effect into the formalism of ideal MHD theory. Experimentally, the importance of this effect was recognized by Hiro Takahashi and Eric Fredrickson (Nucl. Fusion, v.44 1075 (2004))*
- 3. The sign of toroidal asymmetry of the plasma current on JET during the disruptions was explained for the first time, thus confirming that the “Hiro” currents generated by the mode, rather than the “halo” are shared with the wall.*

**By introducing the formalism for TKM stability we only initiated their understanding and understanding of the plasma edge**