Intense Lithium Streams in Tokamaks[1]

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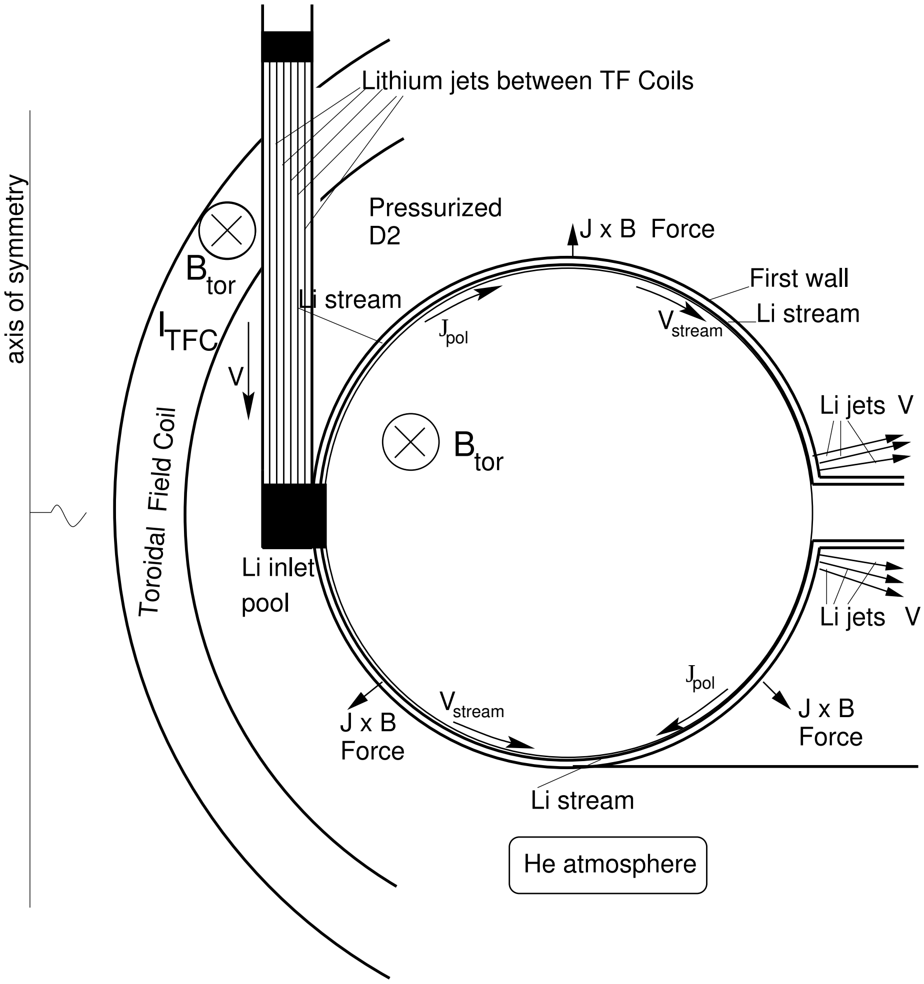
1. Temperature of the streams



Pumps are problematic inside the toroidal field of tokamaks.

$$p_w|_{outlet} > 1\ atm \tag{1.1}$$

Propulsion uses inhomogeneity of toroidal field

$$\rho\frac{\langle V_{Li}^2(s)\rangle}{2} = \rho\frac{\langle V_{Li}^2\rangle_{inlet}}{2} + \frac{I_{pol}}{I_{TFC}}\frac{B_{tor,inlet}^2 - B_{tor}^2(s)}{2\mu_0}, \qquad (1.2)$$

where I_{TFC} is the total current in TFC.

Necessary condition for expulsion of lithium from the vacuum chamber is

$$p_{outlet} > 1\ atm, \quad p_{inlet} = p_{outlet}\frac{r_{outlet}^2}{r_{inlet}^2}. \qquad (1.3)$$

For aspect ratios $A \equiv R/a < 5.8$ this translates into the pressure drop between inlet and outlet points

$$\begin{aligned}\Delta p \equiv p_{inlet} - p_{outlet} &= p_{outlet}\frac{r_{outlet}^2 - r_{inlet}^2}{r_{inlet}^2} \\ &= p_{outlet}\frac{4A}{(A-1)^2}\bigg|_{A<5.8} > p_{outlet} > 1\ atm.\end{aligned} \qquad (1.4)$$

With negligible viscosity, characteristic stream velocity

$$V_{Li} > 19.2\sqrt{\frac{\Delta p}{1\ atm}}\ \frac{m}{sec}. \qquad (1.5)$$

Evaporation imposes a restriction that the lithium surface temperature

$$\Gamma_{Li}^{evap} \simeq 3 \cdot 10^{18} \cdot 10^{\frac{(T-300)^o C}{50}} \frac{1}{m^2 sec} \tag{1.6}$$

cannot be higher than $\simeq 400^o$ C.

The temperature T of the flowing lithium surface is determined by equation of thermo-conduction

$$\begin{aligned} &\rho c_p \frac{\partial T}{\partial t} = \kappa T''_{nn}, \quad t \equiv \int_0^s \frac{ds}{V_{Li}}, \\ &\kappa T'_n|_{n=0} = q_{wall}, \quad \kappa T'_n|_{n=\infty} = T_{inlet}. \end{aligned} \tag{1.7}$$

This equation has a solution

$$\begin{aligned} &T = T_{inlet} + \frac{q_{wall}}{\sqrt{\pi \kappa \rho c_p}} \int_0^t \frac{d\tau}{\sqrt{\tau}} exp\left(-\frac{x^2}{4D\tau}\right), \\ &D \equiv \frac{\kappa}{\rho c_p} \simeq 0.21 \, \frac{cm^2}{sec}. \end{aligned} \tag{1.8}$$

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Surface temperature evolves in accordance with

$$T_{surface} - T_{inlet} = 2q_{wall}\sqrt{\frac{t}{\pi\kappa\rho c_p}}. \tag{1.9}$$

The limitations on the surface temperature translates into a transit time of the lithium as (Moir 97)

$$\begin{aligned} \Delta T_{Li} &= 200^o \frac{q_{wall}}{3.5\ \frac{MW}{m^2}}\sqrt{\frac{t_{transit}}{0.25\ sec}} < 200^o, \\ t_{transit} &\equiv \int_{inlet}^{outlet} \frac{ds}{V_{Li}}. \end{aligned} \tag{1.10}$$

Without violating evaporation limit, lithium with a speed of 20 m/sec in the reactor with the minor radius of 1.6 m can absorb about 3 MW/m^2 of the thermal power, while transmitting 12 MW/m^2 in neutrons through the first wall.

Only a thin surface layer of lithium

$$d_{Li} \simeq \sqrt{\frac{4Dt_{transit}}{\pi}} = 2.6\sqrt{\frac{t_{transit}}{0.25\ sec}}\ mm \tag{1.11}$$

participates in consumption of the thermal energy flux.

Even thin streams

$$h \simeq 0.5 - 1\ cm \tag{1.12}$$

are sufficient for power extraction.

Magnetic propulsion is consistent with requirements on power extraction.

2. Lithium jets

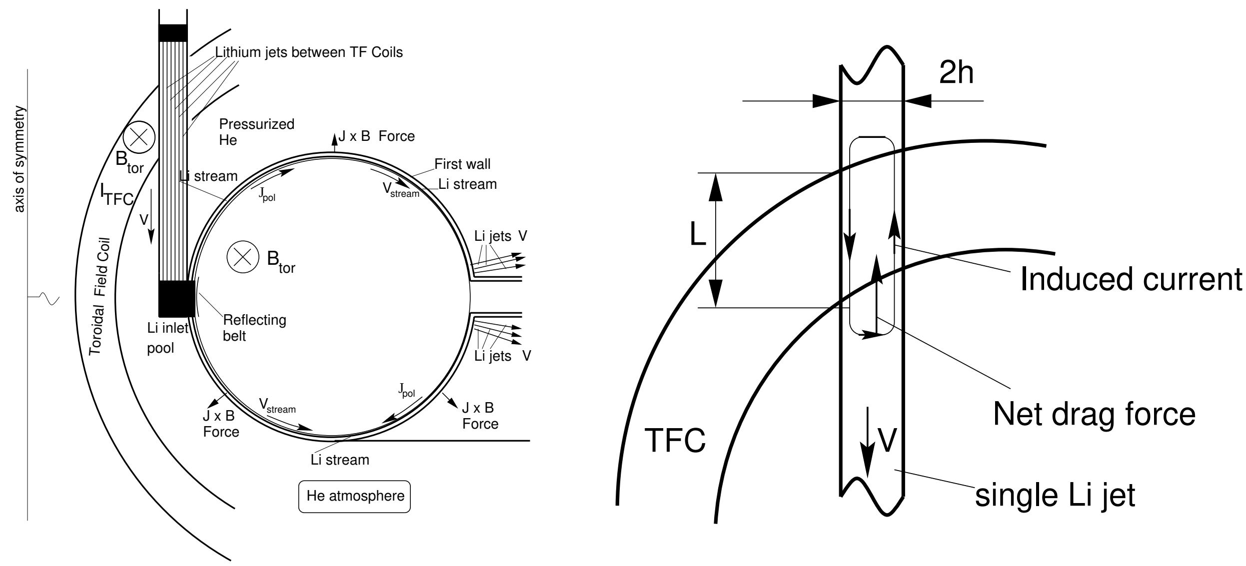
Fast and thin ($h = 2 - 3\ mm$) lithium jets seems to be a practical way of delivering lithium to the inlet of the vacuum chamber.

Induced electric current results in loss in kinetic energy

$$\Delta \frac{\rho V_z^2}{2} = - \langle P_d \rangle \frac{B_0^2}{2\mu_0} \tag{2.1}$$

leads to a fundamental condition

$$\boxed{P_d = \frac{\mu_0 \sigma h^2 V}{L} \ll 1} \tag{2.2}$$



The dynamics of the liquid metal is determined by a standard set of MHD equations

$$\begin{aligned} &\rho\frac{\partial \mathbf{V}}{\partial t} + \rho(\mathbf{V}\cdot\nabla)\mathbf{V} = -\nabla p + (\mathbf{j}\times\mathbf{B}) + \nu\Delta\mathbf{V}, \\ &\frac{\partial \mathbf{A}}{\partial t} - \nabla\varphi_E + (\mathbf{V}\times\mathbf{B}) = \frac{\mathbf{j}}{\sigma}. \end{aligned} \tag{2.3}$$

In local rectangular coordinates x, y, s with s directed along the jet and y oriented along the field, the magnetic field, the velocity and the current density can be written as

$$\mathbf{B} = B(s)\mathbf{e}_y, \quad \mathbf{V} = V\mathbf{e}_s, \quad \mathbf{j} = (\nabla i \times \mathbf{e}_y) = -i'_s\mathbf{e}_x + i'_x\mathbf{e}_s, \tag{2.4}$$

where $i(x, y, s)$ is the stream function of the eddy currents inside the jet.

Similar representation is valid for a thin metal film, coplanar to the magnetic field.

The equation for i follows from the Ohm law

$$-\,(\nabla\times VB\mathbf{e}_x)=\frac{1}{\sigma}((\nabla\times(\nabla i\times\mathbf{e}_y)),\quad i''_{xx}=\sigma(VB)'_s. \tag{2.5}$$

Its solution for both thin jets and films is

$$\boxed{\begin{aligned} i_{jet} &= -\sigma\frac{h^2-y^2-x^2}{2}(VB)'_s,\\ i_{film} &= -\sigma\frac{h^2-x^2}{2}(VB)'_s, \end{aligned} \qquad (2.6)}$$

where h is the radius of the jet (or half the thickness of the film).

The drag force

$$\begin{aligned} \frac{\rho}{2}(V_s^2)'_s &= ((\mathbf{j}\times\mathbf{B})\cdot\mathbf{e}_s)=j_xB=-i'_sB\\ &=\sigma\frac{h^2-y^2-x^2}{2}[(VB)'_s]'_sB, \end{aligned} \tag{2.7}$$

is localized at the point of the highest second derivative in the field amplitude.

For both injection and ejection of the jet this drag force leads to the following loss in the dynamic pressure of the fluid

$$\Delta\frac{\rho V_s^2}{2} = -\int_0^L i'_s B ds \simeq (iB)_{B=B_{tor}}. \tag{2.8}$$

The fraction of the total magnetic pressure acting against the metal motion is determined by a dimensionless parameter P_d (modified Reynolds number)

$$\Delta\frac{\rho V_z^2}{2}\bigg|_{jet} = -P_d\frac{h^2-y^2-x^2}{h^2}\frac{B_0^2}{2\mu_0}, \quad P_d = \frac{\mu_0\sigma h^2 V}{L}, \tag{2.9}$$

where L is the characteristic gradient length of the magnetic field amplitude. Averaged over cross-section, the dynamic pressure drop is then given by

$$\left\langle\Delta\frac{\rho V_z^2}{2}\right\rangle_{jet} = -\frac{1}{2}P_d\frac{B_0^2}{2\mu_0}, \quad \left\langle\Delta\frac{\rho V_z^2}{2}\right\rangle_{film} = -\frac{2}{3}P_d\frac{B_0^2}{2\mu_0}. \tag{2.10}$$

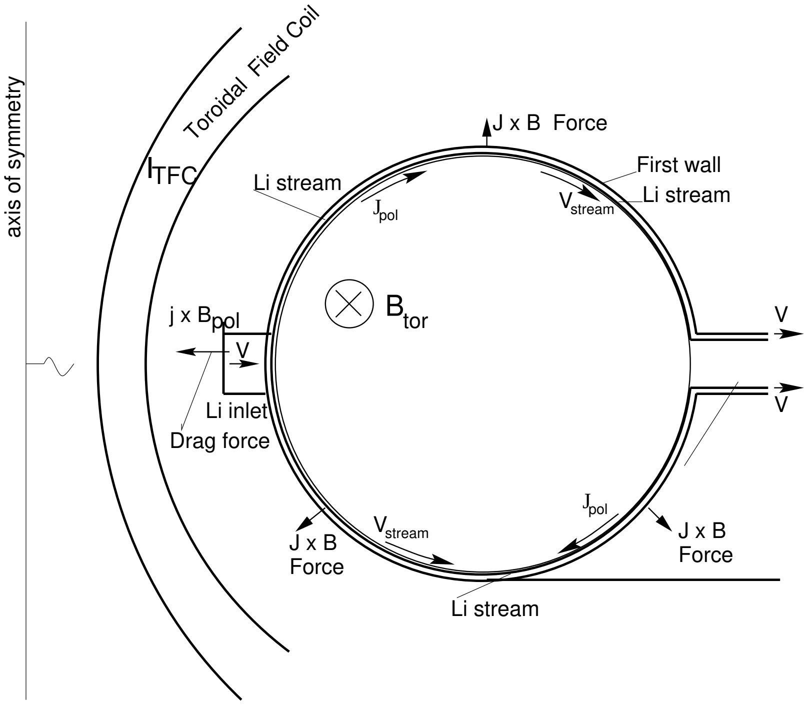
For the tokamak toroidal field $B > 5\ T$ only free-surface jets of the sub-centimeter range in cross-section are acceptable for transporting the lithium into the toroidal magnetic.

3. Injection into vacuum chamber

Inside the inlet to the vacuum chamber, the lithium flows across the poloidal magnetic field B_{pol} of the plasma current. This induces the toroidal current in the lithium

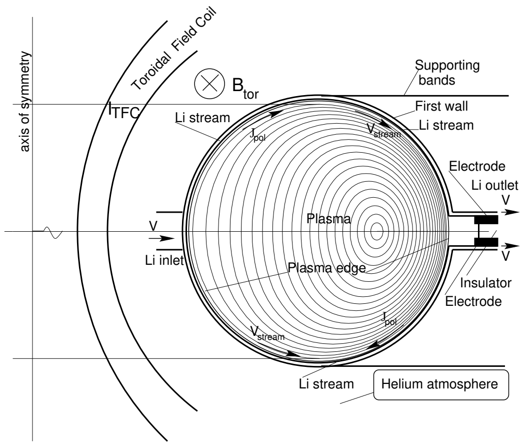
$$j_\varphi = \sigma V_\perp B_\perp, \quad \Delta p \simeq 2Re_m \frac{B_{pol}^2}{2\mu_0}, \quad Re_m \equiv \mu_0 \sigma L_\perp V_\perp. \quad (3.1)$$

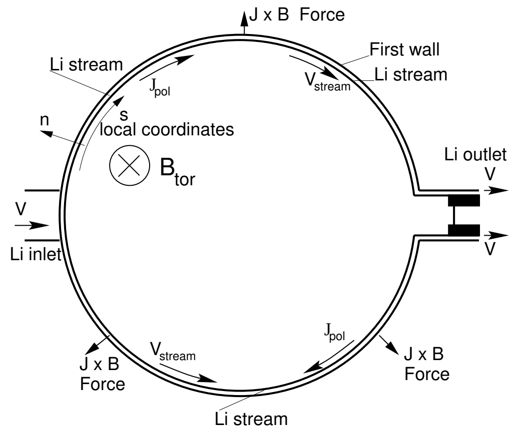
Poloidal magnetic field stops even a fast stream at 1-2 cm and redirects it along the magnetic surfaces.



4. Propulsion inside the vacuum chamber

Inside the chamber lithium moves along magnetic surfaces





Flow is externally controlled

$$\rho\frac{V_{Li}^2(n,s)}{2}=\rho\frac{V_{Li,inlet}^2(n)}{2}+p_{inlet}\left(1-\frac{r_{inlet}^2}{r^2}\right)\frac{n+h(s)}{h(s)},$$
$$h(s)=\frac{\dot{M}}{2\pi r\bar{V}_{Li}(s)},\qquad \bar{V}_{Li}(s)\equiv\int_{-h}^{0}\frac{V_{Li}(h,s)dn}{h}. \tag{4.1}$$

Averaging over the height of the layer leads to expression

$$\rho\frac{\langle V_{Li}^2(s)\rangle}{2}=\rho\frac{\langle V_{Li}^2\rangle_{inlet}}{2}+\frac{I_{pol}}{I_{TFC}}\frac{B_{tor,inlet}^2-B_{tor}^2(s)}{2\mu_0}, \tag{4.2}$$

where I_{TFC} is the total current in TFC.

Drag force due to magnetic pumping is negligible if

$$P_D \equiv \frac{\pi}{24}\frac{\mu_0 \sigma h^2 V}{r}\frac{I_{TFC}}{I_{stream}} \ll 1. \tag{4.3}$$

Inhomogeneity of the toroidal field generates the eddy currents inside the stream

$$\mu_0 \mathbf{j}_{eddy} = (\nabla i_{eddy} \times \nabla \varphi), \tag{4.4}$$

which can be described by the stream function $i_{eddy}(n, s)$. The normal component of Ohm's law in the main approximation gives the distribution of the scalar potential

$$\varphi'_{E,n} = -\frac{V\bar{F}}{r^2}, \quad \varphi_E \simeq \varphi_0(s) - \frac{U\bar{F}_w}{r^2}, \tag{4.5}$$

while the longitudinal component determines the stream function $i(\bar{n}, s)$ of the eddy currents

$$\begin{aligned} &\varphi'_0 - U\bar{F}_w \left(\frac{1}{r^2}\right)'_s = \frac{1}{r\mu_0\sigma} i'_n, \\ &\bar{F}_w \left(\bar{n}\int_{-1}^{0} U d\bar{n} - \int_0^{\bar{n}} U d\bar{n}\right)\left(\frac{1}{r^2}\right)'_s = \frac{i}{r\mu_0\sigma h}. \end{aligned} \tag{4.6}$$

The normal component of the eddy currents

$$j^n = \frac{i_s'}{\mu_0 r} = -\sigma h \bar{F}_w \left(\frac{1}{r^2}\right)''_{ss} \int_{\bar{n}}^0 (\langle U \rangle - U) d\bar{n}, \quad \langle U \rangle \equiv \int_{-1}^0 U d\bar{n} \tag{4.7}$$

creates the diamagnetic drag force. When included in the longitudinal equation of motion it gives the two-dimensional integro-differential magnetic propulsion equation in coordinates $\bar{n}, s$

$$\begin{aligned} &\rho \frac{\partial V}{\partial t} + \rho \left(\frac{V^2}{2}\right)'_s = \\ &- 2\frac{[I_{pol} - I(\bar{n})]}{I_{TFC}} \frac{(rB_{tor})^2}{2\mu_0} \left(\frac{1}{r^2}\right)'_s - \rho g z_s' \\ &+ \frac{\sigma h \bar{F}_w^2}{r} \left(\frac{1}{r^2}\right)''_{ss} \int_{\bar{n}}^0 (\langle U \rangle - U) d\bar{n} + \frac{\nu V''_{\bar{n}\bar{n}}}{h^2}, \end{aligned} \tag{4.8}$$

which together with the kinematic relations describes the dynamics of the intense lithium streams.

The factor which determines dependence on s in the force associated with the eddy currents, has two components

$$\frac{h}{r}\left(\frac{1}{r^2}\right)''_{ss} = \left[\frac{h}{r}\left(\frac{1}{r^2}\right)'_s\right]'_s - \left(\frac{h}{r}\right)'_s\left(\frac{1}{r^2}\right)'_s. \tag{4.9}$$

The first acts against the flow from the inlet up to the top (or bottom) point of the cross-section and then accelerates the flow without contributing to the net effect of propulsion on the whole path. It determines the maximum retardation due to the eddy currents

$$\begin{aligned} &\int_i^{top} \sigma \bar{F}_w^2 \left[\frac{h}{r}\left(\frac{1}{r^2}\right)'_s\right]'_s ds \int_{\bar{n}}^0 (\langle U\rangle - U)d\bar{n} = \\ &-2\frac{h\sigma\bar{F}_w^2}{r^4}\int_{\bar{n}}^0(\langle U\rangle - U)d\bar{n}. \end{aligned} \tag{4.10}$$

Averaged over depth of the stream assuming uniform velocity this part of the drag force would give loss of the dynamic pressure

$$\Delta\rho\left\langle\frac{V^2}{2}\right\rangle = -\frac{\mu_0\sigma h^2 V}{3R}\frac{B_{tor}^2}{2\mu_0}, \tag{4.11}$$

where the right hand side is calculated at the top (or bottom) point of the flow.

The second component of the drag force results in an irreversible loss of the dynamic pressure, which, e.g., for a constant velocity and a circular cross-section of the guide wall

$$V = const, \quad r_w = R - a\cos\frac{s}{a}, \quad r_s' = \sin\frac{s}{a} \tag{4.12}$$

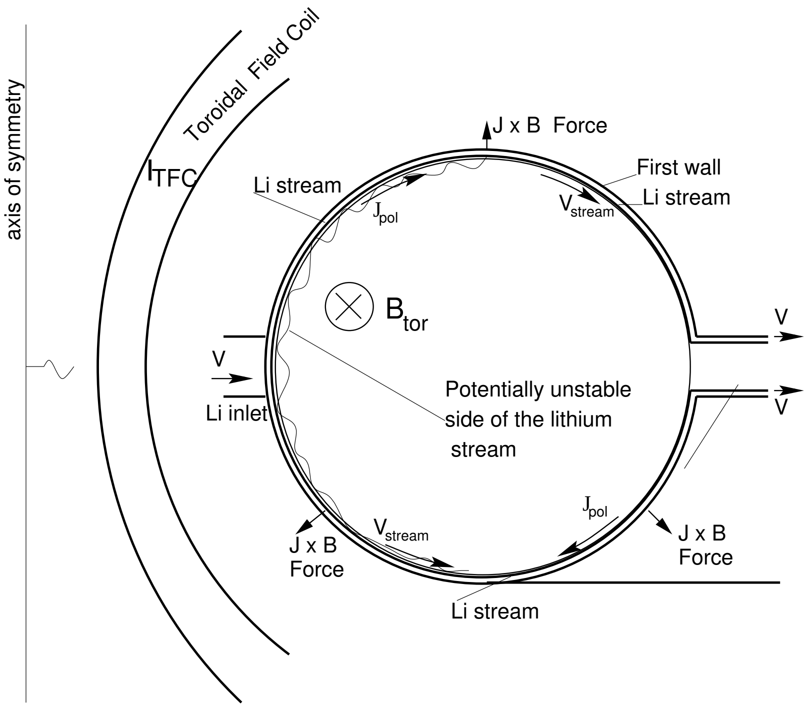
can be estimated as

$$\begin{aligned} \rho \left\langle \frac{V^2}{2} \right\rangle_s' &= -\frac{\sigma h_i^2 r_i^2 V \bar{F}_w^2}{3} \left(\frac{\sin^2 \frac{s}{a}}{r^3} \right)^2, \\ \Delta \left\langle \frac{V^2}{2} \right\rangle &= -\frac{\mu_0 \sigma h^2 V}{R} \frac{\bar{F}_w^2}{2\mu_0 R^2} \frac{\pi a}{3R} \left\langle \frac{2\sin^2\tau}{\left(1 - \frac{a}{R}\cos\tau\right)^6} \right\rangle_\tau . \end{aligned} \tag{4.13}$$

5. Stability of the lithium streams

The centrifugal force robustly stabilizes the sausage instability of the stream if

$$\int_{-1}^{\bar{n}} \left(\rho \kappa_w V^2 - 2p \frac{z'_w}{r} \right) d\bar{n} > 0, \quad \kappa_w r \rho \frac{\langle V^2 \rangle}{2} > \frac{p_w}{2} z'_w, \qquad (5.1)$$



6. Expulsion of the lithium

Expulsion can be done only in the form of thin jets.

Necessary condition for expulsion of lithium from the vacuum chamber is

$$p_{outlet} > 1\ atm. \tag{6.1}$$

Expulsion remains to be analyzed in details.

Experiments are necessary for studies jet formations and expulsion of the lithium across the magnetic field.

7. Summary

The theory of magnetic propulsion of the intense liquid lithium streams is now well developed.

(a) It shows the way of resolving some of most critical issues for driving a liquid metal in the strong magnetic field of tokamaks.

(b) It gives the foundation for development of the technology of intense lithium streams.

(c) It shows, that while thin lithium streams are fully compatible with the strong tokamaks magnetic field, any increase in thickness of the liquid metal walls is associated with numerous problems due to increase in the parameter

$$\boxed{P_d \simeq \mu_0 \sigma \frac{h^2}{L} V \propto h^2 \qquad (7.1)}$$