Magnetic propulsion of liquid Li in tokamaks[1]

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Abstract

Interaction of poloidal current, externally driven through the layer of liquid lithium, with the tokamak magnetic field is discussed as a mechanism a) for creating fast moving (plasma facing) Li streams along the inner wall of tokamak and b) for mixing the liquid in a closed volume (i.e., inside the Li blanket). Magnetic force j x B of interaction of poloidal current with toroidal magnetic field pushes liquid metal toward the walls and, in addition, creates inhomogeneous pressure distribution inside the fluid. Resulting pressure, which is larger on the inner side of the torus (where the magnetic field and the current density are biggest), propels fluid along the walls towards the outer side of the torus.

This effect of magnetic propulsion can be used for developing new approaches for protecting the first wall and the plasma facing components in tokamaks as well as for designing the reactor blanket. E.g., two plasma facing liquid lithium streams can be created along the inner wall of tokamaks if the metal is supplied from the inner circumference of the torus and is taken out from its outer circumference.

On the other hand, when the liquid lithium is held in a closed volume, magnetic propulsion generates a convection of the fluid, thus, making thermal contact between the strong toroidal field side of the blanket, which is hardly accessible, and its outer part.

Attractiveness of using liquid (Li or $LiBeF_3$), rather than solid elements, in the neutron flux zone of tokamaks:

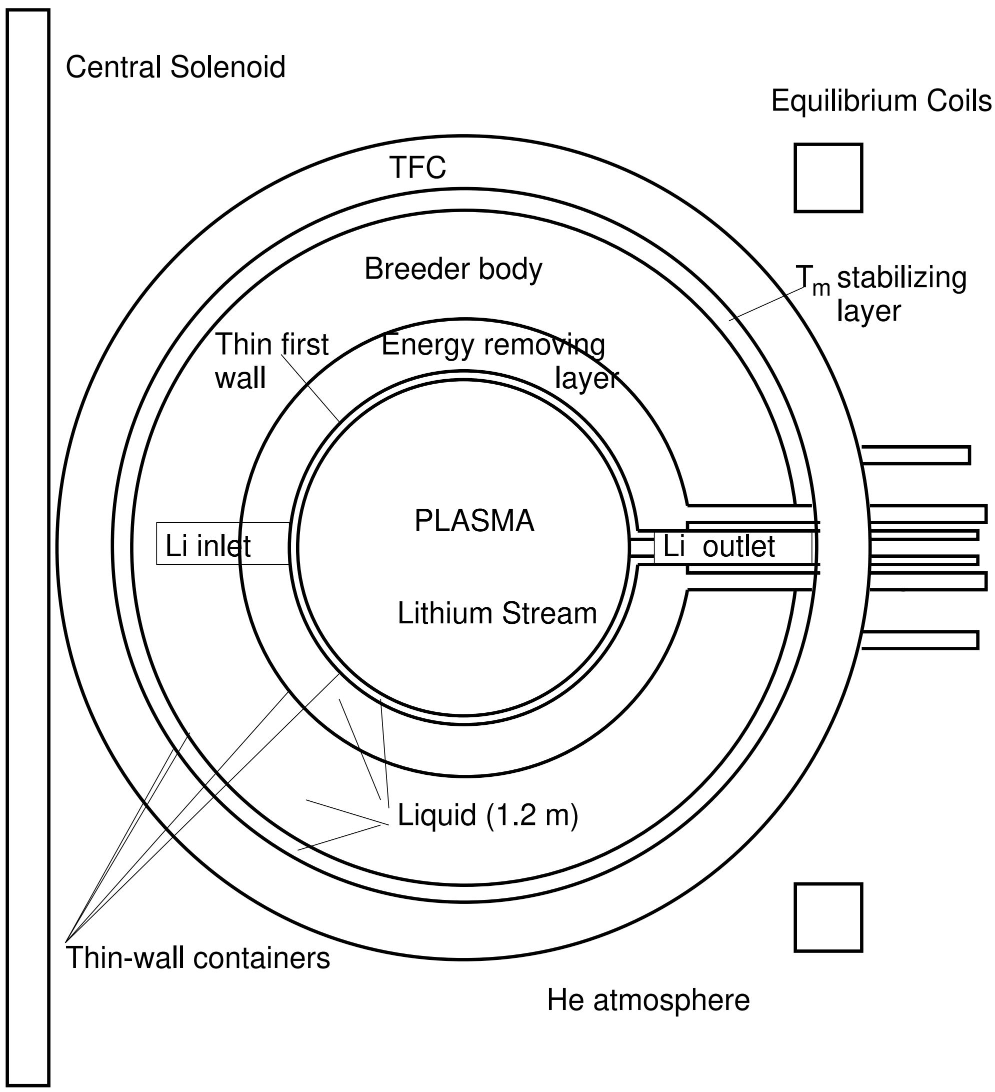
- absence of damage by neutrons
- absence of activation
- absence of damage by disruptions
- no needs in stationary regimes; pulsed regime is possible

Problems due to tokamak toroidal geometry have been noticed earlier, and pump driven liquid walls have been considered mostly for other MFE configurations (R. W. Moir, Nuclear Fus., 1997).

Strong toroidal magnetic field of tokamaks creates problems also in using conducting liquid in the blanket (I. R. Kirillov at al, Fus. Engineering and Design, 1995).

The effect of magnetic propulsion may help in solving tokamak specific problems for tokamak-reactors.

Liquid Li tokamak



OUTLINE

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1 Effect of magnetic propulsion

Both electro-technique and MHD use the Lorenz force

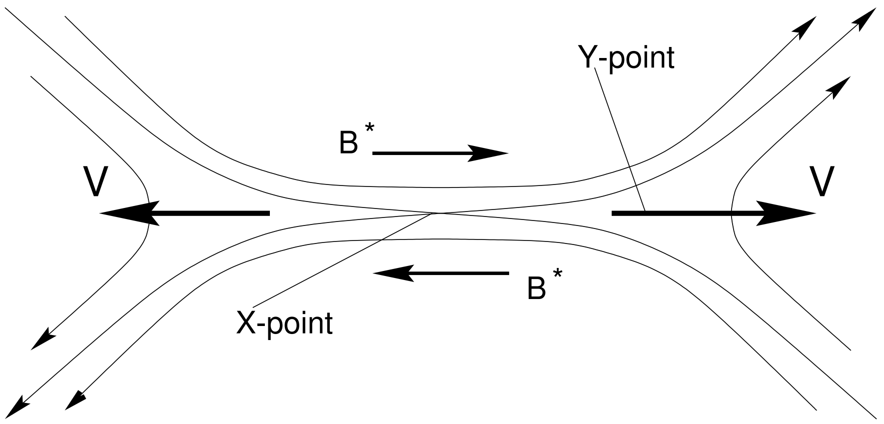
$$\mathsf{j} \times \mathsf{B} \tag{1.1}$$

in order to create a motion.

In electro-technique it creates a motion perpendicular to both j and B.

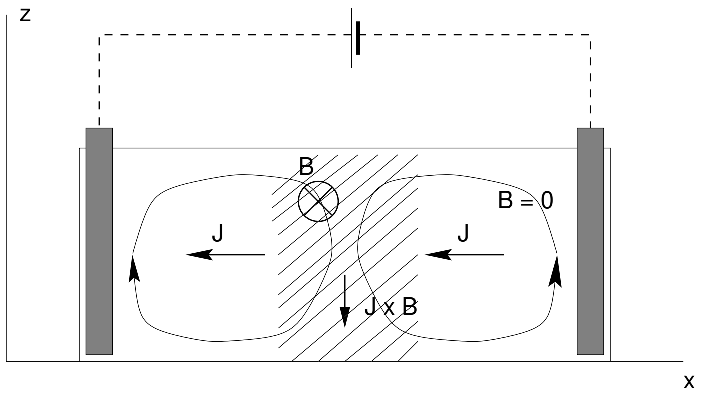
In MHD it is possible to creates a motion either along j or B.

In plasma physics, the plasma propulsion along B^* is well-known in magnetic reconnection.



$$V_\parallel^2 = \frac{B_X^{*2} - B_Y^{*2}}{4\pi n m_i}. \tag{1.2}$$

Magnetic propulsion is based on nonuniformity of $\mathsf{j} \times \mathsf{B}$.



In the simplest case of rectangular aquarium with electrodes,

$$\mathsf{j} = j\mathbf{e}_x, \quad \mathsf{B} = B\mathbf{e}_y \tag{1.3}$$

the vertical Lorentz force

$$\mathsf{j} \times \mathsf{B} = jB\mathbf{e}_z \tag{1.4}$$

is balanced by one component of a pressure gradient

$$p'_z = jB, \quad p = (z - h)jB. \tag{1.5}$$

Then, the horizontal component of the pressure gradient

$$-\,p'_x = -\left[(z - h)jB\right]'_x \tag{1.6}$$

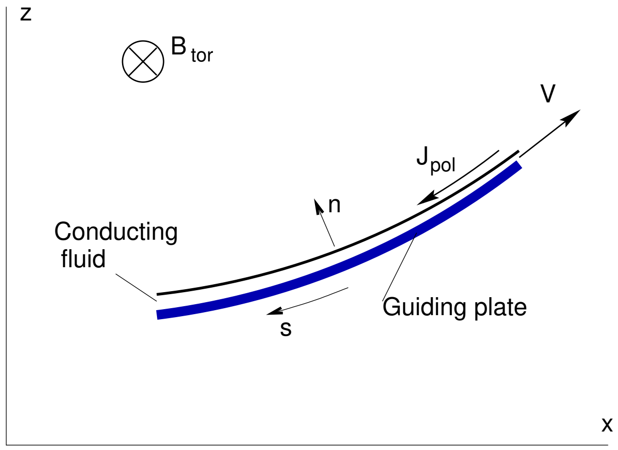
creates a horizontal propulsion force.

2 MHD equations for a thin layer

We consider a simple configuration of the only toroidal magnetic field

$$B_\varphi \equiv B = \frac{0.2 I_{pol}}{r}, \tag{2.1}$$

The geometry of the guiding plate surface, contacting with the fluid,



is described by parametric equations

$$r = r(s), \quad z = z(s), \tag{2.2}$$

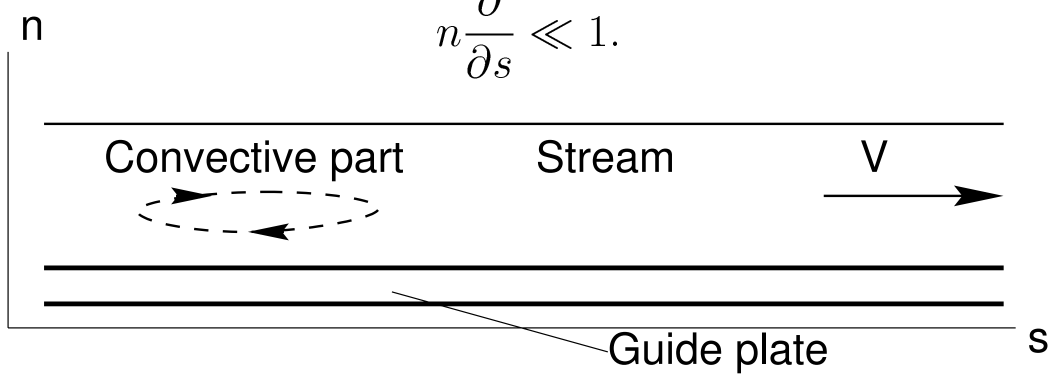
where r, z are the cylindrical coordinates, and s is a coordinate along the plate, measuring the distance from some reference point

$$r = r_i, \tag{2.3}$$

and n is a perpendicular coordinate.

In the thin layer approximation

$$n\frac{\partial}{\partial s} \ll 1. \tag{2.4}$$



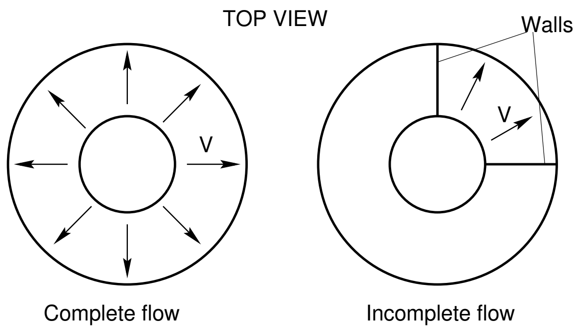
Kinematics of incompressible fluid is described with use of the stream function $f(s,n)$

$$\mathbf{v} = V\mathbf{s} + w^{-1}\nabla f \times \mathbf{e}_\varphi = V\mathbf{s} - w^{-1}f'_n\mathbf{s} + w^{-1}f'_s\mathbf{n}, \tag{2.5}$$

where w is the width of the flow. In a complete axisymmetric flow it is simply

$$w = 2\pi r(s), \tag{2.6}$$

while it can be intentionally designed by limiting flow by special channels.



In the same way, the electric current density can be presented in terms of the current stream function $i(s, n)$

$$\mathsf{j} = J\mathsf{s} + w^{-1}\nabla i \times \mathsf{e}_\varphi = J\mathsf{s} - w^{-1}i'_n\mathsf{s} + w^{-1}i'_s\mathsf{n} \tag{2.7}$$

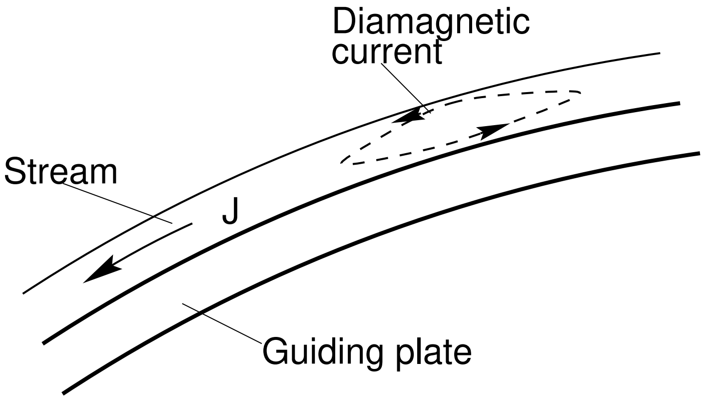
and the net longitudinal current density J, which for simplicity is assumed to be uniform across the stream $J = J(s)$. It is determined by the height $h(s)$ of the fluid and the total electric current I as

$$J = \frac{I}{w(s)h(s)}, \tag{2.8}$$

which reduces simply to

$$J = \frac{I}{2\pi r h} \tag{2.9}$$

for a complete flow covering full axisymmetric plate.



The Ohm's law

$$E\mathsf{s} - \nabla\varphi + \mathsf{v}\times\mathsf{B} = \frac{\mathsf{j}}{\sigma}, \quad E \equiv \frac{J}{\sigma} \tag{2.10}$$

determines the current distribution in the fluid, while the equation of motion

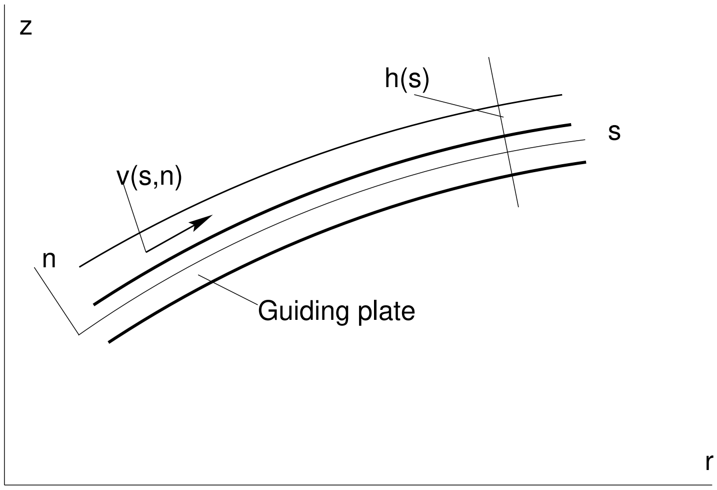
$$\begin{aligned} \rho\frac{\partial \mathsf{v}}{\partial t} + \rho v_s v_s'\mathsf{s} + \nabla p &= \mathsf{j}\times\mathsf{B} + \nu v''_{s,nn}\mathsf{s}, \\ \rho\frac{\partial \mathsf{v}}{\partial t} + \rho v_s v_s'\mathsf{s} + \nabla p &= \mathsf{j}\times\mathsf{B} + \nu V''_{nn}\mathsf{s} - \nu w^{-1} f'''_{nnn}\mathsf{s} \end{aligned} \tag{2.11}$$

determines the velocity profile with ν being viscosity of the fluid.

The height $h(s)$ of the fluid is determined by the continuity condition

$$\frac{dM}{dt} = \rho 2\pi R h V. \tag{2.12}$$

3 Free-surface stream



For the thin stream, assuming fluid supply such that

$$V >> |\nabla f|, \tag{3.1}$$

we can neglect the convective motion of the fluid

$$\mathbf{v} = V\mathbf{s}. \tag{3.2}$$

Perpendicular component of the Ohm's law

$$-\varphi'_n + VB = \frac{j_n}{\sigma}, \tag{3.3}$$

gives an expression for electric potential

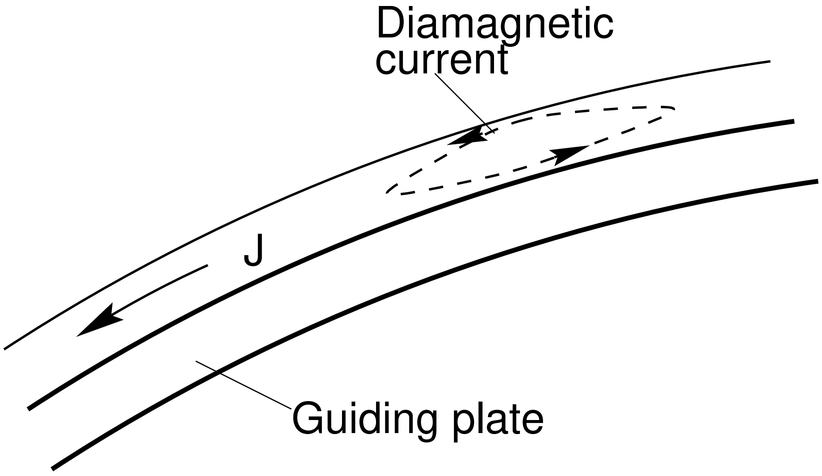
$$\varphi'_n = VB, \quad \varphi = nVB + \varphi_0(s). \tag{3.4}$$

Longitudinal component of the Ohm law

$$E - \varphi'_s = \frac{j_s}{\sigma} = \frac{J}{\sigma} - \frac{i'_n}{w\sigma}, \quad E \equiv \frac{J}{\sigma}, \quad n(VB)'_s + \varphi'_0 = \frac{i'_n}{w\sigma} \tag{3.5}$$

determines the stream function i of electric current

$$i = w\sigma\frac{n^2 - nh}{2}(VB)'_s, \quad \varphi'_0 = -\frac{nh}{2}(VB)'_s. \tag{3.6}$$

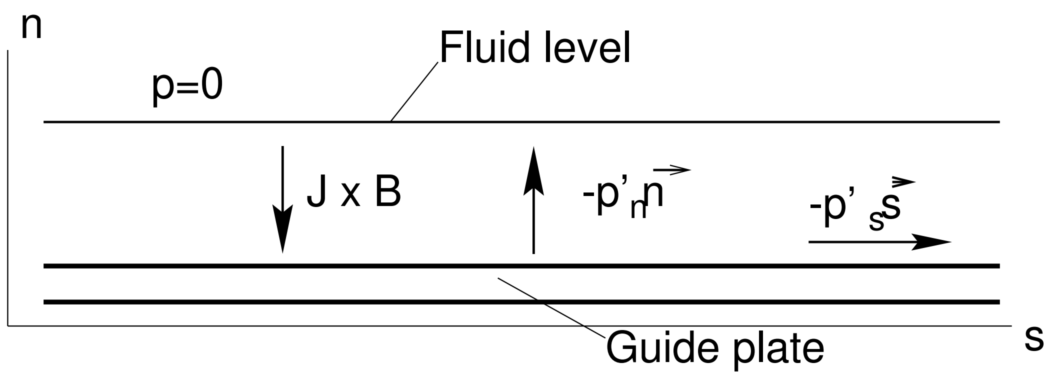


It gives the expression for the perpendicular current density

$$j_n = w^{-1}i'_s = w^{-1}\left[\frac{n^2 - nh}{2}\sigma(VB)'_s\right]'_s. \tag{3.7}$$

This current density creates a “diamagnetic drag” force for the longitudinal motion of the fluid (possibly locally negative).

Now, it is possible to consider the equation of motion.



Perpendicular force balance

$$p'_n = JB, \quad p = (n-h)JB = -(1-\overline{n})hJB, \quad \overline{n} \equiv \frac{n}{h} \tag{3.8}$$

determines the pressure profile in the fluid. We introduced here the locally normalized perpendicular coordinate $\overline{n}$.

The longitudinal equation of motion gives the longitudinal velocity profile

$$\begin{aligned} \rho V V'_s + p'_s &= -j_n B + \nu V''_{nn}, \\ \left(\frac{\rho V^2}{2} + p\right)'_s &= -j_n B + \nu V''_{nn}. \end{aligned} \tag{3.9}$$

Neglecting fluid viscosity, we have for the stream velocity the expression

$$\begin{aligned}\frac{1}{2}\rho(V^2 - V_i^2) + p - p_i &= -\int_0^s j_n B ds \\ &= -\int_0^s w^{-1}\left[\frac{n^2 - nh}{2}\sigma(VB)'_s\right]'_s B ds,\end{aligned} \tag{3.10}$$

where V_i and p_i are the incoming velocity and the pressure in the reference point.

In explicit form magnetic propulsion equation is

$$\begin{aligned}\frac{1}{2}\rho(V^2 - V_i^2) = &-(1 - \overline{n})(h_i J_i B_i - hJB) \\ &-\int_0^s w^{-1}\left[\frac{n^2 - nh}{2}\sigma(VB)'_s\right]'_s B ds,\end{aligned} \tag{3.11}$$

where perpendicular coordinate acts as a parameter.

Neglecting diamagnetic dragging term

$$\frac{1}{2}\rho V^2 = \frac{1}{2}\rho V_i^2 - (1-\overline{n})h_i J_i B_i \left(1 - \frac{w_i^2}{w^2}\right), \tag{3.12}$$

or for a complete flow

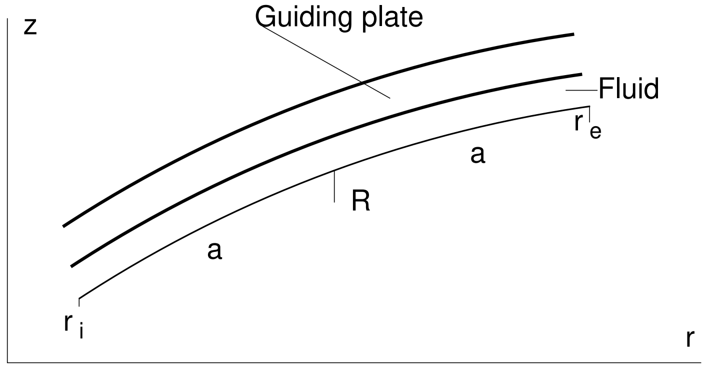
$$\frac{1}{2}\rho V^2 = \frac{1}{2}\rho V_i^2 - (1-\overline{n})h_i J_i B_i \left(1 - \frac{r_i^2}{r^2}\right). \tag{3.13}$$

Propulsion factor P_M (for referencing to the middle point)

$$\frac{1}{2}\rho V^2 = \frac{1}{2}\rho V_i^2 + (1-\overline{n})P_P \frac{IB_{tor}}{2\pi R}, \tag{3.14}$$

where

$$P_P \equiv \frac{4aR^3}{(R^2-a^2)^2}, \quad R \equiv \frac{r_i + r_e}{2}, \quad a \equiv \frac{r_e - r_i}{2}. \tag{3.15}$$



Comparison of the averaged propulsion effect

$$\frac{1}{2}h_i J_i B_i \left(1-\frac{r_i^2}{r^2}\right) = \frac{\delta B B}{2\mu_0}\left(1-\frac{r_i^2}{r^2}\right) = \frac{0.1 I I_{pol}}{2\pi r_i^2}\left(1-\frac{R^2}{r^2}\right) \quad (3.16)$$

with the averaged drag force term

$$\frac{1}{h_i}\int_0^s w^{-1}\left[\frac{h^3}{12}\sigma(VB)'_s\right]'_s Bds \simeq \frac{1}{h_i}\int_{r_i}^r w^{-1}\left[\frac{h^3}{12}\sigma(VB)'_s\right]'_r Bdr \quad (3.17)$$

leads to diamagnetic drag parameter P_D

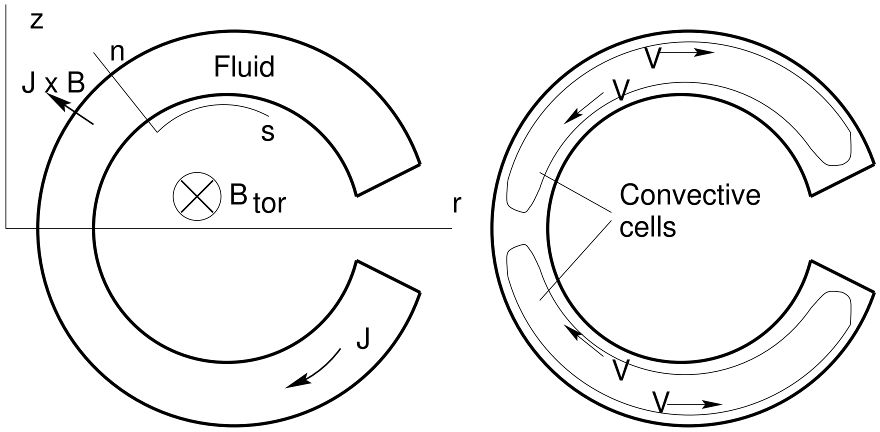
$$P_D \equiv \frac{\mu_0 \sigma h^2 V}{12r}\frac{I_{pol}}{I}, \quad (3.18)$$

relevant for magnetic propulsion.

P_D is controlled by the inflow velocity and the total electric current in fluid.

For thin layers it can be made much less than 1.

4 Convection in a closed volume



Closed volume has the contour, prescribed by equation

$$r = r(s), \quad z = z(s), \tag{4.1}$$

with cross-section given by

$$h = h(s), \quad w = w(s), \tag{4.2}$$

where h is the total height (size along n) and w is the total width (size along φ). In this case there is no net velocity and only convection described by

$$\mathsf{v} = w^{-1}\nabla f \times \mathsf{e}_\varphi = -w^{-1}f'_n\mathsf{s} + w^{-1}f'_s\mathsf{n}. \tag{4.3}$$

Its vorticity $\mathsf{U} \equiv \nabla \times \mathsf{v}$ is given by

$$\mathsf{U} = [(w^{-1}f'_s)'_s + w^{-1}f''_{nn}]\mathsf{e}_\varphi \simeq f''_{nn}\mathsf{e}_\varphi = f''_{nn}\mathsf{e}_\varphi, \quad U \equiv f''_{nn}. \tag{4.4}$$

The equation of motion (with viscosity neglected), rewritten in the form

$$\rho\frac{\partial \mathsf{v}}{\partial t}+\frac{1}{2}\rho\nabla \mathsf{v}^2-\rho \mathsf{v}\times U\mathsf{e}_\varphi+\nabla p=\mathsf{j}\times \mathsf{B}, \tag{4.5}$$

gives for vorticity

$$\rho\frac{\partial U}{\partial t}+\rho \mathsf{v}\nabla U=-(\mathsf{j}\nabla)B\simeq -j_r B_r'. \tag{4.6}$$

The second term in the LHS can be written as

$$\rho \mathsf{v}\nabla U=\nabla(\rho \mathsf{v} U), \tag{4.7}$$

which indicates that the equation for vorticity has no stationary solution, if the current is prescribed.

By integrating it over volume of the fluid, we get the rate of accumulation of vorticity in the fluid

$$\rho\frac{d}{dt}\int U r ds dh d\varphi=-\int j_r B_r' r ds dh d\varphi. \tag{4.8}$$

- Stationary solution for a given Voltage

$$\mathbf{v} = w^{-1}\nabla f \times \mathbf{e}_\varphi = -w^{-1} f'_n \mathbf{s} + w^{-1} f'_s \mathbf{n}. \tag{4.9}$$

From the Ohm's law

$$\varphi'_n = -w^{-1} f'_n B, \quad \varphi = -w^{-1} f B. \tag{4.10}$$

Relation between the stream function and Voltage

$$\varphi'_n = -w^{-1} f'_n B, \quad \varphi_e - \varphi_i = \left(\frac{B_i}{w_i} - \frac{B_e}{w_e}\right) \int_0^h v_s dn. \tag{4.11}$$

- Viscous stationary solution

$$-\left(nJB\right)'_s = \nu v''_{nn}, \tag{4.12}$$

which gives the velocity profile in the form

$$v = \frac{h^3}{\nu}\frac{\overline{n} - \overline{n}^3}{6}(JB)'_s. \tag{4.13}$$

When diamagnetic drag parameter P_D is greater than unity, current and velocity distributions should be determined from magnetic propulsion equation (??).

5 Application to tokamak fusion reactor

Approach, based on solid massive structure elements did not lead yet to competitive solution for tokamak-reactor.

It also imposes severe constraints on tokamak plasma regimes (absence of disruptions, complete stationarity).

a) Basic data on Lithium

1. Mass density	$500\ \frac{kg}{m^3}$	half of water
2. Melting temperature	$\simeq 200\ C^o$	
3. Boiling temperature	$\simeq 1300\ C^o$	
4. c_p	$4200\ \frac{J}{kg \cdot K}$	like water
5. Conductivity σ	$2.2 \cdot 10^6\ \frac{1}{Om \cdot m}$	20 times less than copper
6. Viscosity ν	$0.5 \cdot 10{-}3\ Pa \cdot s$	like water
7. Heat of melting Q_{melt}	$Q = 0.43\ MJ/kg$	25 % more than water
8. Thermal conductivity	$k_T \simeq 47\ W/(m \cdot K)$	1 MW with T'=215 $^o/cm$

Magic numbers.

- In order to absorb the power $P_{th}$of 1 GW with increase in T^o of 250 C^o, it is necessary to propel 1 T/sec of Li (2 m^3).
- For fusion reactor with R=5 m, and Li layer of 1 cm thick the propulsion velocity is 7 m/sec.

$$V = \frac{10^{-3}\frac{dM}{dt}}{\pi Rh} \tag{5.1}$$

- Magnetic Reynolds number

$$R_m \equiv 3hV = 3\frac{10^{-3}\frac{dM}{dt}}{\pi R} = \frac{P_{th}[GW]}{R[m]} = 0.2 \qquad (5.2)$$

No effect on equilibrium fields.

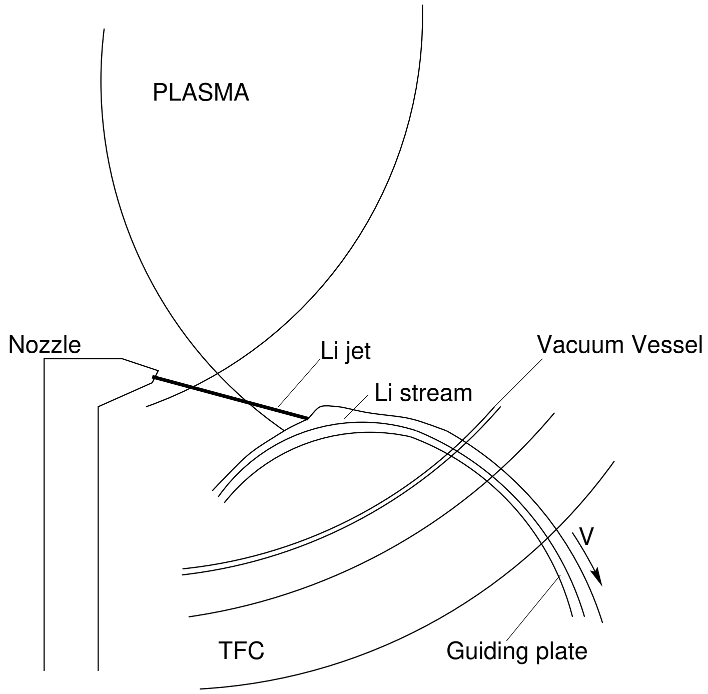
- Dynamic pressure of the fluid

$$\begin{aligned} P_d =& \rho\frac{V^2}{2} = 250V^2\ [Pa] = 2.5\cdot 10^{-4}V^2\ [MPa] \\ &= 25\cdot 10^{-4}V^2\ [Atm], \\ V =& 10, \quad P_d = 0.025\ [MPa] = 0.25\ [Atm], \\ V =& 20, \quad P_d = 1\ [Atm], \\ V =& 100, \quad P_d = 2.5\ [MPa] = 25\ [Atm] \end{aligned} \qquad (5.3)$$

b) Lithium jet for the tokamak divertor

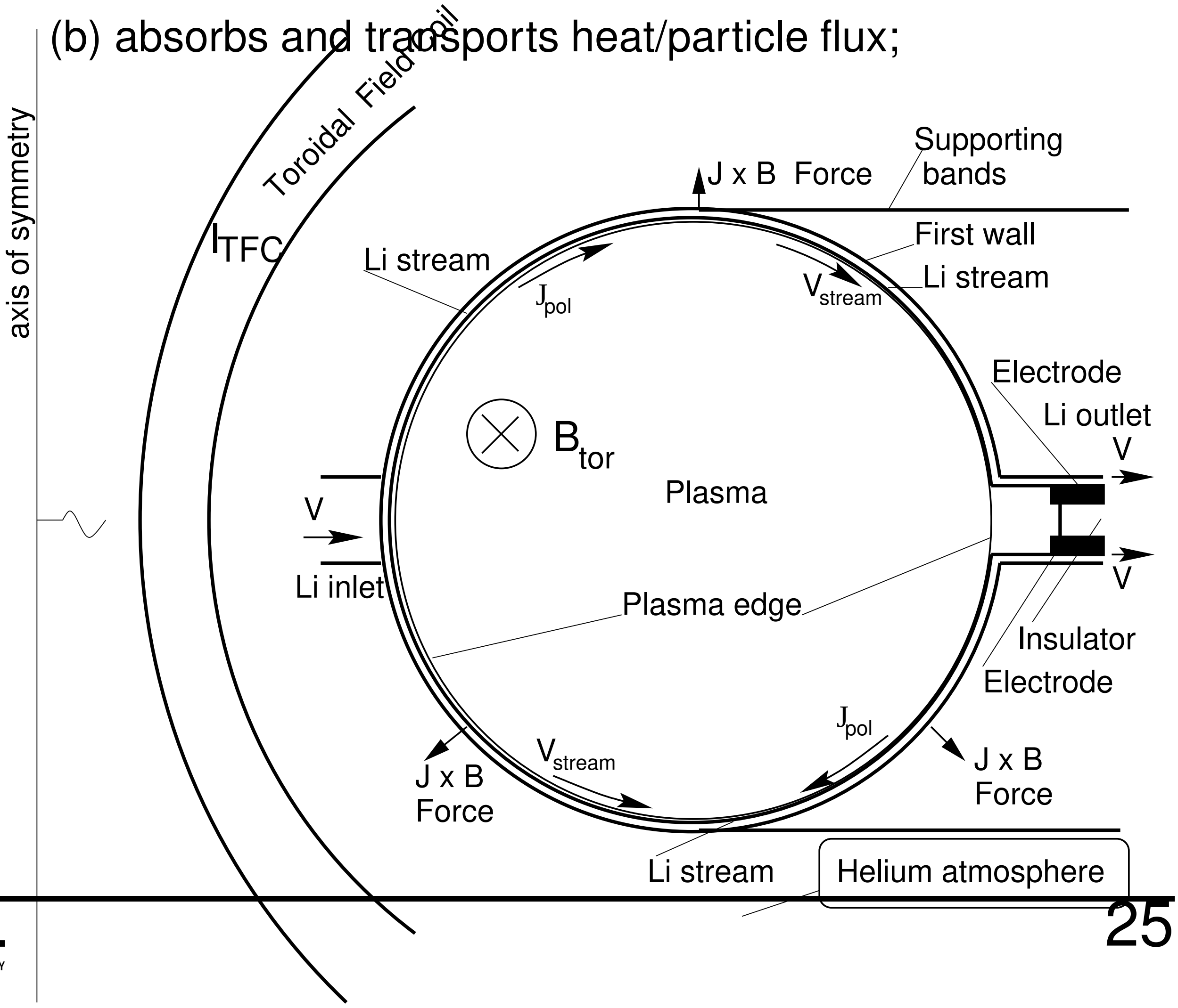
Liquid metal approach to the tokamak divertor problem:

1. Ordinary high-speed liquid Lithium jet ($V = 100$, $h = 1\ mm$)
2. Magnetically propulsed free-surface liquid Lithium stream ($V = 10$, $h = 1\ cm$) for transportation of absorbed power outside Vacuum Vessel and TFC



c) First wall with a Li stream

1. Thin (about 1 mm) toroidal chamber.
2. Magnetically propulsed free-surface liquid Lithium stream ($V = 10$, $h = 1\ cm$) which
   (a) withstands the atmospheric pressure;
   (b) absorbs and transports heat/particle flux;



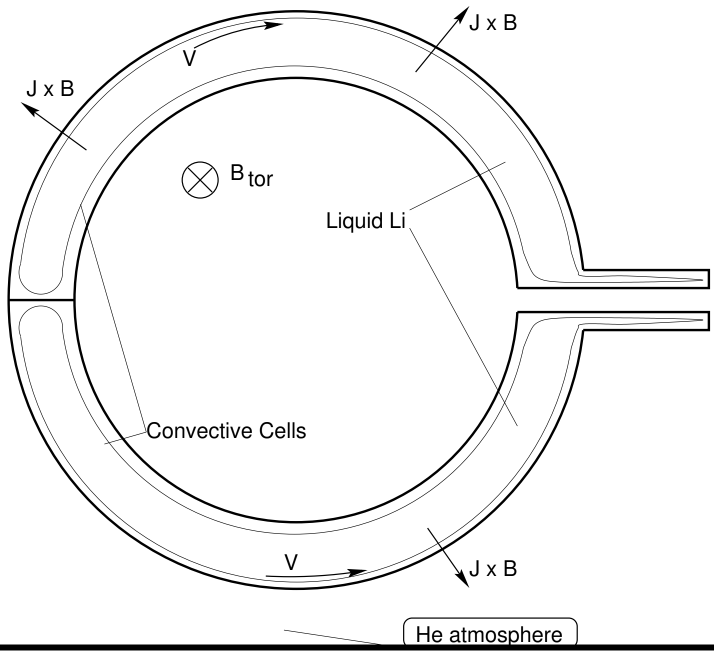
Attractive features:

1. This wall serves as a vacuum barrier.
2. Full access to any point behind 2-5 cm from the plasma boundary;
3. Thin wall, insensitive to thermal deformations, mechanical deformations, cracks or possible disruptions

$$\frac{I_{stream}}{I_{TFC}} > 0.1\frac{B_{pl}^2}{B_{tor}^2}\frac{h}{\delta}, \quad \delta \simeq \sqrt{\frac{\tau_{disruption}}{3}}. \tag{5.4}$$

d) Heat/Lithium mixer (breeder)

1. Thin (about 1 mm) toroidal chambers with C-shaped cross-section.
2. Magnetically propulsed convection of the liquid Lithium ($V = 1$, $h = 10 - 20\ cm$) which
   (a) provides most of the force only on the outer surface;
   (b) absorbs neutron flux and reproduces tritium;
   (c) mixes temperature uniformly;
   (d) provides heat exchange outside TFC;



Attractive features:

1. Thin-wall breeder, insensitive to thermal deformations or mechanical deformations, insensitive to cracks, holes;
2. No welding is needed for container walls. Leaks of Lithium are permissible.
3. Easy repair or replacement. No needs in 30 years of the lifetime. About 2 years of the lifetime could be admissible.

4. Ratio of weights of non-Lithium (activated) materials to Lithium (non-activated) ones can be easily made less than 10 % in the neutron zone.
5. No needs in stationary regimes for DT reactor. Pulsed tokamak regime with hours/10 min burn/re-charging is compatible with the machine.
6. “Water” bag design is insensitive to plasma disruptions or machine control failure.

e) Effect on plasma regime

Plasma containment regimes can be deeply affectred by hydrogen absorption properties of Li walls.

In low recycling regime, plasma profiles are determined by central refueling

$$T_e = T_i \simeq const, \quad n_e = n_e(r), \quad n_e(a) \simeq 0. \tag{5.5}$$

Attractiveness:

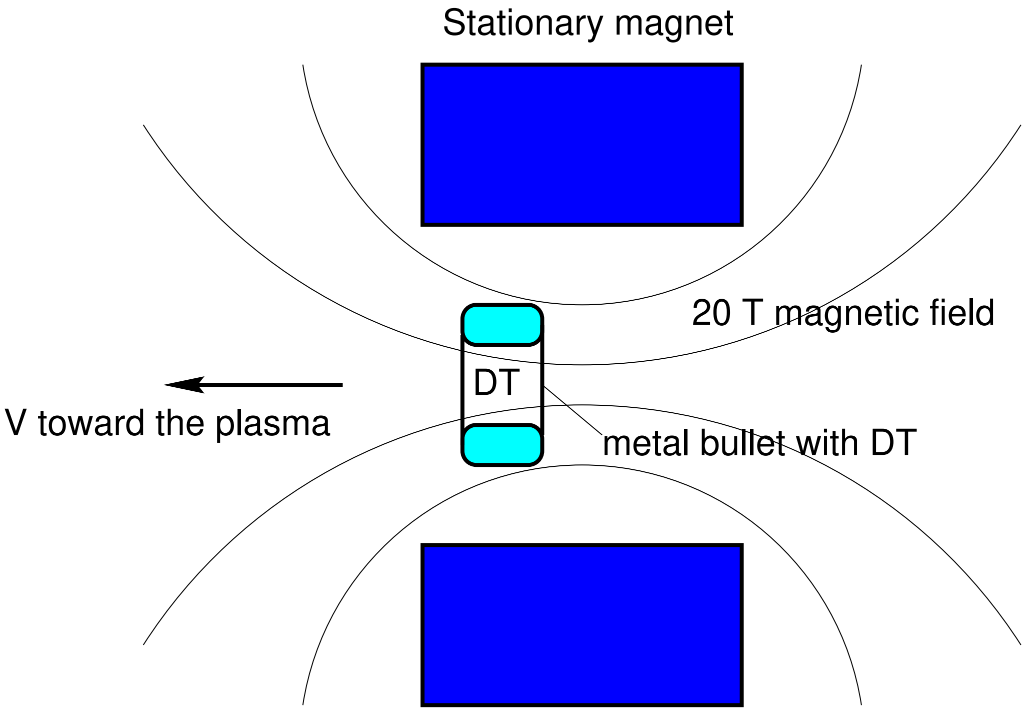
1. Significant elimination of anomalous energy transport (associated with the temperature gradient). More compact reactors.
2. Entire plasma cross-section (not only its central part) burns.
3. Current density profile is nearly uniform and is controlled by refueling. Slightly positive or negative shear regimes are easily possible. No sawteeth.

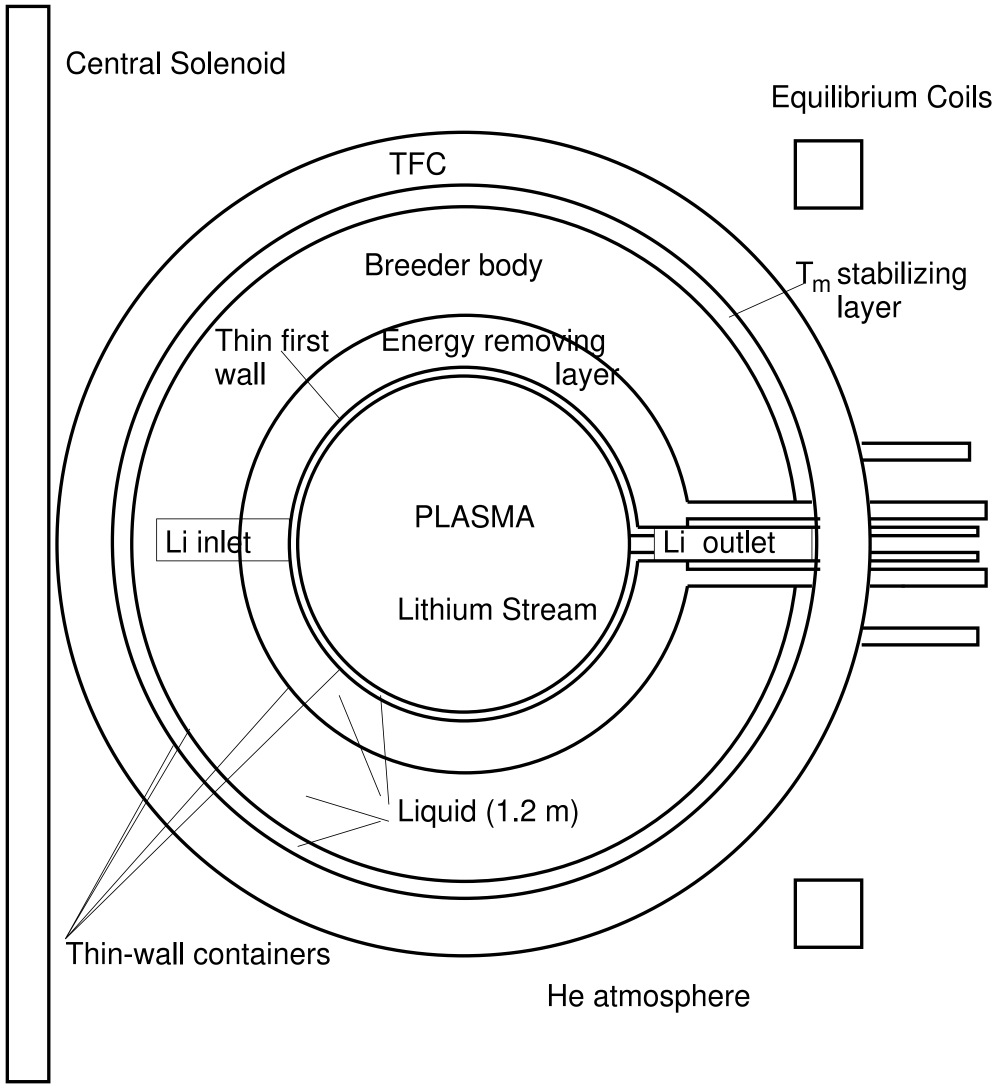
4. Plasma is stable for fast free boundary modes. For typical disruption time
$$\tau_I = 0.0001\ sec \tag{5.6}$$
the effective distance of the conductive wall is
$$\Delta a = \sqrt{\frac{\tau_I}{3}} < 1\ cm, \qquad \tau_\eta = 3(\Delta a)^2 = \tau_I. \tag{5.7}$$
With moving metal outside plasma boundary, it might be possible to stabilized also resistive wall modes.

5. No Troyon limits on β. Instead, indefinite second stability zone.

Plasma refueling

Magnetically self-insulated Morozov's bullets (Sov. J. Plasma Phys., p. 71, 1991) can solve the problem of central refuelling





Central Solenoid
Equilibrium Coils
TFC
Breeder body
T_m stabilizing layer
Thin first wall
Energy removing layer
PLASMA
Li inlet
Li outlet
Lithium Stream
Liquid (1.2 m)
Thin-wall containers
He atmosphere

6 Summary

While emphasizing a number of problems associated with use of liquid Li, such as

- handling large amounts of Li and T
- electrical insulation in intensive neutron flux
- corrosion of structural elements by Li in presentce of electric current,
- MHD problems

magnetic propulsion gives new attractive options in designing tokamak-reactors.

It can be considered for non-circular and spherical tokamaks as well as for other MFE and ICF reactor applications.

It also raises many R&D issues for both theoretical and experimental studies.