Stabilization of tokamak plasma by lithium streams

Leonid E. Zakharov

Princeton Plasma Physics Laboratory, Princeton, NJ 08543-0451

(July 24, 2000)

Abstract

The theory of stabilizing free-boundary magnetohydrodynamic instabilities in tokamaks by liquid lithium streams driven by magnetic propulsion is formulated. It is shown that even for uniform current distribution lithium streams open stability windows. Also, it was found that while the conventional, wall-locked, resistive wall mode can be well suppressed by the flow, a new, stream-locked mode determines the limits of the flow stabilization.

52.55.Fa, 52.30.Jb, 52.35.Py, 52.65.Kj

Recently, the author proposed a mechanism of magnetic propulsion for driving fast lithium streams along the plasma facing surface of the tokamak vacuum chamber (Fig.1). The metal is maintained at the first-wall surface by the electromagnetic force $\mathbf{J} \times \mathbf{B}$, where $\mathbf{J}$ is the poloidal current, driven through the liquid metal by an external power source and $\mathbf{B}$ is the tokamak magnetic field. The magnetic pressure, created in the fluid, is nonuniform along the lithium layer and the fluid is propelled from the high-field side of the tokamak to the low-field side in the form of two streams along the top and bottom halves of the vacuum chamber, respectively.

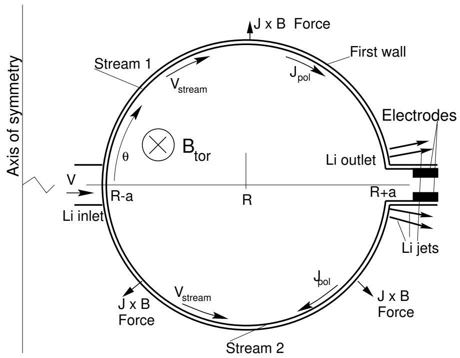


Fig. 1. Cross section of the tokamak with two lithium streams at the top and bottom halves of the vacuum chamber.

In tokamaks, the $\mathbf{J} \times \mathbf{B}$ force can be made sufficient to create a pressure to the wall exceeding the external pressure, e.g., 1 atm. This enables expulsion of the lithium from the vacuum chamber. Accordingly, for conventional aspect ratios $3-5$, the pressure drop along the streams exceeds $1-2$ atm, thus, making velocities of the order of 20 m/sec typical for the lithium streams.

These intense lithium streams have been proposed primarily as a new power extraction scheme compatible with a fusion reactor environment. Remarkably, it was found that due to centrifugal force, the streams themselves can be made robustly stable with respect to the sausage instability associated with the electric current in the metal as well as to the much weaker gravitational instability. The big, externally controlled magnetic pressure, holding streams on the wall, makes liquid metal insensitive to other electromagnetic forces even in the most violent disruption event in tokamaks. We leave discussion of numerous issues specific for magnetic propulsion and magnetohydrodynamics of the lithium streams for a separate publication. In this paper, the flow is considered as given and unperturbed.

There are two stabilizing effects associated with lithium streams. First, streams enable an effective conducting shell right at the plasma boundary. Second, the axisymmetric $m = 1$, $n = 0$ (m, n are the poloidal and toroidal wave numbers, respectively) pattern of the fast metal flow affects explicitly the resistive wall modes [1,2].

The first effect is straightforward and can be considered within the existing theory of resistive (solid) wall modes. In the magnetohydrodynamic model the resistive walls only slow down the growth of the free boundary instabilities without stabilizing them. Because of this, the theory is now focused on nonideal effects which together with the plasma rotation may lead to a stabilization (see, e.g., References [3–9]). The second effect, which relies on a specific flow pattern is considered for the first time. It provides a stabilization mechanism independent of the plasma properties.

Here, we formulate the general theory of stabilization by the lithium steams and then, give examples illustrating the new mechanism of stabilization.

If an internally stable plasma is surrounded by a conducting shell (with or without metal flow), then the remaining instabilities are the free boundary, so-called "resistive wall modes" (RWM) [1,2]. Their growth rate γ is determined by the resistive penetration time τ_{res}

$$\gamma \simeq \tau_{res}^{-1}, \quad \tau_{res} \equiv \mu_0 \sigma a h, \tag{1}$$

where σ is the conductivity of the wall, a a characteristic minor radius, and h the thickness of the metal shell (we dropped the poloidal wave number from the definition of τ_{res}). With such a small growth rate, the plasma inertia can be neglected and the growth rate is determined solely by the time evolution of the currents in the shell.

The metal flow and associated $\mathbf{V} \times \mathbf{B}$ electromotive force ($\mathbf{V}$ is the flow velocity) affect the excitation and evolution of eddy currents and, thus, also stability. The stabilization depends significantly on the flow pattern. Thus, just a rotating metal shell is not effective because the RWM can be locked into rotation [2]. On the other hand, the two stream pattern of magnetic propulsion (Fig. 1) eliminates the possibility of good mode locking. Instead, the mode can be locked only into one of the streams, thus, making the other stream equivalent to a perfectly conducting shell.

This physics consideration can be formalized and linked to the existing stability theory. Perturbations in the magnetic configuration can be expressed in terms of the perturbations of the vector potential $\tilde{\mathbf{A}}\exp(\gamma t)$ where γ is the linear (complex) growth rate. We will use a toroidal flux coordinate system ρ, θ, φ corresponding to the straight field-line representation of magnetic field with axisymmetric azimuth φ and with poloidal angle $\theta = 0$ on the high-field side of the tokamak chamber. The inner (plasma facing) and outer surface of the metal shell are specified by equations $\rho = a$ and $\rho = a + \delta a$ ($\delta a \ll a$). Outside the shell the coordinates represent a continuous extension of internal coordinates with, e.g., $\rho = \infty$ corresponding to infinity and to the axis of symmetry.

Vector potential can be taken in the following covariant form

$$\tilde{\mathbf{A}} \equiv \eta(\rho,\theta,\varphi)\nabla\rho + \phi(\rho,\varphi)\nabla\theta + \psi(\rho,\theta,\varphi)\nabla\varphi, \tag{2}$$

where ϕ does not depend on θ, and ψ is an oscillatory function of θ

$$(\psi)_0 = 0, \tag{3}$$

where $(\ldots)_0$ means averaging over θ. The perturbation of the magnetic field can be expressed either in contravariant form

$$\begin{aligned}\tilde{\mathbf{B}} = &(\psi'_\theta - \phi'_\varphi)(\nabla\theta\times\nabla\varphi) + (\eta'_\varphi - \psi'_\rho)(\nabla\varphi\times\nabla\rho)\\ &+(\phi'_\rho - \eta'_\theta)(\nabla\rho\times\nabla\theta)\end{aligned} \tag{4}$$

or in covariant form

$$\tilde{\mathbf{B}} = \tilde{B}_\rho\nabla\rho + \tilde{B}_\theta\nabla\theta + \tilde{B}_\varphi\nabla\varphi. \tag{5}$$

Covariant components, which will enter into matching conditions across the metal layer can be expressed in terms of η, ϕ, ψ using standard relationships between co- and contravariant components of vectors.

We consider two lithium streams of magnetic propulsion flowing along the plasma boundary from the high-field side of the torus to the low-field side. The velocity of the flow is assumed to be purely poloidal, $\mathbf{V} = V(\theta)\mathbf{e}_{pol}$, where $\mathbf{e}_{pol}$ is the unit vector in the poloidal direction. For each stream, the velocity $V(\theta)$ and the height $h(\theta)$ of the stream are related by equations

$$
\begin{aligned}
V(\theta)|_{0<\theta<\pi} &= V\frac{Rh}{r_w(\theta)h(\theta)}, \\
V(\theta)|_{-\pi<\theta<0} &= -V\frac{Rh}{r_w(\theta)h(\theta)},
\end{aligned} \tag{6}
$$

where $r = r_w(\theta)$ together with $z = z_w(\theta)$ is a parametric representation of the wall contour, R is a characteristic major radius, and V and h are the velocity and height of the flow at this radius, respectively.

The current density in the thin shell can be written in terms of the stream function $I(\theta, \varphi)$:

$$
\begin{aligned}
&\mu_0\tilde{\mathbf{j}}_w \equiv -\frac{I'_\varphi}{J_w}\mathbf{e}_\theta + \frac{I'_\theta}{J_w}\mathbf{e}_\varphi, \quad |\mathbf{e}_\theta| = \sqrt{g_{\theta\theta}}, \quad |\mathbf{e}_\varphi| = 1, \\
&J_w \equiv r_w(\theta)h(\theta)\sqrt{g_{\theta\theta}}, \quad g_{\theta\theta} = (r_w)'^2_\theta + (z_w)'^2_\theta.
\end{aligned} \tag{7}
$$

Here, J_w plays the role of the Jacobian for an infinitesimally thin shell. With this representation the matching conditions across the shell can be written as:

$$
\left(\tilde{B}_\varphi|_{a+\delta a} - \tilde{B}_\varphi|_a\right)_0 = \left(I'_\varphi\right)_0, \quad \tilde{B}_\theta|_{a+\delta a} - \tilde{B}_\theta|_a = I'_\theta. \tag{8}
$$

Ohm'law in the metal

$$
\tilde{\mathbf{j}}_w = \sigma[-\gamma\tilde{\mathbf{A}} - \nabla\phi_E + (\mathbf{V}\times\tilde{\mathbf{B}})], \tag{9}
$$

after elimination of the scalar potential ϕ_E, gives an equation for I:

$$
\begin{aligned}
\frac{hg_{\theta\theta}}{J_w}I''_{\varphi\varphi} + \left(\frac{hr^2}{J_w}I'_\theta\right)'_\theta &= -\mu_0 h\sigma\gamma(\psi'_\theta - \phi'_\varphi) \\
&\quad -R_m\left[\frac{v(\theta)(\psi'_\theta - \phi'_\varphi)}{\sqrt{g_{\theta\theta}}}\right]'_\theta.
\end{aligned} \tag{10}
$$

In Eq.(10), R_m is the magnetic Reynolds number for the flow calculated based on the height of the stream and $v(\theta)$ is the normalized velocity profile

$$R_m \equiv \mu_0 \sigma V h, \quad v(\theta) \equiv \frac{V(\theta)}{V}. \tag{11}$$

For lithium, $\mu_0\sigma \simeq 4$ sec/m^2. Thus, with characteristic velocities of the order of 20 m/sec, it is possible to obtain Reynolds numbers of the order of unity even for thin lithium streams $h \simeq 1$ cm.

Because of the gap in the wall at the outlet point, I should satisfy boundary conditions

$$I(\theta = \pi \pm \alpha, \varphi) = const \tag{12}$$

at the gap sides $\theta = \pi \pm \alpha$, where the angle 2α specifies the extend of the gap.

Equation (8) represents the matching conditions between plasma and vacuum regions and, thus, introduce the effect of metal flow on stability of the free boundary modes.

In Fourier space, conventional for numerical stability codes, it is possible to define a vector function $\vec{\psi}(\rho)$

$$\psi = \sum_{m \neq 0} (\vec{\psi})_m e^{im\theta - in\varphi}, \quad \phi = -(\vec{\psi})_0 e^{-in\varphi}, \tag{13}$$

which is composed from the Fourier harmonics of the components of the vector potential. The full set of independent solutions to the perturbed equilibrium equations (relevant to the resistive wall modes porblem) inside the plasma as well as to the field equations in the vacuum region can be represented in the form of the superposition of the normalized fundamental solutions

$$\vec{\psi}_{p,v}(\rho) = \mathtt{U}_{p,v}(\rho)\vec{\psi}(a), \quad \mathtt{U}_p(0) = 0, \quad \mathtt{U}_v(\infty) = 0, \tag{14}$$

where each column $\mathtt{U}^{m'}_{p,v}$ in matrix functions $\mathtt{U}_{p,v}$ represents an independent solution either in the plasma (index "p") or in the vacuum (index "v") regions. At the plasma boundary $\rho = a$, the $\mathtt{U}_{p,v}$ are the identity matrices, $\mathtt{U}_{p,v}(a) = \mathtt{I}$.

In addition to $\mathtt{U}_{p,v}$, it is possible to introduce related matrices $\mathtt{D}_{p,v}$, which for every fundamental solution $\mathtt{U}^{m'}_{p,v}$ represent the left-hand side of the matching conditions (8)

$$(\mathtt{D})_{m\neq 0,m'} \equiv -\tilde{B}^{m'}_{\theta,m\neq 0} \quad \text{and} \quad (\mathtt{D})_{0,m'} \equiv -\tilde{B}^{m'}_{\varphi,0}. \tag{15}$$

Here, $\tilde{B}^{m'}_{\theta,m}$, $\tilde{B}^{m'}_{\varphi,0}$ are Fourier coefficients of $\tilde{B}_\theta(a,\theta,\varphi)$, $\tilde{B}_\varphi(a,\theta,\varphi)$ corresponding to $\mathtt{U}^{m'}_{p,v}$. For any plasma equilibrium, $\mathtt{D}_v$, $\mathtt{D}_p$ can be calculated using standard stability codes for free-boundary MHD modes.

Using the matrices $\mathtt{D}_{p,v}$, the matching conditions (8) can be written in the form

$$(\mathtt{D}_v - \mathtt{D}_p)\vec{\psi}(a) = -i\mathtt{M}\vec{I}, \tag{16}$$

where vector $\vec{I}$ represents Fourier expansion of $I(a,\theta,\varphi)$ and the matrix $\mathtt{M}$ is defined as

$$\mathtt{M}_{m\neq 0,m'} \equiv m\delta_{mm'} \quad \text{and} \quad \mathtt{M}_{0,m'} \equiv -n\delta_{0m'}. \tag{17}$$

Eq. (10) for $\vec{I}$ can be written in the matrix form as

$$\mathtt{S}\vec{I} = i\mu_0 h\sigma\gamma\mathtt{M}\vec{\psi} - R_m\mathtt{VM}\vec{\psi}, \tag{18}$$

where the matrices $\mathtt{S}$ and $\mathtt{V}$ are defined as

$$\begin{aligned} (\mathtt{S})_{mm'} &= mm'\left(h\frac{r^2}{J_w}\right)_{m-m'} + n^2\left(\frac{hg_{\theta\theta}}{J_w}\right)_{m-m'}, \\ (\mathtt{V})_{mm'} &\equiv m\left(\frac{v(\theta)}{\sqrt{g_{\theta\theta}}}\right)_{m-m'}. \end{aligned} \tag{19}$$

Its solution, after subsitution into Eq. (16) gives a dispersion relation for γ

$$(\mathtt{D}_v - \mathtt{D}_p)\vec{\psi}(a) = \mu_0 h\sigma\gamma(\mathtt{MS}^{-1}\mathtt{M})\vec{\psi} + iR_m(\mathtt{MS}^{-1}\mathtt{VM})\vec{\psi}. \tag{20}$$

This final equqtion of the present theory has the following structure. The matrices $\mathtt{D}_v - \mathtt{D}_p$ and $\mathtt{MS}^{-1}\mathtt{M}$ are Hermitian. Their off-diagonal terms containing, in particular, the toroidal ballooning mode coupling, make a destabilizing contribution. The matrix $iR_m\mathtt{MS}^{-1}\mathtt{VM}$ describing the flow effect is essentially anti-Hermitian. Its off-diagonal terms, producing the mode coupling, contribute positively to stability.

As an example, we consider a tokamak plasma with a circular cross section $(a/R \ll 1)$ and a constant velocity of each stream $v(\theta)|_{0<\theta<\pi} = 1$, $v(\theta)|_{-\pi<\theta<0} = -1$ in Eq.(10). Neglecting effect of the gap in the flow, Fourier expansion of $v(\theta)$ is given by

$$v(\theta) = \sum_k v_{2k+1} e^{i(2k+1)\theta}, \quad v_{2k+1} = \frac{2}{i\pi(2k+1)}. \tag{21}$$

The dispersion relation (20) can be written as an eigenvalue problem for a system of linear equations

$$\begin{aligned}(a\Delta'_{mm} - \tau_{res}\gamma)\psi_m = &- \sum_{m' \neq m} a\Delta'_{mm'}\psi_{m'} \\ &+ iR_m \sum_k (m + 2k + 1) v^*_{2k+1} \psi_{m+2k+1},\end{aligned} \tag{22}$$

where $a\Delta'_{mm'} \equiv R(\mathtt{D}_v - \mathtt{D}_p)_{mm'}$.

In the case with $B_{pol}/B_{tor} \ll 1$ and $2\mu_0 p/B^2_{tor} \ll 1$ (p is the plasma pressure) matrix elements $a\Delta'_{mm'}$ can be calculated from the approximate form of the δW energy principle of Refs. [10,11]

$$\begin{aligned}\delta W = \sum_m \frac{\pi}{R} \int_0^\infty \Bigg\{ &\rho|\psi'_m|^2 + \frac{m^2}{\rho}|\psi_m|^2 + \frac{j'R}{B(\mu - \mu_m)}|\psi_m|^2 + 4\frac{\rho}{R^2}\beta_{pol} m^2 \frac{2\beta_{pol} + 1 - \frac{n^2}{m^2}}{(m - nq)^2}|\psi_m|^2 \\ &- 2\frac{\beta_{pol}}{R}\frac{m(m+1)\psi_m\psi^*_{m+1}}{(m - nq)(m + 1 - nq)} - 2\frac{\beta_{pol}}{R}\frac{m(m-1)\psi_m\psi^*_{m-1}}{(m - nq)(m - 1 - nq)} \\ &- 2\frac{\beta_{pol}}{R}\frac{m\psi_m(\rho\psi'^*_{m+1} + \rho\psi'^*_{m-1})}{m - nq} - 2\frac{\beta_{pol}}{R}\frac{(m+1)\rho\psi'_m\psi^*_{m+1}}{m + 1 - nq} - 2\frac{\beta_{pol}}{R}\frac{(m-1)\rho\psi'_m\psi^*_{m-1}}{m - 1 - nq}\Bigg\} d\rho.\end{aligned} \tag{23}$$

Here, q is the safety factor and β_{pol} is defined as:

$$\beta_{pol}(\rho) \equiv -\frac{\mu_0 \rho p'(\rho)}{2B^2_{pol}(\rho)}. \tag{24}$$

In particular, for a classical example of a plasma with a uniform current distribution $q(\rho) =$ const [12] and parabolic pressure profile $\beta_{pol}(\rho) =$ const the matrix $a\Delta'_{mm'}$ has an explicit three-diagonal form

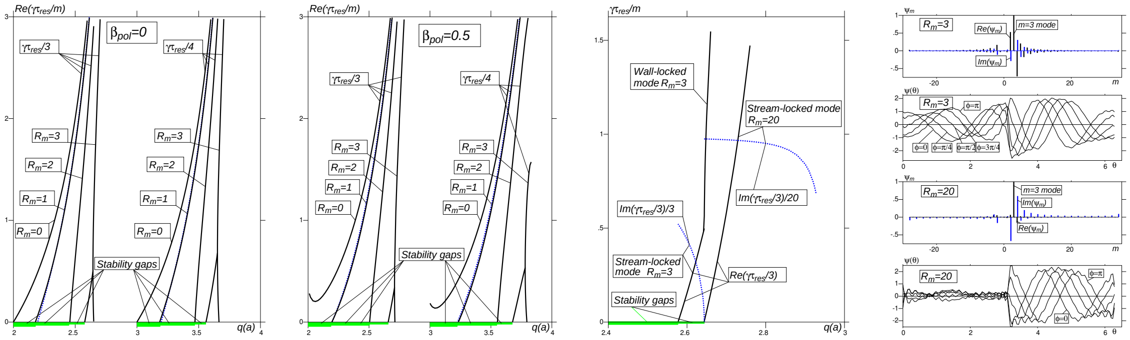
$$\begin{aligned}a\Delta'_{m,m} &= -2m\frac{m - 1 - nq}{m - nq} \\ &\quad - \beta_{pol}\frac{a^2}{R^2}\frac{m+2}{m+1}\left(\beta_{pol} + 1 - \frac{n^2}{m^2}\right)\frac{m^2}{(m - nq)^2}, \\ a\Delta'_{m-1,m} &= a\Delta'_{m,m-1} = 2\beta_{pol}\frac{a}{R}\frac{m-1}{m - 1 - nq}.\end{aligned} \tag{25}$$

In order to make stabilization by the flow evident, let us consider the case $\beta_{pol} = 0$ and $R_m < 1$, when only the coupling of the satelite harmonics ($m' \neq m$) with the main harmonic m is essential. Then Eq.(22) can be reduced to an algebraic equation for γ:

$$
\begin{aligned}
a\Delta'_{mm} - \tau_{res}\gamma + mR_m^2 \sum_k \frac{m'|v_{2k+1}|^2}{a\Delta'_{m'm'} - \tau_{res}\gamma} &= 0 \\
m' \equiv m + 2k + 1. &
\end{aligned} \qquad (26)
$$

In a typical situation of an unstable low m, n mode with $a\Delta'_{mm} > 0$, the satellite modes are stable, i.e., $a\Delta'_{m'm'} < 0$. Because of this, all terms in the sum are negative and have a stabilizing effect on the non-rotating RWM. Mutual coupling of the satelite harmonics would lead to additional terms in this equations with higher powers of R_m and should be taken into account numerically.

Growth rates of RWM for a unifrom current distribution are shown in Fig. 2a ($\beta_{pol} = 0$) and Fig. 2b ($\beta_{pol} = 0.5$). In the absence of flow the configuration is unstable [1,2] over the entire region $2 \leq q_a \leq 4$. With no resonant surfaces inside the plasma, it cannot be stabilized by the nonideal plasma effects considered in the literature [3–9]. The metal flow creates stability gaps depending on the flow rate.



(a) (b) (c) (d)

Fig. 2. Real part of the growth rate of the wall- and stream-locked RWM modes for a circular plasma with a uniform current distribution, (a) $\beta_{pol} = 0$, (b) $R/a = 4$, $\beta_{pol} = 0.5$. (c) Growth rates and rotation frequency of the $m = 3$ mode for $R_m = 3$ and $R_m = 20$. (d) Angular harmonic spectrum and waveforms of the marinally stable stream-locked modes for $R_m = 3$ and $R_m = 20$.

It possible also to notice from both the structure of Eq. (20) and from Figs. 2a,b that for a given n, the size of the stability gaps is not sensitive to the poloidal mode number m. Thus, both low- and high-m modes can be stabilized by the lithium flow.

For $R_m = 1$ on Fig.2a,b there are superimposed (almost undistinguishable) curves corresponding to a 3 harmonic approximation in Eq.(20). Thus, such a simple approximation in the theory is valid for all range $R_m \leq 1$.

For $R_m = 3$ in addition to the conventional wall-locked mode, which has a purely real growth rate $\Im(\gamma) = 0$, Eq. (20) contains additional roots corresponding to a new mode which rotates and tends to be "locked" into one of the streams. For this mode another stream acts like a perfectly conducting wall. The stream-locked mode is insensitive to the flow velocity and determines the limits for the flow stabilization. For $m = 3$ mode, Fig.2c shows both real and imaginary parts of the growth rate for $R_m = 3$ and $R_m = 20$. Normalization of imaginary part is such that 1 corresponds to rotaion synchronized with one of the streams. At the marginal stability point for $R_m = 3$ ($q = 2.59$) the $m = 3$ mode is still partially locked, while for an extreme case $R_m = 20$ it rotates with one of the stream. Fig.2d shows the Fourier spectrum (rather broad for $R_m = 20$) and waveforms of the stream-locked modes. It is seen that for the very fast flow the perturbation is significantly suppressed on the side of the slipping stream $0 < \theta < \pi$.

Summarizing, the intense lithium streams, driven by magnetic propulsion, lead to improved stability of the tokamak free-boundary modes. The stabilizing becomes essential when R_m (11) exceeds 1. The general theoretical formalism presented in this paper is applicable for any flow pattern of the steams and geometry of the guide wall. It allows to calculate the eddy currents as well as to use the existing numerical codes for evaluation of growth rates and stability conditions for resistive wall instabilities in the presence of flow.

This work was supported by United States Department of Energy Contract No. DE-AC02-76-CHO-3073.

REFERENCES

[1] P. H. Rutherford, *Basic Physical Processes of Toroidal Fusion Plasmas (Proceedings of the Courses Workshop held at Varenna, 1985)* (CEC, Brussels, 1986), Vol. 2, p. 531.

[2] L. E. Zakharov and S. V. Putvinskii, Sov. J. Plasma Phys. **13**, 68 (1987).

[3] A. Bondeson and D. J. Ward, Phys. Rev. Lett. **72**, 2709 (1994).

[4] J. M. Finn, Phys. Plasmas **2**, 3782 (1995).

[5] R. Betti and J. P. Freidberg, Phys. Rev. Lett. **74**, 2949 (1995).

[6] J. M. Finn and R. A. Gerwin, Phys. Plasmas **3**, 2344 (1996).

[7] A. Bondeson and D. X. Xie, Phys. Plasmas **4**, 2081 (1997).

[8] J. M. Finn and C. P. Sovenic, Phys. Plasmas **5**, 461 (1998).

[9] A. Bondeson, C. G. Gimblett, and R. J. Hastie, Phys. Plasmas **6**, 637 (1999).

[10] L. E. Zakharov, Nucl. Fusion **18**, 335 (1978).

[11] O. P. Pogutse and E. I. Yurchenko, Nucl. Fusion **18**, 1629 (1978).

[12] V. D. Shafranov, Sov. Phys. - Techn. Phys. **40**, 241 (1970).

FIGURES

FIG. 1. (a) Cross section of the tokamak with two lithium streams at the top and bottom halves of the vacuum chamber.

FIG. 2. Real part of the growth rate of the wall- and stream-locked RWM modes for a circular plasma with a uniform current distribution, (a) $\beta_{pol} = 0$, (b) $R/a = 4$, $\beta_{pol} = 0.5$. (c) Growth rates and rotation frequency of the $m = 3$ mode for $R_m = 3$ and $R_m = 20$. (d) Angular harmonic spectrum and waveforms of the marinally stable stream-locked modes for $R_m = 3$ and $R_m = 20$.