Tokamak MHD (TMHD) - the theory/simulation model of tokamak VDE disruptions

***Leonid E. Zakharov* (PPPL)**

in collaboration with

***Sergei Gerasimov, JET Contributors*[1] (CCFE, UK) and *Xujing Li* (ICM&SEC, China)**

[1]See the Appendix of F. Romanelli et al., Proceedings of the 25th IAEA Fusion Energy Conference 2014, Saint Petersburg, Russia

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Abstract

Since 2007, a great progress was made in understanding VDEs: the theory of the Wall Touching Kink and Vertical Modes (WTK, WTVM) was created. It has explained the sideways forces and toroidal asymmetries in plasma current and diamagnetic signal in JET tokamak, which is equipped with uniquely capable magnetic diagnostics. The Hiro currents, generated by the plasma flow to the wall, was introduced as the main inductive effect in disruptions. The installation of a special diagnostics for the Hiro current measurements in pure axisymmetric VDE on the EAST tokamak was motivated. Being consistent in direction with the wall current measurements on JET and EAST, the Hiro currents undermined the conventional concept of "halo" currents, which would have the opposite direction, as the source of sideways forces. Instead, the theory revealed the real role of "halo" currents in the form of Evans currents (introduced by the theory instead of the proper name but compromised by association with a non-existent poloidal current drive and sideways forces). These currents are limited by the ion saturation current and can explain the earlier measurements of currents to the tile surfaces in tokamaks without mysteries of a poloidal current drive.

Finally, the theory of VDE disruptions was formalized as a set of Tokamak MHD (TMHD) equations. In contrast to the existing MHD codes, essentially hydrodynamical, the TMHD model gives the basis for development of the plasma physics based numerical codes, consistent with the exceptional anisotropy of the high temperature tokamak plasma. In 2014, the 2-D VDE code was created in PPPL based on TMHD model. For the first time, it shown 5 VDE regimes and demonstrated the generation of Hiro currents in axisymmetric vertical instabilities. VDE code simulations motivated the design and future installation of a special tile diagnostics for measurements of the Hiro and Evans currents on NSTX-U.

Contents

1 Large VDE on JET (Aug.10,1996, 16:54:12, #38070)

In 2007, ITER project was alarmed by prediction of 40-50 MN sideways forces due to excitation of the m/n=1/1 kink mode scaled from JET

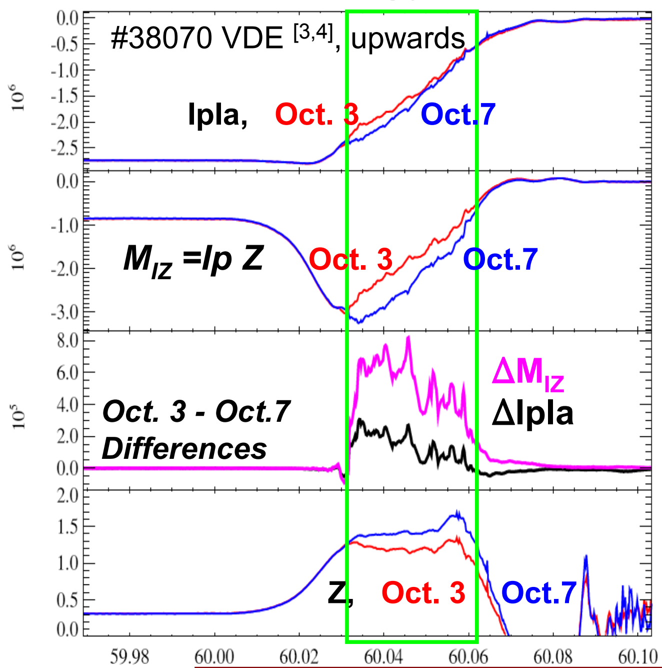
Long 30 ms duration of

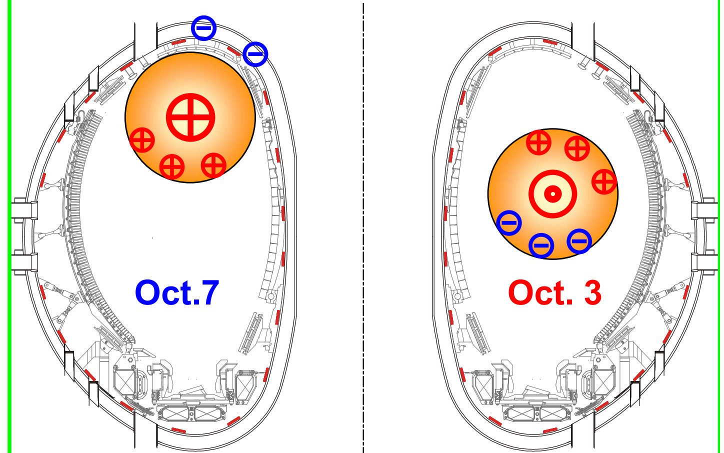
would be the fastest tokamak kink mode
$m/n = 1/1$

was a puzzle for theory in 2007

EFDA JET

Vessel current during VDE, #38070





- In octant 7 the plasma is closer to top of the vessel than in octant 3.
- **The current from plasma flows on vessel in octant 7.**

The measured Ipla in octant 7 is higher then in octant 3 ➔ the missing vessel current in octant 7 is OPPOSITE to Ipla!

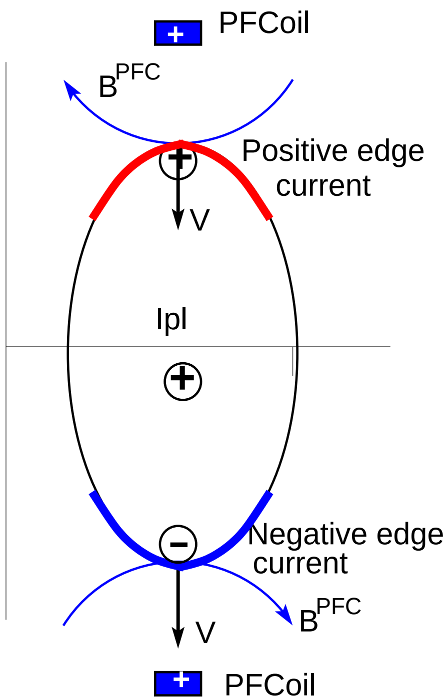
The “halo” current based interpretation predicts the opposite sign of asymmetry in the current measurement and contradicts JET Ipla’s.

UKAEA

S N Gerasimov et al, Scaling JET Disruption Data to ITER. W70 7/10/09 8/24

2 TMHD essence: VDE is a fast equilibrium evolution

$$\tau_{MHD} \simeq \frac{R}{V_A} = \underbrace{\frac{R}{2.18\cdot 10^6 B/\sqrt{n}}}_{<1\ \mu s} \ll \underbrace{\tau_{VDE}}_{\simeq 1-50\ ms} \ll \underbrace{\tau_{transport}}_{\simeq 0.1\ s} \ll \underbrace{\tau_{resistive}}_{\simeq 1\ s} \qquad (2.1)$$



VDE is a special FAST equilibrium evolution,
which preserves the magnetic fluxes

TMHD predicts the direction and value or the currents generated at the plasma surface (edge)

- ***negative** (opposite to the plasma current) at the leading side*
- ***positive** at the trailing side*

$$-\frac{\partial \vec{A}^{i,surf}}{\partial t} \underbrace{- \frac{\partial \vec{A}^{pl,core}}{\partial t} + V B_{pl} \vec{e}_\varphi}_{vanishes\ \ for\ \ m/n=1/0} + \underbrace{\vec{V}\times\vec{B}^{PFC}}_{Driving\ EMF} - \nabla\phi_E^{surf} = \frac{\vec{\jmath}}{\sigma} \qquad (2.2)$$

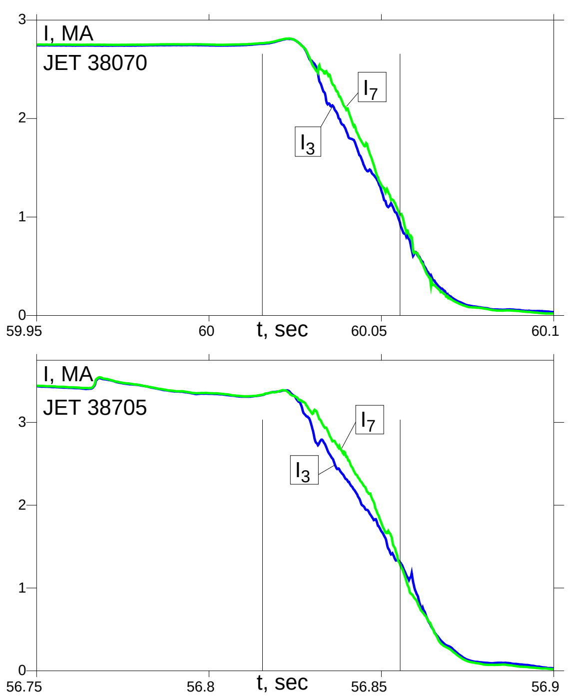
The edge negative currents are converted into the wall Hiro currents when plasma touches the tiles or wall surface.

This (first) fundamental TMHD result is supported by the entire JET disruption database (more than 2000 relevant cases).

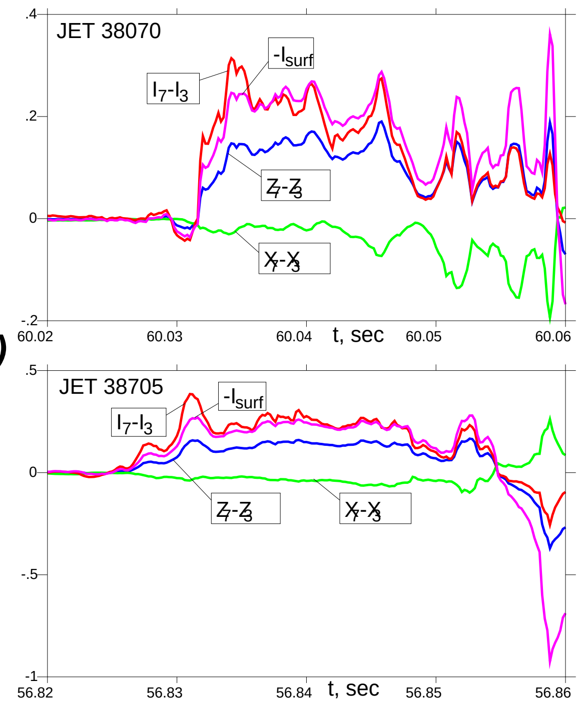
Theory resolved the puzzle and confirmed JET scaling

The unexpected direction of wall currents was explained by theory of Wall Touching Kink Mode and Hiro currents.

a)



b)



(a) Plasma currents $I_{3,7}$ in octants 3,7 on JET during the disruptions.
(b) $Z_7 - Z_3$ and $R_7 - R_3$, $I_7 - I_3$ and its prediction $-I^{surf}$ from the present theory.

$$\mu_0 \vec{\imath}_{11} = -2\xi_{11}\frac{B_\varphi}{R}\left(\mathrm{e}_\varphi + \frac{a}{R}\mathrm{e}_\theta\right), \quad I_{Hiro} \simeq I_{surf} = -4a\frac{B_\varphi}{R\mu_0}\frac{\delta Z_{7,3}}{2}. \tag{2.3}$$

The second outstanding result of TMHD is the amazing consistency of waveforms of Hiro currents with the measured wall currents on JET.

Force balance versus plasma inertia in disruptions

All existing 3-D MHD codes are driven by the plasma inertia term in equation of motion

$$\rho\frac{d\vec{V}}{dt} = -\nabla p + (\vec{j}\times\vec{B}) + \textit{(hyper-) viscosity}, \qquad (2.4)$$

which causes a severe problem of Courant time step limitation, unsolved for 4 decades.

JET discharge parameters of 38070:

n_e	$\simeq 3\cdot 10^{19}$	*plasma density*
V_{olume}	$\simeq 60$ *m*^{3}	*plasma volume*
ξ	< 0.3 *m*	*amplitude of the m/n=1/1 perturbation*
Δt	*25 ms*	*duration of m/n=1/1 perturbation*
F_x	$\simeq 2.4$ *MN*	*the sideways force*

The force of plasma inertia vs MHD forces

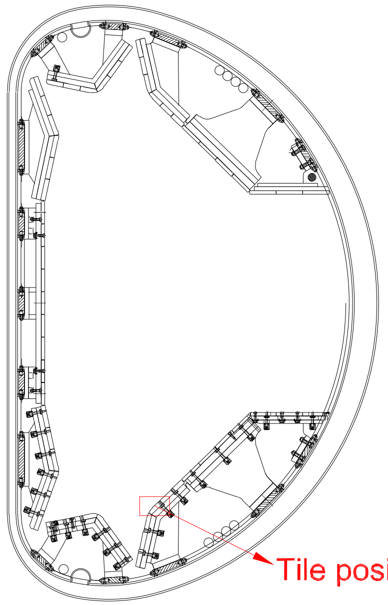
$$F_\rho \simeq m_i n_i \cdot V_{olume} \cdot \frac{2\xi}{(\Delta t)^2} \simeq 0.006\ [N] \quad \ll 2.4\cdot 10^6 [N]. \qquad (2.5)$$

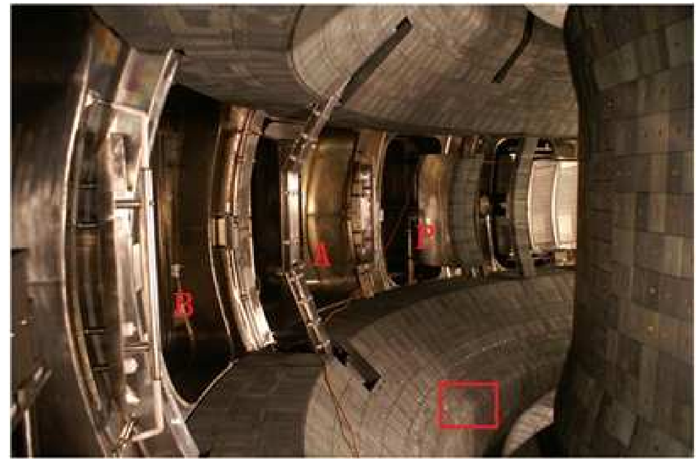
The third fundamental result of TMHD is the revelation of the outstanding 10^8 mismatch in driving forces or 10^4 in the time scale between 3-D codes and VDEs in tokamaks.

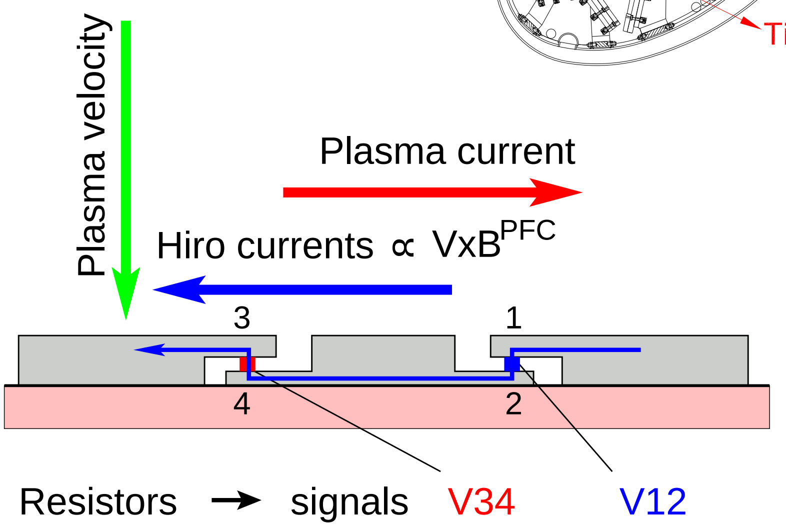
The JET VDE data devaluate all existing plasma inertia driven MHD codes in VDE applications.

3 Hiro currents for n=0 VDE. Xiong tiles on EAST

The 4th result of TMHD is prediction of Hiro currents in n=0 VDE and motivation of their measurements on EAST.





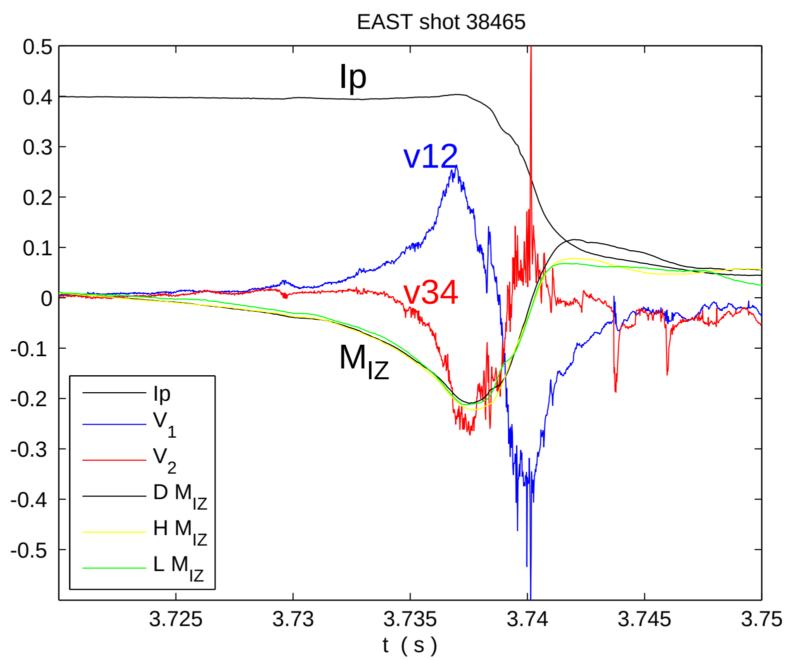


Two resistors between 3 shaped Mo tiles mounted through the thermal contact on a 2 cm thick copper heat sink plate, point-wise grounded

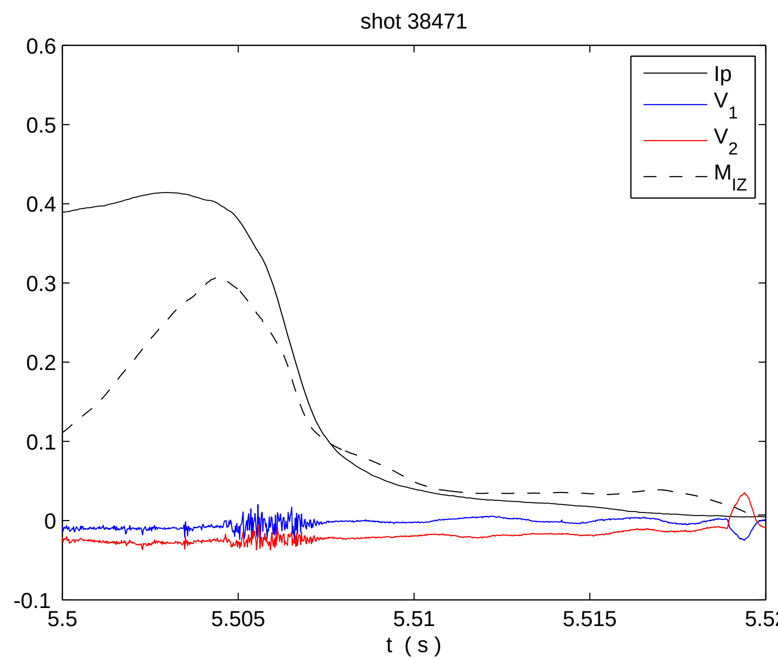
4 types of currents can be distinguished by the Xiong tiles.

First measurements of Hiro currents in n=0 VDE

Toroidal Hiro currents ($\simeq 0.8kA$), opposite to the plasma current, were measured on EAST in May 2012 for the first time in an axisymmetric VDE



Downward VDE



Upward VDE

Pure n=0 VDE: Miz, Ipl and Mirnov signals from three cross-sections are identical

Hiro currents in n=0 VDE are NOT SHARED between plasma and the tiles.
They cannot be confused with the "halo" currents
as Boozer tried to do.

The Tokamak MHD is presented by the following set of equations

1. ***Equation of motion is split into an equilibrium equation***

$$\nabla p = (\vec{j} \times \vec{B}), \quad \bar{\Psi} = \bar{\Psi}(\bar{\Phi}), \tag{4.1}$$

and the plasma boundary advancing equation

$$\lambda \vec{\xi} = -\frac{\bar{F}}{r^2} \nabla \tilde{F}, \quad \left(\nabla \cdot \frac{\bar{F}^2}{r^4} \nabla \tilde{F} \right) = 0. \tag{4.2}$$

2. ***Faraday's (Ohm's) law in plasma and the wall (with no $\vec{V}$)***

$$-\frac{\partial \vec{A}}{\partial t} - \nabla \varphi^E + (\vec{V} \times \vec{B}) = \frac{\vec{j}}{\sigma}, \quad \vec{V} \equiv \frac{d\vec{\xi}}{dt}. \tag{4.3}$$

3. ***Plasma anisotropy***

$$\sigma = \sigma(\bar{\Phi}), \quad (\vec{B} \cdot \nabla) \simeq 0. \tag{4.4}$$

4. ***Boundary condition at the wetting zone (determines plasma V_{normal} to the wall)***

$$(\vec{V} \times \vec{B}) - \nabla \phi^E = \frac{\vec{j}^{wall}}{\sigma^{wall}}. \tag{4.5}$$

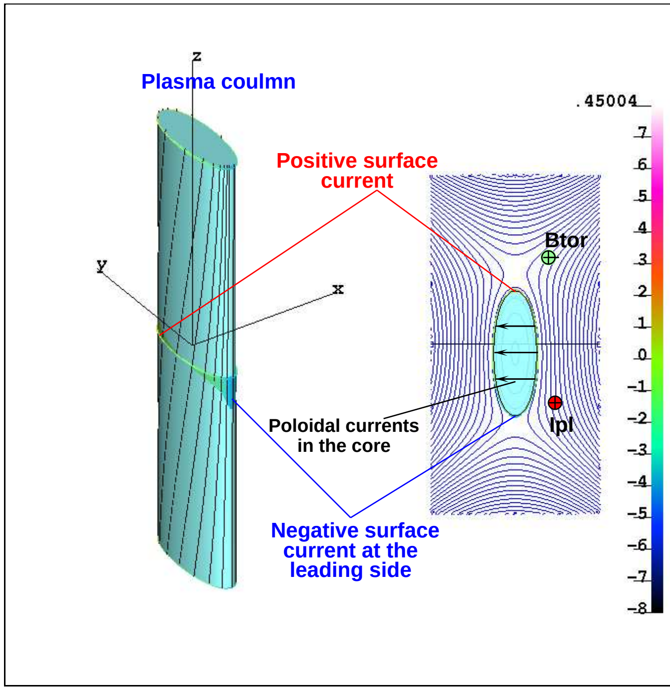
Force balance across the free plasma surface

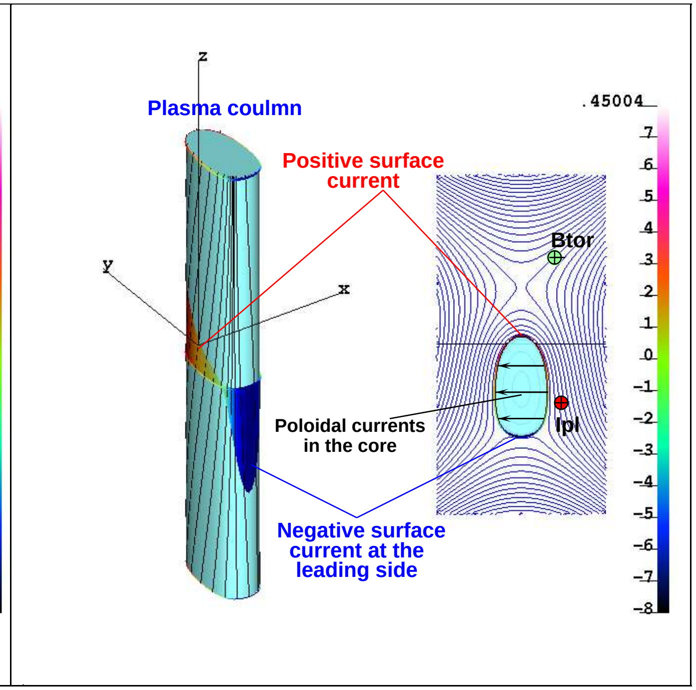
$$\left(p + \frac{|\vec{B}|^2}{2\mu_0} + \frac{\bar{F}\tilde{F}}{r^2 \mu_0} \right)_i = \left(\frac{|\vec{B}|^2}{2\mu_0} \right)_e, \tag{4.6}$$

where subscripts 'i, e' specify the inner and outer sides of the plasma surface.

5 Xujing Li discovery of a new VDE-reconnection pattern

2-D (n=0) version of VDE code was created in 2014 by X.Li.
The first run for the simplest case resulted in discovery of a new reconnection pattern (the 5th outstanding result of TMHD)





Elliptic plasma cylinder,

j=const,
strong Bz,
quadrupole external field Bext,

no walls.

Initial downward plasma displacement

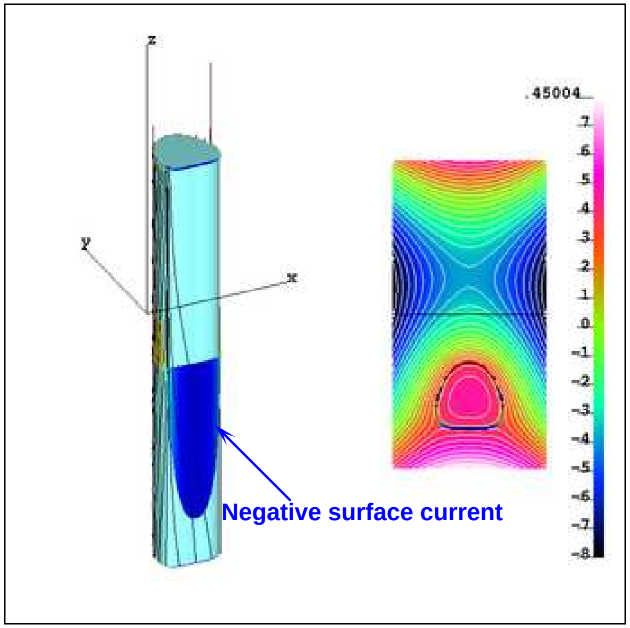
Nonlinear phase of instability. Negative surface current at the leading edge

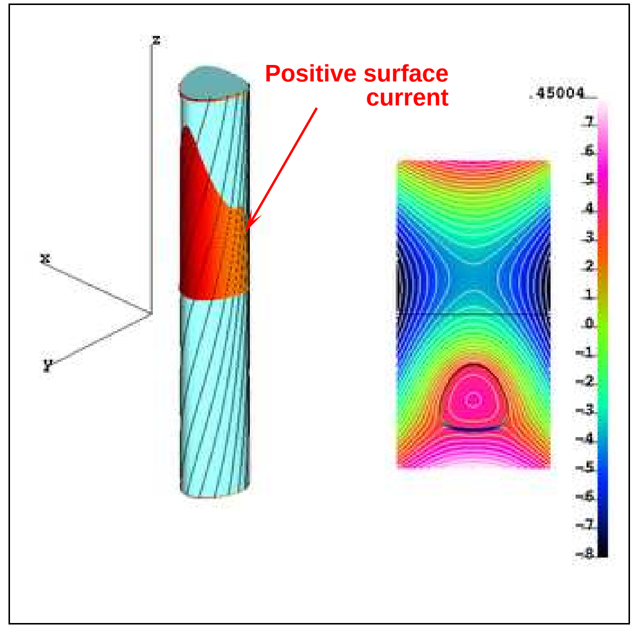
Surface currents are generated at the plasma boundary due to flux conservation

Neither DINA nor TSC, used for ITER and NSTX VDE simulations, were tested by this simplest case.

Strong external field stops vertical plasma motion

New geometry for reconnection is discovered: unstable plasma reconnects with the vacuum magnetic field





1. Strong negative current is generated at the leading plasma edge
2. Plasma cross-section becomes triangle-like

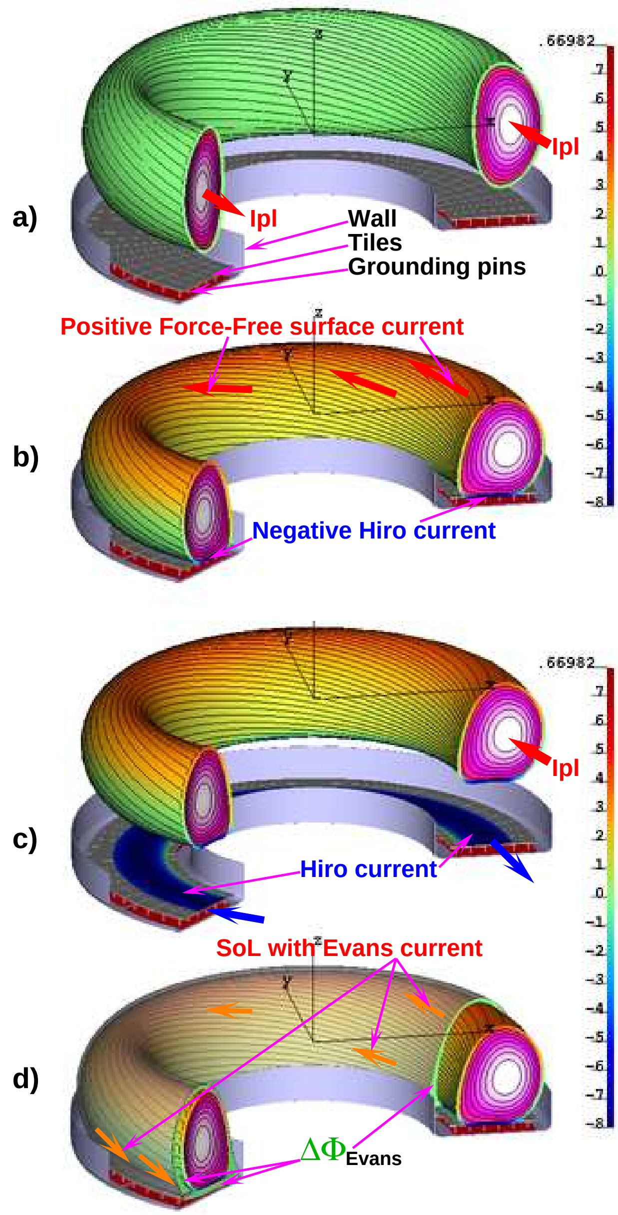
1. Strong positive current is generated at the free plasma edge
2. Plasma stops moving in vertical direction.

(a) opposite poloidal field $B_\theta^{vac} \simeq -B_\theta^{core}$ across the leading plasma edge;

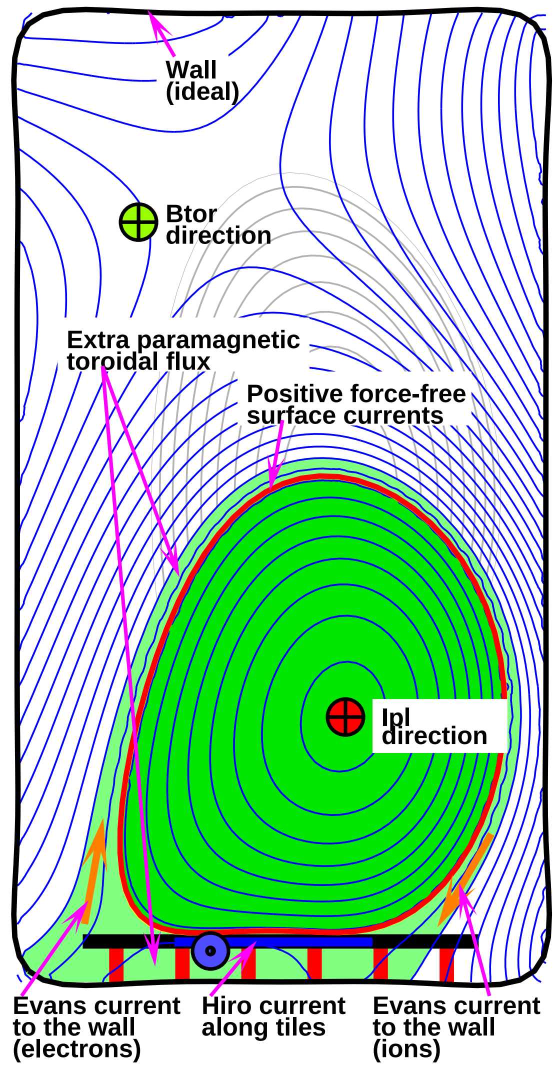
(b) two Null Y-points of poloidal field in two vertices of plasma cross-section.

Plasma should be leaked through the Y-point untill full disappearance. The simulation of reconnection is beyond the scope of VDE-code.

6 TMHD and understanding of VDE in tokamaks



5 regimes found for VDE in tokamaks and VDE-code.
TMHD gives the proper understanding of the halo zone



The shrinking plasma core releases the plasma particles and creates the halo zone.

Loop voltage drives the Evans currents in the halo zone which are limited by particle source

$$I_{pol}^{Evans} < 2e\frac{dN_e}{dt}$$

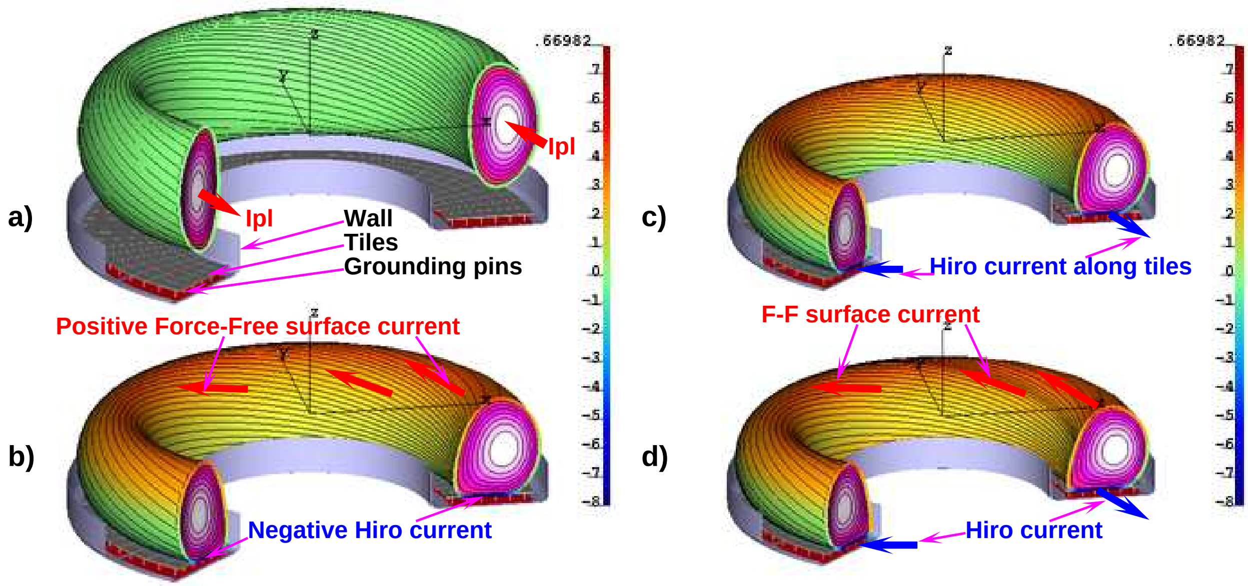
Because the right name "halo" was compromised by A.Boozer, TMHD uses the term Evans currents.

Evans currents are force-free

They are observed as the currents to the tiles surface.

The 6th result of TMHD is prediction of source limitation of the halo (Evans) currents

Plasma shrinking due to decay of Hiro currents

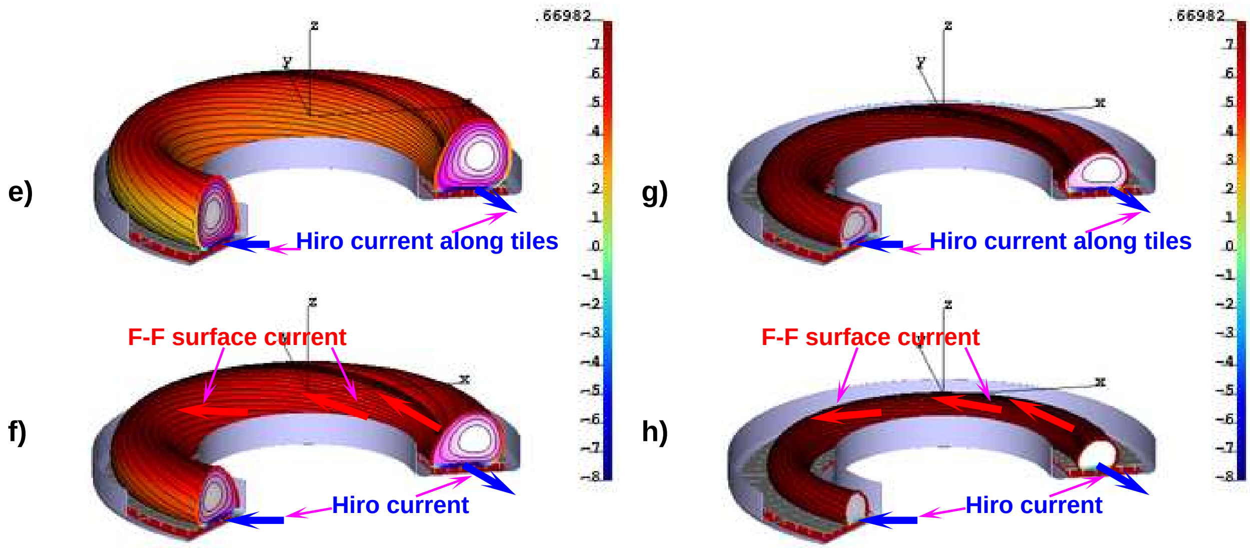


During plasma shrinking I_{pl} decays

The poloidally symmetric compensation current at the plasma edge

$$I^{FF} \simeq -I^{Hiro} + I^{V_{loop}}$$

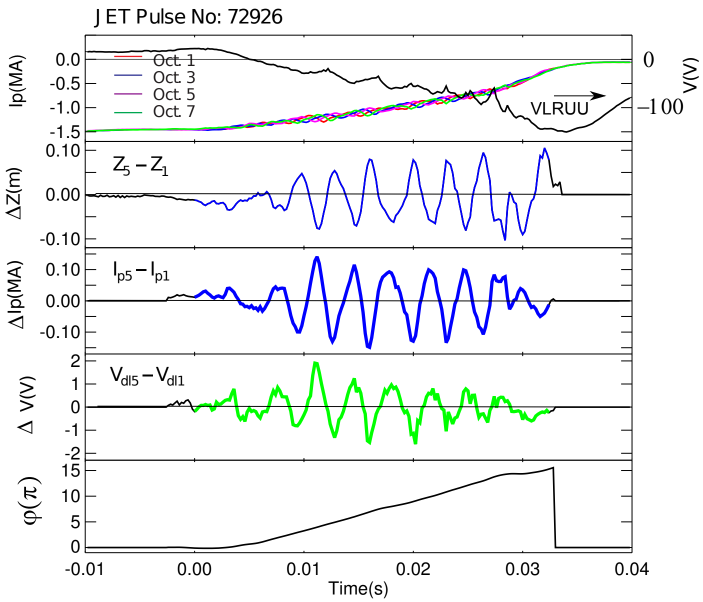
Magnetic VDE-type of reconnection happens at 2 Y-points in the Hiro current layer



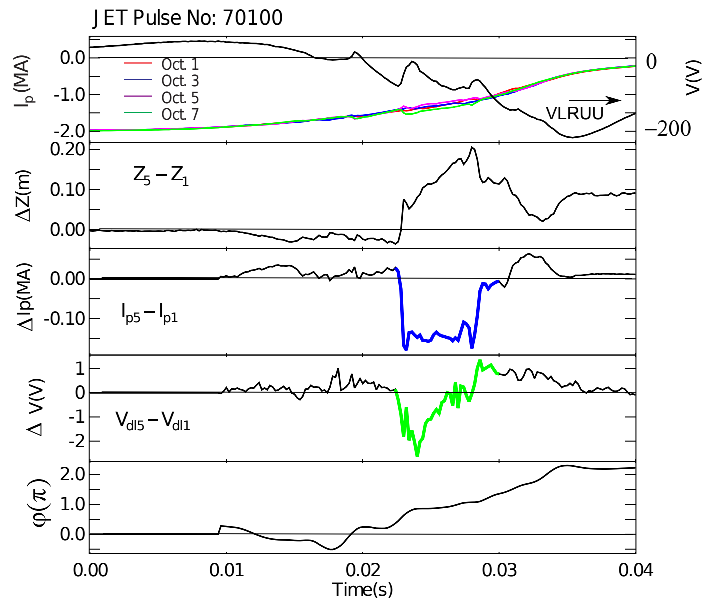
At the later stage the contribution of $I^{V_{loop}}$ enhances the darkness of the color of the total edge current I^{FF}

On the way, the kink mode m/n=1/1 can be developed

7 JET asymmetry in the Diamagnetic signal



Rotating 1/1 kink mode



Locked 1/1 kink mode

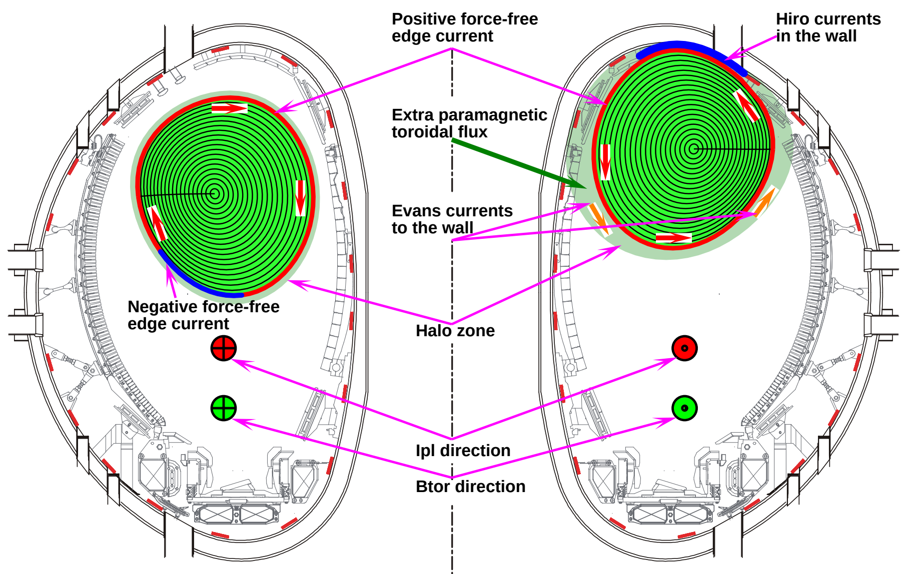
Ip, Loop Voltage

Kink 1/1 amplitude

Hiro currents

Paramagnetic Voltage

Toroidal phase



1. ***The halo-zone is toroidally localized in the vicinity of the wetting zone***
2. ***Both Hiro and Evans currents enter the wall structure (Hiro currents escape magnetic probes)***
3. ***The Hiro currents are situated right after the plasma core edge which have the same Φ in all cross-sections***
4. ***The Evans currents have a larger footprint and generate an extra paramagnetic flux***

The theory interpretation of $\delta\dot{\Phi}$ asymmetry

$< n_e >$	$3 \cdot 10^{19}$	- average core plasma density
V_{olume}	60 m^3	- plasma volume
t^{CQ}	$\simeq 25$ ms	- current quench time for assessment of $\dot{N}_e$
$Q = 2en_eV_{olume}$	600 A·s	- total particle charge

The source limitation on the Evans current

$$I^{Evans} \leq I^{S-Limit} \equiv 2e\frac{dN}{dt} = \frac{600}{0.025} \simeq 0.024 \text{ MA}. \quad (7.1)$$

For rotating mode 72926 $I^{S-Limit}$ is sufficient necessary)

$$I^{Evans}_{MA} \simeq 0.02 \cdot \frac{0.2}{\delta S^{Evans}_{m^2}} \cdot \frac{L^{wet}_m}{5} \text{ MA} < I^{S-Limit} \quad (7.2)$$

For the locked mode 70100 the limit is restrictive

$$I^{Evans}_{MA} \simeq \frac{\int V_{pol}dt}{\mu_0 \delta S^{Evans}} L^{wet} \qquad \simeq 0.04 \cdot \frac{0.2}{\delta S^{Evans}_{m^2}} \cdot \frac{L^{wet}_m}{5} \text{ MA} > I^{S-Limit} \quad (7.3)$$

I^{Evans} may have reached the saturation level causing the drop in dimagnetic signal $\delta\dot{\Phi}$

The outstanding 7th result of TMHD is interpretation of the toroidal asymmetry of diamagnetic signal on JET and explanation of its difference in rotating and locked kink modes.

8 TMHD duration time of MHD modes in VDE

Hiro currents i^{Hiro} are automatically generated in order to maintain the equilibrium during fast evolution

The Faraday law at the wall/tile surface

$$-\frac{\partial \mathrm{A}}{\partial t} - \nabla\phi^E = (\vec{V} \times \vec{B}_{pol}) - \nabla\phi^E = \frac{\mathrm{i}^{Hiro}}{h\sigma^{wall}} \tag{8.1}$$

determines the plasma V_{normal} velocity as

$$V_{normal} \simeq \frac{1}{\mu_0 h\sigma} \cdot \frac{\mu_0 \mathrm{i}^{Hiro}}{B^i_{pol}} \tag{8.2}$$

and the mode duration time Δt_{VDE} as

$$\Delta t_{VDE} \simeq \tau^{wall} \cdot \frac{B^i_{pol}}{\mu_0 \mathrm{i}^{Hiro}} > \tau^{wall}, \quad \tau^{wall} \equiv \mu_0 \sigma^{wall} ah. \tag{8.3}$$

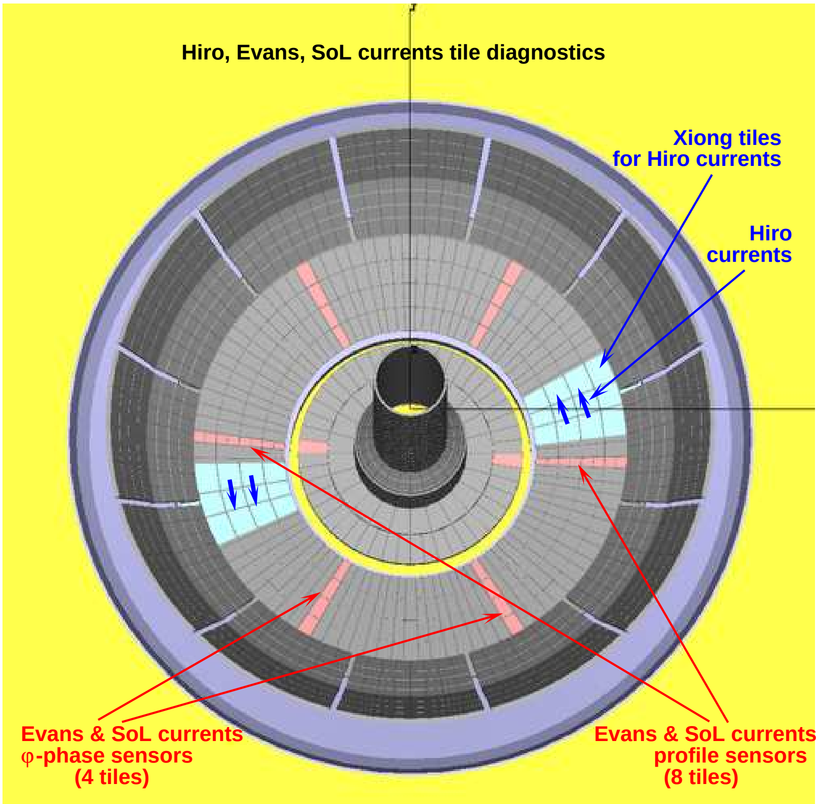
In fact, σ^{wall} includes the plasma-wall interaction physics, which is beyond TMHD.

The 8th outstanding result of TMHD is the prediction of the decay time consistent with DIII-D and JET data.

TMHD considers the C-mod VDEs as an exception, with no appreciable Hiro currents (due to specifics of VV) and controlled by the halo currents (still source limited) driven by the loop voltage.

9 TMHD and VDE-code motivate new diagnostics

The 9th result of TMHD is the suggestion of a comprehensive set of innovative tile diagnostics for Hiro, Evans and SoL current measurements on NSTX-U.



Tile sensors for measuring Hiro, Evans, and SoL currents and different kinds of diagnostics including

1. ***Hiro current diagnostics***
2. ***Evans current profile diagnostics with enhanced radial resolution***
3. ***Evans current φ-phase diagnostics***
4. ***SoL current measurements***

Evans currents carry important information on plasma-PFC interactions, never touched

The 10th outstanding result of TMHD is the energy principles for all TMHD equations.

***3-D equilibrium* (3-D Hermit elements, block tri-diagonal)**

$$W^{\vec{j}\times\vec{B}} \equiv \frac{1}{2}\int\left(\frac{|\vec{B}|^2}{2\mu_0} - (\vec{A}\cdot\vec{j})\right)d^3\mathrm{r}$$

$$\equiv \frac{1}{2\mu_0}\int\Big\{K(\bar{\Psi}'+\psi'_a+\bar{\Phi}'\eta'_\zeta)^2 - 2N(\bar{\Psi}'+\psi'_a+\bar{\Phi}'\eta'_\zeta)(\psi'_\theta-\phi'_\zeta) + M(\psi'_\theta-\phi'_\zeta)^2 \quad (10.1)$$

$$+Q(\bar{\Phi}'+\phi'_a+\bar{\Phi}'\eta'_\theta)^2 - 2\tilde{N}(\bar{\Psi}'+\psi'_a+\bar{\Phi}'\eta'_\zeta)(\bar{\Phi}'+\phi'_a+\bar{\Phi}'\eta'_\theta)$$

$$+2\tilde{M}(\psi'_\theta-\phi'_\zeta)(\bar{\Phi}'+\phi'_a+\bar{\Phi}'\eta'_\theta) - (\bar{\Phi}+\phi)\hat{F}'_a + (\bar{\Psi}+\psi)(\hat{J}'_a+\nu'_\theta)\Big\}\,da\,d\theta\,d\zeta.$$

***Plasma advancing* (3-D Hermit elements, block tri-diagonal)**

$$W^F = \frac{1}{2}\int \bar{F}^2\frac{g^{aa}\tilde{F}'^2_a + 2g^{a\theta}\tilde{F}'_a\tilde{F}'_\theta + g^{\theta\theta}\tilde{F}'^2_\theta + 2g^{a\zeta}\tilde{F}'_a\tilde{F}'_\zeta + 2g^{\theta\zeta}\tilde{F}'_\theta\tilde{F}'_\zeta + g^{\zeta\zeta}\tilde{F}'^2_\zeta}{r^4}J\,da\,d\theta\,d\zeta. \quad (10.2)$$

***Faraday's law* (3-D Hermit elements, block tri-diagonal)**

$$W^t = \frac{1}{2}\int\left\{\frac{\partial}{\partial t}\left(KB^\theta B^\theta + 2\tilde{M}B^\theta B^\zeta + QB^\zeta B^\zeta\right) + \eta^{pl}\left(Kj^\theta j^\theta + 2\tilde{M}j^\theta j^\zeta + Qj^\zeta j^\zeta\right)\right\}d^3r. \quad (10.3)$$

***Sink/source wall current from the plasma* (triangle based wall model, small sparse matrix)**

$$W^S = \int\left\{\frac{\bar{\sigma}(\nabla\phi^S)^2}{2} + j_\perp\phi^S\right\}dS - \frac{1}{2}\oint\phi^S\bar{\sigma}[(\vec{n}\times\nabla\phi^S)\cdot d\vec{l}]. \quad (10.4)$$

***Hiro, eddy currents in the wall* (triangle based wall model, stationary matrix)**

$$W^I \equiv \frac{1}{2}\int\left\{\frac{\partial(\vec{\tau}\cdot\vec{A}^I)}{\partial t} + \bar{\eta}|\nabla I|^2 + 2\left(\vec{\tau}\cdot\frac{\partial\vec{A}^{ext}}{\partial t}\right)\right\}dS - \oint(\phi^E-\phi^S)\frac{\partial I}{\partial l}dl. \quad (10.5)$$

They specify the revolutionary transition from the present hydro-dynamic schemes to straightforward and stable numerical schemes, based on plasma physics.

10 specified outstanding results manifest the unmatched success of TMHD (comparable only to the Li Wall Fusion).

(The side 11th result of TMHD is a new energy principle for 3-D equilibrium codes - much more powerful than the Kruskal-Kulsrud one used for stellarators)

The proposed near term research includes:

1. *Conversion of the present 2-D VDE code to a research tool for EAST, DIII-D, NSTX-U, JET, C-mod.*
2. *Designing the new tile diagnostics on NSTX-U for Hiro currents (similar to EAST) and for Evans (halo) currents profile studies.*
3. *TMHD extension, including a realistic plasma-wall interaction models based on calibration of VDE simulations by experimental data.*
4. *Parallel work on development of an equilibrium reconstruction code with variance analysis, linked to fast ASTRA transport simulator, for the Real Time Forecast for disruption prediction.*
5. *Return to PPPL of X.Li as a PostDoc. She is the absolute best in FE code developments.*
6. *Extension of 2-D version of VDE-code to 3-D VDE for simulation of JET and ITER disruptions.*

I would suggest to DoE to create a separate project for funding this work on disruption theory and simulations, covering me, two PostDocs (one is coming, the second will be X.Li in a year) and collaborations.

Implementation of TMHD promises the real progress in understanding of disruptions and resolving the remaining apparent inconsistencies.

7.5 years since 2007 shown how dramatic is the situation in VDE disruption simulations:

In applications to VDE, the existing MHD codes (with M3D as a leader), by W.Pauli qualification and JET data, are "not even wrong", they are just uncorrectable.