Abstract

Potential variances in $q$- and $p$- profiles have been calculated for different sets of external and internal measurements envisioned for equilibrium reconstruction in ITER. It was shown that complementing the external magnetic measurements with either Stark line polarization signals (MSE-LP) or with recently proposed line shift signals (MSE-LS) can significantly improve the reliability of the reconstructed plasma profiles and the magnetic configuration.

Capabilities of calculating variances, incorporated into the numerical code ESC, have completed the theory of reconstruction, which for a long time had a significant gap in ability to evaluate the quality of the presently used equilibrium reconstruction technique.

March 20, 2007, PPPL, Princeton NJ

PPPL Experimental Seminar

Nova Photonics

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Leonid Zakharov
One of unique features of ITER is its 1 MeV neutral beam injection.

Center line of 1 MeV NBI in ITER.

Potential set of signals for equilibrium reconstruction in ITER.

ITER B=5.3 T, I_p=15 MA, \( \beta = 2.8\% \) equilibrium configuration.

Center line of 1 MeV NBI in ITER.

Center line of 1 MeV NBI in ITER.

Summary

1 Potential set of signals for equilibrium reconstruction in ITER.

4.6 Curious case. No D-signals. \( L \neq 0 \). No Bcoil MSE-LP & MSE-LS.

4.5 Free boundary. Magnetic signals & both MSE-LP & MSE-LS.

4.4 Magnetic signals & both MSE-LP & MSE-LS.

4.3 Magnetic signals & MSE-LS.

4.2 Magnetic signals & MSE-LP.

4.1 Good looking magnetic only reconstruction.

4 Capabilities of diagnostics for equilibrium reconstruction.

3 "Rigorous" theory for "non-rigorous" reality

2 Variances in tokamak equilibrium reconstruction.

1 Potential set of signals for equilibrium reconstruction in ITER.
Potential set of signals for equilibrium reconstruction in ITER (cont.)

Measurements of the Line Shift due to MSE was proposed by Nova Photonics as a diagnostics of ITER configuration.

<table>
<thead>
<tr>
<th>Signal name</th>
<th>Relative $\varepsilon$</th>
<th>Absolute $\varepsilon$</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>B-coils</td>
<td>0.01</td>
<td>0.01 T</td>
<td>local probes</td>
</tr>
<tr>
<td>$\Psi$-loops</td>
<td>0.01</td>
<td>0.001 Vsec</td>
<td></td>
</tr>
<tr>
<td>$\Phi$-loop</td>
<td>0.01</td>
<td>0.001 Vsec</td>
<td></td>
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<tr>
<td>Diamagnetic loop</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MSE-LP</td>
<td>0.01</td>
<td>0.1</td>
<td>$\vec{B}<em>z/\vec{B}</em>\phi$ from MSE line polarization</td>
</tr>
<tr>
<td>MSE-LS</td>
<td>0.01</td>
<td>0.05 T</td>
<td>$\sqrt{</td>
</tr>
</tbody>
</table>

The question, neglected by present practice, is what level of perturbations can-
The theory of variances has been created in 2006 by L. Zakharov, J. Levanowksi, V. Drozdov and D. McDonald. ESC can use an extended set of basis functions

The problem is reduced to solving the linearized equilibrium problem

\[ \bar{\Psi} = \bar{\Psi}_0 + \psi, \]

\[ \Delta^* \psi + T' \bar{\Psi} \psi + P' \bar{\Psi} \psi = -\delta T(a) - \delta P(a) r^2 \]

for \( N \) possible perturbations

\[ \xi_n = \sum_{n=0}^{N} A_n \xi_n, \quad \delta T = \sum_{n=0}^{N} T_n f_n, \quad \delta P = \sum_{n=0}^{N} P_n f_n, \]

\( N = N_{\xi} + N_T + N_P, \]

where \( \xi_n(l) \), and \( 0 \leq a \leq 1 \) is the square root from the normalized toroidal flux.

The response of the diagnostics to each of \( N \) solutions can be calculated in a straightforward way.

ESC is based on linearization of the GSE equation. It was complemented with a routine for analysis of variances.

The problem is reduced to solving the linearized equilibrium problem.

The theory of variances has been created in 2006 by L. Zakharov, J. Levanowski, V. Drozdov and D. McDonald.
"Rigorous" theory for "non-rigorous" reality

After solving the perturbed GSh equation, the problem is reduced to a matrix problem.

Let vector $\mathbf{X}$ contains the amplitudes of perturbations:

$$\mathbf{X} = \begin{cases} A_0, A_1, \ldots, A_N \end{cases}$$

and vector $\mathbf{S}$ represents the signals:

$$\mathbf{S} = \begin{cases} \delta \Psi_0, \ldots, \delta \Psi_M, \delta B_0, \ldots, \delta B_M, \delta S_0, \ldots, \delta S_M \end{cases}$$

where $\mathbf{S}$ is the error in the signal.

ESC calculates the response matrix $\mathbf{A}$ relating $\mathbf{S}$ and perturbations $\mathbf{X}$:

$$\mathbf{A} \mathbf{X} = \mathbf{S}$$

The working matrix $\mathbf{A}$ weights $\mathbf{S}$ based on their accuracy:

$$\mathbf{A} = \mathbf{U} \cdot \mathbf{W} \cdot \mathbf{V}^T$$

where $\mathbf{w}_n$ are the eigenvalues of the matrix problem.

Eq. (3.6) gives variances and normalized RMS $\bar{\sigma}$ in an explicit form:

$$\bar{\sigma}_k \equiv \sqrt{\frac{1}{M} \sum_{m=0}^{M} (\mathbf{A} \mathbf{X}_k)^2}$$

Eq. (3.4) gives variances and normalized RMS $\bar{\sigma}$ in an explicit form. The resulting vector of variances can be represented as a linear combination of "eigenvectors", which are the columns of matrix $\mathbf{V}$.

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KiloGb's of reconstructions "data" can be easily generated for $k_J + k_P = 5$. Typically used, the reconstruction looks very good for $k_J \geq 3$, $k_P \geq 2$.

Plasma boundary is well specified, $\Phi$-loop, $B$-coils are used.

For $k_J + k_P = 5$, perturbations $j_k > s$, $j_k > s$ are invisible and cannot be reconstructed.
Plasma boundary is well specified, \( \Phi \)-loop, \( B \)-coils are used.

4.1 Good looking magnetic only reconstruction (cont.)

\[ q_{i0} = 0, \quad i_{1} = 21, \quad k_{0} = 0, \quad k_{1} = 8 \]

\[ p \ [\text{MPa}] \]

\[ a \quad 0 \quad 0.2 \quad 0.4 \quad 0.6 \quad 0.8 \]

\[ 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \]

\[ g_{d} \text{ Signal} \quad k_{0} = 0, \quad k_{1} = 8 \]

\[ m \quad 0 \quad 20 \quad 40 \quad 60 \]

\[ -4 \quad -2 \quad 0 \quad 2 \quad 4 \]

\[ q \] profile and variances for \( k_{J} \leq 4, \ k_{P} \leq 3 \).

\[ p \] profile and variances as functions of \( a \).

Signals \( \delta S_{m}/\epsilon_{m} \) generated by perturbations.

Testing \( k_{J} + k_{P} = 7 \) shows that the reconstruction is, in fact, not so good.

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Use of MSE-LP drops largest RMS. 4 makes 12 perturbations visible, and dramatically improves reconstruction of q, p.

Fixed plasma boundary with (Φ & B & MSE-LP) signals

4.2 Magnetic signals & MSE-LP

Test of $k_J + k_P = 16$ shows that with no constraints the reconstruction has no scientific value and is a sort of "beliefs". Use of MSE-LP drops largest RMS, makes 12 perturbations visible, and dramatically improves reconstruction of q, p. 4 makes 12 perturbations visible, and is a sort of "beliefs". Use of MSE-LP drops largest RMS, makes 12 perturbations visible, and dramatically improves reconstruction of q, p.

Plasma boundary is well specified, Φ-loop, B-coils are used.

4.1 Good looking magnetic only reconstruction (cont.)
Only perturbations with \( k < 14 \) might be potentially troublesome. For all \( k \), the signals \( \delta S_m / \epsilon_m \) are perturbations generated as functions of \( q \) and \( p \) profiles and their variances.

Testing \( N = 12 \) shows that MSE-LP allows to reconstruct both \( q \)- and \( p \)-profiles. Only perturbations with \( k \geq 14 \) might be potentially troublesome.
4.3 Magnetic signals & line shift

**MSE-LS**

- **Fixed plasma boundary with** $(\Phi \& B \& MSE-LS)$ **signals**

- **log** of Errors in $j, q, p$ Variance = $1.000e+02$

- $k$ 0 5 10 15

- $a$ 0 .2 .4 .6 .8

- $p$ [MPa] $i_0=0$ $i_1=21$ $k_0=0$ $k_1=12$

- $m$ 0 20 40 60 80

- **Signals** $\delta S^m/\epsilon^m$ generated by perturbations $Perturbations$ with $k \leq 12$ can be reconstructed using MSE-LS

- Use of MSE-LS can compete with MSE-LP in its value for reconstruction

**Fixed plasma boundary with** $(\Phi \& B \& MSE-LS)$ **signals**

4.3 Magnetic signals & line shift MSE-LS (cont.)
MSE-LS can pick up the details of the current drive for all \( a \) and non-monotonic \( q \) profile and variances as functions of \( a \). Signals \( \delta S \) and \( \epsilon / \epsilon_m \) generated by perturbations.

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The pressure profile reconstruction.

A realistic reduction of relative error \( \epsilon_{\text{relative}} \) from 0.1% improves MSE-LS and non-monotonic \( q \) and \( \bar{J} \) profile and variances as functions of \( a \). Signals \( \delta S \) and \( \epsilon / \epsilon_m \) generated by perturbations.

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4.3 Magnetic signals & line shift MSE-LS (cont.)

- **Back to reference fixed boundary and** \( (\Phi \& B \& MSE-LS) \)

**Displaced profiles and variances** for all \( k \) signals & both MSE-LP \& MSE-LS

**With MSE-LS only perturbations with** \( k \geq 13 \) might be potentially troublesome

**Fixed plasma boundary with** \( (\Phi \& B \& MSE-LP \& MSE-LS) \) signals

- **Magnetic signals & both MSE-LP \& MSE-LS**
Free boundary expands the $k$ range but does not affect the reconstruction.

\[
0 = \mathcal{J} \neq 0
\]

\[
0 = \mathcal{J} \text{ in case of } (\Phi, B, \text{MSE-LP, MSE-LS})
\]

\[
0 \neq \mathcal{J} \text{ in case of } (\Phi, B, \text{MSE-LP, MSE-LS})
\]

\[
\log_{10} \left\{ \bar{\sigma}_k, \bar{\sigma}_q, \bar{\sigma}_p \right\} \text{ in case of } (\Phi, B, \text{MSE-LP, MSE-LS})
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Free boundary expands the $k$ range but does not affect the reconstruction.

$\mathcal{J}$- and $q$-profiles can be reconstructed in all spectrum of $k$ and $\mathcal{J}$-profiles as functions of $a$ generically.

$\mathcal{J}$-profile and variance $q$-profile and its variance signals.

4.5 Free boundary, magnetic signals & both MSE-LP & MSE-LS signals.

4.4 Magnetic signals & both MSE-LP & MSE-LS (cont).
Free boundary, plasma with (\( \Phi \& B \& \text{MSE-LP} \& \text{MSE-LS} \)), signals

\[ q_{i0}=0 \quad i_{1}=21 \quad k_{0}=0 \quad k_{1}=32 \]

\begin{align*}
\begin{array}{cccc}
\alpha & 0 & 0.2 & 0.4 \\
\log_{10} p & 0 & 0.5 & 1.0 \\
\end{array}
\end{align*}

\begin{align*}
\begin{array}{cccc}
\alpha & 0 & 0.2 & 0.4 \\
\log_{10} gd & 0 & 2 & 4 \\
\end{array}
\end{align*}

\( q^{-1} \) profile and variances for all extended spectrum of \( k \) and \( p \) profiles and its variances as functions of \( \alpha \).

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4.5 Free boundary plasma with (\( \Phi \& B \& \text{MSE-LP} \& \text{MSE-LS} \)), signals
The extension of the theory of equilibrium reconstruction capability in NER

The equilibrium reconstruction capability in NER, energy in NER for extraction of MSE-LS signals would significantly enhance

even with NO \( B \)-coil signals, NO \( B \)-signals, NO \( B \)-signals,

for all extended spectrum of \( a \) and \( b \)-profiles, can be reconstructed over extended spectrum of \( a \) and \( b \) for all extended spectrum of \( a \) and \( b \) and its variances.

\( q \)-profile and variances for all extended spectrum of \( a \) and \( b \) and its variances.

\( b \) and \( d \)-profiles can be reconstructed over extended spectrum of \( a \) and \( b \) and its variances.

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