# Power extraction by liquid metal jets

*Leonid E. Zakharov,*

*Princeton University, Princeton Plasma Physics Laboratory*

*with appreciation to Richard Nygren (SNL) for important comments.*

Presented at ALPS/APEX Meeting

Sandia National Laboratory

November 13, 2000, Albuquerque, NM

[1] **This work is supported by US DoE contract No. DE–AC020–76–CHO–3073.**

# Web page

Web-page of mini-conference at APS 2000 (October 25-26, 2000, Quebec, Canada) "Lithium covered walls and low recycling regimes in tokamaks"

http://www.pppl.gov $\Longrightarrow$ Meetings $\Longrightarrow$ Lithium Walls 2000

Titles of the mini-conference talks
will be linked with
presentations and supporting stuff.

# Abstract

*Power extraction from the Scrape-Off Layer using fast liquid metal jets is being analyzed. A numerical code solving thermal diffusion equation inside the body of the jets has been created. The results of calculations are expressed in terms of the form factor in formula for the surface heating by a unifirm ehenrgy flux. The form factor depends on ratio of heat penetration distance and the diameter of the jets, as well as spacing between jets and inclination of the magnetic field ines. In the tokamak case when the energy flux follows magnetic field lines, discreetness of the jets is dramatically reduced with respect to continuous flow. It was found that the optimal size of jets is crutial for heat extraction by metal jets.*

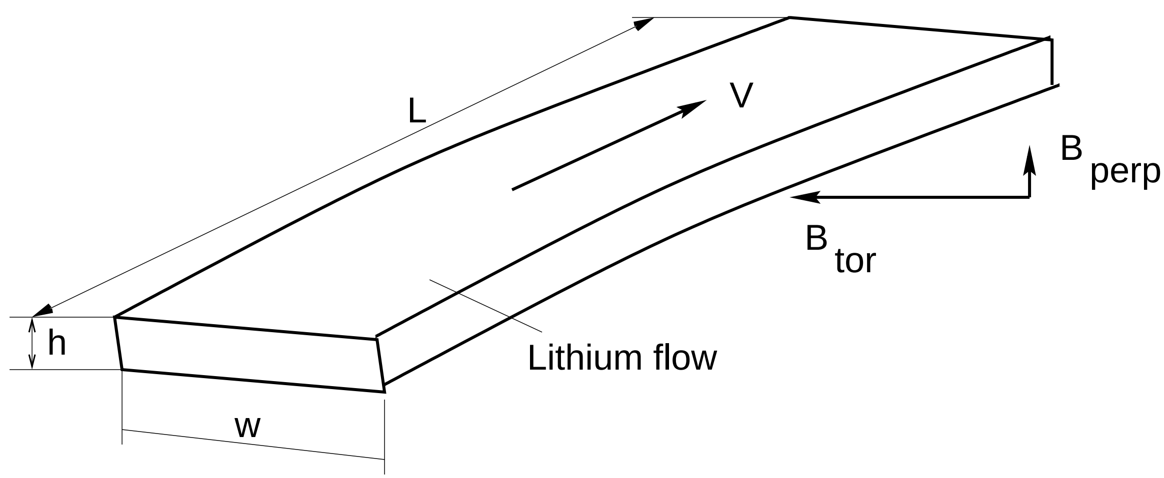
# OUTLINE

1. Basics of liquid lithium MHD.
2. Metal jets in a divertor.
3. Dynamics of metal jets.
4. Surface tension instability.
5. Power extraction.
6. Temperature distribution and optimal size of jets.
7. Summary.

# 1 Basics of liquid lithium MHD

There 3 magnetic Reynolds numbers which control lithium MHD in tokamaks

$$\begin{aligned} \text{dynamics}: &\quad \Re_0 \equiv \mu_0\sigma L V, \\ \text{electro-dynamics}: &\quad \Re_1 \equiv \mu_0\sigma h V, \\ \text{dynamics}: &\quad \Re_2 \equiv \mu_0\sigma \frac{h^2}{L} V, \\ \text{for lithium}: &\quad \mu_0\sigma \simeq 4 \ \left[\frac{sec}{m^2}\right]. \end{aligned} \tag{1.1}$$



Dynamic pressure losses are determined by $\Re_0$ and $\Re_2$

$$\begin{aligned} \Re_0: &\quad \Delta\rho\frac{V^2}{2} = \mu_0\sigma L V \frac{B_\perp^2}{2\mu_0}, \\ \Re_2: &\quad \Delta\rho\frac{V^2}{2} = \mu_0\sigma \frac{h^2}{L} V \Delta\frac{B_\parallel^2}{2\mu_0}. \end{aligned} \tag{1.2}$$

Magnetic fields from the currents in the stream are determined by $\Re_1$

$$\Re_1: \quad B_{||out} - B_{||in} = \mu_0\sigma h V B_\perp. \tag{1.3}$$

Magnetic pressure:

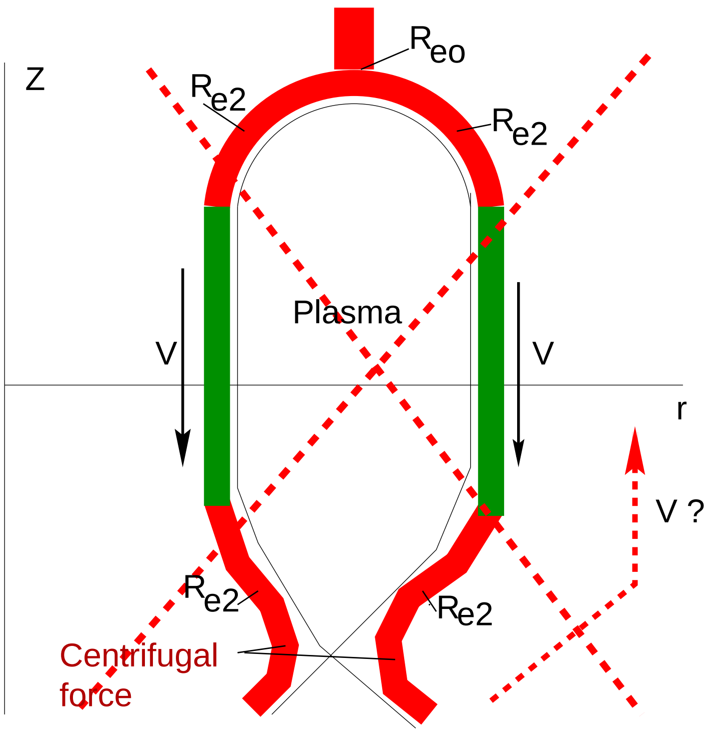
$$B = 1\ T \rightarrow \frac{B^2}{2\mu_0} = 4\ [atm], \qquad B = 5\ T \rightarrow \frac{B^2}{2\mu_0} = 100\ [atm]. \tag{1.4}$$

| Characteristic flow parameters | | | Li | Ga | SnLi | |
|---|---|---|---|---|---|---|
| $\rho\frac{V^2}{2} = 1$ | [atm] | $V$ | 19.23 | 5.67 | 5.37 | [m/sec] |
| $V = 20$ | [m/sec] | $\rho\frac{V^2}{2}$ | 1.08 | 12.45 | 13.88 | [atm] |
| | | $\mu_0\sigma$ | 4 | ? | ? | [sec/m$^2$] |

Lithium is most susceptible to MHD effects

Lithium “water-falls” are incompatible at the basic level with the tokamak strong toroidal field

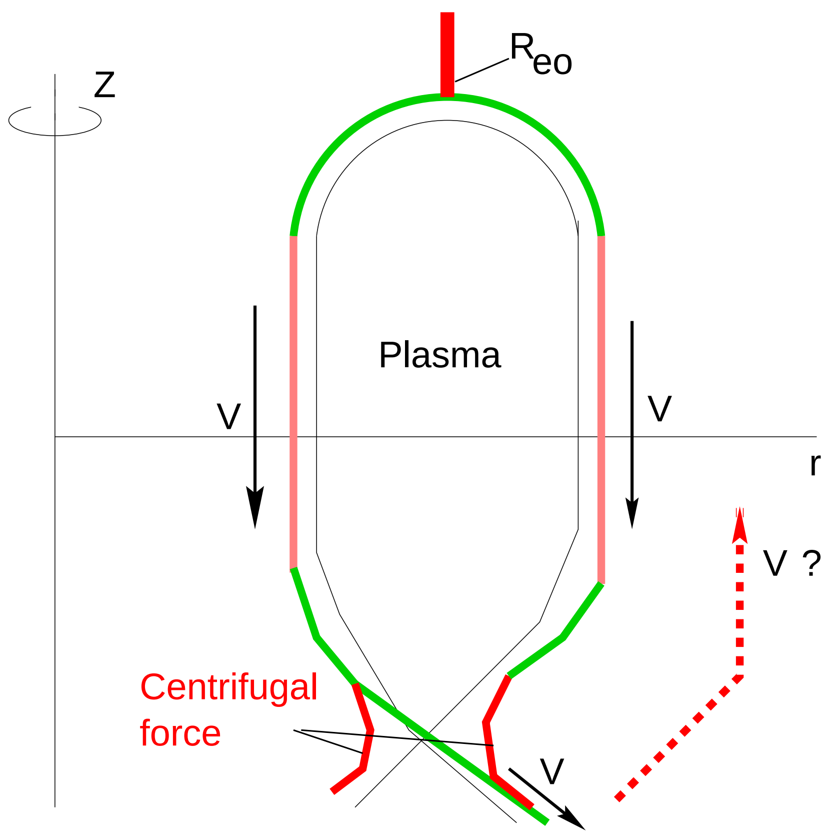
$$\begin{aligned} &h = 0.1\ m, \quad L_1 \simeq 0.2\ m, \quad L_2 \simeq 3\ m, \quad V > 2-5\ [m/sec], \\ &\Re_0 = 4L_1 V => 1.6, \\ &\Re_2 = 4\frac{h^2}{L_2}V = 4\frac{h}{L_2}(hV) \simeq 0.01 - 0.025. \end{aligned} \tag{1.5}$$




$$\boxed{\rho\frac{V^2}{2} \ll \Re_2 \Delta \frac{B_{tor}^2}{2\mu_0}}$$

## Momentum driven thin walls have a lot of unresolved problems in lithium MHD

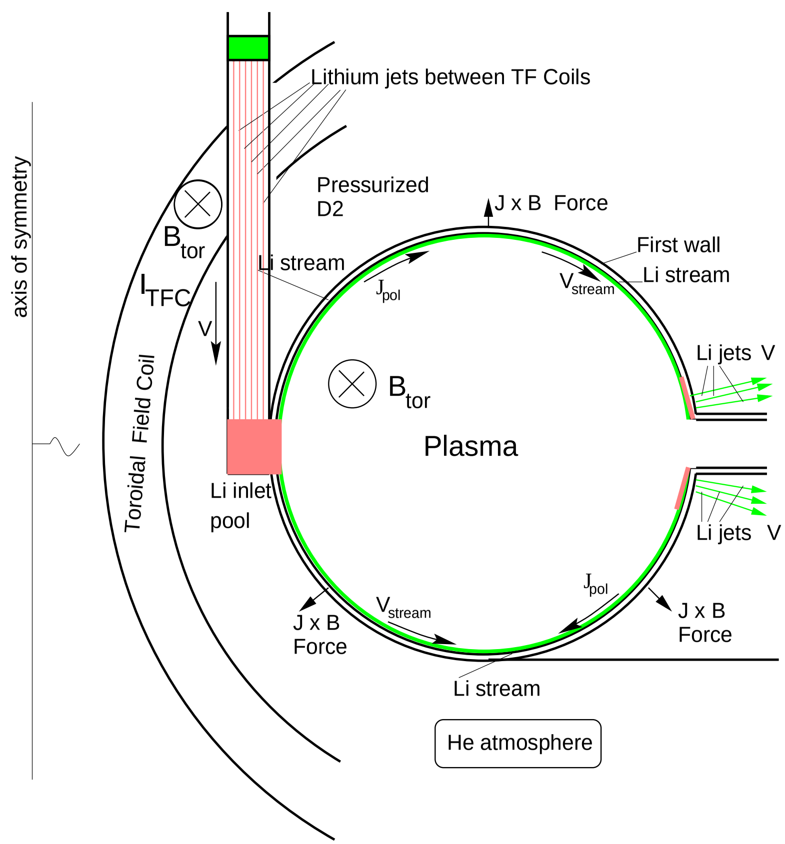
$$h = 0.01\ m, \quad L_1 \simeq 0.02\ m, \quad L_2 \simeq 3\ m, \quad V \simeq 20\ [m/sec],$$
$$\Re_2 = 4\frac{h^2}{L_2}V \simeq 1.3 \cdot 10^{-4}. \tag{1.6}$$




$$\boxed{\Re_0 = 1.6, \quad \rho\frac{V^2}{2} < \Re_0\frac{B_{pol}^2}{2\mu_0}}$$

Magnetic propulsion makes MHD of intense lithium streams compatible with tokamaks (at least at the basic level).

$$p_{\mathbf{j}\times\mathbf{B}}\big|_{inlet} - p_{\mathbf{j}\times\mathbf{B}}\big|_{outlet} \gg \Re_2 \frac{B_{tor}^2}{2\mu_0}, \qquad \Re_2 \equiv \mu_0 \sigma \frac{h^2}{R} V \simeq 0.0015$$




- Driving electromagnetic pressure $p_{\mathbf{j}\times\mathbf{B}}\big|_{outlet} > 1\ atm$

$$p_{\mathbf{j}\times\mathbf{B}}\big|_{inlet} - p_{\mathbf{j}\times\mathbf{B}}\big|_{outlet} \simeq 1.5 - 3\ [atm]$$

- Flow parameters

$$V \simeq 20\ m/sec, \qquad h \simeq 0.01\ m$$

- Magnetic Reynolds numbers

$$\Re_1 \equiv \mu_0 \sigma h V \simeq 0.8, \qquad \Re_2 \simeq 0.0015$$

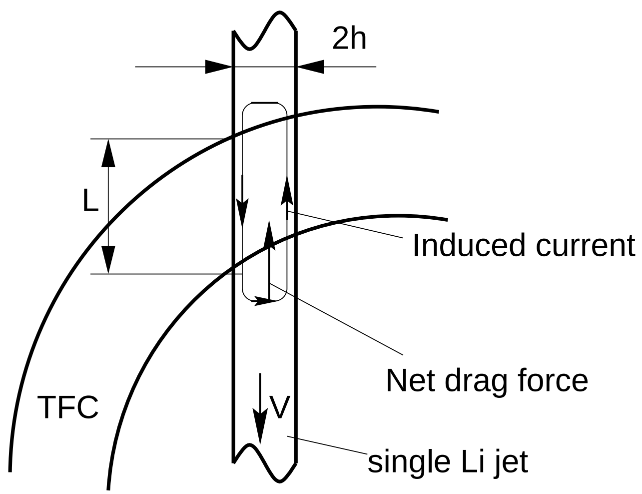
- Stream are robustly stable due to centrifugal force

$$\rho \frac{\langle V^2 \rangle}{2} > \frac{a}{2R} p_{wall} n_r$$

# 2 MHD of metal jets

Electric current in fast jets is excited because of inhomogeneity of the magnetic field. Its interaction with the magnetic field leads to losses in kinetic energy

$$\left\langle\Delta\frac{\rho V_z^2}{2}\right\rangle_{jet} = -\frac{1}{2}\Re_2\Delta\frac{B_0^2}{2\mu_0}, \quad \left\langle\Delta\frac{\rho V_z^2}{2}\right\rangle_{film} = -\frac{2}{3}\Re_2\Delta\frac{B_0^2}{2\mu_0}, \quad \Re_2 \equiv \frac{\mu_0\sigma h^2 V}{L} \ll 1 \qquad (2.1)$$




$$\rho\frac{\partial \mathbf{V}}{\partial t} + \rho(\mathbf{V}\cdot\nabla)\mathbf{V} = -\nabla p + (\mathbf{j}\times\mathbf{B}) + \nu\Delta\mathbf{V},$$
$$\frac{\partial \mathbf{A}}{\partial t} - \nabla\varphi_E + (\mathbf{V}\times\mathbf{B}) = \frac{\mathbf{j}}{\sigma},$$
$$\mathbf{B} = B(s)\mathbf{e}_y, \qquad \mathbf{V} = V\mathbf{e}_s,$$
$$\mathbf{j} = (\nabla i \times \mathbf{e}_y) = -i'_s\mathbf{e}_x + i'_x\mathbf{e}_s,$$
$$i_{jet} = -\sigma\frac{h^2 - y^2 - x^2}{2}(VB)'_s,$$
$$i_{film} = -\sigma\frac{h^2 - x^2}{2}(VB)'_s$$
$$\frac{\rho}{2}(V_s^2)'_s = ((\mathbf{j}\times\mathbf{B})\cdot\mathbf{e}_s) = j_x B = -i'_s B$$
$$\frac{\rho}{2}(V_s^2)'_s = \sigma\frac{h^2 - y^2 - x^2}{2}[(VB)'_s]'_s B.$$

# 3 Surface tension instability

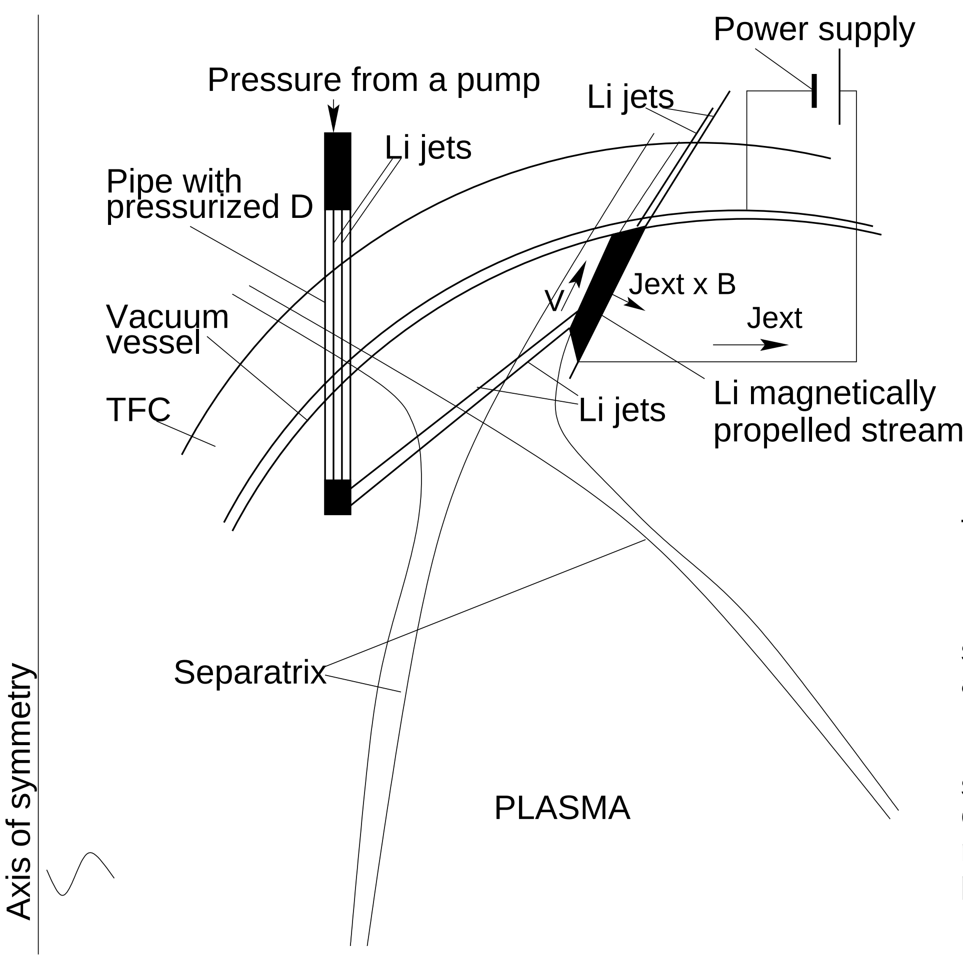
Metal jets exhibits a “sausage”-like instability due to the high surface tension of the liquid metals

$$\gamma = \sqrt{\frac{T}{h^3\rho}\frac{khI_0'(kh)}{I_0(kh)}(1-k^2h^2)}, \qquad \gamma_{max}|_{kh=0.697} = 0.3433\sqrt{\frac{T}{h^3\rho}} = 0.97\sqrt{\frac{T}{d^3\rho}}, \tag{3.1}$$

where $T$ is the surface tension, $k$ is the wave-vector, $h$ is the radius and $d$ is the diameter of the jet.

| Characteristics of instability | | Li $300^o$ C | Ga $300^o$ C | SnLi $300^o$ C | |
|---|---|---|---|---|---|
| $\rho$ | | 0.53 | 6.1 | 6.8 | [g/cm$^3$] |
| $T$ | | 380 | 700 | 500 ??? | $10^{-3}$[N/m] |
| $\gamma = .97\sqrt{\frac{T}{d^3\rho}}$ | $h = 0.5\ cm$ | 26.0 | 10.4 | 8.3 ??? | 1/sec |

# 4 Metal jets in a divertor




- Fast jets are injected into a pipe with a pressurized gas (e.g. D).
- Gravity collects liquid metal at the bottom.
- Metal jets are ejected into X-point region.
- Metal jets are collected at the guide plate (parallel to poloidal magnetic field) behind SOL.
- Then, metal is magnetically propelled the guide plate toward the wall of VV.
- $\mathbf{J} \times \mathbf{B}$ force should be large enough to expell the metal outside VV.

This scheme has no problems with lithium MHD.

Li velocities of the order of $20\ m/sec$, corresponding to pressure drop of $1\ atm$ can be realistically obtained.

For lithium, all pressure drops, required for this scheme, are of the order of several $atms$ only. Other liquid metals, like gallium or SnLi would require at least an order of magnitude higher pressure drops.

# 5 Power extraction

The temperature inside the body of the jet is determined by the thermo-conduction equation

$$\rho c_p V_{jet} T'_s = \kappa \nabla_\perp^2 T, \tag{5.1}$$

where $\rho$, $c_p$, $\kappa$ are the density, heat capacity and thermo-conduction of the lithium, correspondingly, $V_{jet}$ is the speed of the jet and $s$ is a coordinate along the jet.

For lithium

$$\rho \simeq 0.53\ g/cm^3, \quad c_p \simeq 4.3\ \frac{J}{g \cdot K^o}, \quad \kappa \simeq 0.47\ \frac{W}{cm \cdot K^o}, \tag{5.2}$$

for gallium ($300 - 700^o C$)

$$\rho \simeq 6.1\ g/cm^3, \quad c_p \simeq 0.38\ \frac{J}{g \cdot K^o}, \quad \kappa \simeq 0.4\ \frac{W}{cm \cdot K^o}, \tag{5.3}$$

and for SnLi

$$\rho \simeq 6.8\ g/cm^3, \quad c_p \simeq 0.26\ \frac{J}{g \cdot K^o}, \quad \kappa \simeq 0.33\ \frac{W}{cm \cdot K^o}. \tag{5.4}$$

Assuming one-side energy flux $\mathbf{q}$ and R.Nygren comments on importance of inclination of the magnetic field lines to the jets plane

$$\mathbf{q} = q_{pol}\frac{\mathbf{B}_{pol}}{|\mathbf{B}|} + q_{tor}\frac{\mathbf{B}_{tor}}{|\mathbf{B}|} \tag{5.5}$$

(indexes “pol” and “tor” stand for poloidal and toroidal components of vectors) the final maximum temperature raise at the jet surface can be written in the form

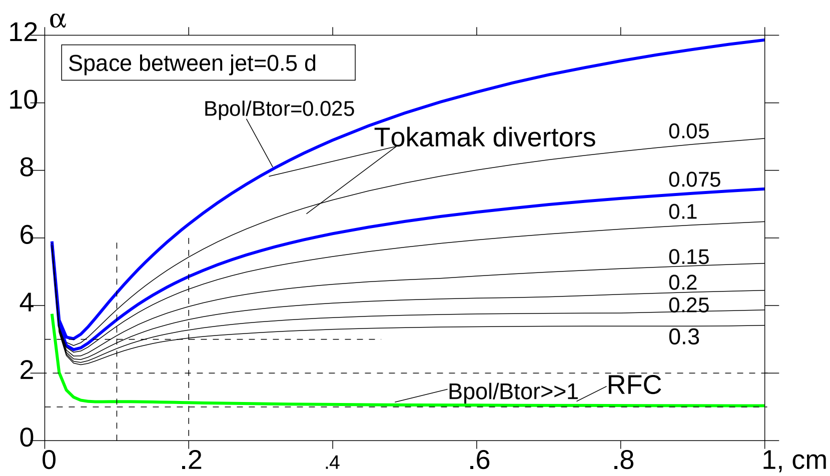
$$\Delta T_{max} = \alpha q_{pol}\sqrt{\frac{4t_{transit}}{\pi\kappa\rho c_p}}, \qquad t_{transit} \equiv \frac{L}{V_{jet}}, \tag{5.6}$$

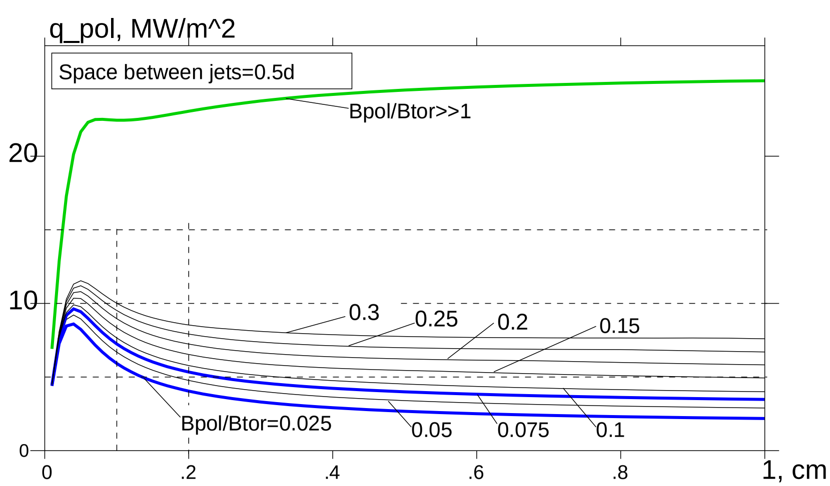
where $L$ is the energy deposition length, and factor $\alpha$ is a function of ratio of the jet diameter to the thermal skin depth $\delta_{skin}$

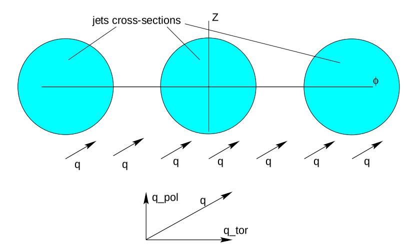
$$\alpha = \alpha\left(\frac{d}{\delta_{skin}}, \frac{B_{pol}}{B_{tor}}, spacing\ factor\right), \qquad \delta_{skin} \equiv \sqrt{Dt_{transit}}, \qquad D \equiv \sqrt{\frac{\kappa}{\rho c_p}} \tag{5.7}$$

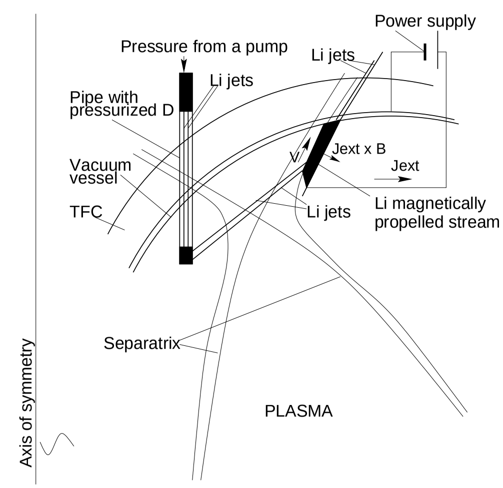
($D$ is the heat diffusion coefficient, $D_{Li} = 0.21\ \frac{cm^2}{sec}$, $D_{Ga} = 0.17\ \frac{cm^2}{sec}$, $D_{SnLi} = 0.19\ \frac{cm^2}{sec}$).

Power extraction by the jets of Lithium, LiSn, Ga, crossing the X-point region in the tokamak.




Concentration of the power deposition is highly unfavorable

Power extraction capability of lithium walls fits well reactor relevant wall loadings, $> 10\ MW/m^2$ in neutrons (distributed over the wall surface)

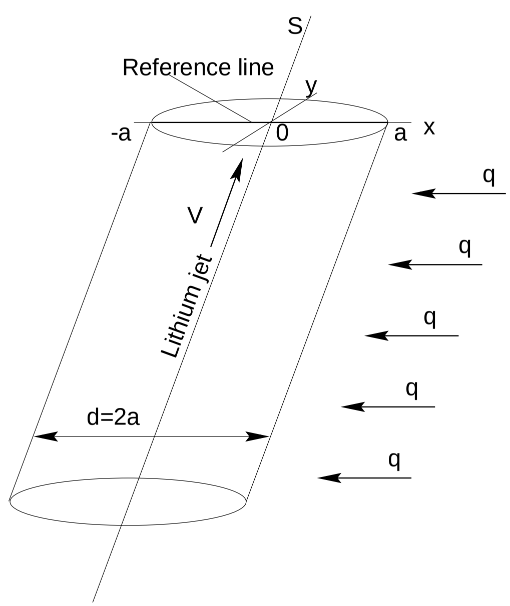
$$\boxed{\Delta T_{max} = q_{wall}\sqrt{\frac{4t_{transit}}{\pi\kappa\rho c_p}}, \qquad d_{skin} \equiv \sqrt{\frac{\kappa t_{transit}}{\rho c_p}}}$$

$$\boxed{R = 6\ m, \qquad a = 1.6\ m, \qquad q_{wall} \simeq 3.5\ \frac{MW}{m^2}, \qquad P_{wall} = 4\pi^2 R a q_{wall} \simeq 1.3\ GW}$$

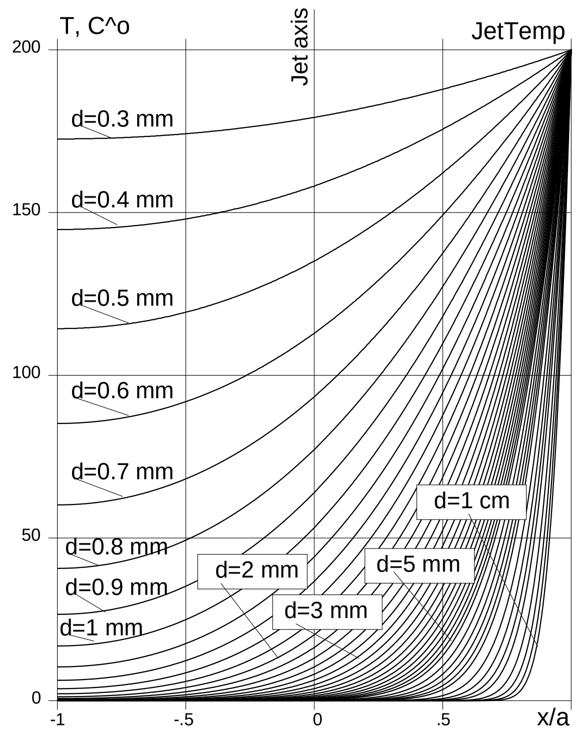
even with no reliance on the vortices in the streams.

# 6 Temperature distribution and optimal size of jets

Fig.2. shows temperature distribution in lithium for $L \simeq 0.1\ m$ and $V = 20\ m/sec$ for different diameters $d = 2a$ of jets in absence of mutual shadowing by jets.



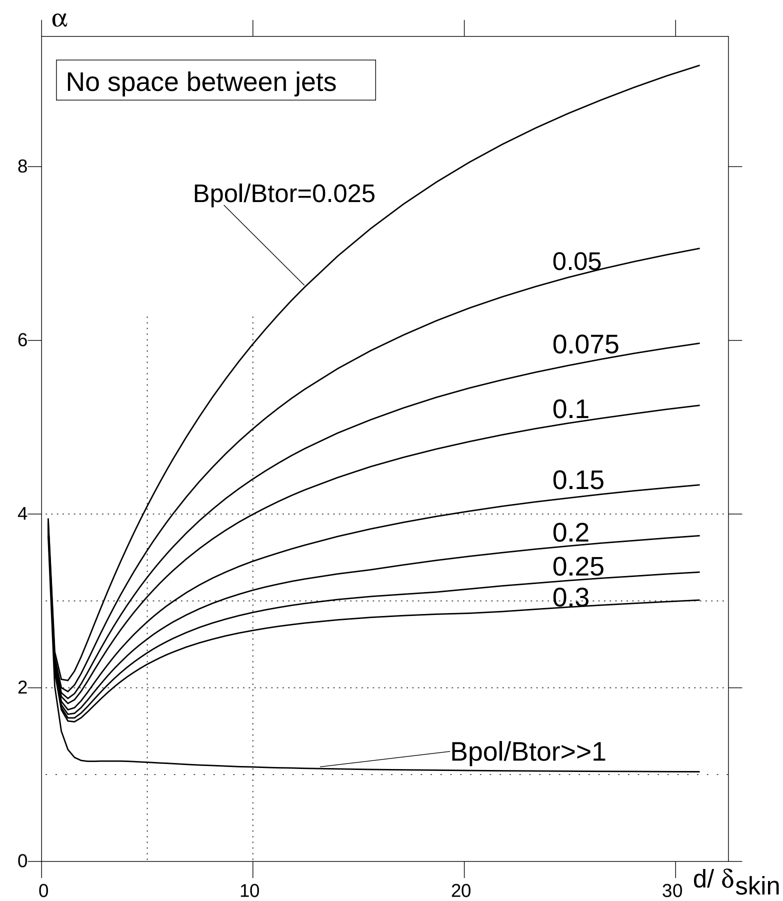

a)




b)

Fig.2. a) Jet geometry and the reference line; b) Temperature profiles along the reference line after passing SOL for $B_{tor} = 0$.

Factor $\alpha(d/\delta_{skin})$ in Eq.(5.6) has been calculated numerically by solving Eq.(5.1). Obliqueness of heat flux leads to dramatic increase in $\alpha$ for the same $q_{pol}$.



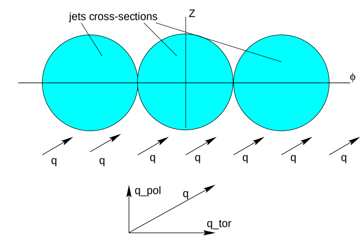
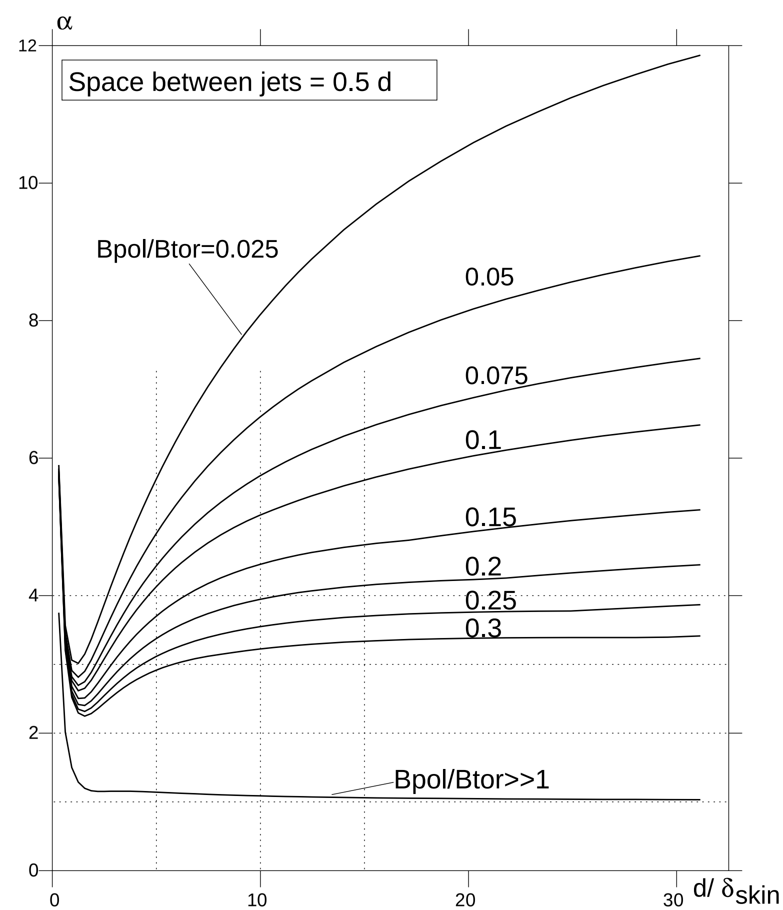




Fig.3a. Factor $\alpha$ for normal and oblique heat fluxes.

Separation of jets results in further increase of the $\alpha$-factor.



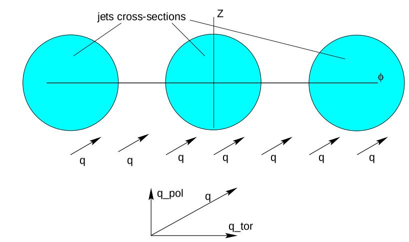
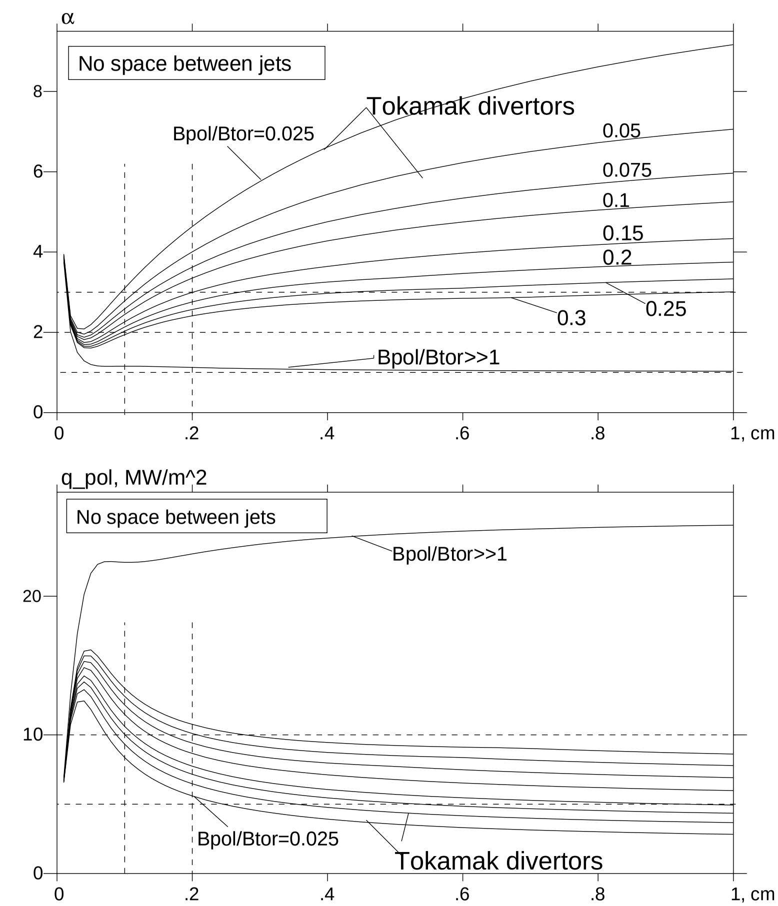




Fig.3b. Factor $\alpha$ for separated jets.

Fig.4a shows results of heat flux calculations for $L = 0.1\ m$, $V = 20\ m/sec$ with no space between lithium jets.



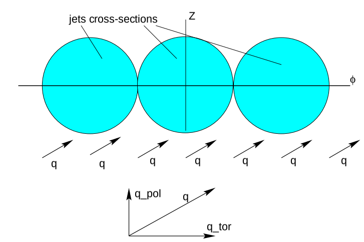
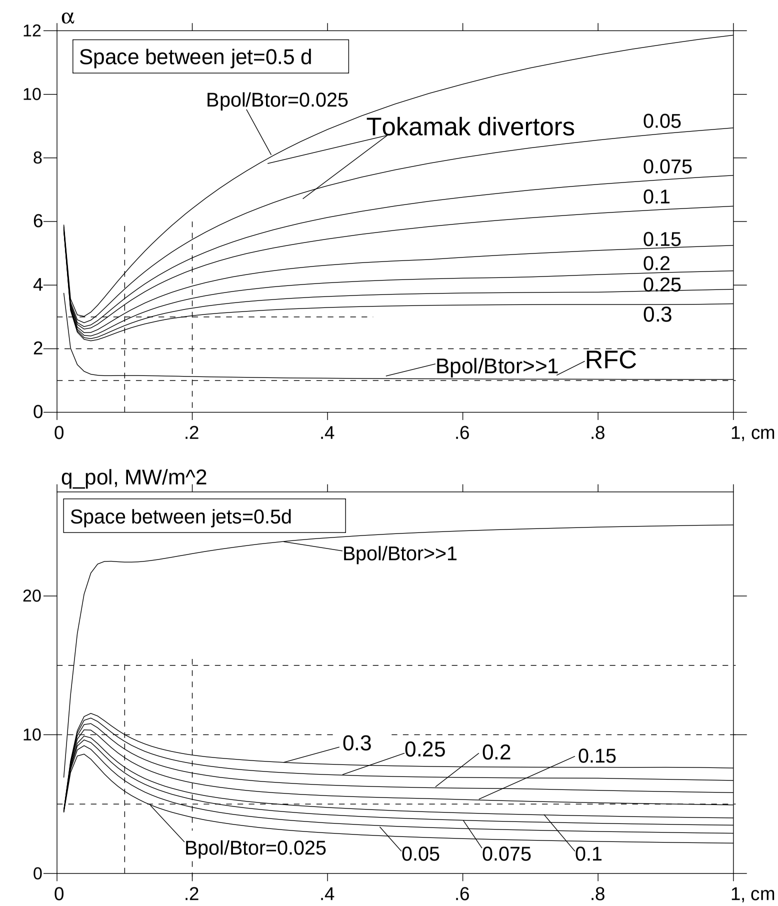




Fig.4a. Factor $\alpha$ of Eq.(5.6) and poloidal energy flux $q_{pol}$, corresponding to $\Delta T_{max} = 200^oC$, as functions of jet diameter.

# Finite separation of lithium jets reduces power extraction.



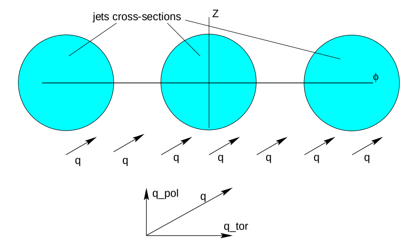




Fig.4b. Factor $\alpha$ and poloidal energy flux $q_{pol}$ for lithium as functions of jet diameter.

Figs.4 indicate that only for $B_{tor} = 0$ (e.g., FRC case) the acceptable energy flux is a weak function of the jet diameter.

For tokamaks, $B_{tor} \gg B_{pol}$, the acceptable heat flux is significantly reduced and depends strongly on the jet diameter.

Lithium, gallium and SnLi have practically the same heat diffusion coefficient $D$. For all of them the optimal diameter of jets is about $0.5\ mm$.

For a typical major radius $R$ of the X-point of the SOL of $6\ m$ and $d \simeq 0.5\ mm$ the mass flow $\dot{M}$ for lithium is

$$\dot{M} = 2\pi\rho RLV \simeq 0.16\ \frac{T}{sec}. \tag{6.1}$$

The mass flows for gallium and SnLi are correspondingly 11 and 13 times larger (due to higher density).

The resulting power extraction capabilities of lithium jets are very modest

$$P_{SOL} \equiv 2\pi RLq \simeq 30\ MW. \tag{6.2}$$

SnLi has the same vapor pressure at $600^oC$ as lithium at its admissible temperature $400^oC$. Assuming that $600^oC$ is a temperature limit for SnLin, its admissible $\Delta T \simeq 300^oC$ would be 1.5 higher than that for lithium. Because of 25 % smaller factor $\sqrt{\kappa\rho c_p}$, SnLi has only slightly (12 %) higher power extraction capability than lithium.

Gallium has the same $\sqrt{\kappa\rho c_p}$ as lithium. Its allowable $\Delta T$ remains yet undetermined.

With the same power deposition length $L$, in order to increase power extraction $P_{SOL}$ (and $q$), e.g. 2 times, the speed of the jets should be increased 4 times (with 16 times increase in the jet injection pressure).

On the other hand, simultaneous increase in $L$ and $V_{jet}$, e.g., by a factor 2, would increase power extraction by the same factor 2.

# 7 Summary

For a scheme of free surface jets, the maximum temperature increase of the jet surface is determined by

$$\Delta T_{max} = \alpha\left(\frac{d}{d_{skin}}, \dots\right) q_{pol}\sqrt{\frac{4t_{transit}}{\pi\kappa\rho c_p}}, \qquad d_{skin} \equiv \sqrt{\frac{\kappa t_{transit}}{\rho c_p}} \tag{7.1}$$

where function $\alpha$ is presented on Figs.3a,3b and for tokamaks is a strong function of jet diameter.

The optimal jet diameter corresponds to $\simeq 2\delta_{skin}$ and for lithium, gallium, SnLi is $0.5\ mm$, where surface tension instability can be very destructive.

For lithium and SnLi jets and for typical tokamak conditions power extraction is, at best, $\simeq 30\ MW$ with equivalent poloidal energy flux of the order of $8\ MW/m^2$.

Power extraction capability of jets is almost 2 orders of magnitude smaller than that of the lithium streams on the inner walls which can handle power fluxes ($\Delta T_{Li\ surface} < 200^0\ C$)

$$q_{wall} \simeq 3.5\ \frac{MW}{m^2}, \qquad P_{wall} = 4\pi^2 Raq_{wall} \tag{7.2}$$

(distributed over the wall surface) and reactor relevant wall loadings $P_{wall}$ of the order of $GW s$.