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Equilibrium reconstruction of q- and p- profiles in ITER using different external and internal measurements¹

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Nova Photonics

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Abstract

The talk presents a theory of uncertainties in the reconstructions of the plasma current density and pressure profiles in the Grad-Shafranov equation. The associated technique was incorporated into the ESC code.

Potential variances in q - and p - profiles have been calculated for different sets of external and internal measurements envisioned for equilibrium reconstruction in ITER.

It was shown that complementing the external magnetic measurements with either Stark line polarization signals (MSE-LP) or with recently proposed for ITER by Nova Photonics line shift signals (MSE-LS) can significantly improve the reliability of the reconstructed plasma profiles and the magnetic configuration.

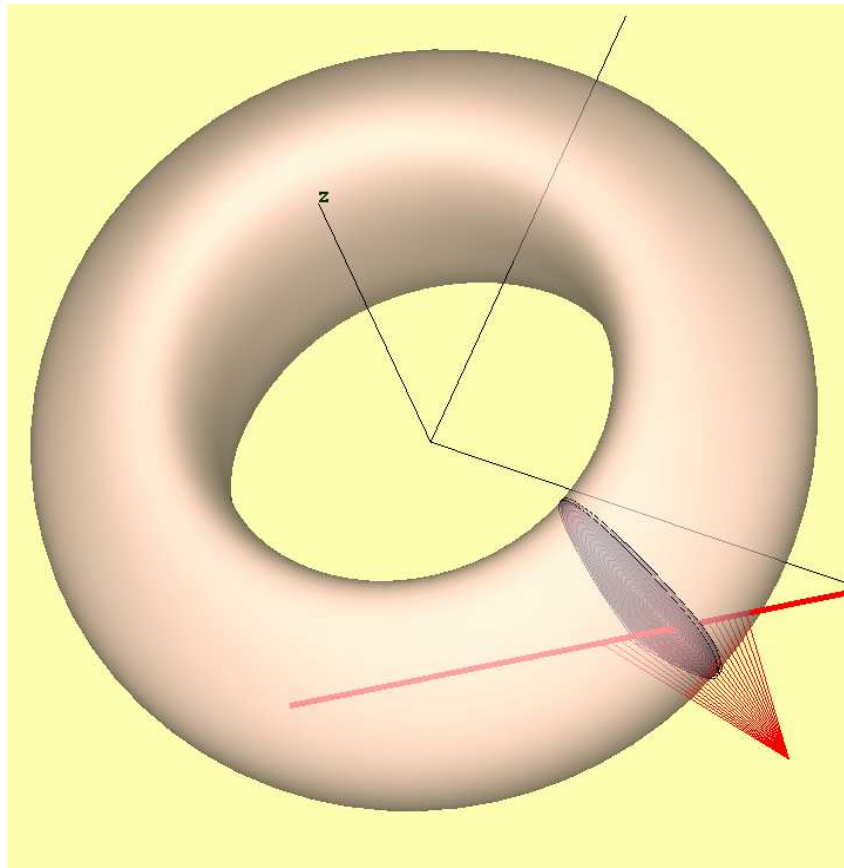
Capabilities of calculating variances, incorporated into the numerical code ESC, have completed the theory of reconstruction, which for a long time had a significant gap in ability to evaluate the quality of the presently widely used equilibrium reconstruction technique.

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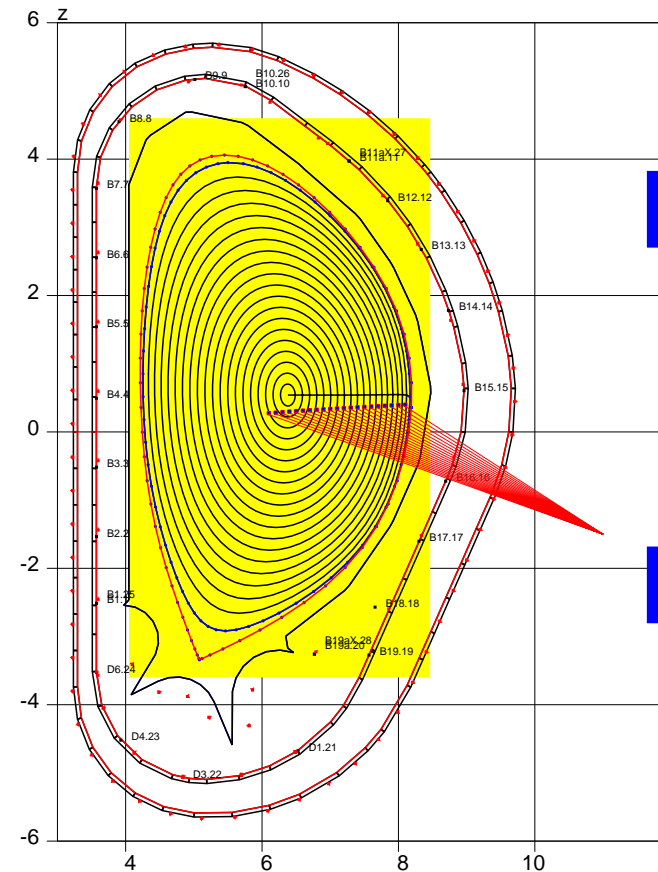
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1 Set of signals for equilibrium reconstruction

ITER $B=5.3$ T, $I_{pl}=15$ MA $\beta = 2.8\%$ equilibrium configuration



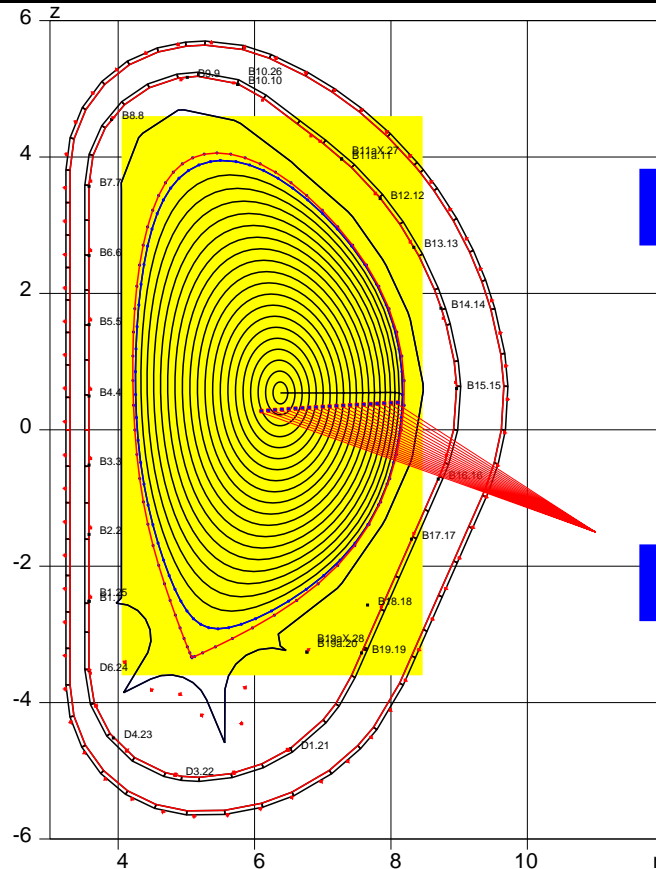
Center line of 1 MeV NBI in ITER



Ψ -loops, B -coils, pickup points of MSE

One of unique features of ITER is its 1 MeV neutral beam injection

Measurements of the Line Shift due to MSE was proposed by Nova Photonics as a diagnostics of ITER configuration



Reference signal errors ϵ used here for calculating variances in equilibrium reconstruction in ITER:

Signal name	$\epsilon^{relative}$	$\epsilon^{absolute}$	Comment
B-coils	0.01	0.01 T	local probes
Ψ -loops	0.01	0.001 Vsec	
Φ -loop	0.01	0.001 Vsec	diamagnetic loop
MSE-LP	0.01	0.1°	
			B_z/B_ϕ from MSE line polarization
MSE-LS	0.01	0.05 T	$\sqrt{ \mathbf{B} ^2 - (\mathbf{B} \cdot \mathbf{v})^2}$ from MSE line shift

MSE-LP and MSE-LS signals were assumed to be point-wise. This requires more realistic model from Nova Photonics.

The capabilities of equilibrium reconstruction with such a set of signal is the topic of the talk

2 Variances in tokamak equilibrium reconstruction

The practice typically neglects making analysis of variances in reconstructed equilibria

In tokamaks the Grad-Shafranov (GSh) equation describes the configuration

$$\Delta^* \bar{\Psi} = -T(\bar{\Psi}) - P(\bar{\Psi})r^2, \quad T \equiv \bar{F} \frac{d\bar{F}}{d\bar{\Psi}}, \quad P \equiv \mu_0 \frac{dp}{d\bar{\Psi}}, \quad (2.1)$$

Its solution can be perturbed by

1. perturbation of the plasma shape

$$\xi(a_{pl}, l), \quad \text{and} \quad (2.2)$$

2. perturbation of two 1-D functions

$$\delta T(\bar{\Psi}), \quad \delta P(\bar{\Psi}). \quad (2.3)$$

The question, neglected by present practice, is what level of perturbations cannot be distinguished given the finite accuracy of measurements.

The level of variances $\xi, \delta T, \delta P$ determines the very value of reconstruction and of the entire diagnostics system

The theory of variances has been created in 2006 by L.Zakharov, J.Levandowski, V.Drozдов and D.McDonald

The problem is reduced to solving the linearized equilibrium problem

$$\bar{\Psi} = \bar{\Psi}_0 + \psi, \quad \Delta^* \psi + T'_{\bar{\Psi}} \psi + P'_{\bar{\Psi}} \psi = -\delta T(a) - \delta P(a) r^2 \quad (2.4)$$

for N possible perturbations

$$\xi = \sum_{n=0}^{n < N_{\xi}} A_n \xi^n(l), \quad \delta T = \sum_{n=0}^{n < N_J} T_n f^n, \quad \delta P = \sum_{n=0}^{n < N_P} P_n f^n, \quad (2.5)$$

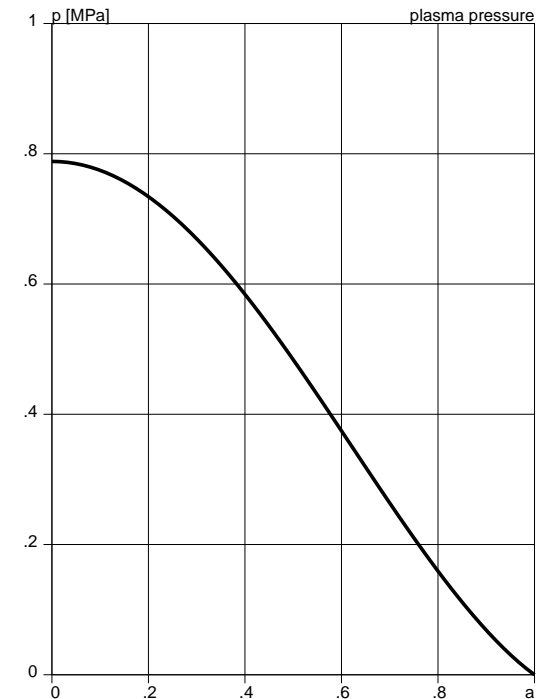
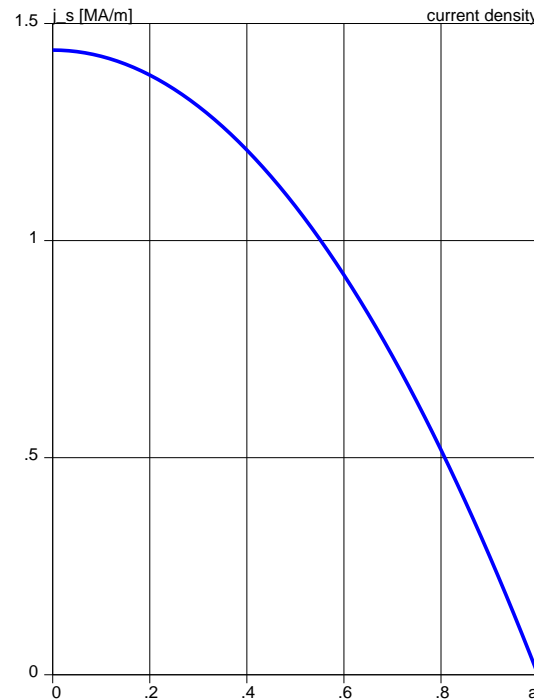
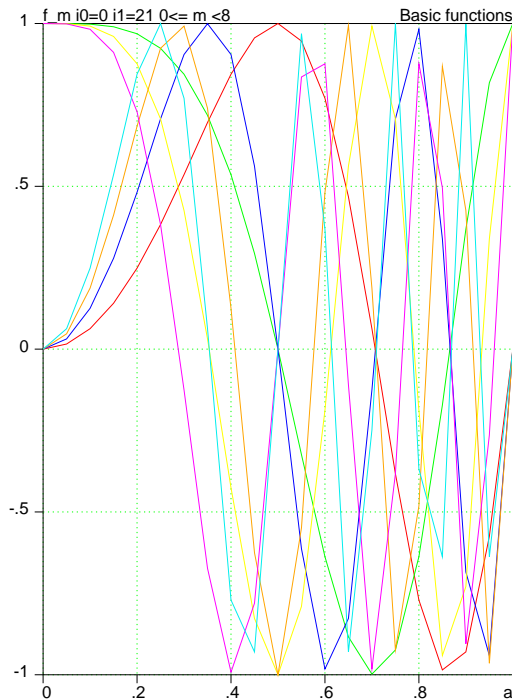
$$N = N_{\xi} + N_J + N_P, \quad f^{2n} = \cos 2\pi n a^2, \quad f^{2n+1} = \sin 2\pi n a^2,$$

where l is the poloidal coordinate at the plasma boundary, and $0 \leq a \leq 1$ is the square root from the normalized toroidal flux.

The response of the diagnostics to each of N solutions ψ^n can be calculated in a straightforward way.

ESC is based on linearization of the GSh equation. It was complemented with a routine for analysis of variances

8 functions $f^n(a^2)$ has been used to perturb $P(\bar{\Psi}), T(\bar{\Psi})$



Trigonometric expansion functions $f^n(a^2)$ background current density profiles $\bar{j}_s(a)$ background pressure profile $\bar{p}(a)$

ESC can use an extended set of basis functions

3 “Rigorous” theory for “non-rigorous” reality

After solving the perturbed GSh equation, the problem is reduced to a matrix problem

Let vector \vec{X} contains the amplitudes of perturbations

$$\vec{X} \equiv \left\{ \underbrace{A_0, A_1, \dots, A_{N_b-1}}_{N_\xi \text{ of } \xi}, \underbrace{T_0, \dots, T_{N_T-1}}_{N_T \text{ of } \delta T}, \underbrace{P_0, \dots, P_{N_P-1}}_{N_P \text{ of } \delta P} \right\} \quad (3.1)$$

and vector $\delta\vec{S}$ represents the signals

$$\delta\vec{S} \equiv \left\{ \underbrace{\delta\Psi_0, \dots, \delta\Psi_{M_\Psi-1}}_{M_\Psi \text{ of } \delta\Psi}, \underbrace{\delta B_0, \dots, \delta B_{M_B-1}}_{M_B \text{ of } \delta B_{pol}}, \underbrace{\delta S_0, \dots, \delta S_{M_S-1}}_{M_S \text{ of } \delta \text{others}} \right\}, \quad (3.2)$$
$$M \equiv M_\Psi + M_B + M_S, \quad M > N.$$

32 Ψ —, 1 $\Phi_{diamagnetic}$ -loops, 64 B -probes, 21 MSE-LP (line polarization) and 21 MSE-LS (line shift) signals (both pointwise) were used in the analysis.

ESC calculates the response matrix A relating $\delta\vec{S}$ and perturbations $\delta\vec{X}$

$$\delta\vec{S} = A\vec{X}, \quad A = A_{M \times N}. \quad (3.3)$$

The working matrix \bar{A} weights δS_m based on their accuracy

$$(\bar{A})_m^n = \frac{1}{\epsilon_m} (A)_m^n, \quad \delta \bar{S}_m = \frac{1}{\epsilon_m} \delta S_m, \quad \bar{A} \vec{X} = \delta \vec{S}, \quad (3.4)$$

where ϵ_m is the error in the signal S_m . SVD expresses the matrix \bar{A} as a product

$$\begin{aligned} \bar{A} &= U \cdot W \cdot V^T, \\ U &= U_{M \times N}, \quad U^T \cdot U = I, \quad I_m^n = \delta_m^n, \\ W &= W_{N \times N}, \quad W_n^k = w^k \delta_n^k, \\ V &= V_{N \times N}, \quad V^T \cdot V = I. \end{aligned} \quad (3.5)$$

Here, w^k are the eigenvalues of the matrix problem. The columns V^k of matrix V represent the normalized eigen-vectors of the problem.

**The solution of matrix problem (3.5) generates a hierarchy
of eigen-perturbations each corresponding
to columns of matrix V**

Eigen-values w^k determine visibility of eigen-perturbations

In terms of columns of matrix V , the eigen-perturbations \vec{X}^k can be defined as

$$\vec{X}^k \equiv \gamma^k V^k, \quad (3.6)$$

where factors γ^k scale each physical perturbation to the most limiting value among characteristic $\xi_{max}, \delta T_{max}, \delta P_{max}$.

Calculation of RMS of the signals \vec{S}^k generated by each \vec{X}^k

$$\delta \vec{S}^k = \bar{A} \vec{X}^k = \gamma^k w^k \vec{U}^k, \quad \sqrt{\frac{1}{M} \sum_{m=0}^{m \leq M} (\delta \bar{S}_m^k)^2} = \frac{\gamma^k w^k}{\sqrt{M}}, \quad (3.7)$$

determines variances $\bar{\sigma}^k$ in reconstruction of each eigen-perturbations as

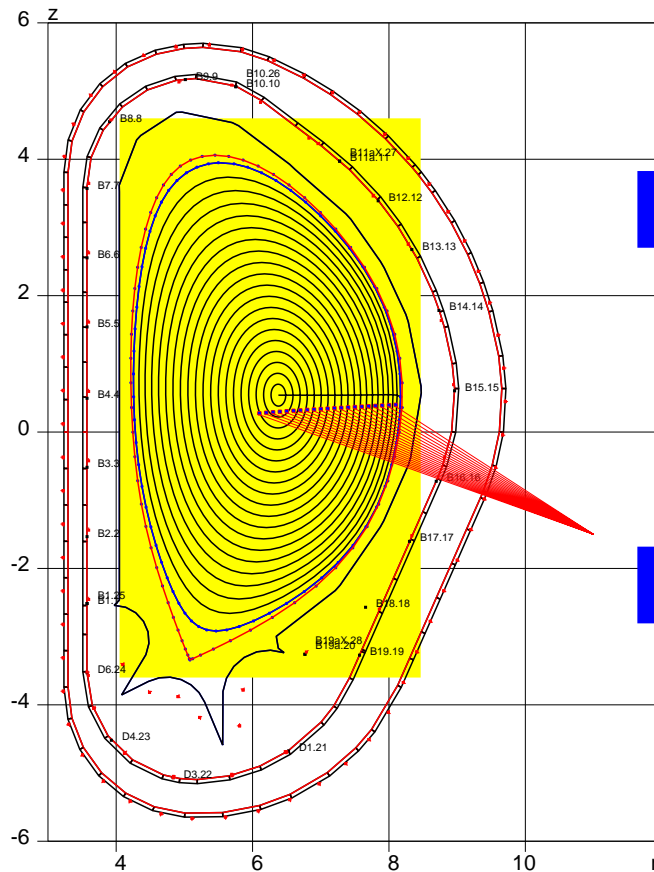
$$\bar{\sigma}^k \equiv \frac{\sqrt{M}}{\gamma^k w^k}, \quad (3.8)$$

The spectrum of $\bar{\sigma}^k$, defined by Eq.(3.8), is a quantitative measure of quality of diagnostic systems

Perturbations \vec{X}^k with $\bar{\sigma}^k > 1$ are “invisible” for diagnostics

4 Capabilities of diagnostics for equilibrium reconstruction

ITER configuration is used for illustrating the technique



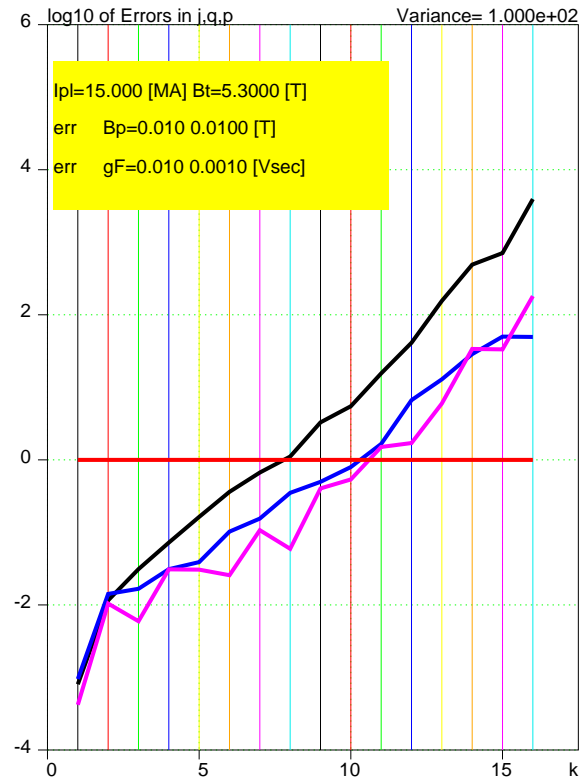
Reference signal errors ϵ used here for calculating variances in equilibrium reconstruction in ITER:

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MSE-LP	0.01	0.1°	B_z/B_ϕ from MSE line polarization
MSE-LS	0.01	0.05 T	$\sqrt{ B ^2 - (B \cdot v)^2}$ from MSE line shift

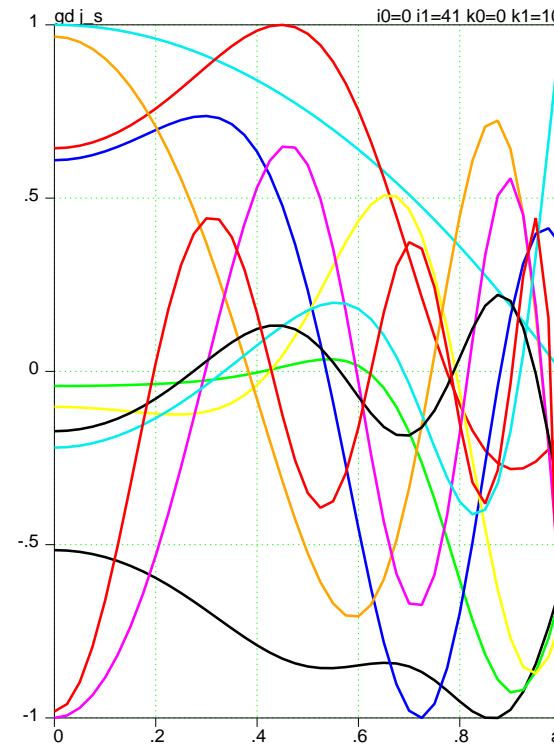
MSE-LP and MSE-LS signals were assumed to be point-wise. This requires more realistic model from Nova Photonics.

Different combinations of signal lead to different residual variances

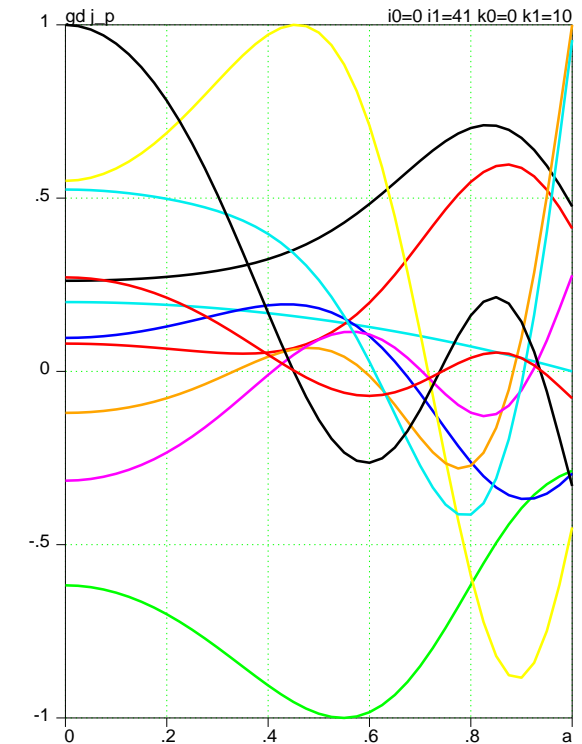
Plasma boundary is well specified, Φ -loop, B -coils are used



$\log_{10} \bar{\sigma}_q^k$, $\log_{10} \bar{\sigma}_p^k$, $\log_{10} \bar{\sigma}_p^k$
($N_J = 8$, $N_P = 8$)



Eigen-variances $\delta j_s^k(a)$.



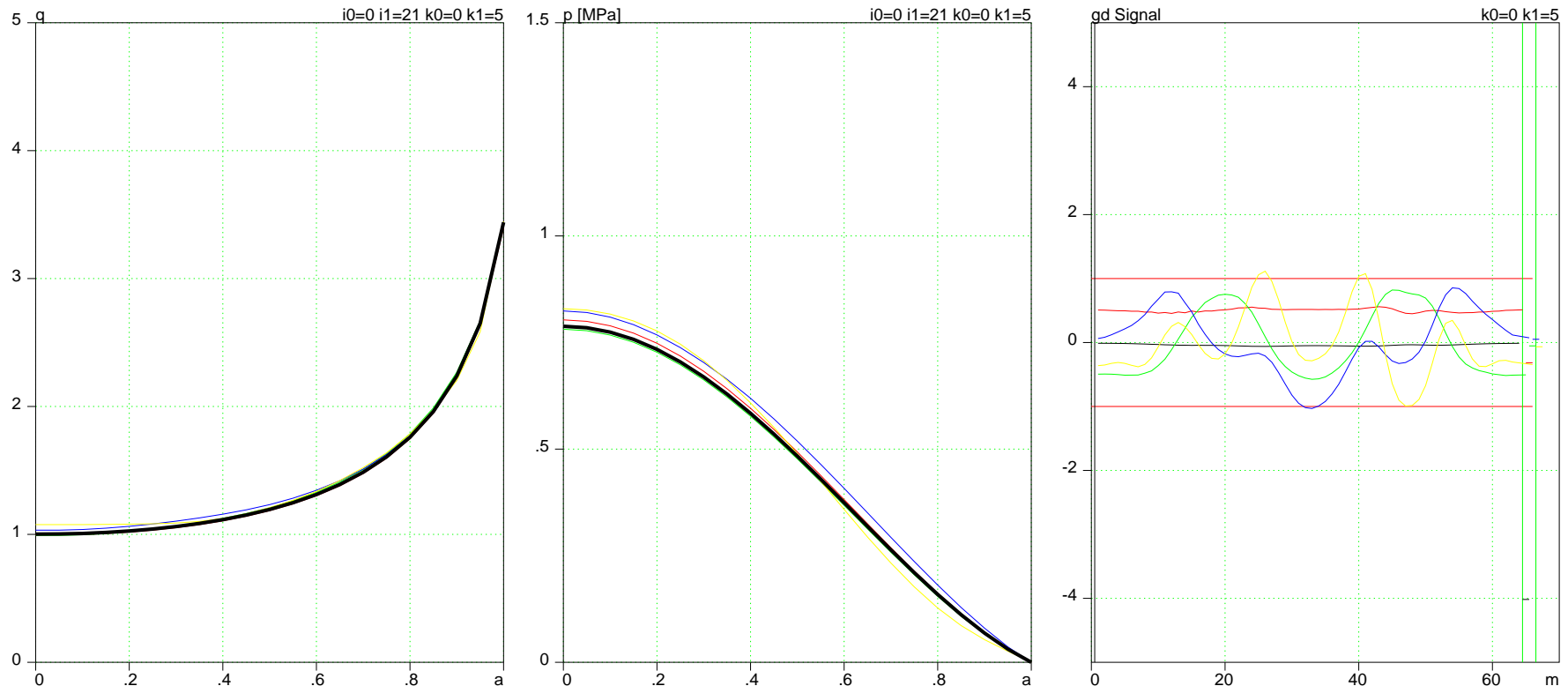
Eigen-variances $\delta j_p^k(a)$,
 $j_p \equiv P/R_0$

$\bar{\sigma}_q$ and $\bar{\sigma}_p^k$ [MPa] on the left plot are RMS for q- and p-profiles

Perturbations $j_s^{k>8}$, $j_p^{k>8}$ are invisible and cannot be reconstructed

4.1 Good looking magnetic only reconstruction

Plasma boundary is well specified, Φ -loop, B -coils are used



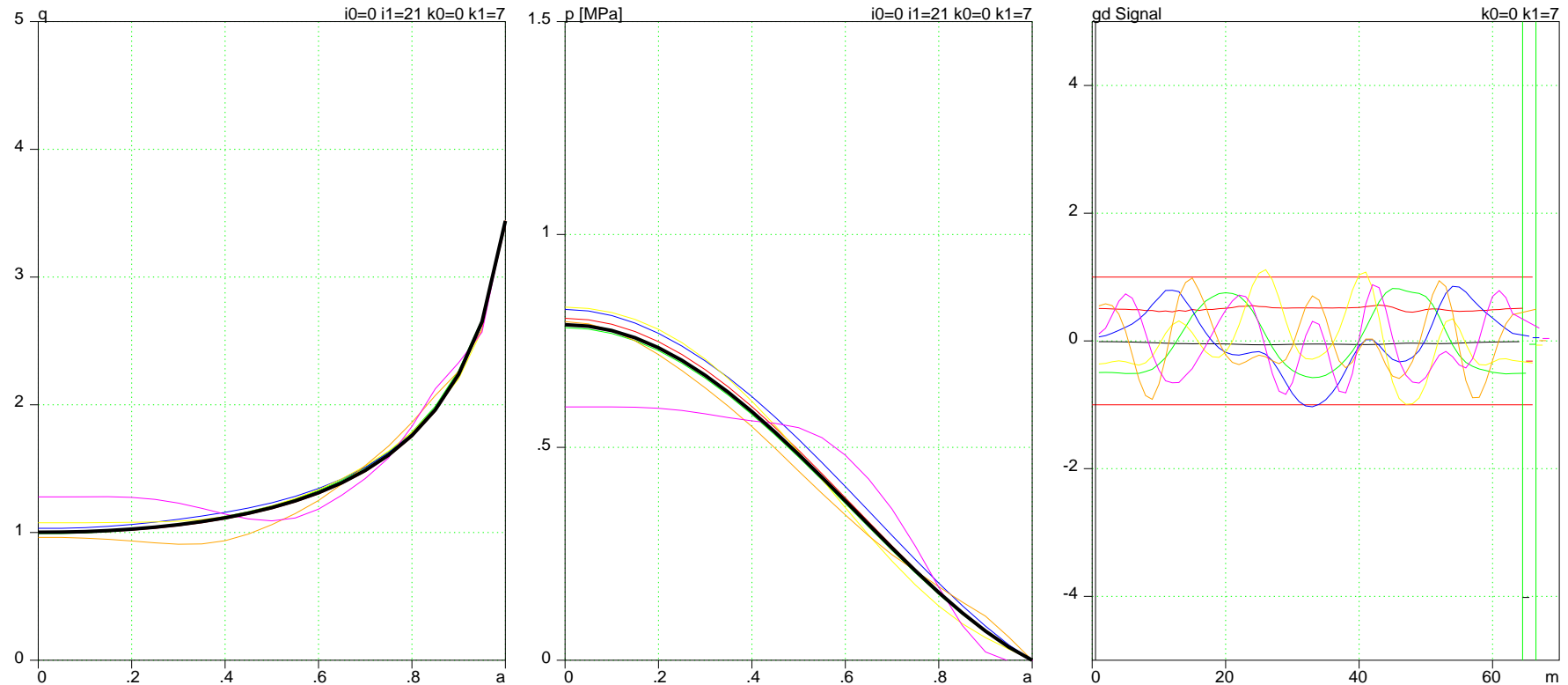
q — *profile and variances for $k_J \leq 3, k_P \leq 2$* *variances in p —profile as functions of a* *Signals $\delta S_m / \epsilon_m$ generated by perturbations*

For $k_J + k_P = 5$, typically used, the reconstruction looks very good

KiloGb's of reconstructions “data” can be easily generated

4.1 Good looking magnetic only reconstruction (cont.)

Plasma boundary is well specified, Φ -loop, B -coils are used

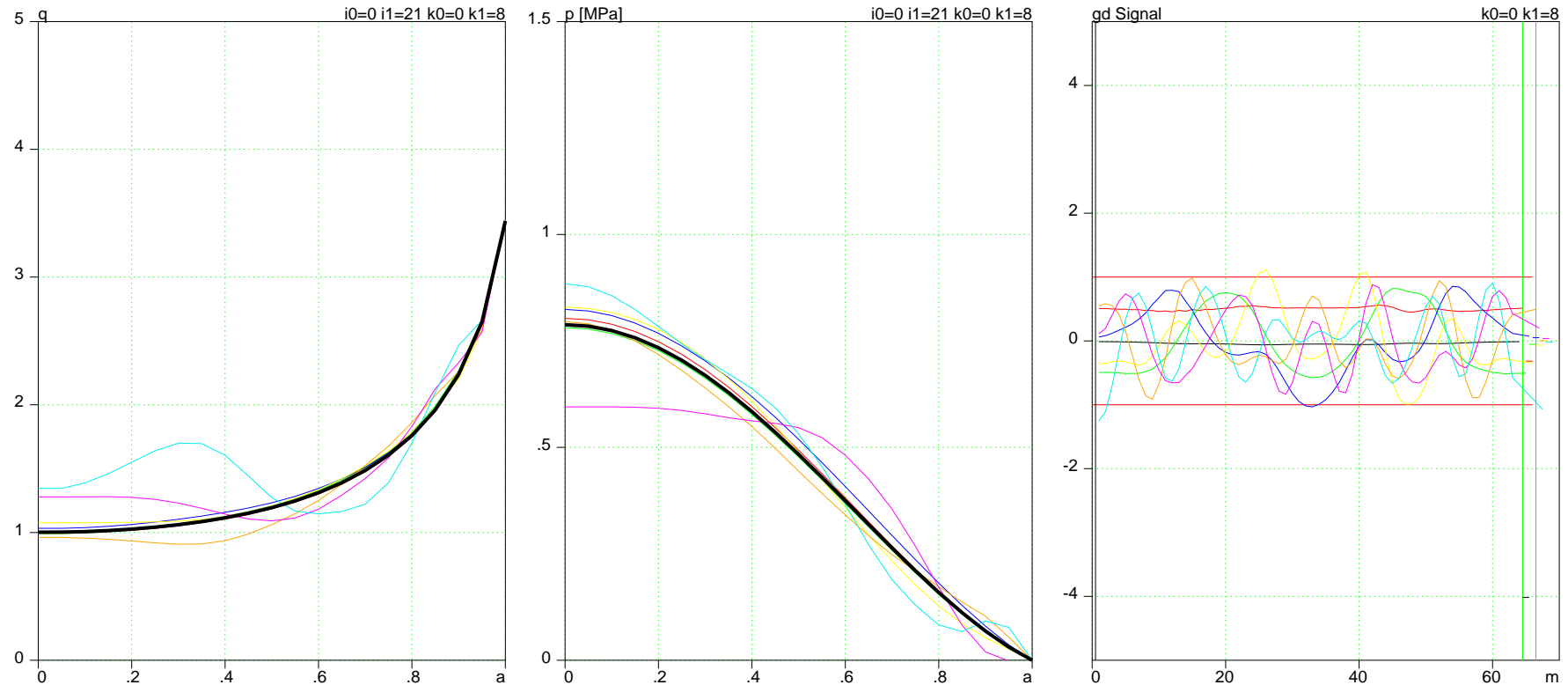


q — profile and variances for $k_J \leq 4, k_P \leq 3$. p — profile and its variances as functions of a Signals $\delta S_m / \epsilon_m$ generated by perturbations

Testing $k_J + k_P = 7$ shows that the reconstruction is, in fact, not so good

4.1 Good looking magnetic only reconstruction (cont.)

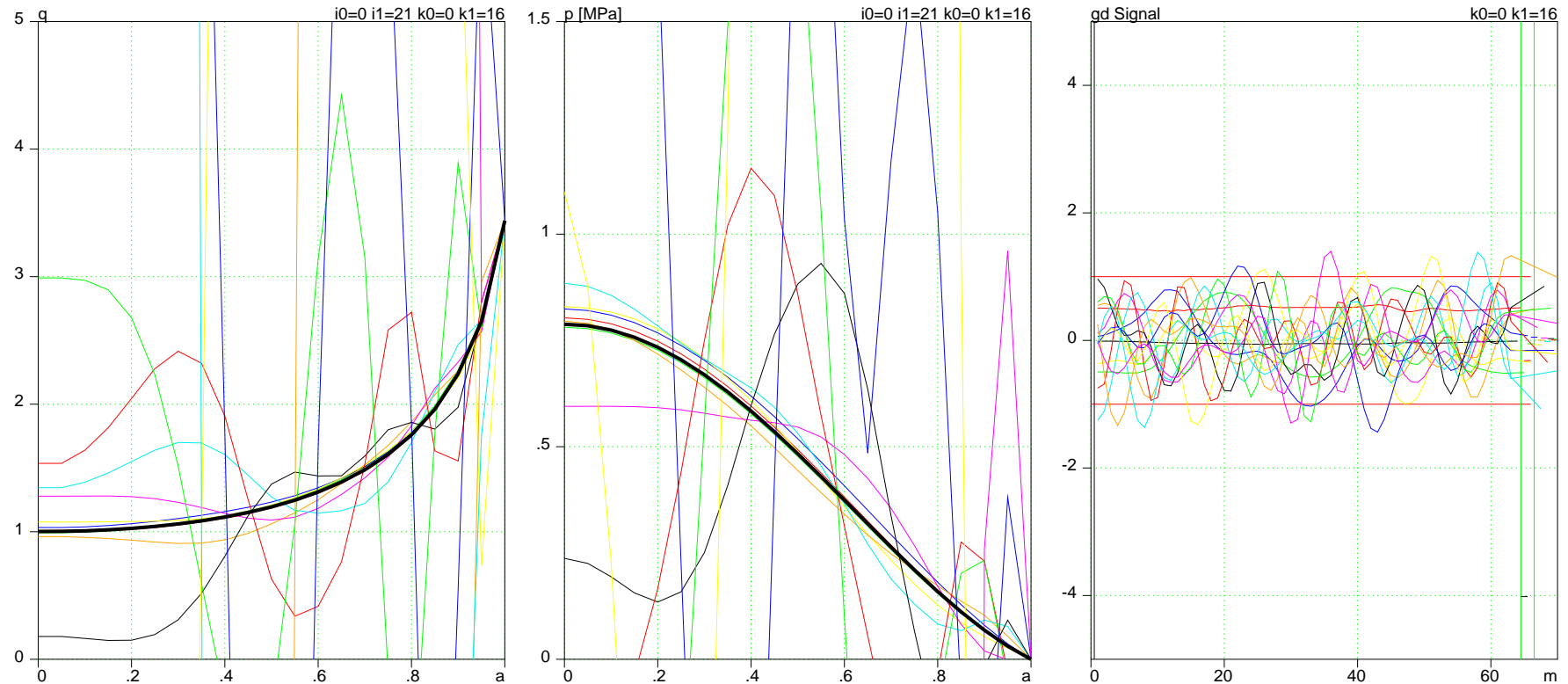
Plasma boundary is well specified, Φ -loop, B -coils are used



q — *profile and variances for $k_J \leq 4, k_P \leq 4$* p — *profile and its variances as functions of a* Signals $\delta S_m / \epsilon_m$ *generated by perturbations*

Testing $k_J + k_P = 8$ shows that even the q reconstruction is doubtful

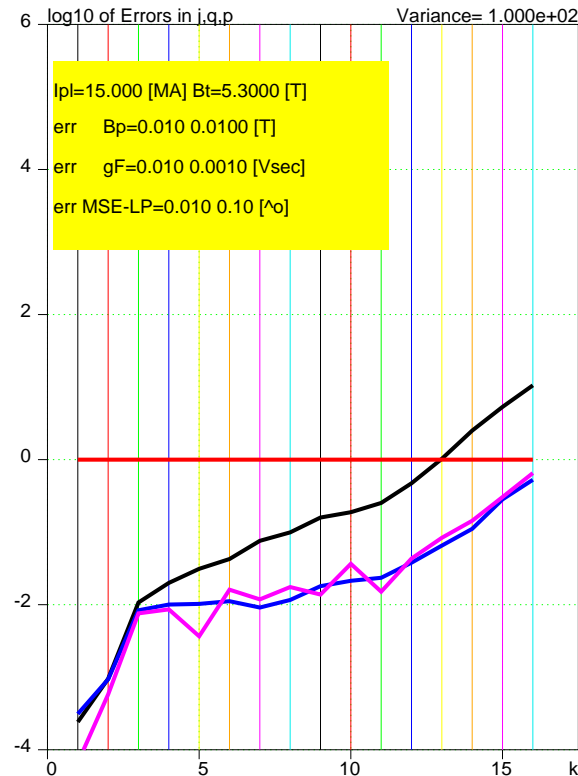
Plasma boundary is well specified, Φ -loop, B -coils are used



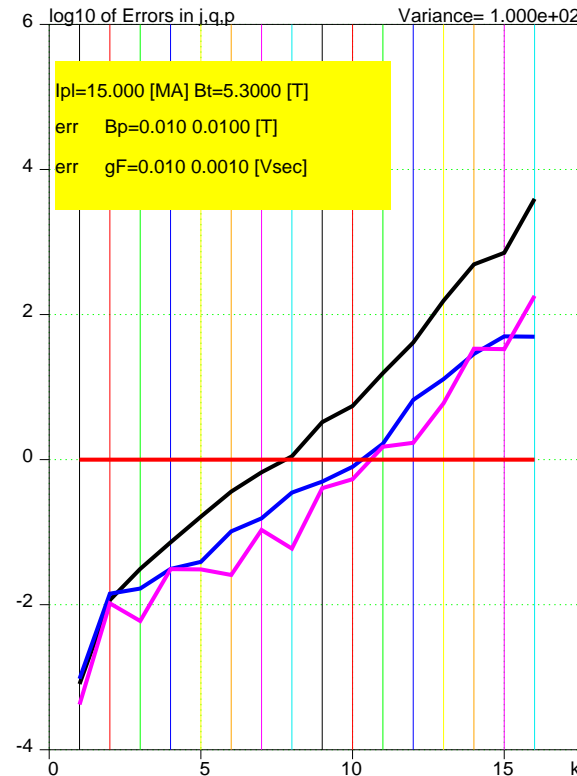
q — *profile and variances for $k_J \leq 8, k_P \leq 8$* p — *profile and its variances as functions of a* Signals $\delta S_m / \epsilon_m$ *generated by perturbations*

Test of $k_J + k_P = 16$ shows that with no constraints the reconstruction has no scientific value and is a sort of “beliefs”

Fixed plasma boundary with (Φ & B & MSE-LP) signals



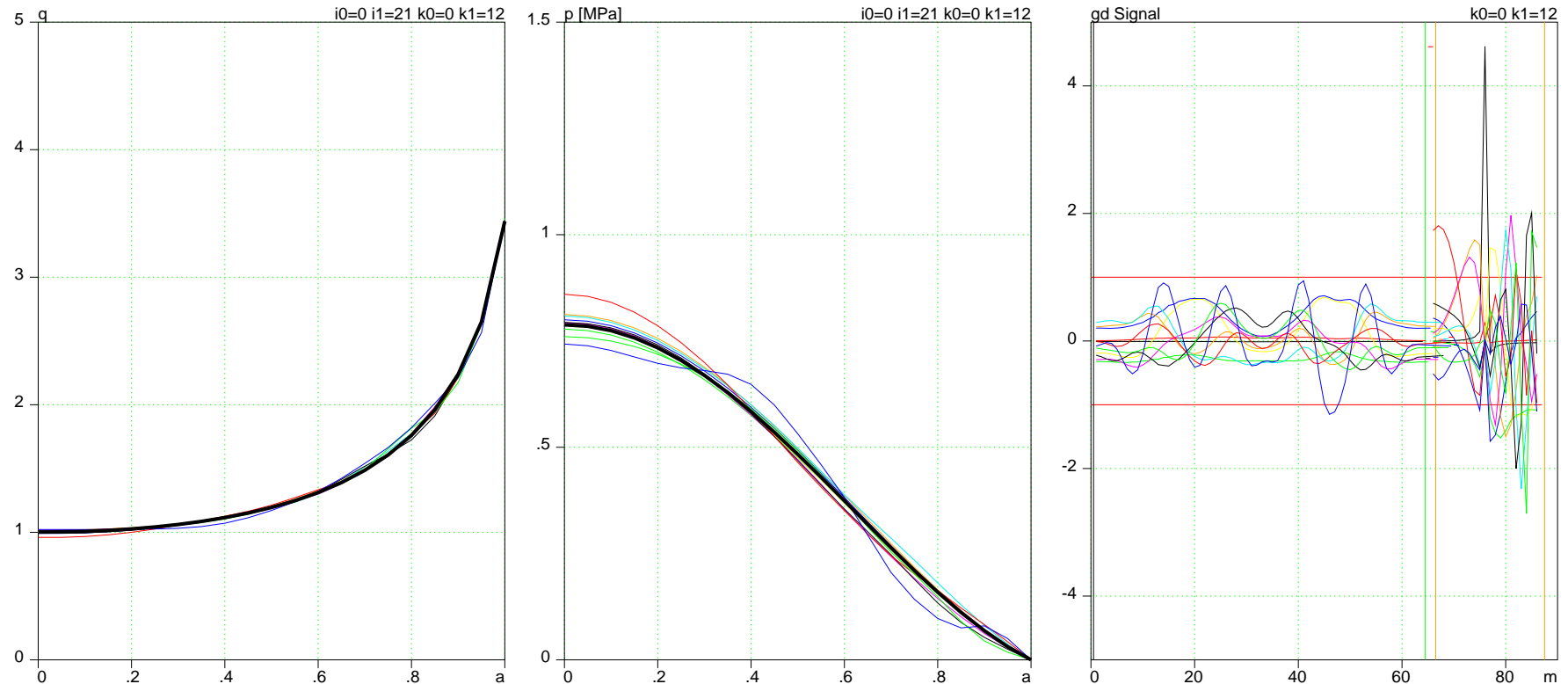
$\log_{10} \bar{\sigma}^k$, $\log_{10} \bar{\sigma}_q^k$, $\log_{10} \bar{\sigma}_p^k$
in case of Φ & B & MSE-LP



$\log_{10} \bar{\sigma}^k$, $\log_{10} \bar{\sigma}_q^k$, $\log_{10} \bar{\sigma}_p^k$
in case of Φ & B only

Use of MSE-LP drops largest RMS $\bar{\sigma}$, makes 12 perturbations visible, and dramatically improves reconstruction of q, p

Fixed plasma boundary with (Φ & B & MSE-LP) signals



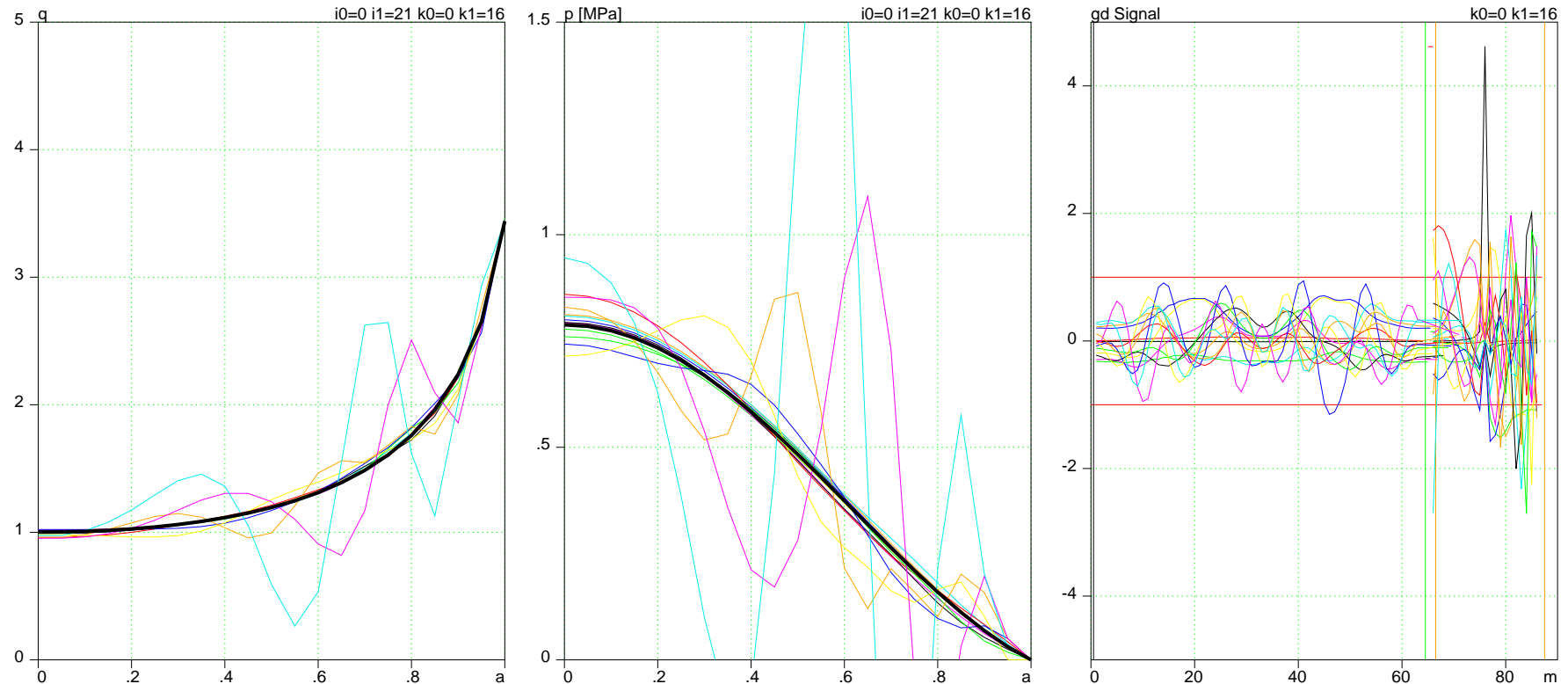
q — *profile and variances for $k_J \leq 6$, $k_P \leq 6$*

p — *profile and its variances as functions of a*

Signals $\delta S_m / \epsilon_m$ generated by perturbations

Testing $N = 12$ shows that MSE-LP allows to reconstruct both q - and p -profiles

Fixed plasma boundary with (Φ & B & MSE-LP) signals



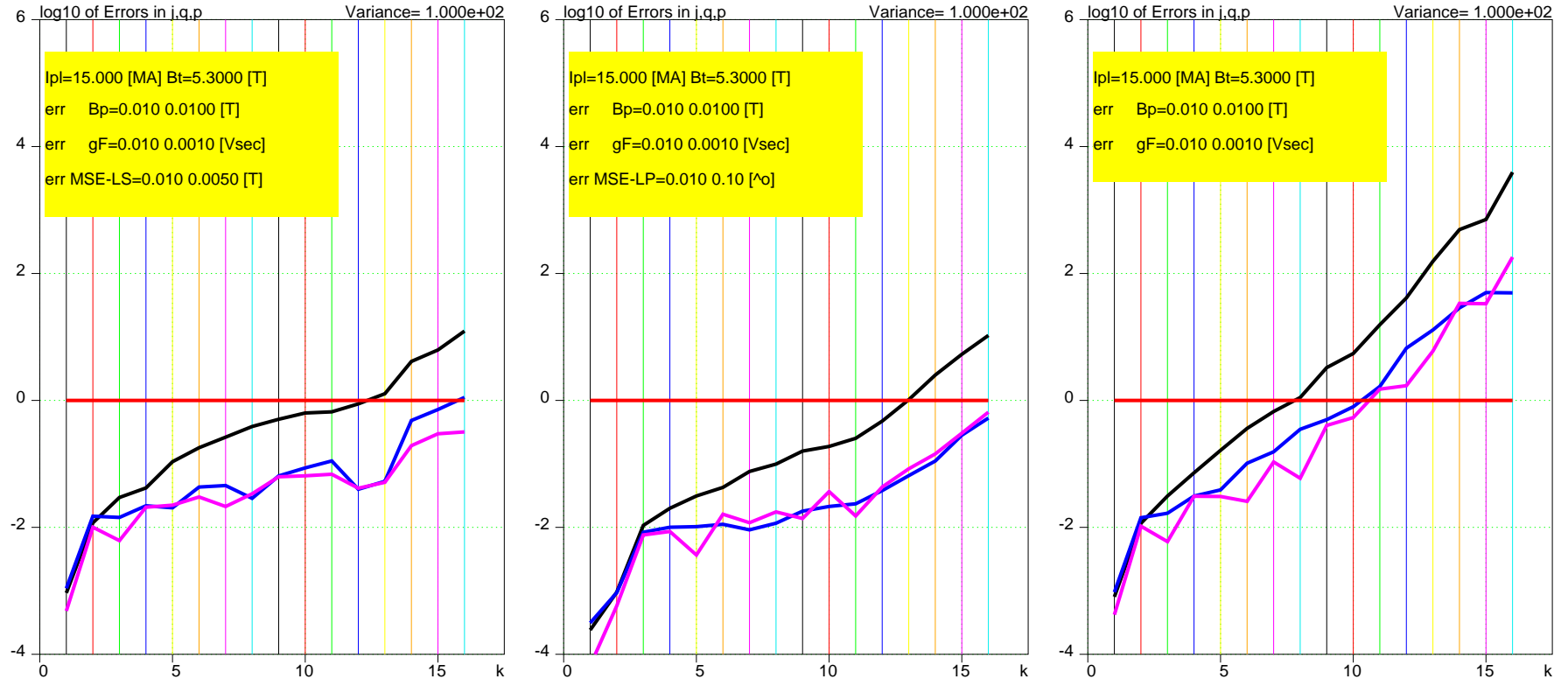
q — profile and variances for all k

p — profile and its variances as functions of a

Signals $\delta S_m / \epsilon_m$ generated by perturbations

Only perturbations with $k \geq 14$ might be potentially troublesome

Fixed plasma boundary with (Φ & B & MSE-LS) signals



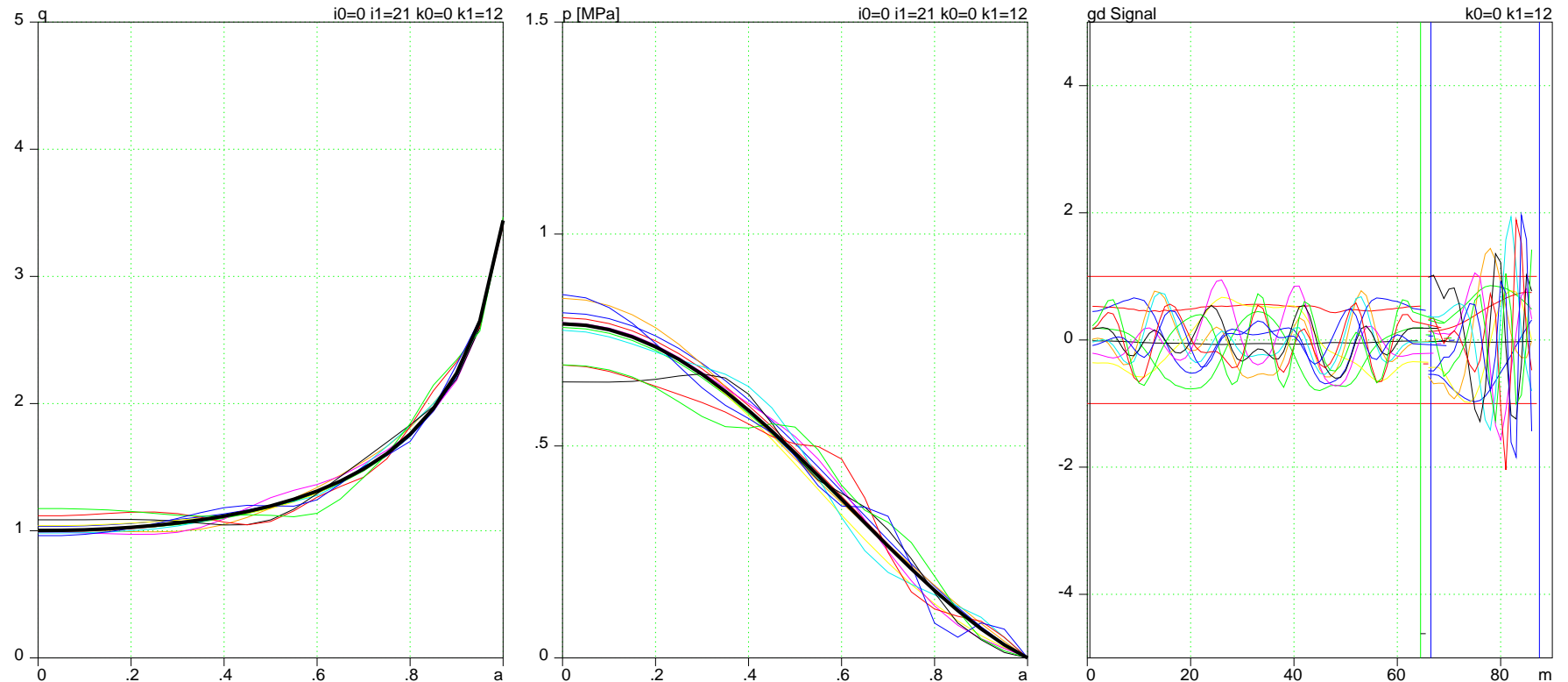
$\log_{10}\{\bar{\sigma}^k, \bar{\sigma}_q^k, \bar{\sigma}_p^k\}$ in case of (Φ & B & MSE-LS)

$\log_{10}\{\bar{\sigma}^k, \bar{\sigma}_q^k, \bar{\sigma}_p^k\}$ in case of (Φ & B & MSE-LP)

$\log_{10}\{\bar{\sigma}^k, \bar{\sigma}_q^k, \bar{\sigma}_p^k\}$ in case of (Φ & B) only

Use of MSE-LS can compete with MSE-LP in its value for reconstruction

Fixed plasma boundary with (Φ & B & MSE-LS) signals



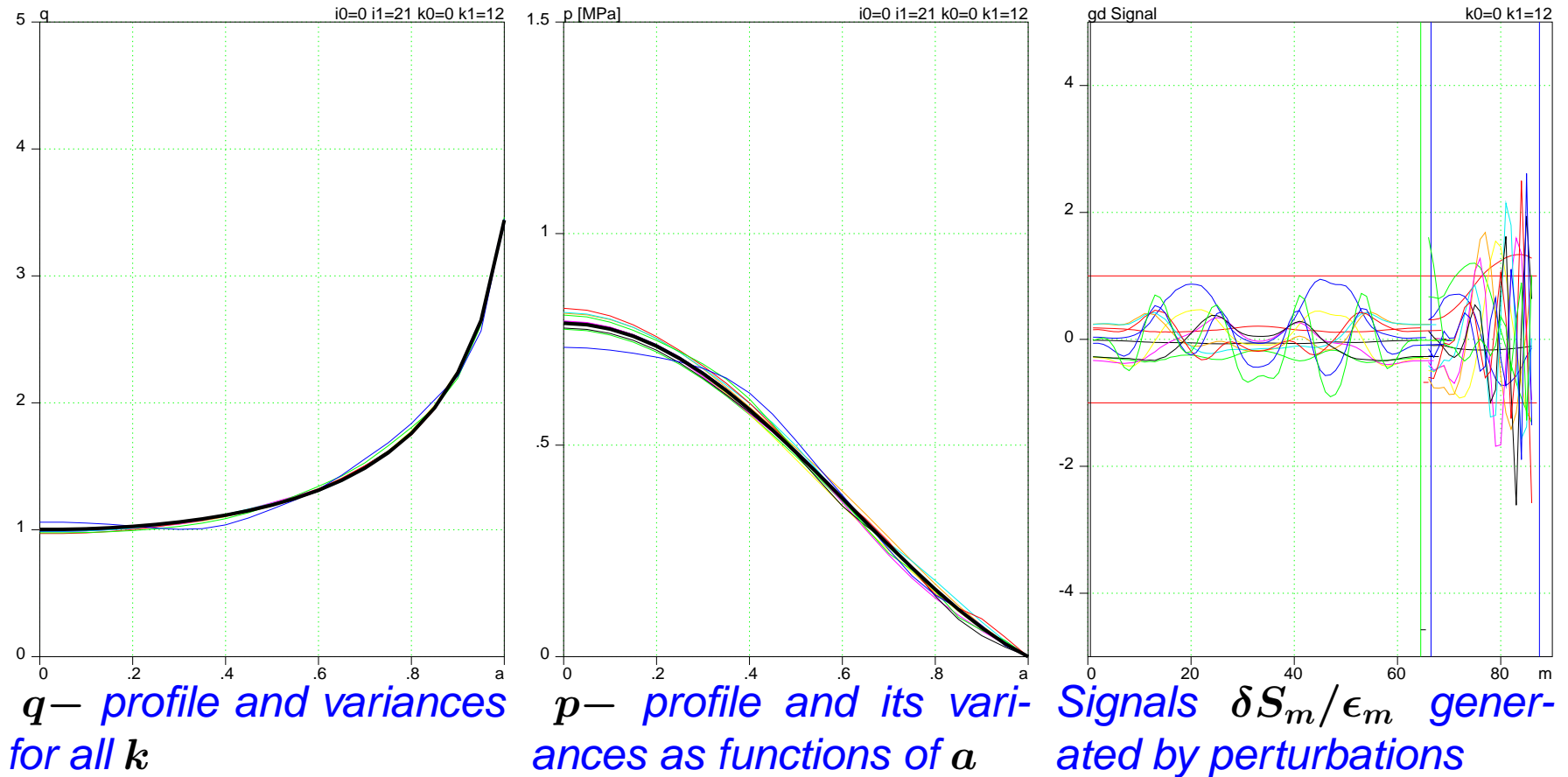
q — profile and variances for all k

p — profile and its variances as functions of a

Signals $\delta S_m / \epsilon_m$ generated by perturbations

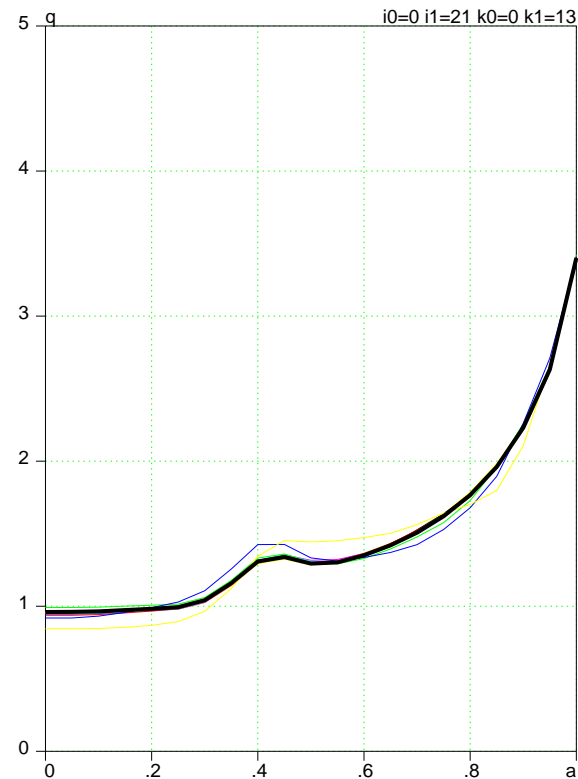
Perturbations with $k \leq 12$ can be reconstructed using MSE-LS

Same case with the improved relative accuracy of MSE-LS

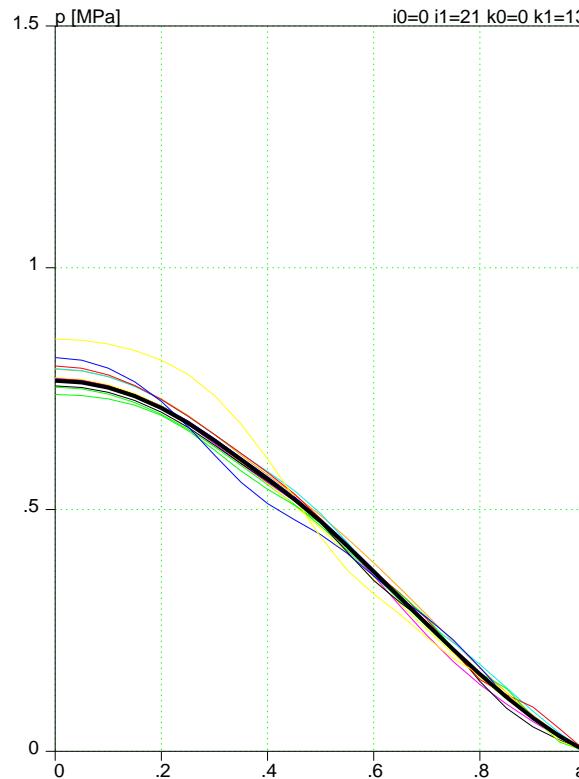


A realistic reduction of relative error $\epsilon_{MSE-LS}^{relative} \rightarrow 0.1\%$ improves the pressure profile reconstruction

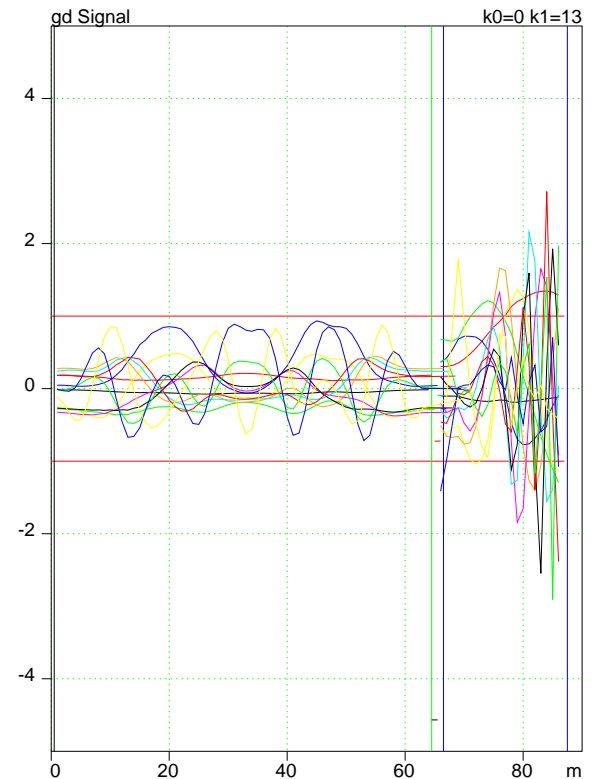
Same case with $\epsilon_{MSE-LS}^{relative} \rightarrow 0.1\%$ and non-monotonic \bar{j}_s



q — profile and variances for all k



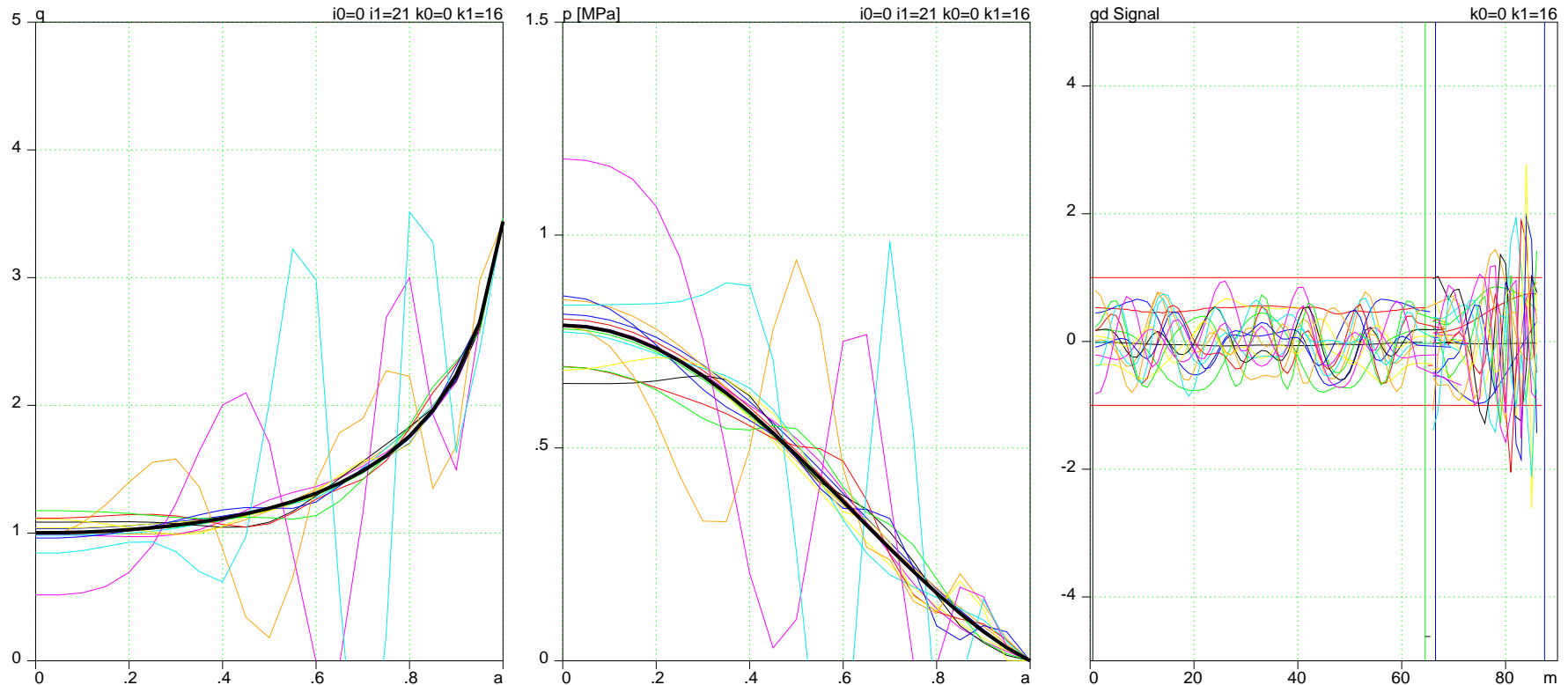
p — profile and its variances as functions of a



Signals $\delta S_m / \epsilon_m$ generated by perturbations

MSE-LS can pick up the details of the current drive

Back to reference fixed boundary and (Φ & B & MSE-LS)



q — profile and variances for all k

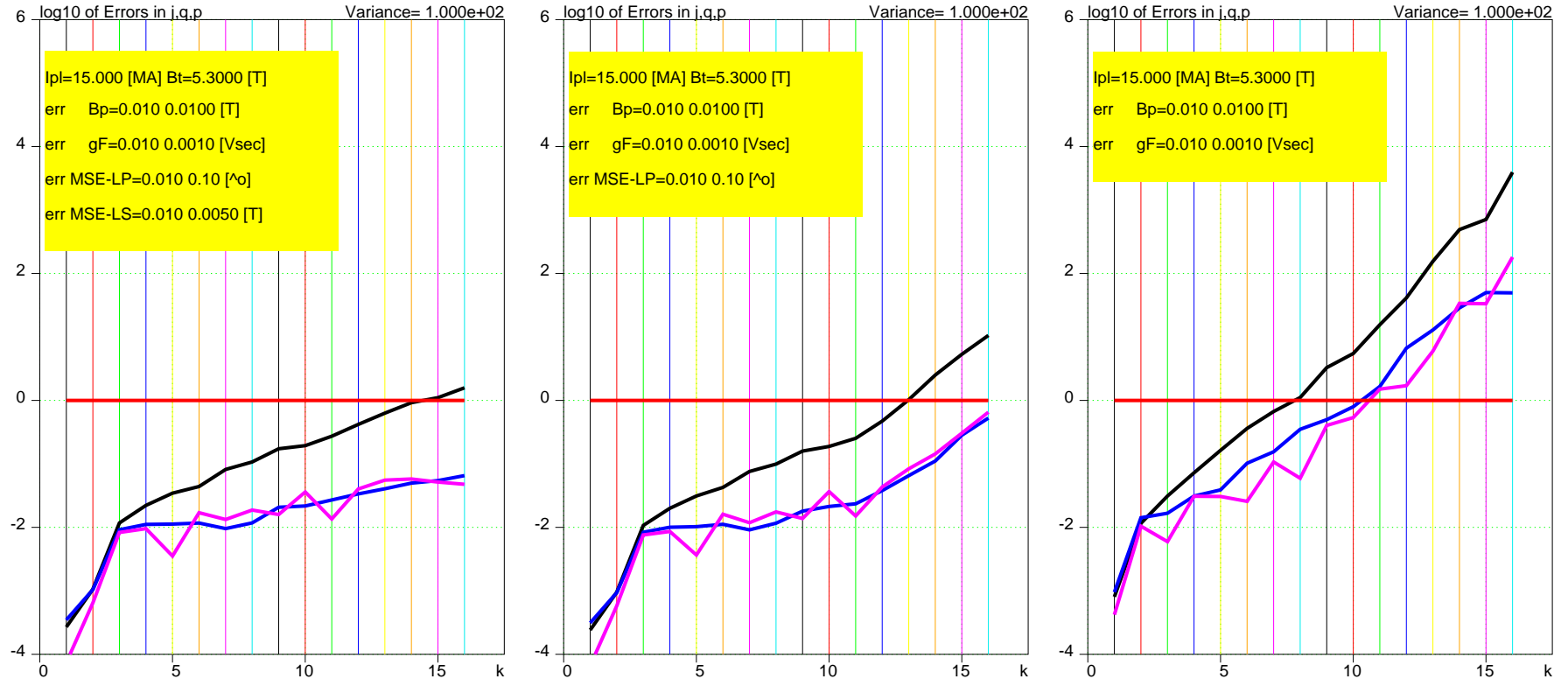
p — profile and its variances as functions of a

Signals $\delta S_m / \epsilon_m$ generated by perturbations

With MSE-LS only perturbations with $k \geq 13$ might be potentially troublesome

4.4 Magnetic signals & both MSE-LP & MSE-LS

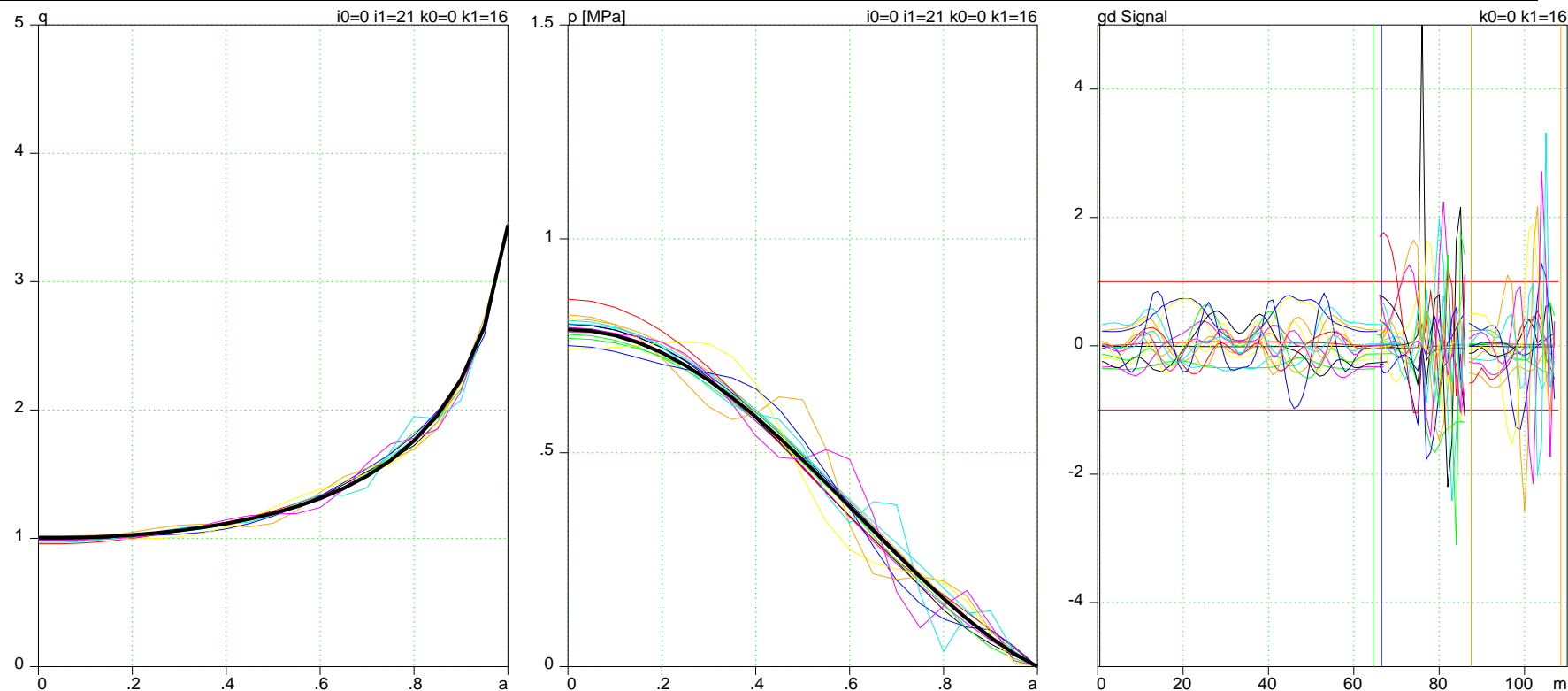
Fixed plasma boundary with (Φ & B & MSE-LP & -LS) signals



$\log_{10}\{\bar{\sigma}^k, \bar{\sigma}_q^k, \bar{\sigma}_p^k\}$ in case of (Φ & B & MSE-LP & MSE-LS)
 $\log_{10}\{\bar{\sigma}^k, \bar{\sigma}_q^k, \bar{\sigma}_p^k\}$ in case of (Φ & B & MSE-LP)
 $\log_{10}\{\bar{\sigma}^k, \bar{\sigma}_q^k, \bar{\sigma}_p^k\}$ in case of (Φ & B) only

Both MSE-LP & LS allows for a reliable reconstruction of q - and p -profiles

Fixed plasma boundary with (Φ & B & MSE-LP&LS) signals

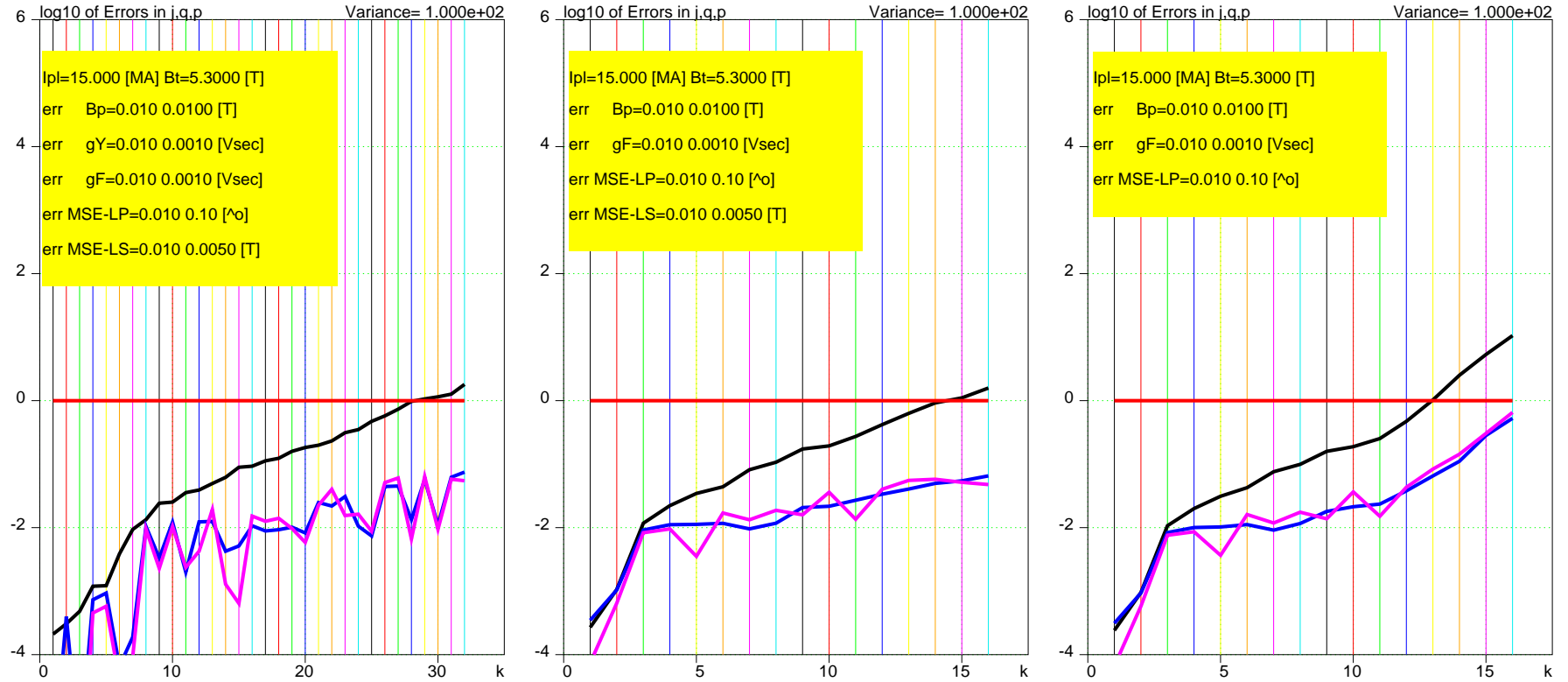


q — profile and variances for all k p -profile and its variances as functions of a Signals $\delta S_m / \epsilon_m$ generated by perturbations

q - and p -profiles can be reconstructed in all spectrum of k

4.5 Free boundary, magnetic signals & both MSE-LP & MSE-LS

Free boundary plasma with (Φ & B & MSE-LP & -LS) signals



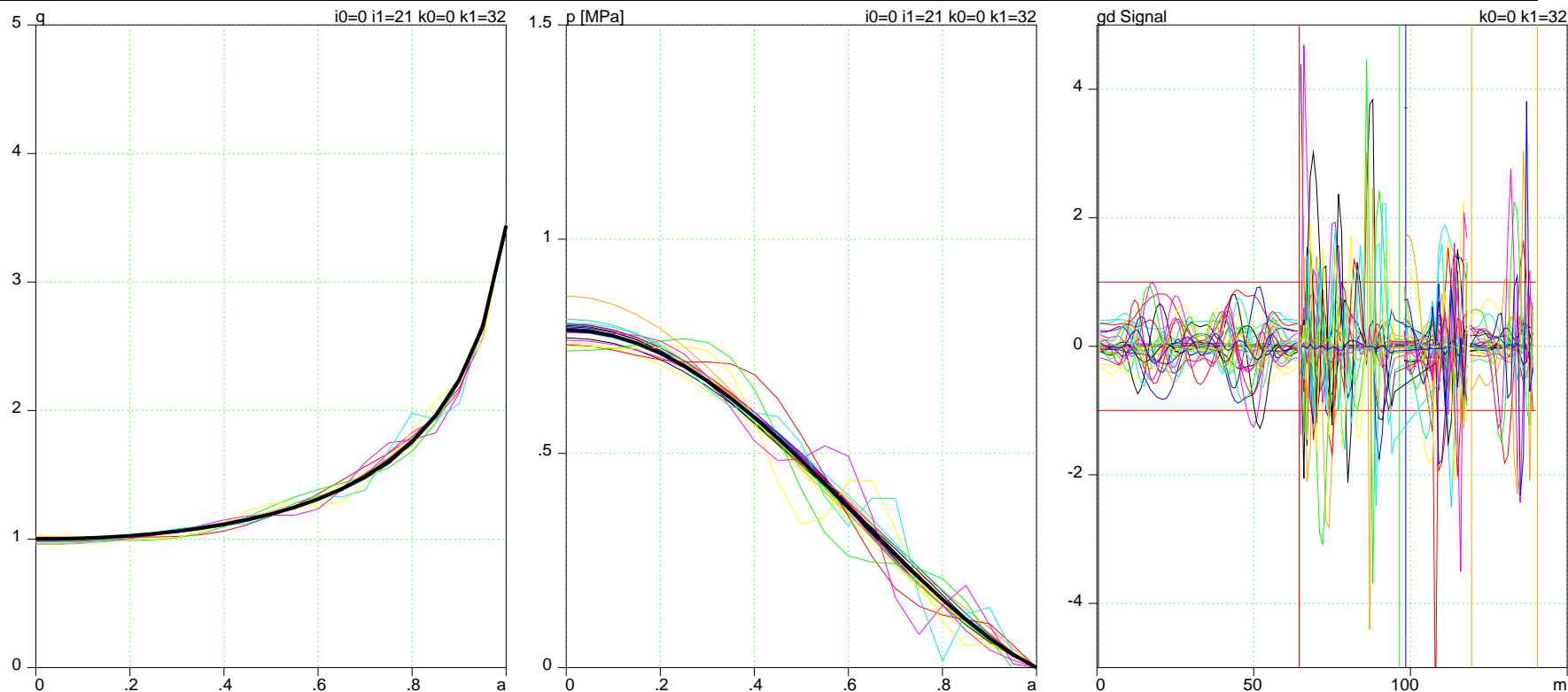
$\log_{10}\{\bar{\sigma}^k, \bar{\sigma}_q^k, \bar{\sigma}_p^k\}$ in case of
 (Φ & B & MSE-LP & MSE-LS),
 $\vec{\xi} \neq 0$

$\log_{10}\{\bar{\sigma}^k, \bar{\sigma}_q^k, \bar{\sigma}_p^k\}$ in case of
 (Φ & B & MSE-LP & MSE-LS),
 $\vec{\xi} = 0$

$\log_{10}\{\bar{\sigma}^k, \bar{\sigma}_q^k, \bar{\sigma}_p^k\}$ in case of
 (Φ & B & MSE-LP), $\vec{\xi} = 0$

Free boundary expands the k range but does not affect the reconstruction

Free boundary plasma with (Φ & B & MSE-LP & -LS) signals



q — profile and variances
for all extended k

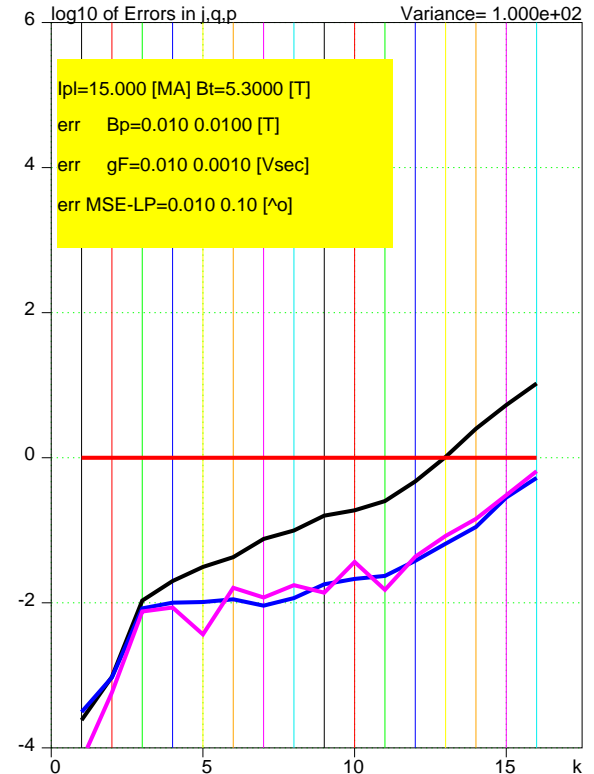
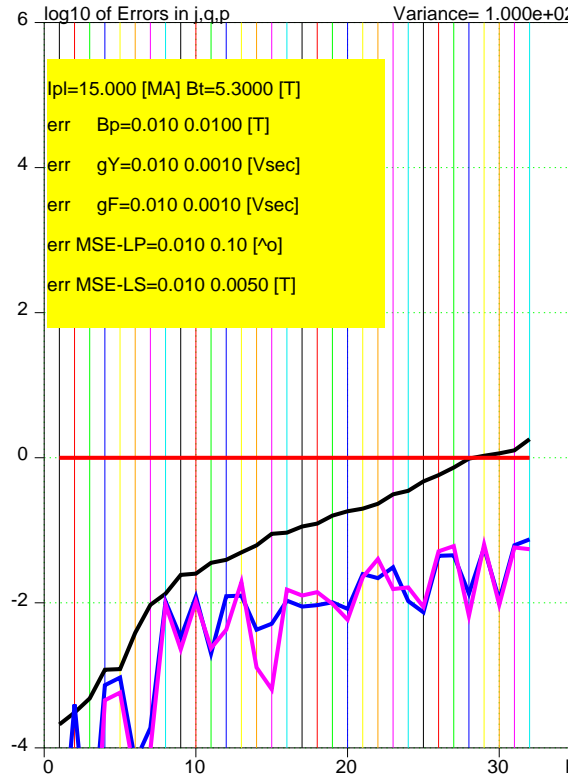
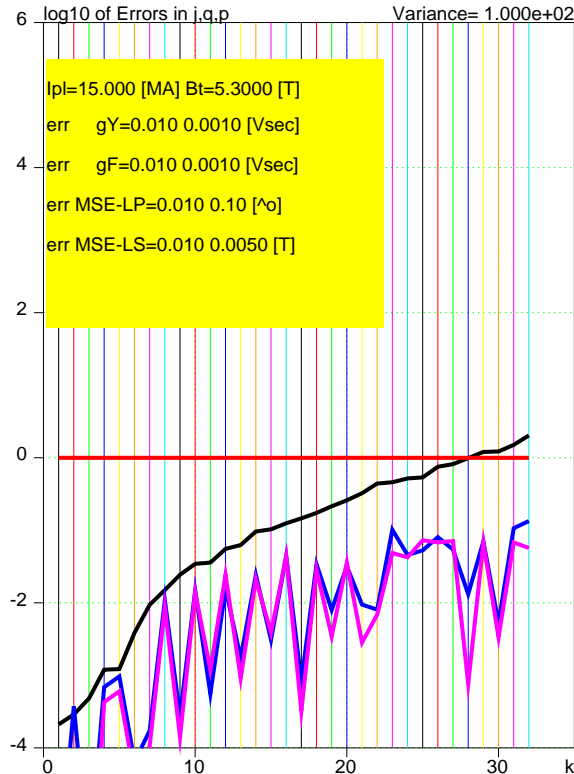
p -profile and its variances
as functions of a

Signals $\delta S_m / \epsilon_m$ gener-
ated by perturbations

q - and p -profiles can be reconstructed in all extended spectrum of k

4.6 Curious case, NO B -signals, $\xi \neq 0$, Φ & both MSE-LP & MSE-LS

Free boundary, (Φ & MSE-LP & -LS) signals, NO B -signals



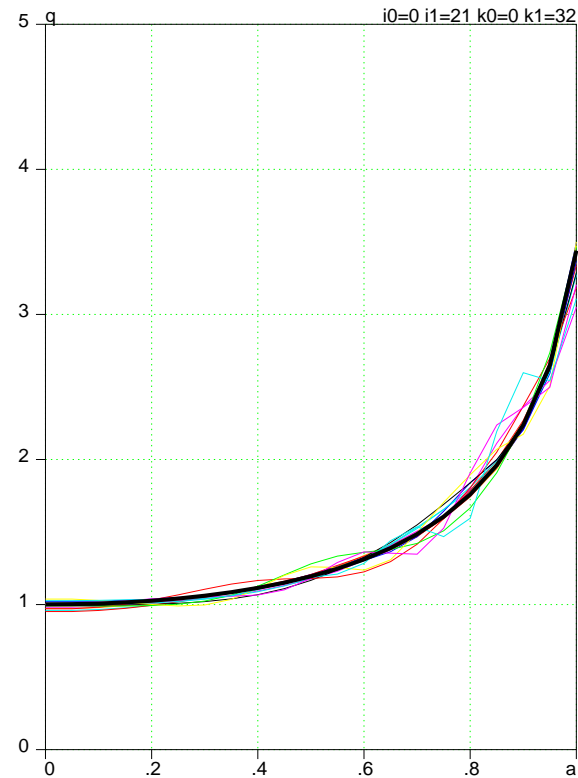
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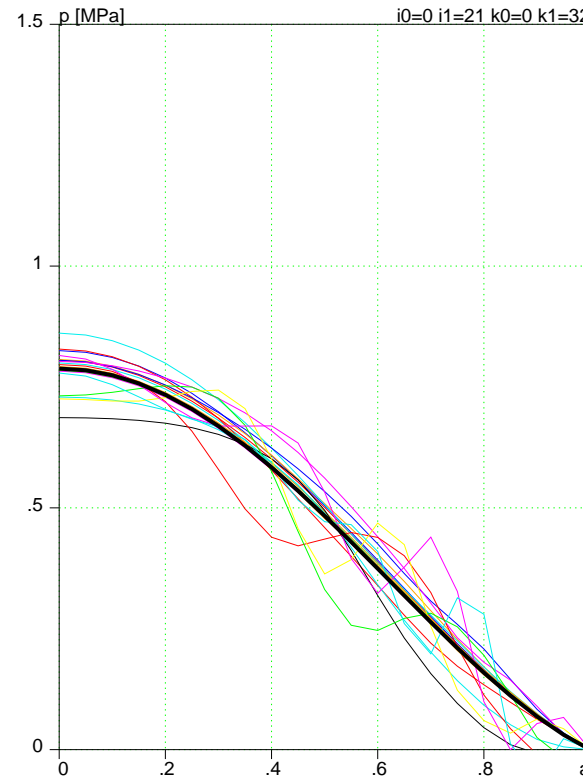
$\log_{10}\{\bar{\sigma}^k, \bar{\sigma}_q^k, \bar{\sigma}_p^k\}$ in case of
 (Φ & B & MSE-LP), and $\vec{\xi} = 0$

(MSE-LP & MSE-LS) together can do the job for external B -coils

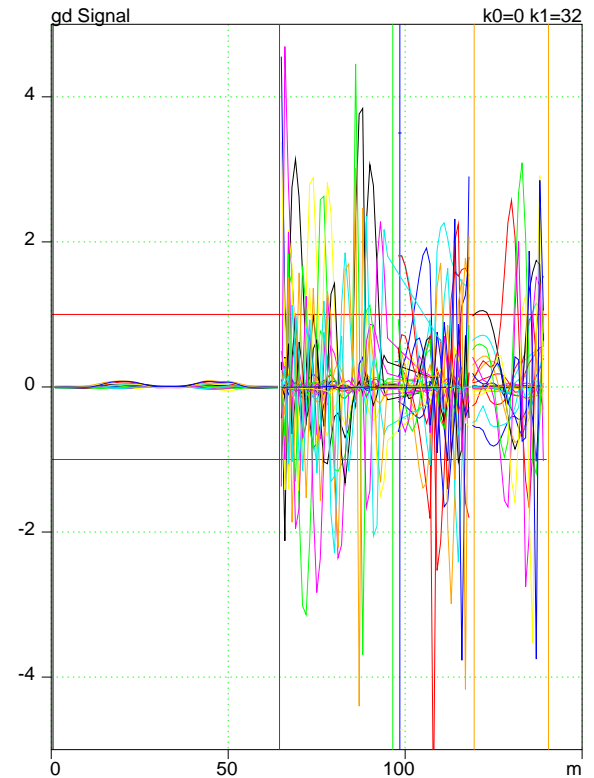
Free boundary, (Φ & MSE-LP & -LS) signals, NO B -signals



q — profile and variances for all extended k



p -profile and its variances as functions of a



Signals $\delta S_m / \epsilon_m$ generated by perturbations

q - and p -profiles can be reconstructed over extended spectrum of k
even with NO B -coil signals

The capability of calculating variances, now developed, has completed the theory of equilibrium reconstruction

- 1. The quantitative evaluation of the quality of diagnostics systems on existing and future machines can be done based on spectrum of “visible” perturbations ($\bar{\sigma}$ -curves)*
- 2. It was confirmed that the internal measurements of the magnetic field are crucial for reconstruction.*
- 3. Either MSE-LP (line polarization) or MSE-LS (line shift) signals from the plasma in addition to external measurements allow for a complete reconstruction (of both q - and p -profiles).*
- 4. The presented technique can be used to optimize the diagnostic set on any tokamaks. Contribution of any signal can be evaluated.*
- 5. The proposal by Nova Photonics to utilize MSE-LS signals would significantly enhance the equilibrium reconstruction capability in ITER.*

The extension of the theory should be focused on realistic simulation of signals used in reconstructions